CP PUBLICATION

SHORT TRICKS
Mathematics
for JEE (Main & Adv)

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- Career Point Editorial

SHORT TRICKS
Mathematics
for JEE (Main & Advanced)

CONTENTS

1

Quadratic Equation

Key Concepts

1. Polynomial

Algebraic expression containing many terms is called Polynomial.

e.g. : $4x^4 + 3x^3 - 7x^2 + 5x + 3$, $3x^3 + x^2 - 3x + 5$

(i) **Real Polynomial :** Let a_0, $a_1, a_2.....a_n$ be real numbers and x is a real variable.
Then $f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$ is called real polynomial of real variable x with real coefficients.

(ii) **Complex Polynomial:** If $a_0, a_1, a_2...a_n$ be complex numbers and x is a varying complex number, then $f(x) = a_0 + a_1x + a_2x^2 + ... a_nx^n$ is called a complex polynomial of complex variable x with complex coefficients.
eg.- $3x^2 - (2 + 4\,i)\,x + (5i\ -4)$,

(iii) **Degree of Polynomial :** Highest Power of variable x in a polynomial is called as a degree of polynomial.
e.g. $f(x) = a_0+a_1x+a_2x^2+a_3x^3+....a_{n-1}x^{n-1}+a_nx^n$ is n degree polynomial.

2. Quadratic Expression

A polynomial of degree two of the form $ax^2 + bx + c$ $(a \neq 0)$ is called a quadratic expression in x.

3. Quadratic Equation

A quadratic Polynomial f(x) when equated to zero is called Quadratic Equation.
$ax^2 + bx + c = 0$
Where, $a, b, c \in C$ and $a \neq 0$

4. Roots or Solution of Quadratic Equation

The values of variable x which satisfy the quadratic equation is called as Roots (also called solutions or zeros) of a Quadratic Equation.

(i) **Factorization Method :**
Let $ax^2 + bx + c = a(x - \alpha)(x - \beta) = 0$
Then $x = \alpha$ and $x = \beta$ will satisfy the given equation.
Hence factorize the equation and equating each to zero gives roots of equation.

(ii) **Hindu Method (Sri Dharacharya Method) :**
Quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has two roots, given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

5. Nature of Roots

The term $b^2 - 4ac$ is called discriminant of the equation. It is denoted by Δ or D.

(i) Suppose $a, b, c \in R$ and $a \neq 0$ then

(a) If $D > 0 \Rightarrow$ roots are real and unequal

(b) If $D = 0 \Rightarrow$ roots are real and equal and each equal to $-b/2a$

(c) If $D < 0 \Rightarrow$ roots are imaginary and unequal or complex conjugate.

(ii) Suppose $a, b, c \in Q$, $a \neq 0$ then

(a) If $D > 0$ & D is perfect square $\Rightarrow$ roots are unequal & rational

(b) If $D > 0$ & D is not perfect square $\Rightarrow$ roots are irrational & unequal

6. Conjugate Roots

The Irrational and complex roots of a quadratic equation are always occurs in pairs. Therefore $(a, b, c, \in Q)$

If One Root	then Other Root
$\alpha + i\beta$	$\alpha - i\beta$
$\alpha + \sqrt{\beta}$	$\alpha - \sqrt{\beta}$

7. Sum and Product of Equation

(i) Quadratic Equation : If the roots of quadratic equation $ax^2 + bx + c$ $(a \neq 0)$ are α and β then

Sum of roots : $S = \alpha + \beta = \frac{-b}{a} = -\frac{\text{Coefficient of } x}{\text{cofficient of } x^2}$

and Product of Roots : $P = \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{coefficient of } x^2}$

(ii) Cubic Equation : If α, β and γ are the roots of cubic equation $ax^3 + bx^2 + cx + d = 0$.

Then, $\Sigma\alpha = \alpha + \beta + \gamma = -\frac{b}{a}$

$\Sigma\alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\alpha\beta\gamma = -\frac{d}{a}$

(iii) Biquadratic Equation :

If α, β, γ and δ are the roots of the biquadratic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$, then

$S_1 = \alpha + \beta + \gamma = -\frac{b}{a}$,

$S_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = (-1)^2 \frac{c}{a} = \frac{c}{a}$

or $S_2 = (\alpha + \beta)(\gamma + \delta) + \alpha\beta + \gamma\delta = \frac{c}{a}$

$S_3 = \alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \alpha\beta\delta = (-1)^3 \frac{d}{a} = -\frac{d}{a}$

or $S_3 = \alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta) = -\frac{d}{a}$

and $S_4 = \alpha\beta\gamma\delta = (-1)^4 = \frac{e}{a} = \frac{e}{a}$

8. Relation between Roots and Coefficients

If roots of quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are α and β then

(i) $(\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \pm \dfrac{\sqrt{b^2 - 4ac}}{a} = \dfrac{\pm\sqrt{D}}{a}$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \dfrac{b^2 - 2ac}{a^2}$

(iii) $\alpha^2 - \beta^2 = (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = -\dfrac{b\sqrt{b^2 - 4ac}}{a^2}$

(iv) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -\dfrac{b(b^2 - 3ac)}{a^3}$

(v) $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\ \{(\alpha+\beta)^2 - \alpha\beta\}$

$$= \frac{(b^2 - ac)\sqrt{b^2 - 4ac}}{a^3}$$

(vi) $\alpha^4 + \beta^4 = \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2\alpha^2\beta^2 = \left(\dfrac{b^2 - 2ac}{a^2}\right)^2 - 2\dfrac{c^2}{a^2}$

(vii) $\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2) = \dfrac{\pm b(b^2 - 2ac)\sqrt{b^2 - 4ac}}{a^4}$

(viii) $\alpha^2 + \alpha\beta + \beta^2 = (\alpha + \beta)^2 - \alpha\beta$

(ix) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

(x) $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta)$

(xi) $\left(\dfrac{\alpha}{\beta}\right)^2 + \left(\dfrac{\beta}{\alpha}\right)^2 = \dfrac{\alpha^4 + \beta^4}{\alpha^2\beta^2} = \dfrac{(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

