Module 1 Diffraction

SYLLABUS	Weightage : 12-15 Marks

Diffraction

(**Prerequisites** : Wave front and Huygen's principle, reflection and refraction, diffraction, Fresnel diffraction and Fraunhoffer diffraction)

Diffraction : Fraunhoffer diffraction at single slit, Diffraction Grating, Resolving power of a grating; Applications of diffraction grating; Determination of wavelength of light using plane transmission grating

Topics :

1.1 FRAUNHOFFER DIFFRACTION GRATING DIFFRACTION

Q.1 What is diffraction? [D-14]

Ans.: **Diffraction**

It is observed in day to day life that any form of wave bends around the obstacles in its path. The amount of bending depends upon the size of the obstacle and the wavelength of the wave. Light being a wave should also bend around obstacles. Normally light propagates in a rectilinear fashion i.e. in a straight line. The deviation due to obstacles is small when wavelength of light waves us less than the dimensions of the obstacle.

Thus, when light falls on obstacles whose size is comparable to wavelength of light then there is a departure from the straight line propagation. The light bends around the corners of the obstacle and the enters in the geometrical shadow. This bending of light is called diffraction. It was found that diffraction produces bright and dark fringes known as diffraction bands.

The diffraction phenomenon is due to mutual interference of secondary wavelengths originating from various points of the wave front which are not blocked by the obstacle. The formation of bright and dark fringes can be predicted by using Huggen's principle of secondary wavelets in relation to the phenomenon of interference.

Q.2 What are the types of diffraction and differentiate between them. [D-15]

Ans.: Diffraction are of two types :

(1) **Fresnel's diffraction :** In this class of diffraction, source and screen are placed at finite distances from the aperture of obstacle having sharp edges. In this case no lenses are used for making the rays parallel or convergent. The incident wave front are either or spherical or cylindrical.

(2) **Fraunhoffer's Diffraction :** In this class of diffraction source and screen or telescope (through which the image is viewed) are placed at infinity or effectively at infinity. In this case the wave front which is incident on the aperture or obstacle is plane.

Difference between Fresnel's & Fraunhoffer's diffraction

	Fresnel diffraction	Fraunhoffer's diffraction
(i)	Distance of source and screen from obstacle is finite.	Source and screen are at infinite distance from obstacle.
(ii)	No lenses are required to study diffraction in the laboratory.	Two biconvex lenses are required to study diffraction in laboratory.
(iii)	The wavefront incident on the aperture is either spherical or cylindrical.	The wavefront incident on aperture is plane.
(iv)	The diffracted wavefront is either spherical or cylindrical.	The diffracted wavefront is plane.
(v)	The initial phase of secondary wavelets is different at different points in the plane of aperture.	The initial phase of secondary wavelets is same at all points in the plane of aperture.
(vi)	It has no importance in optical instruments.	It is very important in optical instruments.

1.2 FRAUNHOFFER DIFFRACTION FOR A SINGLE SLIT

Q.1 Explain Fraunhoffer single slit diffraction quantitatively.

Ans.:

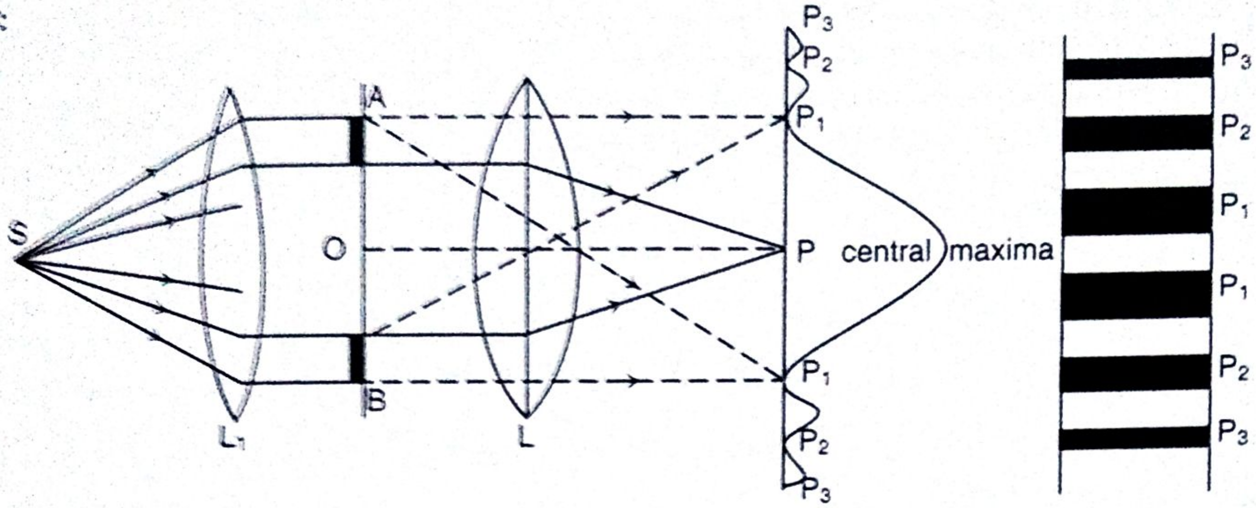

Fig. 1

To obtain a Fraunchoffer diffraction pattern, the incident wavefront must be plane and the diffracted light is collected on the screen with the help of Lens. Thus the source of light should either be at a large distance from the slit or a collimating lens must be used. Let a beam of parallel monochromatic light be incident on an opaque plate having a narrow vertical slit AB as shown in Figure 1. The slit's width is 'd'.

