Performance of Turbo Codes with SOVA Algorithm in DSSS over Channels with non-White Additive Gaussian Noise

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Abstract

In the paper we present performance of turbo codes with SOVA decoder in DSSS over non-white channels. It has been shown, that its performance is similar to this, achieved in AWGN. Moreover, in order to increase its performance a double-matched reception has been employed. Thereby, the structure of a receiver is now matched not only to the useful signal, as it is in all classical receivers, but also to non-white noise. In comparison with a single-matched detection the obtained gains reach a dozen or so dBs depending on the entropy of noise.

1. Introduction

Turbo codes, first presented in 1993, are the most important breakthrough in coding.

Nowadays almost every modern digital radio systems employs turbo codes (like UMTS, CDMA 2000). Since turbo codes have been presented for the first time, many researches dealt with them. Most of them were focused on evaluating performance of turbo codes over AWGN channels.

In the paper we present a novel approach. Our goal is to evaluate its performance in direct spread spectrum systems over non-white, additive, Gaussian channels. Such noise could occur in a few situations: it may be a result of shaping the actual noise source by an antenna or RF filters or it may be an interfering signal, e.g. a BPSK signal from a user of a narrowband system, or a tactical jamming signal in military communications. Particularly, in this study a BPSK narrowband signal (as an example) is considered.

Moreover, in the paper we present a method of a double-matched reception that effectively suppresses such a noise. This method might sometimes give a dozen or so dBs improvements, expressed in Eb/No. Our investigation is based on the fundamental studies of VanTrees [1], Lee and Messerschmitt [2].

The paper is organized as follows. In section 2 we present a layout of non-white detection, in section 3 we describe a model of the considered system, section 4 presents the results of simulations. Our conclusions are drawn in section 5.

2. An outline of non-white detection theory

It has been shown that a receiver filter that yields the maximum signal to noise ratio satisfies the integral equation [2]:

$$as(t_0-\tau)=\int_0^\tau h(v)R(\tau-v)dv \quad (0\le\tau\le \mathrm{T}) \tag{1}$$

where:
$h(\tau)$ – impulse response of a receiver filter,
$s(t_0-\tau)$ - useful signal,
R (.) – autocorrelation function of noise,
a – a real constant.

If noise is white, its autocorrelation function is given by the formula $R(\tau-v)=\sigma^2\delta(\tau-v)$, then from (1) we directly obtain:

$$h(\tau)=\begin{cases}\dfrac{2a}{\delta^2}s(t_0-\tau) & \text{for } 0\le\tau\le T\\ 0 & \text{for } \tau>\mathrm{T}\end{cases} \tag{2}$$

Thus, the response *h(τ)* must be selected in accordance with the signal *s(t)* that is to be filtered.

If noise is non-white, but the assumption *h(τ) = 0* for *τ<0, τ>T* is held, considering (1) we obtain:

$$as(t_0-\tau)=\int_0^T h(v)R(\tau-v)dv=h(\tau)*R(\tau)\Leftrightarrow$$
$$\Leftrightarrow aS^*(\omega)e^{jwt_0}=H(\omega)P(\omega) \tag{3}$$

where:
S(ω) - useful signal spectrum,
$S^*(\omega)$ – its conjugate counterpart,
P(ω) – power spectrum of noise,
H(ω) – transfer function of desired filter.

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The delay factor $exp(j\omega t_0)$ does not affect the transfer function, so we neglect it. The constant a can be put unity. Then the final result is

$$H(\omega) = S^*(\omega)P^{-1}(\omega) \qquad (4)$$

In the case of white noise, equation (4) is still true, because then $P(w) = const$ and matched filter transfer function is equal to $S^*(\omega)$ multiplied by a constant.

Digital approach

The ML principle is:

$$D_m = \sum_{k=1}^{K} | r_k - s^*_{mk} |^2 = \sum | r_k |^2 + \sum | s^*_{mk} |^2 -$$
$$-2\,\mathrm{Re} \sum r_k \cdot s^*_{mk} = \min \qquad (5)$$

where:
r_k – *k-th* sample of incoming signal,
s_{mk} – *k-th* sample of useful signal of *m-th* number in alphabet,
D_m - distance of $\boldsymbol{s}_m$ signal to the vector $\boldsymbol{r}$.

The idea of maximum likelihood is expressed here as a minimum distance rule. We say that $\boldsymbol{s}_m$ signal is the most probable if its product $Re\ \{\sum r_k\ s^*_{mk}\}\ (k=1,...,K)$ reaches the maximum over $m=1,...,M$.

Now, let us expand this scheme to the case of non-white noise [3], [4], [5], [6]. The first step is to factorize the power spectrum $P(\omega)$. As a positively definite function, it can be expressed by:

$$P(\omega) = A^2 G(\omega) G^*(\omega) \qquad (6)$$

or the same in Z transform as:

$$P(z) = A^2 G(z) G^*(1/z^*) \qquad (7)$$

where:
$G(z)$ – minimum-phase function,
$G^*(1/z^*)$ – its maximum-phase counterpart,
A – real constant.