(xii) $nb^2 = ac(1 + n)^2$ when one root is n times of another

9. Formation of an Equation with given Roots

(i) **Quadratic Equation :** A quadratic equation whose roots are α and β is given by
$x^2 -$ (sum of Roots)x + Product of Roots = 0
$\therefore\quad x^2 - (\alpha + \beta)x + \alpha\beta = 0$

(ii) **Cubic Equation :** α, β, γ are the roots of cubic equation then the equation is
$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$

10. Equation in terms of the Roots of another Equation

If are roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are

(i) $-\alpha, -\beta \Rightarrow ax^2 - bx + c = 0$ (Replace x by $-x$)

(ii) $1/\alpha, 1/\beta \Rightarrow cx^2 + bx + a = 0$ (Replace x by $1/x$)

(iii) $\alpha^n, \beta^n; n \in N$ $a(x^{1/n})^2 + b(x^{1/n}) + c = 0$ (Replace x by $x^{1/n}$)

(iv) $k\alpha, k\beta \Rightarrow ax^2 + kbx + k^2c = 0$ (Replace x by x/k)

(v) $k + \alpha, k + \beta \Rightarrow a(x - k)^2 + b(x - k) + c = 0$ (Replace x by $(x - k)$)

(vi) $\dfrac{\alpha}{k}, \dfrac{\beta}{k}$ $k^2ax^2 + kbx + c = 0$ (Replace x by kx)

(vii) $\alpha^{1/n}, \beta^{1/n}; n \in N \Rightarrow a(x^n)^2 + b(x^n) + c = 0$ (Replace x by x^n)

11. Roots under particular cases

For the quadratic equation $ax^2 + bx + c = 0$

(i) If $b = 0$ $\Rightarrow$ roots are of equal magnitude but of opposite sign

(ii) If $c = 0$ $\Rightarrow$ one root is zero other is $-b/a$

(iii) If $b = c = 0$ $\Rightarrow$ both root are zero

(iv) If $a = c$ $\Rightarrow$ roots are reciprocal to each other

(v) If $\left.\begin{matrix} a > 0 & c < 0 \\ a < 0 & c > 0 \end{matrix}\right\}$ $\Rightarrow$ Roots are of opposite signs

(vi) If $\left.\begin{matrix} a > 0, b > 0, c > 0 \\ a < 0, b < 0, c < 0 \end{matrix}\right\}$ $\Rightarrow$ Both roots are negative.

(vii) $\left.\begin{matrix} a > 0, b < 0, c > 0 \\ a < 0, b > 0, c < 0 \end{matrix}\right\}$ $\Rightarrow$ Both roots are positive.

(viii) If sign of a = sign of b $\neq$ sign of c
$\Rightarrow$ Greater root in magnitude is negative.

(ix) If sign of b = sign of c $\neq$ sign of a $\Rightarrow$ Greater root in magnitude is positive.

(x) If $a + b + c = 0 \Rightarrow$ one root is 1 and second root is c/a.

(xi) If $a = b = c = 0$ then equation will become an identity and will be satisfy by every value of x.

12. Condition for Common Roots

(i) Only One Root is Common : Let α be the common root of quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ then

$\therefore$ $a_1\alpha^2 + b_1\alpha + c_1 = 0$

$a_2\alpha^2 + b_2\alpha + c_2 = 0$

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

$\therefore$ The condition for only one Root common is

$(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$

(ii) Both roots are common : Then required conditions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Note :

- To find the common roots of two equation make the coefficient of second degree term in the two equation equal and subtract. The value of x obtained is required common root.
- Two different quadratic equation with rational coefficient cannot have single common root which is complex or irrational, as imaginary and surd roots always occur in pair.

13. Graph of Quadratic Expression

An expression of the form $ax^2 + bx + c$, where $a, b, c \in R$ and $a \neq 0$ is called a quadratic expression in x.

We have $y = f(x) = ax^2 + bx + c \quad (a \neq 0)$

$$y = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right]$$

$$y = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right]$$

$$y + \frac{D}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

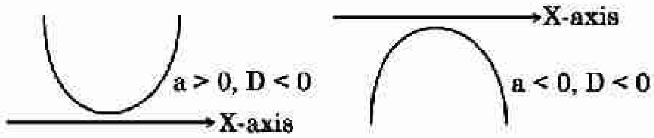
Let $y + \frac{D}{4a} = Y$ and $x + \frac{b}{2a} = X$

$$X^2 = \frac{Y}{a}$$

(i) The graph of the curve $y = f(x)$ is parabolic.

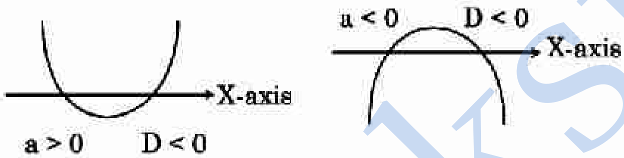
(ii) The axis of the parabola is $X = 0$ or $x + \frac{b}{2a} = 0$

(iii) If $a > 0$, then the parabola opens upward.
If $a < 0$, then the parabola opens downward.

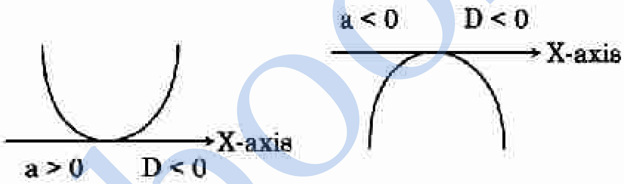


14. Position of Quadratic Equation with respect to axes

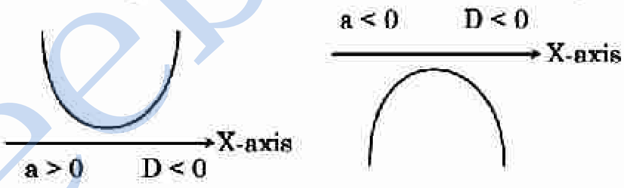
(i) For $D > 0$, parabola cuts X-axis and has two real and distinct points i.e. $x = \frac{-b \pm \sqrt{D}}{2a}$.



