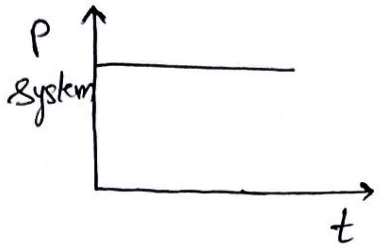
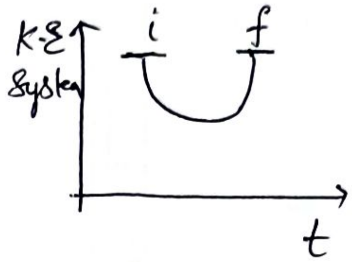
1. Elastic Collision

a) Momentum remains conserved throughout the collision.

b) $K.E_i = K.E_f$

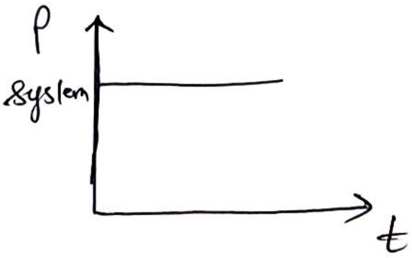


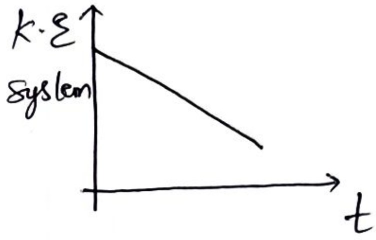


2. Inelastic collision

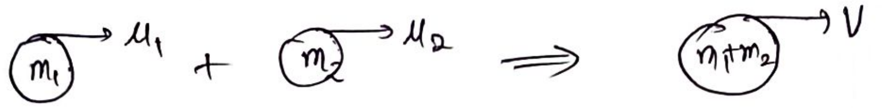
a) Momentum remains conserved throughout the collision.

b) $K.E_i > K.E_f$





Note:- In a perfectly inelastic Collision, the colliding Body sticks to one another & move with Common velocity



$$\boxed{V = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}}$$

Elastic Collision (Head-On) - 1D :

Momentum Conservation : $P_{i\,system} = P_{f\,system}$

$$m_1\vec{u_1} + m_2\vec{u_2} = m_1\vec{v_1} + m_2\vec{v_2} \quad \text{—(i)}$$

$$\underset{System}{K.E_i} = \underset{System}{K.E_f} \quad : \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{—(ii)}$$

Coefficient of Restitution (e)

$$\boxed{e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{\vec{V_2} - \vec{V_1}}{\vec{u_1} - \vec{u_2}}}$$

Elastic collision

$$e = \frac{\vec{V_2} - \vec{V_1}}{\vec{u_1} - \vec{u_2}}$$

$$\boxed{e = 1}$$

Perfectly elastic

i) $P_i = P_f$

ii) $K.E_i = K.E_f$

Inelastic collision

$$e = \frac{\vec{V_2} - \vec{V_1}}{\vec{u_1} - \vec{u_2}}$$

$$\boxed{e = 0}$$

Perfectly inelastic

i) $P_i = P_f$

ii) $K.E_i > K.E_f$

Partially elastic & Inelastic

$$0 < e < 1$$

$$e = \frac{\vec{V_2} - \vec{V_1}}{\vec{u_1} - \vec{u_2}}$$

i) $P_i = P_f$

ii) $K.E_i > K.E_f$

- How to solve Q involving e & if $e \neq 0$ & $e \neq 1$

Partial elastic & inelastic

① $e = \dfrac{\vec{V_2} - \vec{V_1}}{\vec{u_1} - \vec{u_2}}$ ② $P_i = P_f$

- If a Body of mass m_1 moving with velocity u_1 collids inelastically with a stationary mass m_2, then the loss in Kinetic energy is given By

$$\boxed{K.E = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2 (1 - e^2)}$$

In case of perfectly inelastic Collision $e = 0$ then.

$$\boxed{K.E = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2}$$

...ing (i) & (ii)

$$\vec{V_2} - \vec{V_1} = \vec{u_1} - \vec{u_2} \quad \text{-(iii)}$$

Relative velocity of separation — Relative velocity of Approach

Elastic Collision 1-D Head O-N

from (i), (ii) & (iii)

$$V_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$$

$$V_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \left(\frac{2m_1}{m_1 + m_2}\right) u_1$$

