CS 412 Intro. to Data Mining

Chapter 8. Classification: Basic Concepts

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Information Gain: An Attribute Selection Measure

- Select the attribute with the highest information gain (used in typical decision tree induction algorithm: ID3/C4.5)
- Let p_i be the probability that an arbitrary tuple in D belongs to class C_i, estimated by $|C_{i,D}|/|D|$
- Expected information (entropy) needed to classify a tuple in D:

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

- Information needed (after using A to split D into v partitions) to classify D:

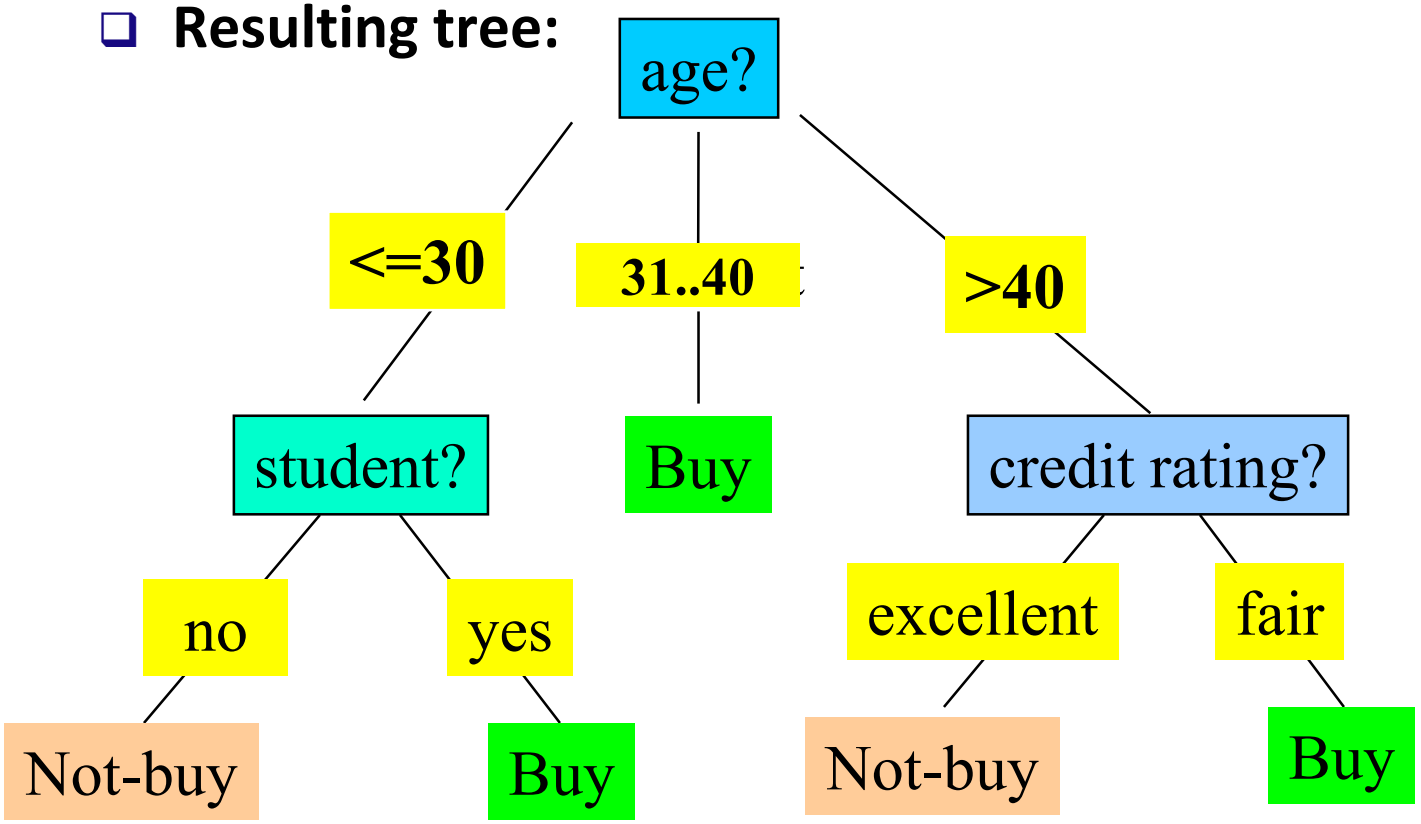
$$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j)$$

- Information gained by branching on attribute A

$$Gain(A) = Info(D) - Info_A(D)$$

Decision Tree Induction: An Example

- **Decision tree construction**:
  - A top-down, recursive, divide-and-conquer process
- **Resulting tree:**



Training data set: Who buys computer?