(ii) For $D = 0$ parabola touch X-axis in one point, $x = -\frac{b}{2a}$.



(iii) For $D < 0$, parabola does not cut X-axis (i.e., imaginary value of x).



Note :

$D < 0$ then,

- If $a > 0$, then $ax^2 + bx + c > 0$ for all x.
- If $a < 0$, then $ax^2 + bx + c < 0$ for all x.

15. Maximum & Minimum value of Quadratic Expression

In a quadratic expression $ax^2 + bx + c$

(i) If $a > 0$, quadratic expression has least value at $x = -\frac{b}{2a}$. This least value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$

(ii) If $a < 0$, quadratic expression has greatest value at $x = -\frac{b}{2a}$. This greatest value is given by

$$\frac{4ac - b^2}{4a} = -\frac{D}{4a}$$

16. Range of an Rational Algebric Expression

To find the value of rational expression of the form $\frac{a_1x^2 + b_1x + c_1}{a_2x^2 + b_2x + c_2}$ for real values of x.

(i) Equate the given rational expression to y.

(ii) Obtain a quadratic equation in x by simplify the expression.

(iii) Obtain the discriminant of the quadratic equation.

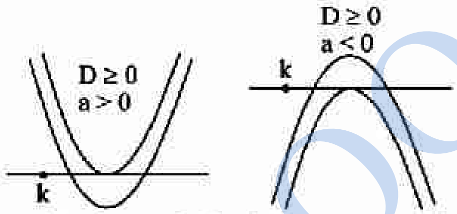
(iv) Put discriminant ≥ 0 and solve the inequation for y.
The value of y, so obtained determines the set of values attained by the given rational expression.

17. Location of Roots of a Quadratic Equation $ax^2+bx+c=0$

Let $f(x) = ax^2 + bx + c$, where $a, b, c \in R$ and $a \neq 0$

(i) Condition for both the roots will be greater than k.

(a) $D \geq 0$ (b) $k < -\frac{b}{2a}$ (c) $af(k) > 0$

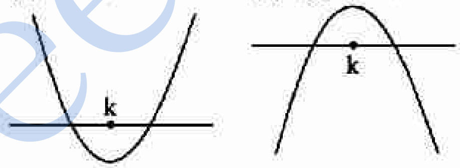


(ii) Condition for both the roots will be less than k.

(a) $D \geq 0$ (b) $k > -\frac{b}{2a}$ (c) $af(k) > 0$

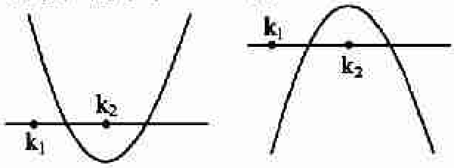
(iii) Condition for k lie between the roots

(a) $D > 0$ (b) $af(k) < 0$



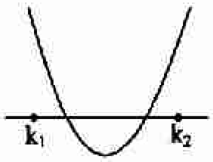
(iv) Condition for exactly one root lie in the interval (k_1, k_2) where $k_1 < k_2$

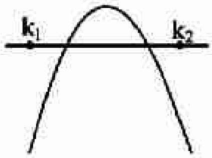
(a) $f(k_1)\ f(k_2) < 0$ (b) $D > 0$



(v) When both roots lie in the interval (k_1, k_2) where $k_1 < k_2$

(a) $D > 0$ (b) $f(k_1).\ f(k_2) > 0$ (c) $k_1 < -\frac{b}{2a} < k_2$





(vi) Any algebraic expression $f(x) = 0$ in interval $[a, b]$ if

(a) sign of $f(a)$ and $f(b)$ are of same then either no roots or even no. of roots exist.

(b) sign of $f(a)$ and $f(b)$ are opposite then $f(x) = 0$ has at least one real root or odd no. of roots.

18. Descartes Rule of Signs

(i) The maximum number of positive real roots of polynomial equation $f(x) = 0$ (arranged in decreasing order of the degree) is the number of changes of signs in $f(x) = 0$ as we move from left to right.

(ii) The maximum number of negative real roots of a polynomial equation $f(x) = 0$ is the number of changes of signs in $f(-x)$.

19. Quadratic Expression in Two Variables

The general form of a quadratic expression in two variables x & y is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$.
The condition that this expression may be resolved into two linear rational factors is

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \ \Rightarrow abc + 2\,fgh - af^2 - bg^2 - ch^2 = 0 \text{ and } h^2 - ab > 0$$

This expression is called discriminant of the above quadratic expression.

Important Points to be Remembered

(1) Every equation of n^{th} degree $(n \geq 1)$ has exactly n roots and if the equation has more than n roots, it is an identity.

(2) If quadratic equations $a_1 x^2 + b_1 x + c_1 = 0$ and

$a_2 x^2 + b_2 x + c_2 = 0$ are in the same ratio $\left(\text{i.e.} \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}\right)$ then $\frac{b_1^2}{b_2^2} = \frac{a_1c_1}{a_2c_2}$

(3) If one root is k times the other root of quadratic equation

$a_1 x^2 + b_1 x + c_1 = 0$ then

$$\frac{(k+1)^2}{k} = \frac{b^2}{ac}$$

SHORT TRICKS

Trick -1 If the roots of equation $Ax^2 + Bx + C = 0$ are real and equal and $A + B + C = 0$, then $A = C$

Q.1 If roots of the equation $(a - b)x^2 + (c - a)x + (b - c) = 0$ are equal, then a, b, c are in -

(A) A.P. (B) H.P. (C) G.P. (D) None of these

Sol. [A]

Proper Method	**Short Trick**
Given $(a - b)x^2 + (c - a)x + (b - c) = 0$ Roots are equal $\therefore D = 0$ $(c - a)^2 - 4(a - b)(b - c) = 0$ $\Rightarrow c^2 + a^2 - 2ac - 4(ab - ac - b^2 + bc) = 0$ $\Rightarrow c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0$ $\Rightarrow (c + a - 2b)^2 = 0$ $\Rightarrow c + a - 2b = 0$ $\Rightarrow 2b = a + c$ $\therefore$ a, b, c are in A.P.	$a - b + c - a + b - c = 0$ $\therefore a - b = b - c$ $\Rightarrow 2b = a + c$ So a, b, c, are in A.P.

