IMPROVING FLIGHT TIME OF A PAPER HELICOPTER USING DESIGN OF EXPERIMENTS PRINCIPLES

ISEN 616
Design & Analysis of Industrial Experiments

Department of Industrial and Systems Engineering
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Submitted to:

Prof. Rui Tuo

Submitted by:
Prabha Kanyal (230002892)

TEXAS A&M UNIVERSITY

CONTENT

1. EXECUTIVE SUMMARY

Design of experiments is being used in various industries for improving the processes in terms of minimizing the variability of processes, as well as to improve its performance. Concept in design of experiments revolves around identifying the effects of various factors on the response and then selecting the best configuration of factors based on the optimizing criteria (i.e. to maximize, minimize or expecting system to perform in selected range).

The goal of this project is to optimize the design of the paper helicopter to increase the flight time by using non regular Orthogonal Array technique. This technique is used in the industries for the advantage of economy of run size and flexibility.

There are 6 main factors to be considered and 7 - 11 are disregarded factors to make resolution of 11 factors for OA (12, 2^{11}) design. Using the principle of Randomization and Replication, the initial experiment was conducted in which 12 paper helicopters were flown from a height of 11 feet and the flight time was noted.

Using 12-run Plackett Burman design in Minitab software, the set of data was analyzed. With the help of Half normal plots the significant main effects were found out. This was followed by an analysis in R using Hamada Wu analysis strategy. The principle of effect hierarchy, effect sparsity and effect heredity has been used in this analysis. The results from the Plackett Burman design analysis and Hamada Wu analysis is concluded and used for achieving optimized output i.e maximum flight time .

Later validation of the has been done by making a paper helicopter with acquired outcome.Throughout this DOE analysis, several challenges were faced which are definite stepping stones for more complex problems in real world.

2. PROJECT DISCRIPTION

This project focuses on applying DOE principles to achieve the maximum flight time possible for a paper helicopter.

The project requires to make a paper helicopter like the one illustrated in the figure below out of a standard US letter-sized paper (8.5 x 11 inches). Tools that aid during the experiment were a pair of scissors, ruler, adhesive tape and stopwatch. The flight time is recorded for 100^{th} of a sec. In the entire experiment, 3 replications of each run was carried out, and the average was taken, so as to have reliability in measurements.

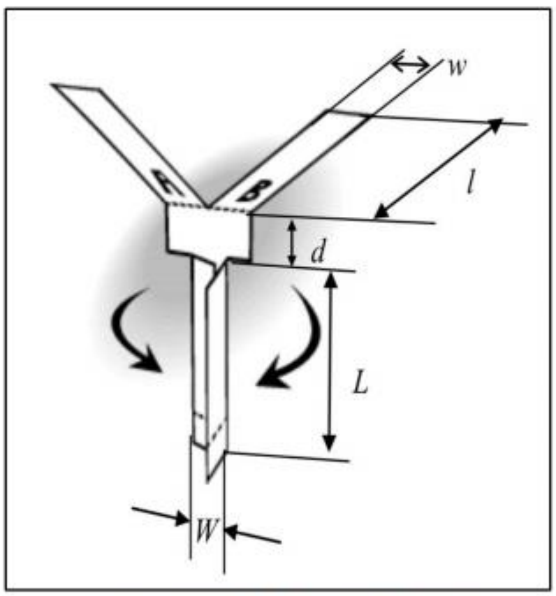


Figure 1 Paper Helicopter Design Template

In this experiment, 12-run Plackett Burman design has been used. The method uses half normal plot for Location and Dispersion to identify the significant effects. This is followed by Hamada Wu analysis strategy which uses stepwise regression model to identify the significant effects.

3. METHODOLOGY AND EXPERIMENTATION

3.1 The Experiment

The following are the six main factors as given in the coursework.

Factors	Symbol	Dimension	
		- level	+ level
Wing length	*l*	3 inches	4.5 inches
Wing width	*w*	1.8 inches	2.4 inches
Body length	*L*	3 inches	4.5 inches
Body width	*W*	1.25 inches	2 inches
Middle body length	*d*	1 inch	1.5 inches
Fold at tip	*F*	no	yes

The following table is prepared in MS Excel with the 3 replicates of Flight time as T1, T2 and T3. Also, the average flight time using the 3 replicates is calculated in column $\bar{Y}$ and the natural log value is taken for standard deviation (ln S^2) of the three replicates. Here 7 -11 factors are taken as unimportant factors for the simplicity of Placket burman design OA (12 ,2^{11}).