V_1 & V_2 → for elastic collision

- If a Body of mass (m_1) moving with velocity u_1 colloids elastically with a stationary mass m_2, then fraction of kinetic energy transfered is given by

$$\boxed{\frac{\Delta K.E}{K.E_i} = \frac{K.E_i - K.E_f}{K.E_i} = \frac{4m_1 m_2}{(m_1+m_2)^2}}$$

⇒ Fraction of K.E retained with the first Body

$$\boxed{1 - \frac{4m_1 m_2}{(m_1+m_2)^2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2}$$

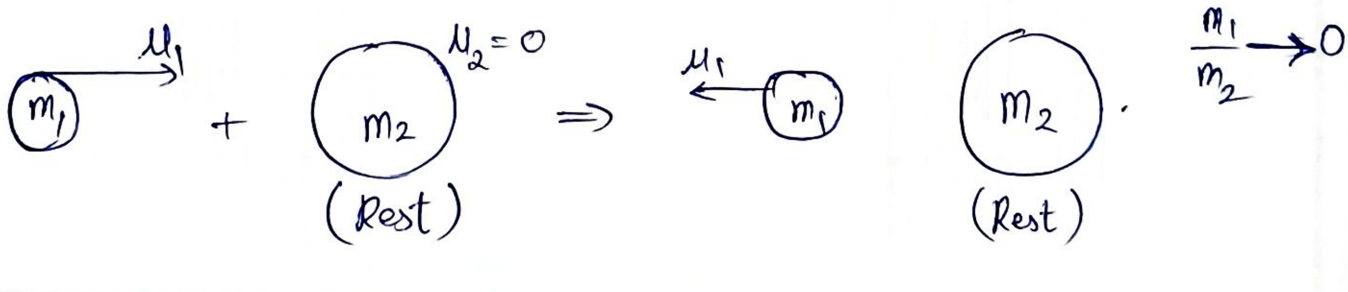
<u>Special Cases of Head-ON Elastic collision</u> :

Case I : $m_1 = m_2$ (identical bodies elastic collision 1-D Head O-N)

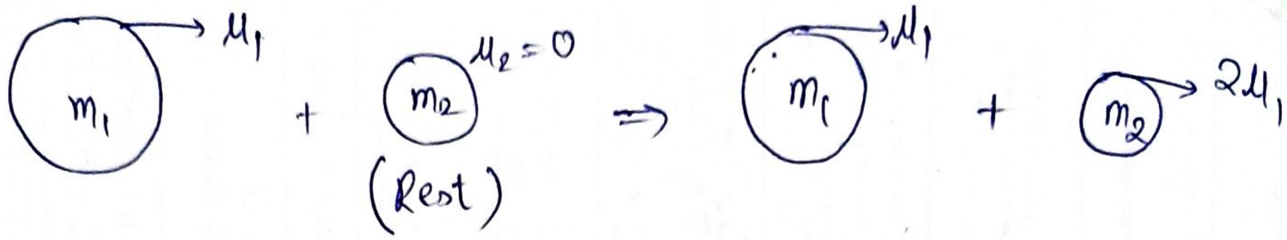
(velocities interchange)

eg.