Q.2 If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then a, b, c are in -

(A) HP (B) GP (C) AP (D) None of these

Sol. [A]

Proper Method	**Short Trick**
Given : $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ Roots are equal $\therefore D = 0$ $\Rightarrow b^2(c - a)^2 - 4a(b - c)c(a - b) = 0$ $\Rightarrow b^2(c^2 + a^2 - 2ac) - 4ac(ba - b^2 - ca + bc) = 0$ $\Rightarrow b^2c^2 + b^2a^2 - b^2 2ac - 4a^2bc + 4acb^2 + 4a^2c^2 - 4abc^2 = 0$ $\Rightarrow b^2c^2 + b^2a^2 + 4a^2c^2 + 2acb^2 - 4a^2bc - 4abc^2 = 0$ $\Rightarrow (bc + ba - 2ac)^2 = 0$ $\Rightarrow bc + ba - 2ac = 0$ $\Rightarrow b = \dfrac{2ac}{a + c}$ $\therefore$ a, b, c are in H.P.	$a(b - c) + b(c - a) + c(a - b) = 0$ $\therefore a(b - c) = c(a - b)$ $\Rightarrow b = \dfrac{2ac}{a + c}$ So, a, b, c are in H.P.

Trick -2 Method of substitution

Q.3 If a, b, c are distinct positive real numbers such that $b(a + c) = 2ac$, then the roots of $ax^2 + 2bx + c = 0$ are:

(A) Real and equal (B) Real and distinct (C) Imaginary (D) None of these

Sol. [C]

Proper Method

$D = 4b^2 - 4ac = 4\left\{\frac{4a^2c^2}{(a+c)^2} - ac\right\}$

$= \frac{16ac}{(a+c)^2}\{4ac - (a+c)^2\}$

$= -\frac{16ac}{(a+c)^2}(a-c)^2 < 0$ [$\because$ a, c > 0]

$\Rightarrow$ roots are imaginary

Short Trick

$b = \frac{2ac}{a+c} \Rightarrow$ a, b, c

are in H.P.

let a = 2, b = 3, c = 6

now equation

$2x^2 + 6x + 6 = 0$

$D = 36 - 48 < 0$

(imaginary roots)

Q.4 If a, b, c, $\in$ R and 1 is a root of the equation $ax^2 + bx + c = 0$, then the equation $4ax^2 + 3bx + 2c = 0$, $c \neq 0$ has roots which are :

(A) Real and equal (B) Real and distinct (C) Imaginary (D) Rational

Sol. [B]

Proper Method

1 is a root of $ax^2 + bx + c = 0 \Rightarrow a + b + c = 0$

D of $4ax^2 + 3bx + 2c = 0$ is

$= 9b^2 - 32ac = 9(a+c)^2 - 32ac$

$= c^2\left\{9\left(\frac{a}{c}\right)^2 - 14\left(\frac{a}{c}\right) + 9\right\}$

$= c^2\left\{\left(3\left(\frac{a}{c}\right) - \left(\frac{7}{3}\right)\right)^2 + 9 - \frac{49}{9}\right\} > 0$

$c^2\left\{3\left(\frac{a}{c} - \frac{7}{3}\right)^2 + \frac{31}{9}\right\} > 0$

$\Rightarrow$ roots are real and distinct.

Short Trick

Let the roots are 1 and 2 then equation

$ax^2 + bx + c = 0$

becomes $x^2 - 3x + 2 = 0$

(a = 1, b = – 3, c = 2)

Now equation

$4ax^2 + 3bx + 2c = 0$

$\Rightarrow 4x^2 - 9x + 4 = 0$

$D = 81 - 64 > 0$

(real and distinct)

Q.5 If α and β are the roots of the equation $x^2 - p(x+1) - q = 0$, then the value of

$\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+q} + \frac{\beta^2+2\beta+1}{\beta^2+2\beta+q}$ is :

(A) 2 (B) 1 (C) 0 (D) None

Sol. [B]

Proper Method

Equation is $x^2 - px - (p+q) = 0$

$\alpha + \beta = p,\ \alpha\beta = -(p+q)$

Now $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$

$= -(p+q) + p + 1 = 1 - q$

The given expression

$= \frac{(\alpha+1)^2}{(\alpha+1)^2+(q-1)} + \frac{(\beta+1)^2}{(\beta+1)^2+(q-1)}$

$= \frac{[2(\alpha+1)^2(\beta+1)^2 + (q-1)\{(\alpha+1)^2+(\beta+1)^2\}]}{[(\alpha+1)^2(\beta+1)^2 + (q-1)\{(\alpha+1)^2+(\beta+1)^2\} + (q-1)^2]}$

$= \frac{2(1-q)^2 + (q-1)[(\alpha+1)^2+(\beta+1)^2]}{2(1-q)^2 + (q-1)[(\alpha+1)^2+(\beta+1)^2]} = 1$

Short Trick

Let $\alpha = 1$ and $\beta = 2$, then equation is

$x^2 - 3x + 2 = 0$

so p = 3 and – p – q = 2

$\Rightarrow q = -5$

$\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+q} + \frac{\beta^2+2\beta+1}{\beta^2+2\beta+q}$

$= \frac{4}{-2} + \frac{9}{3} = 1$

Q.6 If tan A and tan B are the roots of $x^2 + ax + b = 0$, then the value of expression $\sin^2(A + B) + a\sin(A + B)\cos(A + B) + b\cos^2(A + B)$ is equal to -

(A) $\frac{a}{b}$ (B) $\frac{b}{a}$ (C) a (D) b

Sol. [D]

Proper Method

$\tan A + \tan B = -a$ and $\tan A.\tan B = b$

$\Rightarrow \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B} = \frac{-a}{1-b} = \frac{a}{b-1}$