Since the width 'd' is large compared to the wavelength used to illuminate the slit, it would be more appropriate to replace the portion of wavefront AB with a string of point sources following Hygen's principle. The secondary wavelets coming from such a large number of point sources evenly spread out and interfere to give systematic distribution of maxima and minima on various points on screen. They take a shape of alternate dark and bright bands on the screen with a central bright band at e^{th} centre.

The explanation for the various parts on diffraction pattern is given as follows:

Formation of central maxima :

From the Figure 1 it is clear that the secondary waves traveling in a direction parallel to OP comes to a focus at P. All these waves have the same optical lengths as they have travelled the same path. They are all in phase at AB and hence they will be in phase at P also. Therefore they reinforce each other and a bright band appears at P. It is the centre of the diffraction pattern is called zero order central maxima.

Formation of minima :

For any other point on the screen like P_1, P_2 etc. the light from different points on AB travel different distances, and this difference increases as we consider points at increasing distances from P. Hence, there would be a gradual increase in the path difference (or phase difference) between the various waves reaching these points from different parts of the AB.

If we consider point P_1 alone, a point at which waves reach after being diffracted at an angle θ from the original direction. Imagine the aperture AB to be divided into two equal parts AO and OB as shown. Let the path difference AC be λ. Then the path difference between the waves form A and O i.e. OD $= \dfrac{\lambda}{2}$.

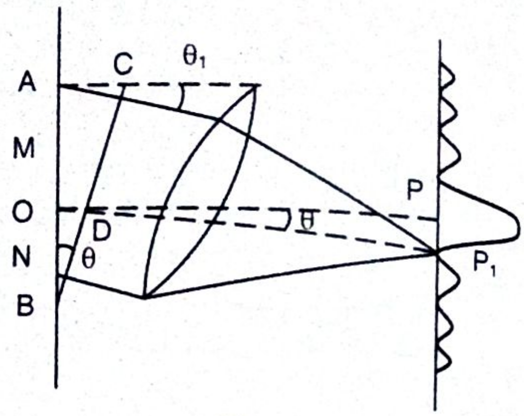

Fig. 2

Therefore the resultant amplitude at P_1 due to them is zero. All other pairs of corresponding points i.e. M and N in the two halves AO and OB also have a $\frac{\lambda}{2}$ path difference. Hence it follows that the intensity at P_1 is zero. Thus there occurs a complete cancellation of waves in e^{th} direction given by angle θ when AC = λ or AD = $\frac{\lambda}{2}$

And $$\sin\theta = \frac{AC}{AB} = \frac{\lambda}{A} \quad \text{... (1)}$$

$$\therefore \quad \lambda = a \sin\theta \quad \text{... (2)}$$

The first minimum or dark band is governed by equation (2) A similar dark band occurs at point P′ on the opposite side above P_1 at an angular distance θ and which is also governed by the same equation (2). The angular width of the central bright band is 2θ. We may divide the slit into 9 quarters, sixths and so on. Using similar arguments it can be proved that a dark band occurs whenever

$$\sin\theta = \frac{\lambda}{d}, \frac{2\lambda}{d}, \frac{3\lambda}{d}, \ldots \quad \text{... (3)}$$

or $$\sin\theta_n = \frac{n\lambda}{d} \text{ where } n = 1, 2, 3, \ldots.$$

If the lens L is very close to the slit or if the screen is very far away from the slit then

$$\sin\theta_1 = \frac{x}{f}$$

But $$\sin\theta_1 = \frac{\lambda}{a}$$

$$\therefore \quad \frac{x}{f} = \frac{\lambda}{a}$$

or $$x = \frac{f\lambda}{a} \quad \text{... (4)}$$

This gives the linear distance of the first minima for the central maxima on the screen. It is also clear that the first order minima occurs at an angle.

$\sin^{-1}\left(\frac{\lambda}{d}\right)$ and the second order minimum at an angle $\sin\theta^{-1}\left(\frac{2\lambda}{d}\right)$. It is natural to expect a bright band half way between them at $\sin\theta = \frac{3\lambda}{2d}$ i.e. at $\sin\theta = \frac{1.5\lambda}{d}$

Formation of maxima :

As mentioned above the first order maxima (bright band) occurs at

$$\sin\theta = \left(\frac{3\lambda}{2d}\right)$$

The first order maxima can be explained if we imagine the wavefront AB to be divided into three equal parts as shown in diagram.

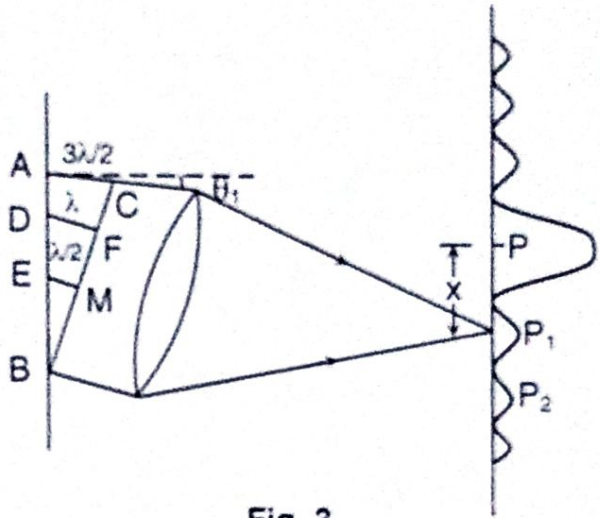

Fig. 3

The slit AB is divided into three equal parts AD, DE, and EB each differing from the adjacent part by $\lambda/2$. Any two adjacent portions (say AD and DE) destroy each other leaving the third portion (i.e. EB) free to produce first maxima at point below.