In the sequel we will introduce a reciprocal of $G(z)$ as $I(z)=1/G(z)$ and assign it a unit impulse response $h_k\ (k=1,2,...,L)$. The filter with such a response will white the incoming noise n_k. Its output signal in a time domain is expressed by a convolution:

$$s'_k = s_k * h_k \qquad (8)$$

From this point the replica s^*_k is no longer matched to it. To achieve matching we have to use:

$$s^{*'}_k = s^*_k * h_k \qquad (9)$$

This is done by the modification filter (Figure 1) of the same unit impulse response h_k, as used in a whitening filter. Now, the only problem is to find the power spectrum of noise or directly its minimum-phase function $I(z)$.

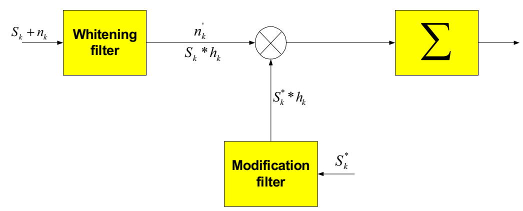

Figure 1. A scheme of a double-matched detector

3. A system model

In this section a system model that was taken into research is described. The scheme of a model is shown in Figure 2. The consecutive abbreviations mean: *So* - source, *TC* – turbo coder, *CI* – channel interleaver, *R* – a replica of PN, *MF* – modeling filter, *WF* – whitening filter, *MF* – modification filter, *ML* – ML detector, *CD* – channel deinterleaver, *TD* – Turbo Decoder SOVA, *Si* – sink of data.

A binary data is generated at pseudo random manner. Next is encoded. To compare different turbo codes performance in non-white channels two turbo codes, denoted by *TC1* and *TC2* with generator matrices $G_1(D)$ and $G_2(D)$ have been chosen. Their generator matrices are given by:

$$G_1(D) = \left[1, \qquad \frac{1+D+D^3}{1+D+D+D^3} \right] \qquad (10)$$

$$G_2(D) = \left[1, \qquad \frac{1+D+D^2}{1+D^2} \right] \qquad (11)$$

Due to puncturing alternate parity bits from the first and the second component encoder, each turbo encoder is half-rate. Encoding is performed in frames. A singular frame is composed of information bits and a tail. A tail is added at the end of each frame in order to drive a turbo encoder to the all-zero state. For comparison purposes two frame lengths have been taken into research:

- 169, which is suitable for speech transmission at approximately 8 kbit/s with a 20 ms frame length,
- 1024, that is suitable for data transmission, for example video transmission isn't so sensitive for delays but requires very low BER. Each encoder employs a pseudorandom interleaver. The length of

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the interleaver is determined by a frame length (it is equal).

Encoded data is spread using 127-chip m-sequence based on the prime number of Mersenne [7]. Hence, the processing gain is 21dB (10log127).

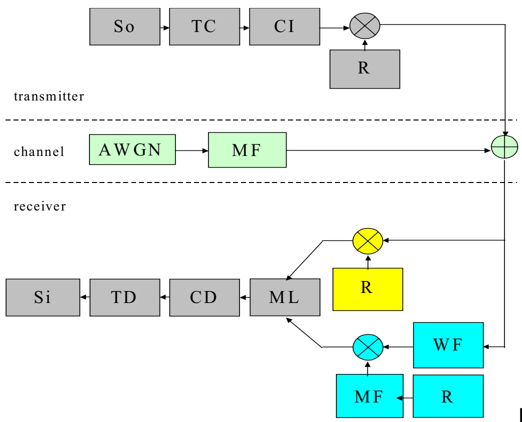

Figure 2. A simplified diagram of simulation model.

A sequence of chips is put into a channel block interleaver with row-wise writing and column-wise reading. The main task of this interleaver is to spread out burst errors that are the effect of transmitting the signals over a correlated noise channel. The size of this interleaver has been settled to 8 columns and 57 rows (the same is in GSM system). The last component of a transmitter is the BPSK modulator. It is assumed that two antipodal signals are transmitted ($s_1(t) = -s_2(t)$). Moreover, the receiver is coherent, exactly knows the carrier and time moments when the phase is changing.

The transmitted signals are subjected to colored, Gaussian, additive noise, which bandwidth is far lower than the bandwidth of useful signal. In the following simulations as an example of such a noise, a narrowband BPSK signal has been chosen. It was achieved as an output of a modeling filter in a response to AWGN. Its power has been normalized to unity. A frequency-response of a BPSK interference produced by this filter is shown on Fig. 4.

In this study we assume a perfect knowledge of BPSK interference at the receiver. Hence, a whitening filter is just a reciprocal of the modeling one. In practice there is a necessity to estimate such an interference. Further investigation of this problem could be reached in [8]. A magnitude response of the whitening filter is given on Fig. 5.

To show the advantage of a double-matched demodulator over a singled-matched demodulator in non-white noise channel, we use two types of a demodulator:

- a classical one, that is matched only to the signal. It is optimal in channels with AWGN in the sense of yielding maximum signal to noise ratio,
- and a double-matched demodulator, matched both to the signal and noise. This one is optimal in channels where noise is not white. The phenomena of a double-matched demodulator is given in section 2.