Runs	wing length (*l*)	wing width (w)	body length (*L*)	body width (*W*)	middle body length (*d*)	fold at tip (*F*)	7	8	9	10	11	flight time (T1)	flight time (T2)	flight time (T3)	$\bar{Y}$	$\ln S^2$
1	4.5	2.4	3	2	1.5	yes	-1	-1	-1	1	-1	3.48	2.96	3.88	3.44	-1.5474
2	3	2.4	4.5	1.25	1.5	yes	1	-1	-1	-1	1	3.73	3.29	3.34	3.45333	-2.84674
3	4.5	1.8	4.5	2	1	yes	1	1	-1	-1	-1	3.49	2.9	3.08	3.15667	-2.39215
4	3	2.4	3	2	1.5	no	1	1	1	-1	-1	3.29	3.08	3.35	3.24	-3.90704
5	3	1.8	4.5	1.25	1.5	yes	-1	1	1	1	-1	3.61	3.15	3.47	3.41	-2.88957
6	3	1.8	3	2	1	yes	1	-1	1	1	1	2.83	3.41	3.6	3.28	-1.82697
7	4.5	1.8	3	1.25	1.5	no	1	1	-1	1	1	3.6	4.14	4.39	4.04333	-1.8138
8	4.5	2.4	3	1.25	1	yes	-1	1	1	-1	1	4.2	3.95	4.07	4.07333	-4.15835
9	4.5	2.4	4.5	1.25	1	no	1	-1	1	1	-1	4.45	3.81	4.2	4.15333	-2.26304
10	3	2.4	4.5	2	1	no	-1	1	-1	1	1	3.33	3.02	3.04	3.13	-3.50323
11	4.5	1.8	4.5	2	1.5	no	-1	-1	1	-1	1	3.16	3	3.09	3.08333	-5.04626
12	3	1.8	3	1.25	1	no	-1	-1	-1	-1	-1	2.57	3.29	3.15	3.00333	-1.92598

3.2 Plackett Burman design

For the 12 run Plackett Burman design in Minitab, the following features were selected:
Type of design: Placket Burman design
No of factor levels: 2
No. of factors : 11
No. of runs: 12
No. of replicates: 3
Keeping in mind the principle of randomization and replication, the following planning matrix is generated in Minitab software:

	StdOrd	RunOrd	PtType	Blocks	wing le	wing wi	body le	body wi	middle	fold at t	G	H	J	K	L	Flight time
1	18	1	1	1	4.5	2.4	3.0	2.00	1.5	yes	-1	-1	-1	1	-1	3.48
2	10	2	1	1	3.0	2.4	4.5	1.25	1.5	yes	1	-1	-1	-1	1	3.73
3	31	3	1	1	4.5	1.8	4.5	2.00	1.0	yes	1	1	-1	-1	-1	3.49
4	5	4	1	1	3.0	2.4	3.0	2.00	1.5	no	1	1	1	-1	-1	3.29
5	8	5	1	1	3.0	1.8	4.5	1.25	1.5	yes	-1	1	1	1	-1	3.61
6	35	6	1	1	3.0	1.8	3.0	2.00	1.0	yes	1	-1	1	1	1	2.83
7	12	7	1	1	4.5	1.8	3.0	1.25	1.5	no	1	1	-1	1	1	3.60
8	16	8	1	1	4.5	2.4	3.0	1.25	1.0	yes	-1	1	1	-1	1	4.20
9	1	9	1	1	4.5	2.4	4.5	1.25	1.0	no	1	-1	1	1	-1	4.45
10	30	10	1	1	3.0	2.4	4.5	2.00	1.0	no	-1	1	-1	1	1	3.33
11	19	11	1	1	4.5	1.8	4.5	2.00	1.5	no	-1	-1	1	-1	1	3.16
12	6	12	1	1	3.0	1.8	3.0	1.25	1.0	no	-1	-1	-1	-1	-1	2.57
13	29	13	1	1	4.5	2.4	3.0	2.00	1.5	yes	-1	-1	-1	1	-1	2.96
14	28	14	1	1	3.0	2.4	4.5	1.25	1.5	yes	1	-1	-1	-1	1	3.29

Plackett Burman (PB) is a non regular orthogonal array technique which is also a combinatorial design. This experiment is of strength 2 which means that each pair of columns are orthogonal to each other. This design is a OA (12 ,2^{11}) design which means that it has only 12 number of runs for maximum 11 factors.

If the number of parametrs (this includes main effects and 2 factor interactions) is more than design points (no. of runs), hen the design is not feasible. This means that having 11 factors in this design, 2 factor interactions cannot be calculated using this technique as we have only 12 degree of freedom (12 runs). But we can use this technique as it is flexible to save extra number of runs and also for its run size economy.