The general condition for secondary maxima is

$$AC = (2n+1)\frac{\lambda}{2},\ n = 1, 2, 3, \ldots\ldots$$

Since $AC = a\sin\theta$

$$a\sin\theta = (2n+1)\frac{\lambda}{2} \quad \ldots (5)$$

The direction in which the n^{th} secondary maxima produced is

$$\sin\theta_n = (2n+1)\frac{\lambda}{2a} \quad \ldots (6)$$

If the lens L is very close to the slit then

$$\sin\theta_1 = \frac{x}{f}$$

but for first order maxima

$$\sin\theta_1 = \frac{3\lambda}{2d}$$

$$\therefore \frac{3\lambda}{2d} = \frac{x}{f}$$

$$\therefore x = \frac{3f\lambda}{2a} \quad \ldots (7)$$

This gives the linear distance of the first order maxima at P_1 from central maxima.

Q.2 Explain Intensity distribution in a single slit diffraction and hence obtain the condition of maxima and minima.

Ans.: Step I

- Consider a single slit of width 'a' illuminated by monochromatic light of wavelength λ as shown in Figure 4.

- The incident plane wavefront is diffracted by the slit and is then focused on the screen by the lens L.
- Every point of the incident wavefront in the plane of the slit acts as a secondary source and sends out secondary waves in all directions.
- The secondary wavelets travelling normally to the slit are brought to focus at point P_0 by the lens.
- All these secondary waves travel the same distance through the lens along the direction $\theta = 0$ and hence produce maximum intensity of light.

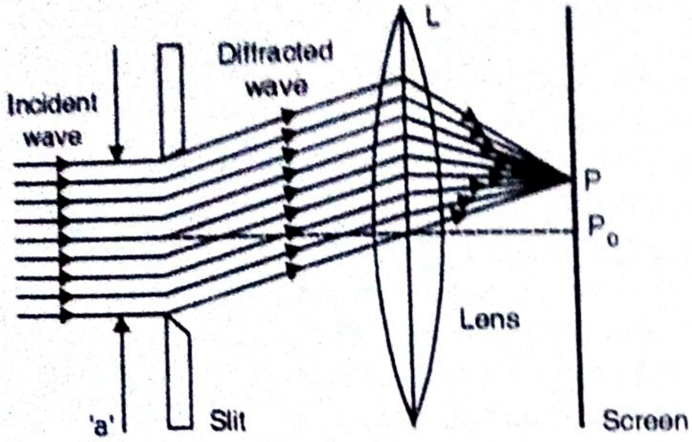

Fig. 4 : Fraunhoffer Diffraction at Single Slit

- Consider a point P on the screen at which the secondary wavelets travelling at an angle θ with the normal are focussed.
- The intensity of light at point P will depend upon the path difference between the secondary wavelets originating from the corresponding points of the wavefront.

Step II

- Consider the given slit to be divided into N parallel slits, each of width dx. See Figure 5(a).

  $\therefore$ a $= dx_1 + dx_2 + dx_3 + \ldots + dx_N$ ...(1)
- One of such slits is shown in Figure 5(b)

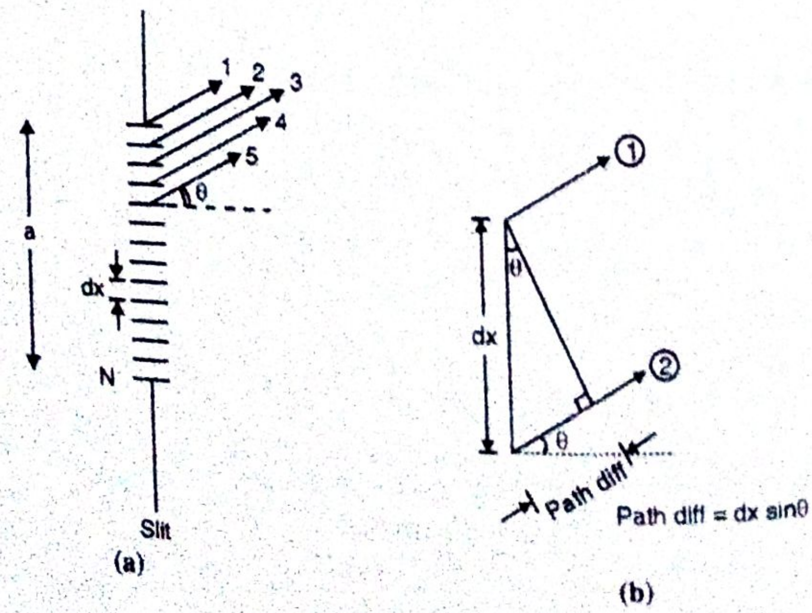

Fig. 5

- The path difference between the rays diffracted from its upper and lower edges is dx . sin θ and hence the corresponding phase difference between them is, (refer Figure 5).

$$\Delta\phi = \frac{2\pi}{\lambda} dx \sin\theta \qquad \text{...(2)}$$

because

Path difference	Phase difference
λ	2π
$dx \sin\theta$	$\Delta\phi$

$$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda} dx \sin\theta$$

- The total path difference between the rays diffracted from the top and bottom edges of the slit of width 'a' is $a\sin\theta$ and the corresponding phase difference between them is,

$$\phi = \frac{2\pi}{\lambda} \cdot a \cdot \sin\theta \qquad \text{...(3)}$$

- Each infinitesimal slit acts as a radiator of Huygen's wavelets and produces a characteristic wave disturbance at point P on the screen.
  This disturbance we represent by a quantity called as phasor. (Concept of phasor is useful when we are dealing in that area where not only the magnitude but also the phase difference is significant.)
- To find out the total amplitude of disturbance at point P, we have to consider N phasors corresponding to N parallel infinitesimal slits.
- Thus at point P, N phasors with same amplitude, same frequency and same phase difference $\Delta\phi$ between the adjacent members combine to produce the resultant disturbance. Figure 5 shows the vector addition of such N phasors.
- The resultant disturbance at P is represented by the arc AB of a circle with radius R.
- Let C be the centre of arc AB.
  $\therefore$ AC = BC = R