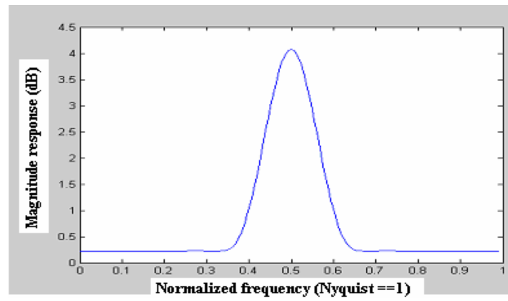

Figure 3. Frequency-response of the BPSK interference.

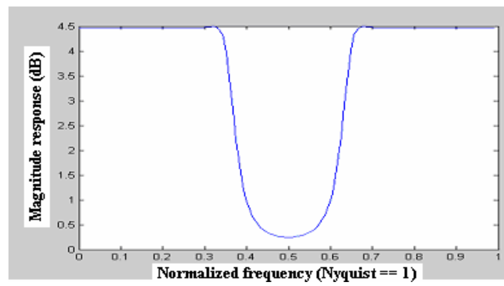

Figure 4. A magnitude response of the whitening filter.

As a decoder the soft output Viterbi decoder has been chosen. It gives, as a matter of fact, a slight worse BER performance than MAP algorithm indeed, but is far easier for implementation and needs less computation. The SOVA is described in [9], [10], [11].

4. Results of simulation

In this section are presented results of simulation runs. All results are plotted in BER versus Eb/No. They are as follows:

- Figure 6. - *TC1* and frame length 1024; curves denoted by *1* and *2* represent a single- and double-matched demodulation, respectively. It can be seen, that since a demodulator is matched not only to the signals but also to the interference the BER performance significantly improves. Gains reach about 12 dBs. For a given reception technique, as it was predicted, the decoder improves Eb/No as the number of iteration increases. Gains coming from coding sums with ones coming from a non-white detection. For comparison purposes on the same figure is also plotted a curve for uncoded BER and a classical demodulator,

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- Figure 7. - TC1 and frame length 169; from this figure we can see that a shorter frame length (equal to the interleaver size) causes BER degradation about 2 dBs,
- Figure 8. - TC2 and frame length 1024; in comparison with TC1 and frame length 1024 we can see that TC2 brings better results then TC1. It is because TC1 is a better code – its component encoders have longer constrain lengths (equal to 4) than component encoders of TC2 (that are equal to 3). Nevertheless, the results are anyway better than for TC1 and frame length 169. It shows how much important is the interleaver size for achieving good BER performance

5. Conclusion

Considering the results of research, we can see that turbo codes in DSSS show similar performance in non-white channels as in AWGN. Nevertheless, unemployment of the receiver structure, apart from the signal as well to noise, results in a significant performance degradation. The relation between non-white noise and achievable gains is as follows: the less entropy of noise, the higher obtained gains.

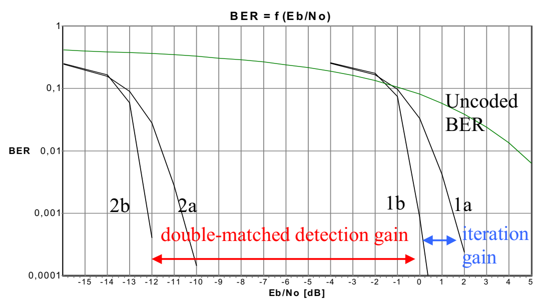

Figure 6. BER performance of TC1. Frame length 1024. Uncoded BER - no coding, single-matched detection, 1a - single-matched detection, 1 iteration, 1b - single-matched detection, 8 iteration, 2a - double-matched detection, 1 iteration, 2b - double-matched detection, 1 iteration.

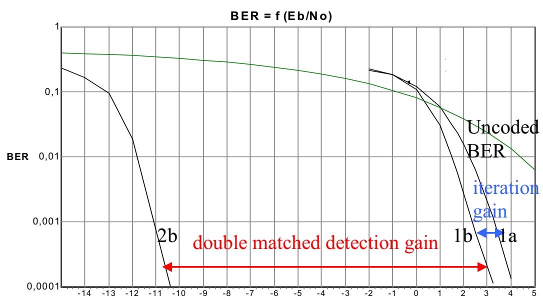

Figure 7. BER performance of TC1. Frame length 196. The meaning of these curves are the same as on figure 6.

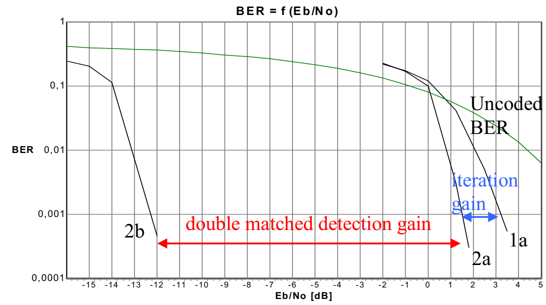

Figure 8. BER performance of TC2. Frame length 1024. The meaning of these curves are the same as on Figure 7.

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6. Literature

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