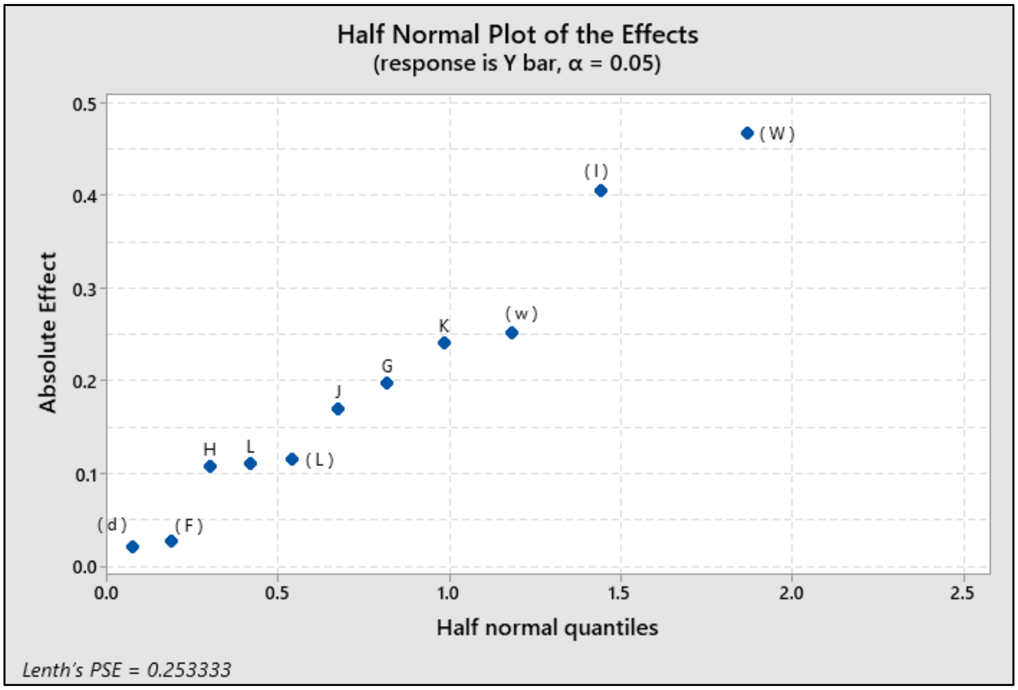
3.2.1 Plackett Burman design: Location Model and analysis

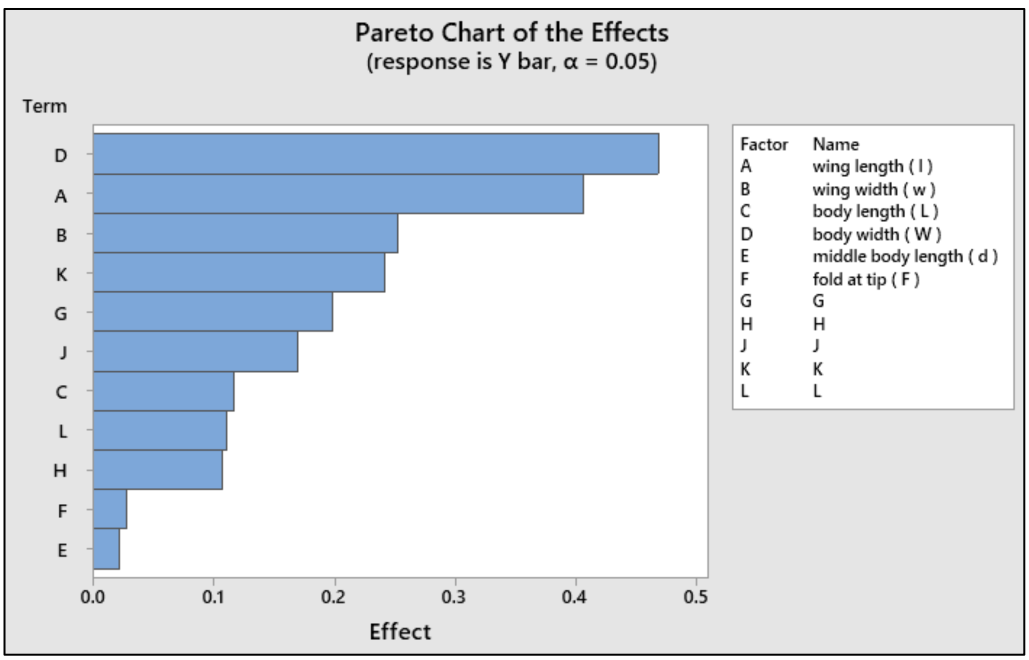
For the location model analysis, main effects using $\bar{Y}$ (average of 3 replicates of flight time) is obtained. Factorial analysis is performed. The following results of half normal plot, regression equation and Pareto chart of the effects is obtained.

Term	Effect	Coef
Constant		3.456
wing length (l)	0.4056	0.2028
wing width (w)	0.2522	0.1261
body length (L)	-0.11556	-0.05778
body width (W)	-0.4678	-0.2339
middle body length (d)	-0.02111	-0.01056
fold at tip (F)	0.02667	0.01333
G	0.19778	0.09889
H	0.10667	0.05333
J	0.16889	0.08444
K	0.2411	0.1206
L	0.11000	0.05500

Regression Equation in Uncoded Units

Y bar = 3.456 + 0.2028 wing length (l) + 0.1261 wing width (w)
- 0.05778 body length (L) - 0.2339 body width (W)
- 0.01056 middle body length (d) + 0.01333 fold at tip (F) + 0.09889 G
+ 0.05333 H + 0.08444 J + 0.1206 K + 0.05500 L





Here Body Width (W) and Wing Length (l) are two main significant effects.