Step III

- Join the chord AB and draw CD perpendicular to AB.
- ϕ is the phase difference between the infinitesimal vectors at the ends of the arc AB i.e. it is the phase difference between the rays coming from the top and bottom edges of the slit of width 'a'.
  $\therefore \angle ACB = \phi$
- Let us represent arc AB = E_m and chord AB = E_θ
  where $E_m \rightarrow$ Amplitude at the centre and $E_\theta \rightarrow$ resultant amplitude at P.
- From the geometry of Figure: 6, we have,
  AD = DB

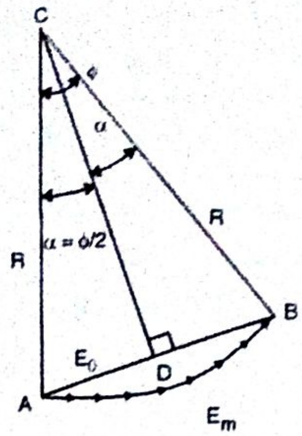

Fig. 6 : Phasor diagram

$$\angle ACD = \angle BCD$$

$$= \frac{\phi}{2} = \frac{\pi}{\lambda} \cdot a \sin\theta = \alpha \qquad ...(4)$$

$$\therefore \quad AD = \frac{E_\theta}{2} = R.\sin\frac{\phi}{2}$$

$$\therefore \quad E_\theta = 2R.\sin\frac{\phi}{2} \qquad ...(5)$$

- Using the relation, angle $= \dfrac{\text{arc}}{\text{radius}}$, we have,

$$\phi = \frac{E_m}{R} \qquad \therefore \quad R = \frac{E_m}{\phi}$$

- Therefore Equation (5) gives,

$$E_\theta = 2.\frac{E_m}{\phi} \cdot \sin\frac{\phi}{2} = E_m \frac{\sin\phi/2}{\phi/2}$$

$$\therefore \quad E_\theta = E_m \cdot \frac{\sin\alpha}{\alpha} \qquad ...(6)$$

- This gives the amplitude of wave disturbance at point P where the rays diffracted at angel θ meet.
- If $I_\theta \rightarrow$ Resultant intensity of light at P and $I_m \rightarrow$ Maximum intensity at P_0 then we have $I_\theta \propto E_\theta^2$ and $I_m \propto E_m^2$
- Hence squaring Equation (6) gives,

$$I_\theta = I_m \cdot \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad ...(7)$$

Step IV

- For E_θ to be maximum; all the phasors must be in phase i.e. α i.e. $\alpha = 0$

$$\therefore \quad \alpha = \frac{\pi}{\lambda}.a \sin\theta = 0$$

$$\Rightarrow \quad \sin\theta = 0 \text{ or } \theta = 0$$

- Thus the maximum value of E_θ is E_m and this principal maximum is formed at $\theta = 0$. i.e. this maximum is formed by the parts of the secondary wavelets which travel normally to the slit. Hence it is produced at P_0.

Step V : Minimum Intensity

- The intensity at point P will be zero when $\sin \alpha = 0$.
- The values of α which satisfy this condition are

$$\alpha = \pm m \pi \qquad \text{where} \quad m = 1, 2, 3, 4, \ldots$$

- Thus the condition for minimum intensity at point P is

$$\alpha = \frac{\pi}{\lambda}.a \sin\theta = \pm m \pi$$

or $a \sin \theta = \pm m \lambda$ ...(8)

where $m = 1, 2, 3, \ldots$

($m = 0$ is not possible because then θ becomes zero which corresponds to principal maximum).

- Thus Equation (8) shows that we have points of minimum intensity on either side of the principal maximum (as there $\pm$ sign on RHS).

Step VI : Secondary Maxima

- In addition to the principal maximum at $\theta = 0$, there are weak secondary maxima on either side of it. They lie approximately halfway between the two minima.
- Hence the secondary maxima can be obtained for

$$\alpha = \pm\left(m + \frac{1}{2}\right)\pi \qquad \text{where} \quad m = 1, 2, 3, \ldots$$

- Putting this condition in Equation (3.4.7) we have the relative intensity of secondary maxima as

$$\frac{I_\theta}{I_m} = \left[\frac{\sin\left(m+\frac{1}{2}\right)\pi}{\left(m+\frac{1}{2}\right)\pi}\right]^2$$

- Putting m = 1, 2, 3, we have

$$\frac{I_\theta}{I_m} = 0.045; 0.016; 0.008; \ldots.$$

- Thus the successive maxima decrease in intensity rapidly.
- The relative intensity distribution in single slit diffraction pattern is shown in Figure 7.

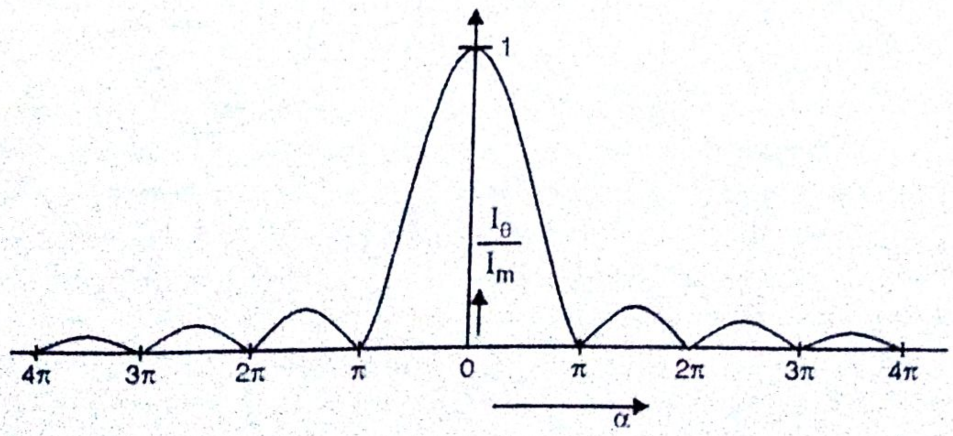

Fig.7 : Relative intensity distribution in single slit diffraction.