$\Rightarrow \sin(A + B) = \frac{a}{\sqrt{a^2 + (b-1)^2}}$

and $\cos(A + B) = \frac{b-1}{\sqrt{a^2 + (b-1)^2}}$

$\sin^2(A + B) + a\sin(A + B)\cos(A + B) + b\cos^2(A + B)$

$= \frac{a^2}{a^2 + (b-1)^2} + \frac{a^2(b-1)}{a^2 + (b-1)^2} + \frac{b(b-1)^2}{a^2 + (b-1)^2}$

$= \frac{a^2 + a^2b - a^2 + b^3 - 2b^2 + b}{a^2 + (b-1)^2}$

$= \frac{b(a^2 + b^2 - 2b + 1)}{(a^2 + b^2 - 2b + 1)}$

$= b$

Short Trick

Let $A = 30°$ and $B = 60°$, then roots are $\frac{1}{\sqrt{3}}, \sqrt{3}$

and equation is $x^2 - \frac{4}{\sqrt{3}}x + 1 = 0$

so, $a = -\frac{4}{\sqrt{3}}$ and $b = 1$

now $\sin^2(A + B) + a\sin(A + B)\cos(A + B) + b\cos^2(A + B) = 1 + 0 + 0 = 1 = b$

Trick -3 Combination of method of substitution and balancing

Q.7 If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$, then $a S_{n+1} + c S_{n-1} =$

(A) bS_n (B) b^2S_n (C) $2bS_n$ (D) $-bS_n$

Sol. [D]

Proper Method

Here α, β are roots

$\therefore \quad a\alpha^2 + b\alpha + c = 0 \quad ...(1)$

$a\beta^2 + b\beta + c = 0 \quad ...(2)$

Now let us consider (Keeping results (1), (2) in mind)

$a S_{n+1} + b S_n + c S_{n-1}$

$= a[\alpha^{n+1} + \beta^{n+1}] + b[\alpha^n + \beta^n] + c[\alpha^{n-1} + \beta^{n-1}]$

$= [a\alpha^{n+1} + b\alpha^n + c\alpha^{n-1}] + [a\beta^{n+1} + b\beta^n + c\beta^{n-1}]$

$= \alpha^{n-1}[a\alpha^2 + b\alpha + c] + \beta^{n-1}[a\beta^2 + b\beta + c]$

$= 0 + 0 = 0$

Hence $aS_{n+1} + cS_{n-1} = -bS_n$.

Short Trick

Let $\alpha = 1, \beta = 2$, then the equation

$x^2 - 3x + 2 = 0$

so, $a = 1, b = -3, c = 2$

and let

$n = 2 \Rightarrow S_2 = \alpha^2 + \beta^2 = 5$

now $aS_{n+1} + cS_{n-1}$

$= S_3 + 2S_1 = 9 + 6 = 15$

by option (D)

$-bS_n = (+3)(5) = 15$ is correct

Q.8 If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2 + bx + 1 = 0$ ($a \neq 0$, $a, b \in R$), then the equation $x(x + b^3) + (a^3 - 3abx) = 0$ has roots :

(A) $\alpha^{\frac{3}{2}}$ and $\beta^{\frac{3}{2}}$ (B) $\alpha\beta^{\frac{1}{2}}$ and $\alpha^{\frac{1}{2}}\beta$ (C) $\sqrt{\alpha\beta}$ and $\alpha\beta$ (D) $\alpha^{\frac{-3}{2}}$ and $\beta^{\frac{-3}{2}}$

[JEE Main Online-2014]

Sol. [A]

Proper Method

$\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of equation $ax^2 + bx + 1 = 0$

$\therefore \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{-b}{a}$

$\Rightarrow \frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}} = \frac{-b}{a}$ ....(1)

Product $\frac{1}{\sqrt{\alpha}} \cdot \frac{1}{\sqrt{\beta}} = \frac{1}{a}$

$\Rightarrow \frac{1}{\sqrt{\alpha\beta}} = \frac{1}{a}$ .... (2)

From (1) $\sqrt{\alpha}+\sqrt{\beta} = -b$ .... (3)

Now the equation

$x(x + b^3) + (a^3 - 3abx) = 0$

$\Rightarrow x^2 + x(b^3 - 3ab) + a^3 = 0$

$\Rightarrow x^2 - x(3ab - b^3) + a^3 = 0$

$\Rightarrow x^2 - x\{-3\sqrt{\alpha\beta}\ (\sqrt{\alpha}+\sqrt{\beta}) + (\sqrt{\alpha}+\sqrt{\beta})^3\} + (\alpha\beta)^{3/2} = 0$

$\Rightarrow x^2 - x\,(\alpha^{3/2} + \beta^{3/2}) + (\alpha\beta)^{3/2} = 0$

$\therefore$ roots are $\alpha^{3/2}$ and $\beta^{3/2}$

Short Trick

Let $\alpha = 4$ and $\beta = 9$ then the equation is

$6x^2 - 5x + 1 = 0$ and

$a = 6$, $b = -5$, $c = 1$

$x(x + b^3) + (a^3 - 3abx) = 0$ becomes

$x^2 - 35x = 216 = 0$

it roots are 8 and 27 satisfy the $\alpha^{3/2}, \beta^{3/2}$

Q.9 If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then the cubic equation whose roots are $\alpha(\beta + \gamma)$, $\beta(\gamma + \alpha)$, $\gamma(\alpha + \beta)$ is :

(A) $x^3 - 2qx^2 + (q^2 + pr)x + r^2 - pqr = 0$ (B) $x^3 - 2px^2 + (p^2 + qr)x + r^2 - pqr = 0$

(C) $x^3 - 2rx^2 + (r^2 + pq)x + r^2 - pqr = 0$ (D) None of these

Sol. [A]

Proper Method

α, β, γ are roots of $x^3 + px^2 + qx + r = 0$

$\therefore \alpha + \beta + \gamma = -p$, $\alpha\beta + \beta\gamma + \gamma\alpha = q$, $\alpha\beta\gamma = r$

$\alpha(\beta + \gamma)$, $\beta(\gamma + \alpha)$, $\gamma(\alpha + \beta)$ are symmetric.