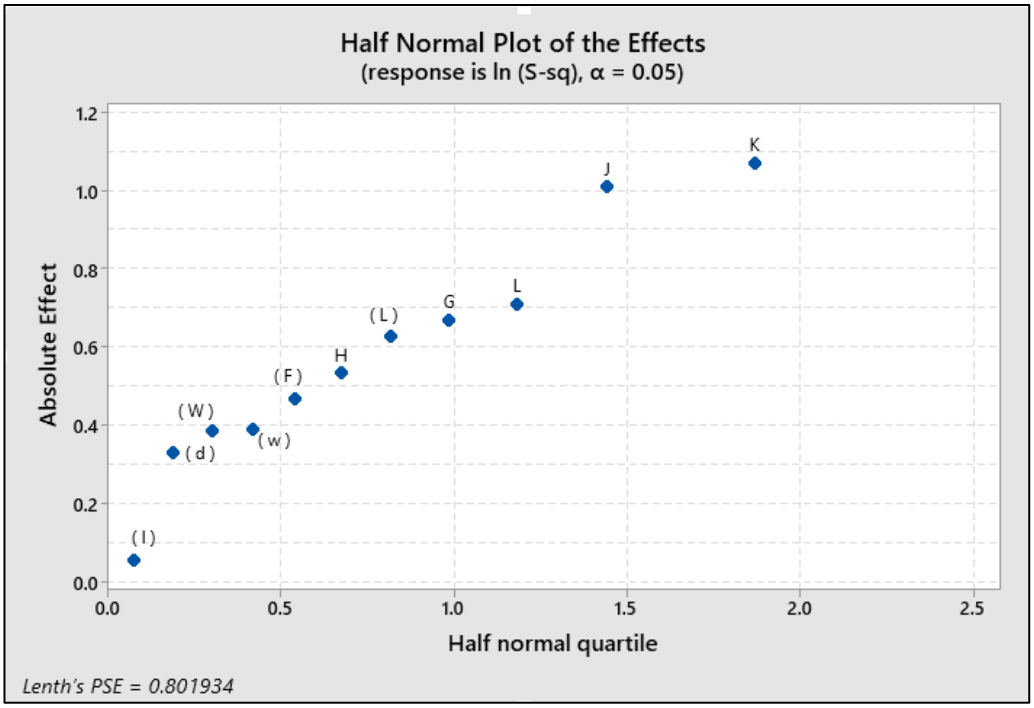
3.2.2 Plackett Burman design : Dispersion Model and analysis

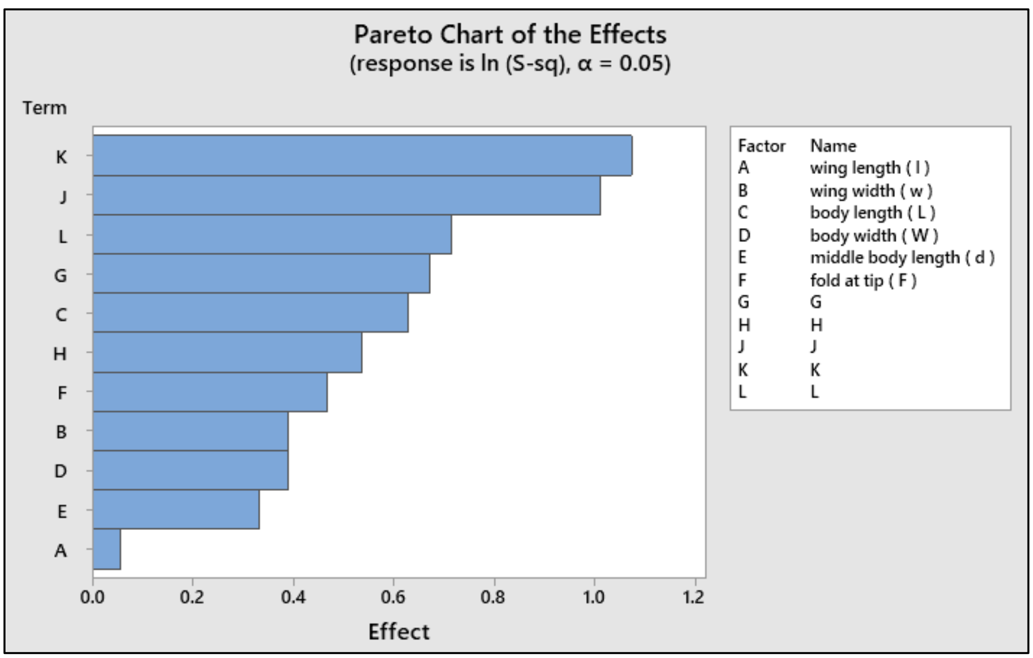
For the Dispersion model analysis, effects using *ln* S^2 obtained. Factorial analysis is performed. The following results of half normal plot, regression equation and pareto chart of the effects is obtained.