1.3 FRAUNHOFFER DIFFRACTION

1.3.1 Plane Diffraction Grating

Q.1 What is diffraction grating?

Ans.: An arrangement which is equivalent in action to a number of parallel, equidistant, small rectangular slits of equal width placed side by side is called diffraction grating.

Since the slits are arranged in a plane, so diffraction grating is also called plane diffraction grating and is a very good and accurate device for study of spectra.

In diffraction grating, the slits are very narrow and are separated by opaque spaces, which do not allow light to pass through them and are so called 'opacities'. Between the opacities, we have the slits which allow the the grating, he was able to study the solar spectrum. It was because light to pass through and so are called 'transparencies'.

Fraunfoher was able to prepare the first grating by making 310 narrow slits in an inch, which he got by ruling lines on glass with fine diamond point. Using of the original formulation by Fraunhofer in the making of parallel slits to study the diffraction phenomenon that his name is associated with this type of diffraction. So, the grating are made by ruling equidistant parallel lines on an optically transparent sheet of material, the line portion being opaque and the space between the lines will be transparent to light. In the construction of a good quality grating, the following consideration must be taken into account.

(i) It should have large number of slits i.e. the number of lines should be very large. Normally this number may be between 12,000 to 30,000 lines/inch.

(ii) The spacing between the lines should be equal.

1.3.2 Formation of Multiple Spectra with Grating

The principal maxima in a grating are formed in directions θ given by

$$(a + b) \sin \theta = \pm n \lambda,$$

where (a + b) is the grating element, n the the maxima and X the wavelength of the incident light.

From the expression we conclude, that

(i) For a particular wavelength λ, the angel of the diffraction θ is different for principal maxima of different orders. When the number of lines grating are large, as usually the case is, the maxima appear sharp, bright lines parallel to the rulings of the grating and are termed as spectral lines.

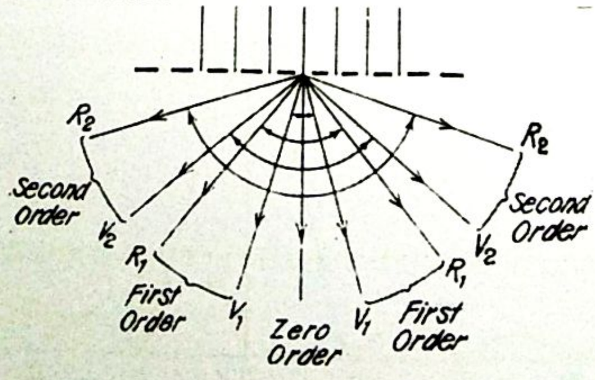

Fig. 8

(ii) For white light (several wavelengths) and for a particular order n, the light of different wavelengths will be diffracted in different directions. The longer the wavelength, greater is the angle of diffraction. So in each order we will get the spectra having as many lines as the wavelengths in the light source. At centre (n =, 0 zero order) $\theta = 0$ which gives the maxima of all wavelengths so here different wavelengths coincide to form the central image of the same colour as that of the light source. The principal maxima of all wavelengths corresponding to n = 1 will form first order spectrum. Similarly, the principal maxima of all wavelengths corresponding to n = 2 will form the second order spectrum and so on. The violet colour being in the innermost position and red colour being in the outermost position. Most of the intensity goes to zero order and rest is distributed among other orders. Thus the spectra become fainter as we go to higher orders.

Chief characteristics of grating spectra.

(i) Spectra of different orders are situated symmetrically on both sides of zero order image.
(ii) Spectral lines are almost straight and quite sharp.
(iii) Spectral colours are in the order from violet to red.
(iv) The spectral lines are more and more dispersed as we go to higher orders.
(v) Most of the incident intensity goes to zero order and rest is distributed among the other orders.

Maximum Number of Orders Available with a Grating

The principle maxima in a grating satisfy the condition

$$(a+b)\sin\theta = n\lambda \qquad \text{or} \qquad n = \frac{(a+b)\sin\theta}{\lambda},$$

where the symbols have their usual meanings,

The maximum angel of deflection is 90°, hence the maximum possible order is given by

$$(n)_{max} = \frac{(a+b)\sin 90^\circ}{\lambda} = \frac{(a+b)b}{\lambda}$$

Now we shall consider the example of a grating having grating element which is less than twice the wavelength of incident light. In this case,

$$(a+b) < 2\lambda \therefore (n)_{max} < \frac{2\lambda}{\lambda} < 2$$

i.e. only the first order is possible

1.4 ABSENT SPECTRA

Q.1 Derive the condition for absent spectra in grating. **[M-13]**

Ans.: Sometimes, for a particular angle of diffraction θ satisfying the relation (a + b) sin $\theta = n\lambda$, there is no visible spectrum. It is because for a given direction θ, the path difference between the diffracted rays from two extreme edges of one space is equal to an even multiple of $\lambda/2$ e.g., if the path difference is λ, the slit can be considered to be divided into two halves and the path difference between the two pairs of corresponding points in the two halves will be $\lambda/2$, which will zero intensity in that direction. So the condition for a minima in a singe slit is given by

$$a\sin\theta = m\lambda \qquad \ldots(1)$$

where m is an integer and is not equal to zero.