Let $y = \alpha(\beta + \gamma) \Rightarrow y = \alpha\beta + \alpha\gamma + \beta\gamma - \beta\gamma$

$\Rightarrow y = q + r/\alpha \Rightarrow \alpha = \frac{r}{y-q}$

α is a solution of given equation

$\therefore \left(\frac{r}{y-q}\right)^3 + p\frac{r^2}{(y-q)^2} + q \cdot \frac{r}{y-q} + r = 0$

Short Trick

Let $\alpha = 1$, $\beta = 2$, $\gamma = 3$ are the roots then

$p = -6$, $q = 11$, $r = -6$

Now the equation whose roots

$\alpha(\beta + \gamma) = 5$, $\beta(\gamma + \alpha) = 8$, $\gamma(\alpha + \beta) = 9$

$x^3 - 22x^2 + 167x - 360 = 0$

now option (A) satisfy the condition.

$\Rightarrow \frac{r}{y-q}\left\{\frac{r^2}{(y-q)^2}+\frac{pr}{y-q}+y\right\}=0$

$\Rightarrow r^2 + pr(y-q) + y(y-q)^2 = 0$

$\Rightarrow y^3 - 2qy^2 + (q^2 + pr)y + r^2 - pqr = 0$

Hence, required equation is

$x^3 - 2qx^2 + (q^2 + pr)x + r^2 - pqr = 0$

Q.10 If α, β, γ are the roots of the equation, $x^3 + ax^2 + bx + c = 0$, then $(1-\alpha^2)(1-\beta^2)(1-\gamma^2)$ is equal to:

(A) $(1+b)^2 - (a+c)^2$ (B) $(1+b)^2 + (a+c)^2$ (C) $(1-b)^2 + (a+c)^2$ (D) None of these

Sol. [A]

Proper Method

α, β, γ are roots of $x^3 + ax^2 + bx + c = 0$

Putting $1 - x^2 = y$, $x^2 = 1 - y$, we have

$x(1-y) + a(1-y) + bx + c = 0$

$\Rightarrow x[1 - y + b] = -[c + a - ay]$

$\Rightarrow x^2[1 + b - y]^2 = [c + a - ay]^2$

$\Rightarrow (1-y)[1 + b - y]^2 - [c + a - ay]^2 = 0.$

This equation in y has roots $1-\alpha^2$, $1-\beta^2$ and $1-\gamma^2$

$\therefore (1-\alpha^2)(1-\beta^2)(1-\gamma^2) = (1+b)^2 - (a+c)^2$

Short Trick

Let $\alpha = 1, \beta = 2, \gamma = 3$

then $a = -6, b = 11, c = -6$

now

$(1-\alpha^2)(1-\beta^2)(1-\gamma^2) = 0$

in options (A)

$(1+b)^2 - (a+c)^2$ is correct option

Q.11 Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$. If α and β are non-zero complex number satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$ then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is : -

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ (B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$ (D) $(p^3 - q)x^2 - (p^3 + 2q)x + (p^3 - q) = 0$

Sol. [B]

Proper Method

$\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$

then $(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = q$

$\Rightarrow -p^3 - 3\alpha\beta(-p) = q$

$\Rightarrow \alpha\beta = \frac{p^3+q}{3p}$

Now equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

Sum $\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{p^2}{\left(\frac{p^3+q}{3p}\right)} - 2 = \frac{p^3-2q}{p^3+q}$

Product $= \frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 1$

So equation $x^2 - x\left(\frac{p^3-2q}{p^3+q}\right) + 1 = 0$

$(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

Short Trick

Let $\alpha = \omega$ and $\beta = \omega^2$

$\therefore \alpha + \beta = \omega + \omega^2 = -1 = -p \Rightarrow p = 1$

and $\alpha^3 + \beta^3 = \omega^3 + (\omega^2)^3 = 2 = q$

so required equation is $x^2\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x + 1 = 0$

$\Rightarrow x^2 + x + 1 = 0$

Put the value of p & q in option then (B) will give required result.

Q.12 If one root of equation $x^2 + px + q = 0$ is square of the other then : -

(A) $p^3 - q(3p - 1) + q^2 = 0$ (B) $p^3 - q(3p + 1) + q^2 = 0$

(C) $p^3 + q(3p - 1) + q^2 = 0$ (D) $p^3 + q(3p + 1) + q^2 = 0$

Sol. [A]

Proper Method

Let the roots of equation $x^2 + px + q = 0$

are α and α^2

then $\alpha + \alpha^2 = -p$ and $\alpha.\alpha^2 = q \Rightarrow \alpha^3 = q$

Now $(\alpha + \alpha^2)^3 = -p^3$

$\Rightarrow \alpha^3 + \alpha^6 + 3\alpha.\alpha^2(\alpha + \alpha^2) = -p^3$

$\Rightarrow q + q^2 + 3q(-p) = -p^3$

$\Rightarrow p^3 - q(3p - 1) + q^2 = 0$

Short Trick

Let $\alpha = 1, \beta = 1$

then the equation is $x^2 - 2x + 1 = 0$

$p = -2, q = 1$

put these value in options then (A) is correct option

Q.13 If α and β are the roots of equation $ax^2 + bx + c = 0$, then the sum of the roots of the equation $a^2x^2 + (b^2 - 2ac)x + b^2 - 4ac = 0$ is given by -

(A) $-(\alpha^2 - \beta^2)$ (B) $(\alpha + \beta)^2 - 2\alpha\beta$ (C) $\alpha^2\beta + \beta^2\alpha - 4\alpha\beta$ (D) $-(\alpha^2 + \beta^2)$

Sol. [D]