Term	Effect	Coef
Constant		-2.843
wing length (l)	-0.05358	-0.02679
wing width (w)	-0.3885	-0.1943
body length (L)	-0.6269	-0.3135
body width (W)	-0.3876	-0.1938
middle body length (d)	-0.3302	-0.1651
fold at tip (F)	0.4664	0.2332
G	0.6702	0.3351
H	-0.5346	-0.2673
J	-1.0103	-0.5052
K	1.0721	0.5360
L	-0.7117	-0.3558

Regression Equation in Uncoded Units

ln (S-sq) = -2.843 - 0.02679 wing length (l) - 0.1943 wing width (w)
- 0.3135 body length (L) - 0.1938 body width (W)
- 0.1651 middle body length (d) + 0.2332 fold at tip (F) + 0.3351 G
- 0.2673 H - 0.5052 J + 0.5360 K - 0.3558 L





Here none of the main effects out of 6 main effects are significant.

As obtained in dispersion model above that none of the main effects are significant (1- 6 factors) which reveals the limitation of PB design of not using 2 factor interaction. As there can be 2 factor interaction that can be significant.

3.3 Hamada Wu Analysis

To overcome the above limitation of PB design, 4 step Hamada Wu analysis strategy is used. this method uses stepwise regression of all the main effects and 2 factor interactions. Here we do not use higher order interactions based on effect Hierarchy principle. Also the Hamada Wu analysis uses effect heredity principle in its analysis.

The given excel table is used for Hamada Wu analysis in R:

Runs	l	w	L	W	d	F	lw	lL	lW	ld	lF	wL	wW	wd	wF	LW	Ld	LF	Wd	WF	dF	$\bar{y}$	$\ln S^2$
1	1	1	-1	1	1	1	1	-1	1	1	1	-1	1	1	1	-1	-1	-1	1	1	1	3.44	-1.55
2	-1	1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1	3.4533	-2.85
3	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	3.1567	-2.39
4	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	1	-1	-1	-1	1	1	-1	-1	3.24	-3.91
5	-1	-1	1	-1	1	1	1	-1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	3.41	-2.89
6	-1	-1	-1	1	-1	1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	1	-1	3.28	-1.83
7	1	-1	-1	-1	1	-1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	1	-1	1	-1	4.0433	-1.81
8	1	1	-1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	1	-1	1	-1	-1	4.0733	-4.16
9	1	1	1	-1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	-1	-1	-1	1	1	1	4.1533	-2.26
10	-1	1	1	1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	1	-1	-1	-1	-1	1	3.13	-3.5
11	1	-1	1	1	1	-1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	1	-1	-1	3.0833	-5.05
12	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.0033	-1.93

3.3.1 Hamada Wu Analysis : Location model

<u>**Step 1:**</u> Stepwise regression of each factor and its 2fi and repeating this for all factors.

```
> ##step 1
> regfit_l1=lm(Ybar~l+lw+lL+lW+ld+lF,data=project_data)
> ols_step_both_p(regfit_l1)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                     Added/                    Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1        l       addition       0.269       0.196    31.8150    13.7350    0.3659
   2        lW      addition       0.526       0.420    19.8530    10.5533    0.3108
-------------------------------------------------------------------------------------
> regfit_w1=lm(Ybar~w+lw+wL+wW+wd+wF,data=project_data)
> ols_step_both_p(regfit_w1)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                     Added/                    Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1        wd      addition       0.245       0.169    -0.3890    14.1308    0.3720
-------------------------------------------------------------------------------------
> regfit_L1=lm(Ybar~L+lL+wL+LW+Ld+LF,data=project_data)
> ols_step_both_p(regfit_L1)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                     Added/                    Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1        lL      addition       0.121       0.033    -2.2150    15.9482    0.4012
-------------------------------------------------------------------------------------
```

```
> regfit_W1=lm(Ybar~W+lW+wW+LW+Wd+WF,data=project_data)
> ols_step_both_p(regfit_W1)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                     Added/                    Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1        W       addition       0.358       0.294     2.6480    12.1766    0.3429
   2        lW      addition       0.615       0.529     0.3960     8.0608    0.2801
-------------------------------------------------------------------------------------
> regfit_d1=lm(Ybar~d+ld+wd+Ld+Wd+dF,data=project_data)
> ols_step_both_p(regfit_d1)

                              Stepwise Selection Summary
--------------------------------------------------------------------------------------
                     Added/                    Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)         AIC        RMSE
--------------------------------------------------------------------------------------
   1        wd      addition       0.245       0.169    -1.7230    14.1308    0.3720
--------------------------------------------------------------------------------------
> regfit_F1=lm(Ybar~F+lF+wF+LF+WF+dF,data=project_data)
> ols_step_both_p(regfit_F1)

                              Stepwise Selection Summary
--------------------------------------------------------------------------------------
                     Added/                    Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)         AIC        RMSE
--------------------------------------------------------------------------------------
   1        lF      addition       0.087      -0.005    -2.4970    16.4133    0.4091
--------------------------------------------------------------------------------------
```

Significant effects obtained in step 1 are l , W , l:W , w:d , l:L , l:F

<u>Step 2 :</u> Stepwise regression of all the factors obtained in step 1 and the main effects.

```
> ##step2:
> regfit_all1=lm(Ybar~l+w+L+W+d+F+lF+wd+lw+lL,data=project_data)
> ols_step_both_p(regfit_all1)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                     Added/                   Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1       W        addition       0.358       0.294    0.2620    12.1766    0.3429
   2       l        addition       0.628       0.545   -1.2060     7.6453    0.2753
-------------------------------------------------------------------------------------
```

Significant effects obtained in step 2 are l and W.