The condition for the principal maxima in the nth order spectrum is given by

$$(a + b)\sin\theta = n\lambda \qquad \text{...(2)}$$

If the conditions (1) and (2) are simultaneously satisfied, the diffracted rays from all the transparencies superimpose upon each other, but the resultant intensity is zero, i.e., the spectrum is absent.

From equation (1) and (2) we have

$$\frac{a+b}{a} = \frac{n}{m}$$

or

$$\frac{b}{a} = \left(\frac{n}{m} - 1\right) \qquad \text{...(3)}$$

If width of opacity is equal to that of transparency i.e., a = b, then from (3)

$$n = 2m$$

So that 2^{nd}, 4^{th}, 6ht etc., orders of the spectra will be missing corresponding to the minima due to a single slit given by m = 1, 2, 3, ...etc.

If width of opacity is twice the width of transparency i.e., b = 2a, then from (3) n = 3m, so that 3^{rd}, 6^{th}, 9^{th} etc. order of the spectra will be missing corresponding to the minima due to a single slit given by m = 1, 2, 3 ....etc.

1.5 RESOLVING POWER OF GRATING

Q.1 What is resolving power at a grating?

Ans.: The objects lying very close to each other cannot be seen as separate by a naked eye but if these are enlarged by magnifying them then these can be seen as separate. This is because an eye can see two objects as separate only if the angle subtended by them at the eye is greater than one minute. Different optical instruments such as telescope, microscope, prism, grating etc.. can be used to increase the angle subtended at the eye so that these objects appear as separate. So this method of seeing two objects or images which are very close to each other as separate using some optical instrument is called resolution. In case of circular aperture of a telescope or a microscope, with a point object, we get diffraction patterns consisting of central bright disc surrounded by dark and bright concentric circular rings of rapidly decreasing intensity. The diffraction patterns of two widely separated sources may be given as under.

If the two point sources are very near, then the diffraction patterns of these sources will overlap and they will appear as one, optical instruments are used to see them separate. So the capacity of an instrument to produce two separate images of very close objects is called resolving power.

In the case of a telescope, the reciprocal of the smallest angle subtended between two objects which can be seen as separate through the telescope measures its resolving power.

In case of microscope, the reciprocal of minimum distance between two point objects which can be seen separately through the microscope measures its resolving power.

The resolution in case of two very close spectral lines is known as spectral resolution.

Spectral resolving power = $\frac{\lambda}{d\lambda}$ where $d\lambda$ is the smallest difference in wavelengths that can be resolved by the instrument from wavelength λ.

1.5.1 Resolving Power of a Diffraction Grating

Q.1 Derive an expression for the resolving power of a grating.

Ans.: The resolving power of a grating is its ability to show two neighbouring lines in a spectrum as separate. It may also be defined as the ratio of the wavelength of any spectral line to its difference of wavelengths between this line and a neighbouring line such that the two spectral lines can by just seen as separate. Two lines of wavelengths λ and $\lambda + d\lambda$ are said to be just resolved if the central maxima due to $\lambda + d\lambda$ falls on the first minima of λ. The resolving power of diffraction grating is given by $\lambda/d\lambda$.

Consider two wavelets of wavelengths λ and $\lambda + d\lambda$ to be incident normally on the surface of the grating. The two wavelengths will give their own diffraction patterns. According to Rayleigh's criteria, the two patterns would be just resolved if the principal maxima of one falls on the first secondary minima of the other in any order.

The condition for the first secondary minimum after the nth order maximum is given by

$$(a + b) \sin (\theta + d\theta) = n\lambda + \frac{\lambda}{N} \qquad ...(1)$$

Now, let the nth order principal maximum corresponding to wavelength $\lambda + d\lambda$ has the same direction as that of the first secondary minimum of the nth order principal maximum corresponding to wavelength λ, then

$$(a + b) \sin (\theta + d\theta) = n(\lambda + d\lambda) \qquad ...(2)$$

From equation (1) and (2)

$$n\lambda + \frac{\lambda}{N} = n\lambda + nd\lambda$$

$$\therefore \quad nd\lambda = \frac{\lambda}{N}$$

or

$$\frac{\lambda}{d\lambda} = N \times n$$

$$= (\text{Total number of lines on grating} \times \text{order of spectrum})$$

which shows that resolving power depends on the order of spectrum and the number of lines on the grating but does not depend on the grating element.

1.5.2 Determination of Wavelength by Diffraction Grating [M-16]

Q.2 How will you determine the wavelength of the source of light by using diffraction grating?

Ans.:
- The diffraction grating is often used in the laboratories for the determination of wavelength of light.

- The grating spectrum of the given source of monochromatic light is obtained by using a spectrometer. The arrangement is as shown in Figure 9

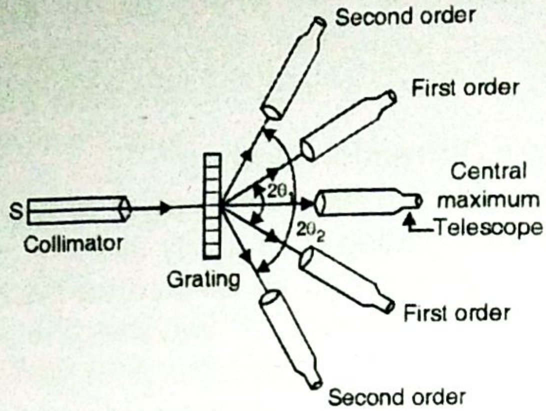

Fig. 9

- The spectrometer is first adjusted for parallel rays. The grating is then placed on the prism table and adjusted for normal incidence.
- In the same direction as that of the incident light, the direct image of the slit or the zero order spectrum can be seen in the telescope.
- On either side of this direct image a symmetrical diffraction pattern consisting of different orders can be seen.
- The angle of diffraction θ for a particular order m of the spectrum is measured.
- The number of lines per inch of grating are written over it by the manufacturers.