Case II : $m_2 >> m_1$, $u_2 = 0$

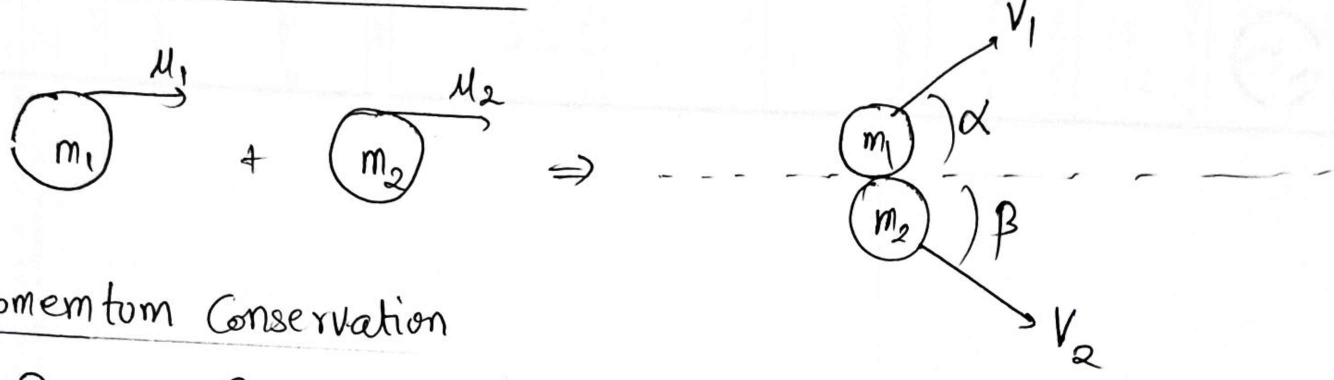


Case III : $m_1 >> m_2$, $u_2 = 0$



Max. Compression $\Rightarrow$ when Both Blocks have equal velocities.

Elastic Collision in 2-D



Momemtum Conservation

$$P_{i\,x} = P_{f\,x}$$
system system

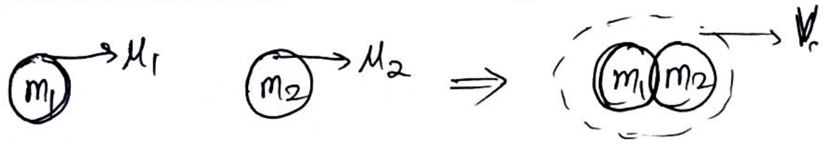
$$m_1 u_1 + m_2 u_2 = m_1 V_1 \cos\alpha + m_2 V_2 \cos\beta$$

$$P_{iy} = P_{fy} \;:\; 0 = m_1 V_1 \sin\alpha + m_2 V_2 \sin\beta$$

Since , Elastic ; $K \cdot E_i = K \cdot E_f$ (System ka)

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

Inelastic Collision (Perfectly)

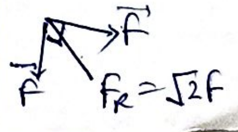


Perfectly inelastic Collision $\Rightarrow$ Bodies Move together after collision with common velocity

Momentum Conserved : $m_1 u_1 + m_2 u_2 = (m_1 + m_2) V$

$K \cdot E_i > K \cdot E_f$

Note



A Body is dropped from a certain height, it stricked the ground with a Velocity 'V' and rebound with a Velocity V_1, the coefficient of restitution

$$e = \sqrt{\frac{2gh_1}{2gh}} = \frac{V_1}{V} \Rightarrow V_1 = Ve$$

First Rebound : $V_1 = Ve$

Second Rebound : $V_2 = e^2 V$

third " : $V_3 = e^3 V$

n^{th} " : $V_n = e^n V$

⇒ A Body is dropped from height 'h', if coefficient of resistution is e, its average speed is

$$V = \frac{\text{Total distance}}{\text{Total time}} = \frac{h(1+e^2)(1-e)}{(1-e^2)\sqrt{\frac{2h}{g}}(1+e)} = \frac{1+e^2}{(1+e)^2}\sqrt{\frac{gh}{2}}$$

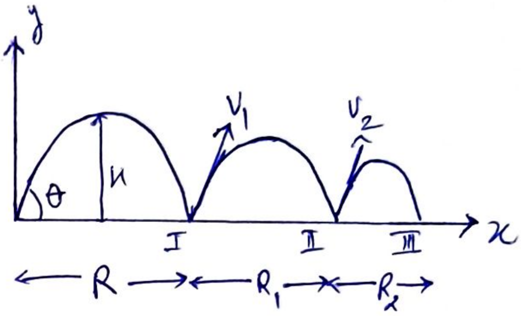
⇒ The time elapse from the moment it is dropped to the second impact with the floor is

$$t = \sqrt{\frac{2h}{g}}(1+2e)$$

⇒ Total change in momentum, Before ball comes to Rest is $\Delta P = m\sqrt{2gh}\,\frac{(1+e)}{(1-e)}$

A Ball projected at an angle θ with COR 'e'

