

$$\sum_{k=0}^{\infty} (-1)^k \frac{(k+1)^2}{k!} = \sum_{k=0}^{\infty} (-1)^k \frac{k^2}{k!} + \sum_{k=0}^{\infty} (-1)^k \frac{2k}{k!} + \sum_{k=0}^{\infty} (-1)^k \frac{1}{k!} = (*)$$

a)

$$\begin{aligned} \sum_{k=0}^{\infty} (-1)^k \frac{k^2}{k!} &= \sum_{k=1}^{\infty} (-1)^k \frac{k}{(k-1)!} = \sum_{k=1}^{\infty} (-1)^k \frac{k-1+1}{(k-1)!} = \\ &= \sum_{k=2}^{\infty} (-1)^k \frac{1}{(k-2)!} + \sum_{k=1}^{\infty} (-1)^k \frac{1}{(k-1)!} = \\ &= (-1)^2 \sum_{l=0}^{\infty} (-1)^l \frac{1}{l!} + (-1) \sum_{l=0}^{\infty} (-1)^l \frac{1}{l!} = 0. \end{aligned}$$

b)

$$\sum_{k=0}^{\infty} (-1)^k \frac{2k}{k!} = 2 \sum_{k=1}^{\infty} (-1)^k \frac{1}{(k-1)!} = -2 \sum_{l=0}^{\infty} (-1)^l \frac{1}{l!} = -\frac{2}{e}.$$

$$(*) = -\frac{2}{e} + \frac{1}{e} = -\frac{1}{e}.$$