Step 3: Using effect heredity principle, Stepwise regression of the factors obtained (i) significant effects obtained in step 2, (ii) 2fi of the significant effect in (i).

(i) W , l

(ii) l:w , l:L , l:W , l:d , l:F , w:W , L:W , W:d , W:F

```
> ##step3:
> regfit_all2=lm(Ybar~l+W+lF+lW+lL+lw+ld+wW+LW+Wd+WF,data=project_data)
> ols_step_both_p(regfit_all2)

                              Stepwise Selection Summary
---------------------------------------------------------------------------------------
                     Added/                   Adj.
Step    Variable    Removed     R-Square    R-Square      C(p)         AIC        RMSE
---------------------------------------------------------------------------------------
   1       W        addition       0.358       0.294    2870.9210    12.1766    0.3429
   2       l        addition       0.628       0.545    1664.5310     7.6453    0.2753
   3       lW       addition       0.884       0.840     516.9980    -4.3365    0.1631
   4       LW       addition       0.970       0.953     132.3130   -18.6034    0.0885
   5       lL       addition       0.995       0.991      22.5260   -38.0294    0.0392
   6       wW       addition       0.998       0.996      11.1560   -46.8329    0.0273
---------------------------------------------------------------------------------------
```

Significant effects obtained in step 3 are W, l , l:W , L:W , l:L , w:W

Step 4: Iteration of step 2 and step 3.

```
> ##Step4:
> regfit_all3=lm(Ybar~l+W+lW+lL+wW+LW,data=project_data)
> ols_step_both_p(regfit_all3)

                              Stepwise Selection Summary
------------------------------------------------------------------------------------------
                       Added/                    Adj.
Step    Variable      Removed     R-Square     R-Square      C(p)          AIC        RMSE
------------------------------------------------------------------------------------------
   1        W         addition       0.358        0.294    1564.2000     12.1766    0.3429
   2        l         addition       0.628        0.545     906.2890      7.6453    0.2753
   3        lW        addition       0.884        0.840     280.5210     -4.3365    0.1631
   4        LW        addition       0.970        0.953      71.3490    -18.6034    0.0885
   5        lL        addition       0.995        0.991      12.3020    -38.0294    0.0392
   6        wW        addition       0.998        0.996       7.0000    -46.8329    0.0273
------------------------------------------------------------------------------------------
> summary(regfit_all3)

Call:
lm(formula = Ybar ~ l + W + lW + lL + wW + LW, data = project_data)

Residuals:
        1         2         3         4         5         6         7         8 
-0.011481  0.002778  0.042407  0.010463  0.008704  0.001204  0.015463 -0.003796 
        9        10        11        12 
-0.011667 -0.011667 -0.030926 -0.011481 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  3.455556   0.007894 437.764 1.18e-12 ***
l            0.234097   0.009043  25.886 1.61e-06 ***
W           -0.209005   0.008450 -24.735 2.01e-06 ***
lW          -0.197778   0.007894 -25.055 1.89e-06 ***
lL          -0.074653   0.009043  -8.255 0.000425 ***
wW          -0.024630   0.009115  -2.702 0.042674 *  
LW          -0.118588   0.008450 -14.035 3.30e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.02734 on 5 degrees of freedom
Multiple R-squared:  0.998,	Adjusted R-squared:  0.9955 
F-statistic: 407.5 on 6 and 5 DF,  p-value: 1.477e-06
```

Significant effects for Location model are Body width (W), Wing length (l), interaction of Wing length (l) : Body width (W), Body length (L) : Body width (W), Wing length (l): Body length (L), and Wing width(w) : Body width (W).