1) Time for 1 collision, $T = \frac{2u\sin\theta}{g}$



2) Height attained, $H = \frac{u^2\sin^2\theta}{2g}$

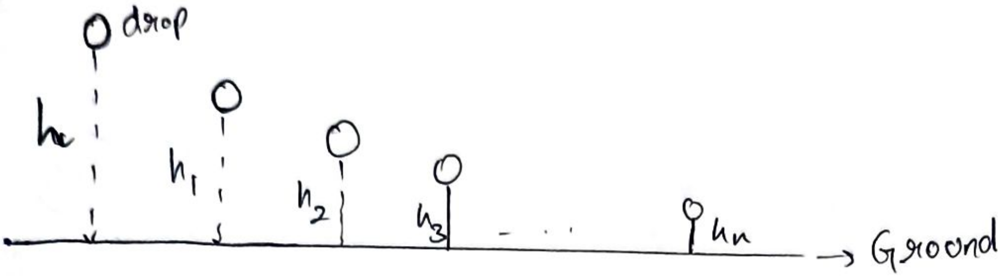
3) Horizontal range, $R = \frac{u^2\sin 2\theta}{g}$

4) Total time of motion till last collision, $T_0 = \frac{T}{(1-e)}$

5) Total Horizontal distance, $R_0 = \frac{R}{(1-e)}$

6) Sum of total maximum height, $H_0 = \frac{H}{(1-e^2)}$

Rebounding of Ball after collision with the ground.



$e \rightarrow$ depends on the nature of the mater. of colliding Bodies.
→ No units & Dimens-
→ Value lie Btw 0 to 1

A Body is dropped from a height 'h' it rebounds to a height h_1. The coefficient of resistution.

$$e = \frac{\vec{V_2} - \vec{V_1}}{\vec{u_1} - \vec{u_2}} = \frac{\sqrt{2gh_1}}{\sqrt{2gh}} \Rightarrow e = \sqrt{\frac{h_1}{h}} \Rightarrow e^2 = \frac{h_1}{h} \Rightarrow h_1 = e^2 h$$

⇒ First rebouncing height $h_1 = e^2 h$
Second " " $h_2 = e^4 h$
Third " " $h_3 = e^6 h$
n^{th} " " $h_n = e^{2n} h$

⇒ Total distance Before 3rd rebound
$S = h + 2h_1 + 2h_2$

⇒ Total distance travelled by the Body Before coming to rest.

$$\boxed{S = h\left(\frac{1+e^2}{1-e^2}\right)}$$

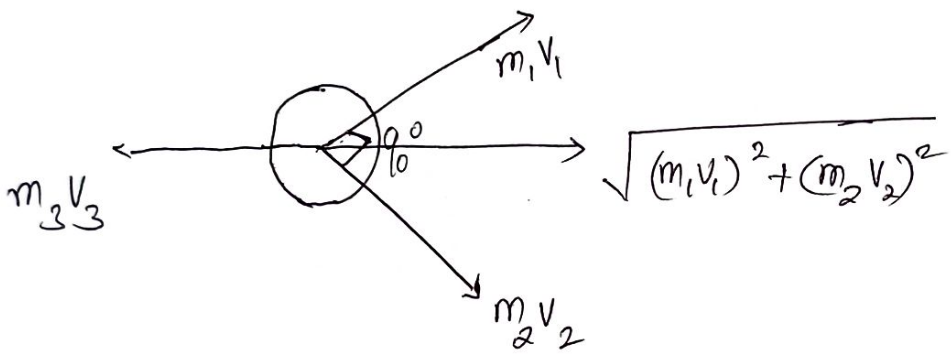
⇒ Total time taken by a Body to come to rest

$$\boxed{t = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)}$$

Note :-

- A shell of Mass 'M' exploded at rest and Breaks into 3 pieces ; Two pieces of masses m_1 & m_2 move at right angles to each other with velocites v_1 & v_2 then the velocity of the third piece

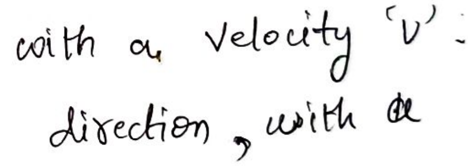
$$m_3 V_3 = \sqrt{(m_1 v_1)^2 + (m_2 v_2)^2}$$