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Note: The data set is adapted from "Playing Tennis" example of R. Quinlan

หา Info(D)

$$\text{Info}(D) = I(9,5) = \underbrace{-\frac{9}{14}\log_{(2)}\left(\frac{9}{14}\right)}_{Y} \underbrace{-\frac{5}{14}\log_{(2)}\left(\frac{5}{14}\right)}_{N}$$

$$= 0.94$$

หา Info age (D)

$$\text{Info}_{age}(D) = \underbrace{\frac{5}{14} I(2,3)}_{\leq 30} + \underbrace{\frac{4}{14} I(4,0)}_{31-40} + \underbrace{\frac{5}{14} I(3,2)}_{>40}$$

$$I(2,3) = -\frac{2}{5}\log_{(2)}\left(\frac{2}{5}\right) - \frac{3}{5}\log_{(2)}\left(\frac{3}{5}\right) = 0.971$$

$$I(4,0) = -\frac{4}{4}\log_{(2)}\left(\frac{4}{4}\right) - \frac{0}{4}\log_{(2)}\left(\frac{0}{4}\right) = 0$$

$$I(3,2) = -\frac{3}{5}\log_{(2)}\left(\frac{3}{5}\right) - \frac{2}{5}\log_{(2)}\left(\frac{2}{5}\right) = 0.971$$

$$\text{Info}_{age}(D) = \frac{5}{14}(0.971) + \frac{4}{14}(0) + \frac{5}{14}(0.971) = 0.694$$

หา Gain (age)

$$\text{Gain}(age) = 0.94 - 0.694 = 0.246$$

an Info income (D)

high medium low

$$\text{Info}_{\text{income}}(D) = \boxed{\frac{4}{14} I(2,2)} + \boxed{\frac{6}{14} I(4,2)} + \boxed{\frac{4}{14} I(3,1)}$$

$$I(2,2) = -\frac{2}{4}\log_2\left(\frac{1}{4}\right) - \frac{2}{4}\log_2\left(\frac{1}{4}\right) = 1$$

$$I(4,2) = -\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log_2\left(\frac{2}{6}\right) = 0.918$$

$$I(3,1) = -\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) = 0.811$$

$$\text{Variation Info income}(D) = \frac{4}{14}(1) + \frac{6}{14}(0.918) + \frac{4}{14}(0.811) = 0.911$$

an Gain (income)

$$\text{Gain}(\text{income}) = 0.94 - 0.911 = 0.029$$

หา Info student (D)

Info student (D) = $\frac{7}{14} I(6,1)$ (Yes: y 6, n 1) + $\frac{7}{14} I(3,4)$ (No: y 3, n 4)

$I(6,1) = -\frac{6}{7} \log_2\left(\frac{6}{7}\right) - \frac{1}{7} \log_2\left(\frac{1}{7}\right) = 0.592$

$I(3,4) = -\frac{3}{7} \log_2\left(\frac{3}{7}\right) - \frac{4}{7} \log_2\left(\frac{4}{7}\right) = 0.985$

Remain Info student (D) = $\frac{7}{14}(0.592) + \frac{7}{14}(0.985) = 0.789$

หา Gain (student)

Gain (student) = $0.94 - 0.789 = 0.151$

หา Info credit_rating (D)

fair excellent

$$\text{Info}_{\text{credit\_rating}}(D) = \boxed{\frac{8}{14} I(6,2)} + \boxed{\frac{6}{14} I(3,3)}$$

$$I(6,2) = -\frac{6}{8}\log_2\left(\frac{6}{8}\right) - \frac{2}{8}\log_2\left(\frac{2}{8}\right) = 0.811$$

$$I(3,3) = -\frac{3}{6}\log_2\left(\frac{3}{6}\right) - \frac{3}{6}\log_2\left(\frac{3}{6}\right) = 1$$

ดังนั้น $\text{Info}_{\text{credit\_rating}}(D) = \frac{8}{14}(0.811) + \frac{6}{14}(1) = 0.892$