3.3.2 Hamada Wu Analysis: Dispersion model

Step 1: Stepwise regression of each factor and it’s 2fi and repeating this for all factors.

```
> ##step 1
> regfit_l=lm(lnssq~l+lw+lL+lW+ld+lF,data=project_data)
> ols_step_both_p(regfit_l)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                      Added/                    Adj.
Step    Variable     Removed      R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1       lw        addition       0.150       0.065    -2.6230    39.3400    1.0633
-------------------------------------------------------------------------------------
> regfit_w=lm(lnssq~w+lw+wL+wW+wd+wF,data=project_data)
> ols_step_both_p(regfit_w)

                             Stepwise Selection Summary
------------------------------------------------------------------------------------
                      Added/                    Adj.
Step    Variable     Removed      R-Square    R-Square     C(p)       AIC        RMSE
------------------------------------------------------------------------------------
   1       wL        addition       0.208       0.129    2.4130    38.4974    1.0266
------------------------------------------------------------------------------------
> regfit_L=lm(lnssq~L+lL+wL+LW+Ld+LF,data=project_data)
> ols_step_both_p(regfit_L)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                      Added/                    Adj.
Step    Variable     Removed      R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1       wL        addition       0.208       0.129    -0.2900    38.4974    1.0266
-------------------------------------------------------------------------------------
> regfit_W=lm(lnssq~W+lW+wW+LW+Wd+WF,data=project_data)
> ols_step_both_p(regfit_W)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                      Added/                    Adj.
Step    Variable     Removed      R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1       WF        addition       0.701       0.671    23.9260    26.7934    0.6304
-------------------------------------------------------------------------------------
```

```
> regfit_d=lm(lnssq~d+ld+wd+Ld+Wd+dF,data=project_data)
> ols_step_both_p(regfit_d)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                      Added/                    Adj.
Step    Variable     Removed      R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1       wd        addition       0.171       0.088    0.2950    39.0407    1.0501
-------------------------------------------------------------------------------------
> regfit_F=lm(lnssq~F+lF+wF+LF+WF+dF,data=project_data)
> ols_step_both_p(regfit_F)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                      Added/                    Adj.
Step    Variable     Removed      R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1       WF        addition       0.701       0.671    7.8870    26.7934    0.6304
   2       dF        addition       0.810       0.768    4.0980    23.3556    0.5298
-------------------------------------------------------------------------------------
```

Significant effects obtained in step 1 are interaction between l:w, w:L, W:F , w:d , W:F and d:F.

Step 2: Stepwise regression of all the factors obtained in step 1 and the main effects.

```
> ##step2:
> regfit_all=lm(lnssq~l+w+L+W+d+F+WF+dF+wd+wL+lw,data=project_data)
> ols_step_both_p(regfit_all)

                           Stepwise Selection Summary
---------------------------------------------------------------------------------
                  Added/                    Adj.
Step    Variable  Removed     R-Square    R-Square     C(p)       AIC        RMSE
---------------------------------------------------------------------------------
   1       WF     addition       0.701       0.671      NaN    26.7934     0.6304
   2       dF     addition       0.810       0.768      NaN    23.3556     0.5298
---------------------------------------------------------------------------------
```

Significant effects obtained in step 2 are interaction between W:F and d:F.

Step 3: Using effect heredity principle, Stepwise regression of the factors obtained (i) significant effects obtained in step 2, (ii) 2fi of the significant effect in (i).
(i) W:F and d:F
(ii) there are no 2fi as there is no main effect in (i)

```
> regfit_all_=lm(lnssq~WF+dF,data=project_data)
> ols_step_both_p(regfit_all_)

                           Stepwise Selection Summary
---------------------------------------------------------------------------------
                  Added/                    Adj.
Step    Variable  Removed     R-Square    R-Square     C(p)       AIC        RMSE
---------------------------------------------------------------------------------
   1       WF     addition       0.701       0.671    6.1590    26.7934    0.6304
   2       dF     addition       0.810       0.768    3.0000    23.3556    0.5298
---------------------------------------------------------------------------------
> summary(regfit_all_)

Call:
lm(formula = lnssq ~ WF + dF, data = project_data)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.97368 -0.17201 -0.00822  0.48354  0.53108 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -2.8434     0.1529 -18.592 1.73e-08 ***
WF            0.8818     0.1529   5.766 0.000271 ***
dF            0.3474     0.1529   2.271 0.049248 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5298 on 9 degrees of freedom
Multiple R-squared:  0.8101,    Adjusted R-squared:  0.768 
F-statistic:  19.2 on 2 and 9 DF,  p-value: 0.0005661
```

Significant effects for Dispersion: Interaction between ,body width (W): Fold at tip (F) and middle body length (d) : Fold at tip (F) are 2 significant effects obtained.

4. CONCLUSION

Comparing the result of Plackett Burman design analysis and Hamada –Wu Analysis in the following table:

Model	***Type of design***	***Significant factors***
Location Model	12- Run Plackett Burman design	W , l
	Hamada Wu Analysis	W , l , l : W , L : W , l : L , w : W
Dispersion Model	12- Run Plackett Burman design	No significant Main effects
	Hamada Wu Analysis	W : F , d : F

Here, it can be noticed that the two designs do not contradict each other in terms of significant effects. While PB design has found the significant main effects, the Hamada Wu analysis includes the significant factors of PB design followed by significant 2 factor interactions. Next, we validate the model using the overall significant effects of the two designs.

5. VALIDATION AND RESULT

Dispersion model significant effects: Body width (W) : Fold at tip (F) and Middle body length (d): Fold at tip (F).