- A shell of Mass 'M' at rest explodes and breaks into two pieces , one piece of mass 'm' moves with a velocity 'v'. The other piece moves in the opposite direction , with a velocity

$$\boxed{v' = \frac{mv}{M - m}}$$

Logic :



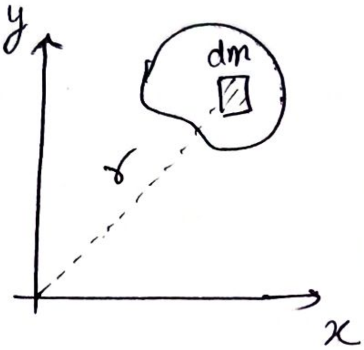
Centre of Mass of a Continuous Mass Distribution.

$$\boxed{r_{cm} = \frac{\int dm\, r}{\int dm}}$$

$$\boxed{x_{com} = \frac{\int dm\, x}{\int dm}}$$

$$\boxed{y_{com} = \frac{\int dm\, y}{\int dm}}$$

$$\boxed{z_{com} = \frac{\int dm\, z}{\int dm}}$$



$$x_{com} = \frac{\int dm\, x}{\int dm} = \frac{\int \lambda dx\, x}{\int \lambda dx}$$

$$= \frac{\lambda \int x\, dx}{\lambda \int dx} = \frac{\cancel{\alpha} x \int x\, dx}{\cancel{\alpha} x \int dx}$$

$$= \frac{\int x^2 dx}{\int x\, dx} = \frac{\left[\frac{x^3}{3}\right]_0^L}{\left[\frac{x^2}{2}\right]_0^L}$$

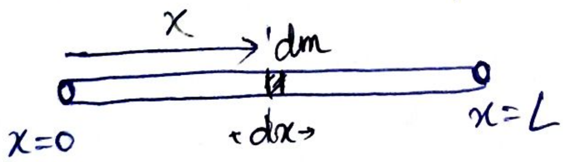
$$= \frac{L^3}{3} \times \frac{2}{L^2} = \frac{2L}{3}$$

$$\therefore \boxed{x_{com} = \frac{2L}{3}}$$

if $\lambda = \alpha x^1$

$\lambda = \alpha x^2$?

Linear mass density $\lambda = \alpha x$, $\alpha = \text{Const.}$ find co-ordinats of C.o.M.



$$\lambda = \frac{\text{Mass}}{\text{Length}} \Rightarrow \lambda = \frac{dm}{dx}$$

$$\therefore \boxed{dm = \lambda dx} \quad \text{—(i)}$$

Linear Mass (1D)

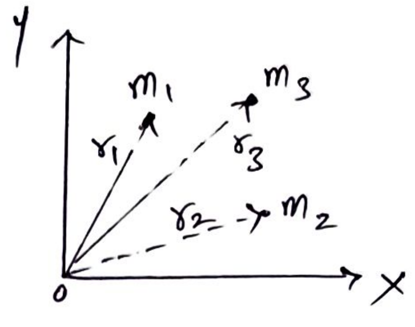
$$\boxed{dm = \lambda dx}$$

Surface Mass (2-D) : $\boxed{dm = \sigma dA}$

Volume Mass (3-D) : $\boxed{dm = \rho dV}$

COM : Note : COM may lie outside the Body.

COM of system of particles



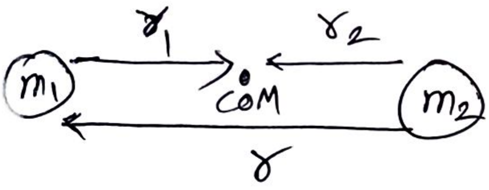
$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$y_{COM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$z_{com} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3}{m_1 + m_2 + m_3}$$

SHORT CUT : for two-mass systems:



$$r_1 = \frac{m_2 r}{m_1 + m_2}$$

$$r_2 = \frac{m_1 r}{m_1 + m_2}$$

For Lamina type (2-D) Body with uniform negligible thickness :