  Hence the grating element is,

$$(a + b) = \frac{1}{\text{Number of lines / cm}}$$
$$= \frac{2.54}{\text{Number of lines / inch}}$$

- Thus using the equation

  $(a + b) \sin \theta = m\lambda$
- The unknown wavelength λ can be calculated by putting the values of the grating element (a + b), the order m and the angle of diffraction θ.

Solved Examples

Ex.1 A plane grating just resolves two lines in the second order. Calculate the grating element if $d\lambda = 6$ Å, $\lambda = 6 \times 10^{-5}$ cm and the width of the ruled surface is 2 cm.

Ans.: $m = 2$, $d\lambda = 6$Å $= 6 \times 10^{-8}$ cm, $\lambda = 6 \times 10^{-5}$ cm, W = 2 cm.

(i) $\text{R.P.} = \frac{\lambda}{d\lambda}$, (ii) R.P. = mN

$$\text{R.P.} = \frac{\lambda}{d\lambda} = mN$$

$$\therefore \; N = \frac{\lambda}{d\lambda} \cdot \frac{1}{m} = \frac{6 \times 10^{-5}}{6 \times 10^{-8}} \times \frac{1}{2} = 500$$

$\therefore$ No. of lines in a width of 2 cm of grating = 500

$\therefore$ No. of lines per cm = $\frac{500}{2}$ = 250

Ex. 2 A parallel beam of sodium light is allowed to be incident normally on a plane grating having 4250 lines per cm and a second order spectral line is observed to be deviated through 30°. Calculated the wavelength of spectral line.

Ans.: We know that $(a+b)\sin\theta = n\lambda$

$$\therefore \lambda = \frac{(a+b)\sin\theta}{n}$$

Here $(a+b) = \frac{1}{4250}$ cm, $\theta = 30°$, and $n = 2$

$$\therefore \lambda = \frac{1\times\sin 30°}{4250\times 2} = \frac{1}{4250\times 2\times 2} = 5883\times 10^{-8}\text{ cm}$$

Ex. 3 Monochromatic light of wavelength 6560×10^{-8} m falls normally on a grating 2 cm wide. The first order spectrum is produced at an angle 18° 14′ from the normal. What is the total number of lines on the grating? Given sin 18° 14′ = 0.3129.

Ans.: We known that $(a+b)\sin\theta = n\lambda$

or $$(a+b) = \frac{n\lambda}{\sin\theta}$$

Here $n = 1$. $\theta = 18° 14'$ or $\sin 18° 14' = 0.3129$

and $\lambda = 6560\times 10^{-8}$ cm

$$\therefore \quad (a+b) = \frac{1\times 6560\times 10^{-8}}{0.3129}\text{ cm}$$

The number of line per cm on the grating

$= \frac{1}{(a+b)} = \frac{0.3129}{6560\times 10^{-8}}$. As the grating is 2 cm wide, hence the total numbers of line N on it is

$$N = 2\times\frac{0.3129}{6560\times 10^{-8}} = 9541$$

Ex.4 A diffraction grating used at normal incidence gives a line (5400 Å in a certain order superposed on the violet line (4050 Å) of the next higher order. If the angle of diffraction is 30°, how many line per cm are there in the grating.

Ans.: We know that $(a+b)\sin\theta = n\lambda$.

Let nth order maxima of λ_1 coincide with (n+1)th order maxima of λ_2, then

$(a+b)\sin\theta = n\lambda_1$ and $(a+b)\sin\theta = (n+1)\lambda_2$

$\therefore n\lambda_1 = (n+1)\lambda_2$ or $\quad n = \frac{\lambda_2}{\lambda_1-\lambda_2}$

Now $(a+b)\sin\theta = n\lambda_1 = \frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}$

No. of line per cm = $\dfrac{1}{(a+b)} = \dfrac{\sin\theta(\lambda_1 - \lambda_2)}{(\lambda_1 \lambda_2)}$

Here $\lambda_1 = 5400 \times 10^{-8}$ cm,
$\lambda_2 = 4050 \times 10^{-8}$ cm, $\theta = 30°$

∴ No. of lines per cm

$$= \frac{\sin 30° \times (5400 - 4050) \times 10^{-8}}{5400 \times 10^{-8} \times 4050 \times 10^{-8}} = 3086$$

Ex.5 A plane transmission grating having 6000 lines per cm is used to obtain a spectrum of light from a sodium light in the second order. Find the angular separation between the two sodium lines whose wavelengths are 5890 Å and 5896 Å respectively.

Ans.: In a plane transmission grating $(a+b)\sin\theta = n\lambda$

Here $a + b = \dfrac{1}{6000}$ $n = 2$

$\lambda_1 = 5890$ Å $= 5890 \times 10^{-8}$ cm

$\lambda_2 = 5896$ Å $= 5896 \times 10^{-8}$ cm

Let $\theta = \theta_1$ for $\lambda = \lambda_1$ and $\theta = \theta_2$ for $\lambda = \lambda_2$

For λ_1

$$(a + b) \sin \theta_1 = n\lambda_1$$

$\sin \theta_1 = \dfrac{n\lambda_1}{(a+b)} = \dfrac{2 \times 5890 \times 10^{-8}}{(1/6000)}$ or $\theta = 44°59'$

For λ_2 ,

$$(a + b) \sin\theta_2 = n\,\lambda_2$$

$\sin \theta_2 = \dfrac{n\lambda_2}{(a+b)} = \dfrac{2 \times 5896 \times 10^{-8}}{1/6000}$ or $\theta_2 = 45°2'$

∴ Angular separation of lines $= \theta_1 - \theta_2 = 3'$

Ex.6 A grating with 15000 rulings per inch is illuminated normally with white light extending from 4000Å – 7000Å. Show that only the first order spectrum is isolated but the second and third order overlap.