Regression Equation for Dispersion is:

$$\ln(S^2) = -2.8434 + 1.4395\, X_W X_F + 0.2551\, X_d X_F$$

- To minimize the dispersion, we need $X_W X_F = (-)$ and $X_d X_F = (-)$.

Location model significant effects : Body width (W), Wing length (l), Wing length (l) : Body width (W), Body length (L) : Body width (W), Wing length (l): Body length (L), and Wing width(w) : Body width (W).

Regression Equation for location is:

$$\bar{Y} = 3.456 + 0.234\, X_l - 0.2090\, X_W - 0.1978\, X_l X_W - 0.0746\, X_l X_L - 0.0246\, X_w X_W - 0.1185\, X_L X_W$$

- To maximize the flight time, we take $l = (+)$ and $W = (-)$ and hence we get $L = (+)$, w = (+) from the above equation.
- Thus placing the above values in the dispersion equation we get, $F = (+)$ and $d = (-)$.

Thus the optimized design parameters to maximize the flight time is:

Factor	***Symbol***	***Levels***
Wing length	(l)	4.5 inches
Wing width	(w)	2.4 inches
Body length	(L)	4.5 inches
Body width	(W)	1.25 inches
Middle body length (d)	(d)	1 inch
Fold at tip	(F)	yes

Based on the optimized output parameters on the location equation, $\bar{Y} = 4.3145$ sec.

So, taking the above parameters in our helicopter, an estimated flight time of 4.46 **seconds** is achieved. Using 3 replicates (4.62 sec, 4.48 sec and 4.32 sec). A deviation of 0.1455 sec from the predicted value has been observed.

6. APPENDIX 1: R-CODE

To calculate significant effects for dispersion model using Hamada Wu analysis strategy (stepwise regression method):

```
library(ISLR)
library(leaps)
library(olsrr)
library(readxl)
project_data  <- read_excel("C:/Users/prabha/Desktop/ISEN 616/Project/prabha_project.xlsx")
View(project_data )

##step 1 (Stepwise regression for each factor and it's 2fi)
regfit_l=lm(lnssq~l+lw+lL+lW+ld+lF,data=project_data)
ols_step_both_p(regfit_l)

regfit_w=lm(lnssq~w+lw+wL+wW+wd+wF,data=project_data)
ols_step_both_p(regfit_w)

regfit_L=lm(lnssq~L+lL+wL+LW+Ld+LF,data=project_data)
ols_step_both_p(regfit_L)

regfit_W=lm(lnssq~W+lW+wW+LW+Wd+WF,data=project_data)
ols_step_both_p(regfit_W)

regfit_d=lm(lnssq~d+ld+wd+Ld+Wd+dF,data=project_data)
ols_step_both_p(regfit_d)

regfit_F=lm(lnssq~F+lF+wF+LF+WF+dF,data=project_data)
ols_step_both_p(regfit_F)

##step 2 (From above significant effects and all the main effects)
regfit_all=lm(lnssq~l+w+L+W+d+F+WF+dF+wd+wL+lw,data=project_data)
ols_step_both_p(regfit_all)
summary(regfitall)

##step 3 (The significant effects from step 2 and 2fi)
regfit_all_=lm(lnssq~WF+dF,data=project_data)
ols_step_both_p(regfit_all_)
```

```
summary(regfit_all_)
```

To calculate significant effects for Location model using Hamada Wu analysis strategy (stepwise regression method):

```
library(readxl)
project_data  <- read_excel("C:/Users/prabha/Desktop/ISEN 616/Project/prabha_project.xlsx")
View(project_data )

##step 1 (Stepwise regression for each factor and it's 2fi)
regfit_l1=lm(Ybar~l+lw+lL+lW+ld+lF,data=project_data)
ols_step_both_p(regfit_l1)

regfit_w1=lm(Ybar~w+lw+wL+wW+wd+wF,data=project_data)
ols_step_both_p(regfit_w1)

regfit_L1=lm(Ybar~L+lL+wL+LW+Ld+LF,data=project_data)
ols_step_both_p(regfit_L1)

regfit_W1=lm(Ybar~W+lW+wW+LW+Wd+WF,data=project_data)
ols_step_both_p(regfit_W1)

regfit_d1=lm(Ybar~d+ld+wd+Ld+Wd+dF,data=project_data)
ols_step_both_p(regfit_d1)

regfit_F1=lm(Ybar~F+lF+wF+LF+WF+dF,data=project_data)
ols_step_both_p(regfit_F1)

##step 2 (From above significant effects and all the main effects)
regfit_all1=lm(Ybar~l+w+L+W+d+F+lF+wd+lw+lL,data=project_data)
ols_step_both_p(regfit_all1)

##step 3 (The significant effects from step 2 and 2fi)
regfit_all2=lm(Ybar~l+W+lF+lW+lL+lw+ld+wW+LW+Wd+WF,data=project_data)
ols_step_both_p(regfit_all2)

##Step4
regfit_all3=lm(Ybar~l+W+lW+lL+wW+LW,data=project_data)
ols_step_both_p(regfit_all3)
summary(regfit_all3)
```