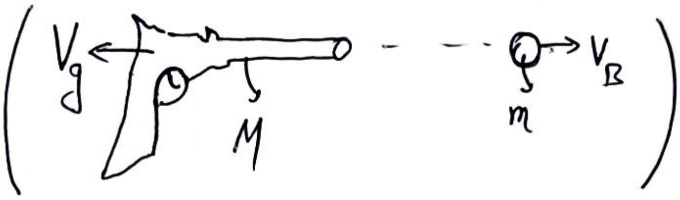
$$\vec{r}_{cm} = \frac{A_1\vec{r}_1 + A_2\vec{r}_2 + A_3\vec{r}_3}{A_1 + A_2 + A_2}$$

$$x_{com} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$y_{com} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

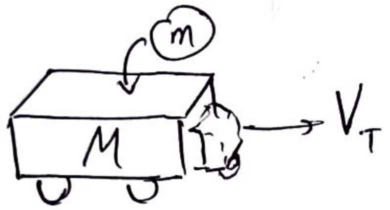
- A Bullet of mass m is fired from a gun of mass 'M' with a velocity 'v', the recoil velocity of the gun is

$$\boxed{V_g = \frac{m V_B}{M}}$$



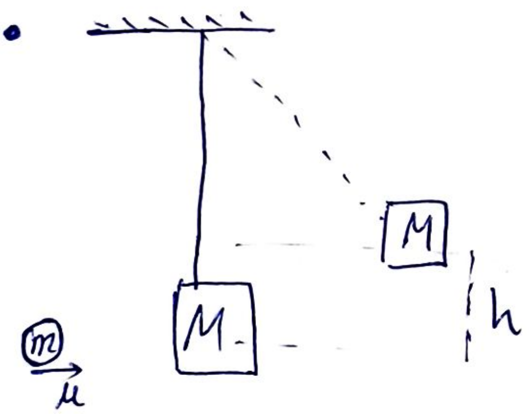
⇒ Momentum Conservation

- A truck of Mass 'M' moves with a velocity 'v'. A packet of mass 'm' is gently dropped into it, the velocity of the truck becomes. $v' = ?$



$$M V_T = (M+m) v'$$

$$\boxed{V' = \frac{M V_T}{M+m}}$$

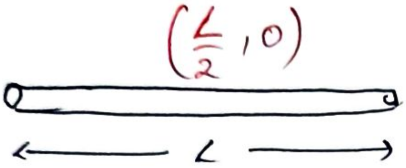
- 

$$m u = (M+m) v \quad \ldots \text{(Momentum Conservation)}$$

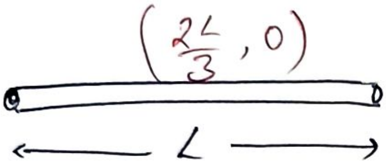
$$v = \sqrt{2gh} \quad \ldots \text{(Conservation of Energy)}$$

$$\boxed{\therefore\ u = \left(\frac{M+m}{m}\right)\sqrt{2gh}}$$

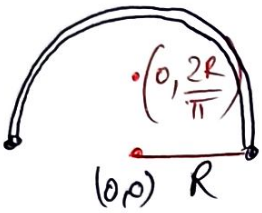
COM of some standard symmetric Bodies

① 

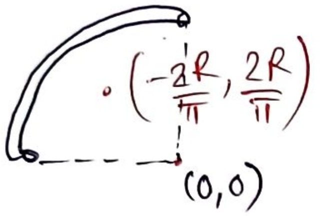
Uniform Rod of Length L

② 

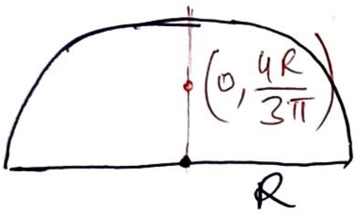
Rod having linear mass density $\lambda = \alpha x$

③ 

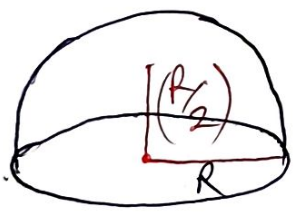
Uniform Circular Ring

④ 

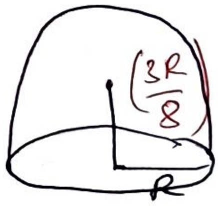
Quadrant of Uniform Ring.

⑤ 

Uniform Semicircular Disc

⑥ 

Uniform Hemispherical Shell or Hollow sphere

⑦ 

Uniform Solid Sphere