Proper Method

Let the roots of equation $ax^2 + bx + c = 0$ are α and β

$\therefore \alpha + \beta = -\dfrac{b}{a}, \alpha\beta = \dfrac{c}{a}$

and let the roots of equation

$a^2x^2 + (b^2 - 2ac)x + b^2 - 4ac = 0$ are γ and δ

then $\gamma + \delta = -\left(\dfrac{b^2 - 2ac}{a^2}\right), \gamma\delta = \dfrac{b^2 - 4ac}{a^2}$

$\gamma + \delta = \dfrac{b^2}{a^2} - \dfrac{2c}{a}$

$\gamma + \delta = -[(\alpha + \beta)^2 - 2\alpha\beta] = -(\alpha^2 + \beta^2)$

Short Trick

Let $\alpha = 1, \beta = 2$, then equation $x^2 - 3x + 2 = 0$

$\Rightarrow a = 1, b = -3, c = 2$

Now the required equation will reduce to $x^2 + 5x + 1 = 0$

$\Rightarrow$ sum of roots $= -5$

Put $\alpha = 1, \beta = 2$ in options then (D) is correct answer

Q.14 If (α, β) are roots of $ax^2 + 2bx - a = 0$ and quadratic equation whose roots are $\left(2\alpha - \dfrac{1}{\beta}\right)$ and $\left(2\beta - \dfrac{1}{\alpha}\right)$ is $px^2 + qx + r = 0$, then $p + q + r =$

(A) $2b$ (B) $6a - 8b$ (C) $6b - 8a$ (D) 0

Sol. [C]

Proper Method

$\alpha + \beta = -\dfrac{2b}{a}, \alpha\beta = -1$

Now sum and product of roots $\left(2\alpha - \dfrac{1}{\beta}\right)$ and $\left(2\beta - \dfrac{1}{\alpha}\right)$

Sum $\Rightarrow 2\alpha - \dfrac{1}{\beta} + 2\beta - \dfrac{1}{\alpha} = 2(\alpha + \beta) - \left(\dfrac{1}{\beta} + \dfrac{1}{\alpha}\right)$

Short Trick

Let $\alpha = 1, \beta = -1$, then equation is $x^2 - 1 = 0$

Compare with $ax^2 + 2bx - a = 0$, we get $a = 1$, $b = 0$ Roots of second equation are $(3, -3)$

then the equation $x^2 - 9 = 0$

$\therefore p + q + r = -8$

$= 2\left(\frac{-2b}{a}\right) - \left(\frac{\alpha+\beta}{\alpha\beta}\right)$

$= \frac{-4b}{a} - \left(\frac{+2b}{a}\right) = \frac{-6b}{a}$

Product $\Rightarrow \left(2\alpha - \frac{1}{\beta}\right)\left(2\beta - \frac{1}{\alpha}\right) = 4\alpha\beta - 4 + \frac{1}{\alpha\beta}$

$= -4 - 4 - 1 = -9$

$\therefore$ equation is $x^2 + \frac{6b}{a}x - 9 = 0 \Rightarrow ax^2 + 6bx - 9a = 0$

Compare with $px^2 + qx + r = 0$

We get $p = a$, $q = 6b$, $r = -9a$

Now $p + q + r = a + 6b - 9a = 6b - 8a$

Q.15 If (α, β) are the roots of equation $x^2 - px + q = 0$ and (α', β') are that of $x^2 - p'x + q' = 0$, then $(\alpha - \alpha')^2 + (\beta - \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2 =$

(A) $2(p^2 - 2q + p'^2 - 2q' - pp')$ (B) $2(p^2 - 2q + p'^2 - 2q' - qq')$

(C) $2(p^2 - 2q - p'^2 - 2q' - pp')$ (D) $2(p^2 - 2q - p'^2 - 2q' - qq')$

Sol. [A]

Proper Method

$\alpha + \beta = p$, $\alpha\beta = q$ and $\alpha' + \beta' = p'$, $\alpha'\beta' = q'$

Now $(\alpha - \alpha')^2 + (\beta - \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2$

$= \alpha^2 + \alpha'^2 - 2\alpha\alpha' + \beta^2 + \alpha'^2 - 2\beta\alpha' + \alpha^2 + \beta'^2 - 2\alpha\beta' + \beta^2 + \beta'^2 - 2\beta\beta'$

$= 2(\alpha^2 + \beta^2) + 2(\alpha'^2 + \beta'^2) - 2\alpha'(\alpha + \beta) - 2\beta'(\alpha + \beta)$

$= 2(p^2 - 2q) + 2(p'^2 - 2q') - 2\alpha'(p) - 2\beta'(p)$

$= 2(p^2 - 2q + p'^2 - 2q' - pp')$

Short Trick

Let $\alpha = 1$, $\beta = 2$, then equation $x^2 - 3x + 2 = 0$

and $\alpha' = -1$, $\beta' = -2$, then equation $x^2 + 3x + 2 = 0$

so, $p = 3$, $q = 2$, $p' = -3$, $q' = -2$

now $(\alpha - \alpha')^2 + (\beta - \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2$

$= 4 + 9 + 9 + 16 = 38$

Putting these values in options then (A) will correct answer

Trick -5 Method of substitution

Q.9 If S_n denotes the sum of n terms of an A.P., then $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ is equal to -

(A) 0 (B) 1 (C) 1/2 (D) 2

Sol. [A]

Proper Method	**Short Trick**
In AP : S_n $\boxed{T_1, T_2, T_3 \ldots\ldots, T_n}$ $T_{n+1}, T_{n+2}, T_{n+3}$ Clearly $S_{n+3} = S_n + T_{n+1} + T_{n+2} + T_{n+3}$ $S_{n+2} = S_n + T_{n+1} + T_{n+2}$ $S_{n+1} = S_n + T_{n+1}$ Putting in $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ We get $= T_{n+1} - 2T_{n+2} + T_{n+3}$ $= (T_{n+1} + T_{n+3}) - 2(T_{n+2}) = 0$	Let the A.P. is 1, 2, 3, 4, 5, 6, ... $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ Put n = 1 $\Rightarrow S_4 - 3S_3 + 3S_2 - S_1$ $= 10 - 3(6) + 3(3) - 1 = 0$