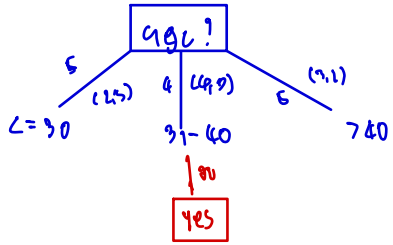
หา Gain (credit_rating)

$$\text{Gain}(\text{credit\_rating}) = 0.94 - 0.892 = 0.048$$

สรุป Gain

Gain (age) = 0.246
Gain (income) = 0.029
Gain (student) = 0.151
Gain (credit_rating) = 0.048

เลือก Gain ที่มีค่ามากที่สุดมาเป็นตัวแรก ซึ่งในที่นี้คือ Gain (age)



age = (<=30)

ขั้น Info(D) ของ age <=30

$Info(D) = I(\overset{y}{2},\overset{n}{3}) = 0.971$

ขั้น $Info_{income}(D)$ ของ age

high medium low

$Info_{income}(D)$ ของ age (<=30) = $\boxed{\frac{2}{5} I(\overset{y}{0},\overset{n}{2})} + \boxed{\frac{2}{5} I(\overset{y}{1},\overset{n}{1})} + \boxed{\frac{1}{5} I(\overset{y}{1},\overset{n}{0})}$

$I(\overset{y}{0},\overset{n}{2}) = -\frac{0}{2}\log_2(\frac{0}{2}) - \frac{2}{2}\log_2(\frac{2}{2}) = 0$

$I(\overset{y}{1},\overset{n}{1}) = -\frac{1}{2}\log_2(\frac{1}{2}) - \frac{1}{2}\log_2(\frac{1}{2}) = 1$

$I(\overset{y}{1},\overset{n}{0}) = -\frac{1}{1}\log_2(\frac{1}{1}) - \frac{0}{1}\log_2(\frac{0}{2}) = 0$

ดังนั้น $Info_{income}(D)$ ของ age (<=30) = $\frac{2}{5}(0) + \frac{2}{5}(1) + \frac{1}{5}(0) = 0.4$

ขั้น Gain (income) ของ age (<=30)

Gain(income) ของ age (<=30) = 0.971 - 0.4 = 0.571

หา Info $_{student}$ (D) ของ age (<=30)

Info $_{student}$ (D) ของ age (<=30) = $\frac{2}{5} I(2,0)$ (yes) + $\frac{3}{5} I(0,3)$ (no)

สิ้นสุด Yes → yes (buy_computer), No → (buy_computer)

เลือกแปลเดียว student เพราะว่า สามารถแปลงข้อมูลได้ครบสมบูรณ์

Age (>40)

หา Info (D) ของ Age (>40)

Info (D) ของ Age (>40) = $I(3,2) = 0.971$

หา Info income (D) ของ age (>40)

Info income (D) ของ age (>40) = $\frac{3}{5} I(2,1)$ (medium) + $\frac{2}{5} I(1,1)$ (low)

$I(2,1) = -\frac{2}{3}\log_2(\frac{2}{3}) - \frac{1}{3}\log_2(\frac{1}{3}) = 0.918$

$I(1,1) = 1$

ดังนั้น Info income (D) ของ age (>40) = $\frac{3}{5}(0.918) + \frac{2}{5}(1) = 0.951$

หา Gain (income) ของ age (>40)

Gain (income) ของ Age (>40) = 0.971 - 0.951 = 0.02

หา Info student (D) for age (>40)

Info student (D) for age (>40) = $\frac{3}{5} I(2,1)$ (yes, no) + $\frac{2}{5} I(1,1)$ (yes, no)

$I(2,1) = -\frac{2}{3}\log_2(\frac{2}{3}) - \frac{1}{3}\log_2(\frac{1}{3}) = 0.918$

$I(1,1) = 1$

แทนค่า Info student (D) for age (>40) = $\frac{3}{5}(0.918) + \frac{2}{5}(1) = 0.951$

หา Gain (student) ของ age (>40)

Gain (student) age (>40) = 0.971 - 0.951 = 0.02

หา Info credit_rating (D) ของ age (>40)

Info credit_rating (D) ของ age (>40) = $\frac{3}{5} I(3,0)$ (fair) + $\frac{2}{5} I(0,2)$ (excellent)

สังเกตุ fair → yes (buy_computer), excellent → no (buy_computer)

เลือกแบ่งด้วย credit_rating เพราะสามารถแบ่งข้อมูลได้ชัดเจน

สรุป