Q.10 Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals – **[AIEEE-2004]**

(A) 0 (B) 1 (C) 1/mn (D) $\frac{1}{m} + \frac{1}{n}$

Sol. [A]

Proper Method	**Short Trick**
$T_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$ ......(i) & $T_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$ ......(ii) (i) – (ii) $(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$ $\therefore \boxed{d = \frac{1}{mn}}$ From (i) $a + \frac{(m-1)}{mn} = \frac{1}{n}$ $\Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$ $\therefore \boxed{a = \frac{1}{mn}}$ $\therefore a - d = 0$	Let m = 1, n = 2, then $T_1 = \frac{1}{2}$ and $T_2 = 1$ $\Rightarrow d = \frac{1}{2}$ $\Rightarrow a - d = 0$

Q.11 If a^2, b^2, c^2 are in A.P. then $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+d}$ are in-

(A) A.P. (B) G.P. (C) H.P. (D) None of these

Proper Method

Let numbers are a & b

$\therefore\ x = \frac{a+b}{2}$

& $\because$ a, y, z, b are in GP

$\therefore\ y^2 = az$ & $z^2 = by$

Now, $\frac{y^3+z^3}{xyz} = \frac{1}{x}\left(\frac{y^2}{z}+\frac{z^2}{y}\right) = \frac{1}{x}(a+b) = 2$

Short Trick

Let the G.P is 1, 2, 4, 8

then $y = 2$, $z = 4$ and $x = \frac{9}{2}$

Now $\frac{y^3+z^3}{xyz}$

$= \frac{8+64}{(2)\left(\frac{9}{2}\right)(4)} = 2$

Q.20 Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are- **[IIT Sc.-1997]**

(A) Not in A.P./G.P./H.P. (B) in A.P. (C) in G.P. (D) in H.P.

Sol. [D]

Proper Method

a, b, c, d : AP

Dividing by abcd, we get

$\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$: AP

$\therefore$ bcd, acd, abd, abc : HP

Short Trick

Let the A.P. is 1, 2, 3, 4,

Now abc = 6, abd = 8,

acd = 12, bcd = 24 are in H.P.

Trick -6 Combination of method of substitution and balancing

Q.21 Let the sequence $a_1, a_2, a_3, \ldots\ldots, a_n$ form an A.P., then $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots\ldots + a_{2n-1}^2 - a_{2n}^2$ is equal to -

(A) $\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$ (B) $\frac{2n}{n-1}(a_{2n}^2 - a_1^2)$ (C) $\frac{n}{n+1}(a_1^2 + a_{2n}^2)$ (D) None of these

Sol. [A]

Proper Method

Given $a_1, a_2, \ldots\ldots, a_{2n}$: AP

$\therefore\ a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots\ldots + a_{2n-1}^2 - a_{2n}^2$

$= (a_1 - a_2)(a_1 + a_2) + (a_3 - a_4)(a_3 + a_4) + \ldots\ldots$

$+ (a_{2n-1} + a_{2n})(a_{2n-1} - a_{2n})$

But $a_1 - a_2 = a_3 - a_4 \ldots\ldots = a_{2n-1} - a_{2n} = -d$

$= -d[a_1 + a_2 + a_3 + a_4 \ldots\ldots\ldots\ldots + a_{2n}]$

$= -d \cdot \frac{2n}{2}[a_1 + a_{2n}]$

$= -nd(a_1 + a_{2n})$

But $a_{2n} = a_1 + (2n-1)d \Rightarrow d = \frac{a_{2n} - a_1}{2n-1}$

$= \frac{-n(a_{2n} - a_1)}{2n-1} \cdot (a_{2n} + a_1)$

$= \frac{n}{2n-1}(a_1^2 - a_{2n-1}^2)$

Short Trick

Let n = 2

$a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 4$

$a_1^2 - a_2^2 + a_3^2 - a_4^2$

$= 1 - 4 + 9 - 16 = -10$

by option (A)

$\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$

$= \frac{2}{3}(1 - 16) = -10$

Q.24 Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 are (repetition allowed) **[AIEEE-2002]**

(A) 216 (B) 375 (C) 400 (D) 720

Sol. **[D]**

Proper Method

0, 1, 2, 3, 5, 7 : Six digits

The last place can be filled in by 1, 3, 5, 7. *i.e.*, 4 ways as the number is to be odd. We have to fill in the remaining 3 places of the 4 digit number *i.e.* I, II, III place. Since repetition is allowed each place can be filled in 6 ways. Hence the 3 place can be filled in $6 \times 6 \times 6 = 216$ ways.

But in case of $0 = 216 - 36 = 180$ ways.

Hence by fundamental theorem, the total number will be $= 180 \times 4 = 720$

Short Trick

			1, 3, 5, 7

$5 \times 6 \times 6 \times 4 = 720$

Q.25 How many numbers lying between 10 and 1000 can be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 (repetition is allowed)

(A) 1024 (B) 810 (C) 2346 (D) 2549

Sol. **[B]**

Proper Method

The total number between 10 and 1000 are 989 but we have to form the numbers by using numerals 1, 2,........9, i.e. 0 is not occuring so the numbers containing any '0' would be excluded *i.e.*,

Required number of ways

$$= 989 - \left\{\begin{matrix} 20, 30, 40, \ldots\ldots 100 = 9 \\ 101, 102, \ldots\ldots 200 = 19 \\ 201, \ldots\ldots 300 = 19 \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \\ 901, \ldots\ldots 990 = 18 \end{matrix}\right\}$$

$= 989 - (9 + 18 + 19 \times 8) = 810$.

Aliter : Between 10 and 1000, the numbers are of 2 digits and 3 digits.

Since repetition is allowed, so each digit can be filled in 9 ways.

Therefore number of 2 digit numbers $= 9 \times 9 = 81$

and number of 3 digit numbers $9 \times 9 \times 9 = 729$

Hence total ways $= 81 + 729 = 810$.

Short Trick

Two digit number

$9 \times 9 = 81$

Three digit number

$9 \times 9 \times 9 = 729$

Total = 810