Ans.: We know that $(a + b) \sin \theta = n\lambda$

∴ $\sin\theta = \dfrac{n\lambda}{(a+b)}$

Here $(a + b) = 2.54 / 15000$

For $\lambda = 4000 \times 10^{-8}$ cm,

$\sin \theta = \dfrac{n \times 4000 \times 10^{-8}}{2.54/15000} = 0.236\,n$

$$\theta = \sin^{-1}(0.236 \times n)$$
$$= \sin^{-1}(0.236 \times 1,\ 2,\ 3...)$$

where n = 1, n = 2, n = 3 correspond to first, second, third etc. orders.

$\therefore \theta \approx 13°, 28°, 45°, ...$

For $\lambda = 7000 \times 10^{-8}$ cm

$$\sin\theta' = \frac{n \times 7000 \times 10^{-8}}{2.54/15000} = 0.413\ n$$

or $\theta' = \sin^{-1}(0.413 \times 1, 2, 3.....) = 24°, 56°, ...$

Thus the angular positions for 4000 Å and 7000 Å lines in the various orders come out as follows:

I order 13° , 24° Isolated

$$\left.\begin{array}{l}\text{II order } 28°, 56° \\ \text{III order } 45°, ...\end{array}\right\} \text{(Overlap)}$$

Ex.7 How many orders will be visible if the wavelength of the incident radiation is 5000 Å and the number of lines on the grating is 2620 in one inch?

Ans.: We known that (a + b) sin θ = nλ

$(a + b) = n\lambda \qquad \text{or } n = \frac{(a+b)}{\lambda}$

Here $(a+b) = \frac{2.54}{2620}$ cm ,

$\lambda = 5000$ Å $= 5\times 10^{-5}$ cm, and n = ?

$\therefore n = \frac{2.54}{2620 \times 5 \times 10}$ or $n > 19$.

Hence the highest order of the spectrum which can be seen is 19.

Ex.8 In relation to a plane transmission grating with 5.0×10^3 lines per cm, answer the following :

1. **For wavelength 6.0×10^{-5} cm what is the highest order of spectrum which may be observed ?**
2. **If opaque spaces are exactly 2.0 times the transparent spaces, which order of spectra will be absent?**

Ans.: The maximum number of orders visible with a grating

$$n = \frac{(a+b)}{\lambda} = \frac{1}{5 \times 10^3 \times 6 \times 10^{-5}} = 3.33$$

Hence the third order is the highest order visible.

The condition for the spectrum of order n to be absent is

$$\frac{(a+b)}{a} = \frac{n}{m} \qquad \text{(where m = 1, 2, 3, ...)}$$

Here b = 2 a, $\therefore \frac{(a+2a)}{a} = \frac{n}{m}$

or $n = 3m$

Therefore 3rd, 6th 9th etc. orders of spectra will be absent.

Ex.9 What is the highest order spectrum which may be seen with light of wavelength 5×10^{-5} cm by means of a grating with 3000 lines/cm. Calculate the wavelengths in the visible spectrum (3.5×10^{-5} cm – 7.0×10^{-5} cm) which coincides with the fifth order spectrum of this light.

Ans.: We know that $n_{max} = (a + b)\lambda$

$$\therefore n_{max} = \frac{1}{3000 \times (5 \times 10^{-5})} = 6.6$$

Hence the highest order which may be seen is 6.

The direction θ of the fifth order (n = 5) spectrum of wavelength 5×10^{-5} cm is given by

$$(a + b) \sin\theta = 5 \times (5 \times 10^{-5})$$

Let λ′ be the other wavelength falling in the same direction θ. Then

$(a + b) \sin\theta = (5 \pm m)\lambda'$ where m = 1, 2, 3…

$\therefore 5 \times (5 \times 10^{-5}) = (5 \pm m)\lambda'$

or $$\lambda' = \frac{25 \times 10^{-5}}{(5 \pm m)} \text{cm}$$

putting m = 1, 2, 3, …

$$\lambda' = (4.1, 6.2; 3.6, 8.3; 3.1, 12.5....)\ 10^{-5} \text{ cm}$$

Following are the wavelengths which lie in the visible region

GRADED QUESTIONS

1. Discuss the phenomenon of Fraunhoffer diffraction at a single slit and obtain condition for first maxima and minima from central maximum.
2. Derive the condition of absent spectra in grating.
3. Obtain an expression for the resolving power of a grating? On what factors does it depend on?
4. Define Resolving Power of a grating and write factors on which the resolving power of the grating depends.
5. Explain the effects on the diffraction pattern of a single slit, if the slit width "a" decreases.
6. What is Rayleigh's criterion for resolution? How can the resolving power of grating be increased?
7. What are the types of diffraction? Differentiate between them.
8. Derive an expression for the Intensity of a single slit diffraction.
9. What is grating and grating element?
10. Explain how grating can be used to determine the wavelength of a monochromatic light.

EXAMINATION QUESTIONS

Dec. '19

1. In a plane transmission grating the angle of diffraction for the first order principal maximum is 20° for a wavelength of 6500 A°. Calculate the number of lines in one cm of the grating surface. **[3]**

   [Ans.: 1.9×10^{-4} m]

2. What is grating element? The visible spectrum ranges from 4000 A° to 5000 A°. Find the angular breadth of the first order visible spectrum produced by a plane grating having 6000 lines/cm when light is incident normally on the grating. **[7]**

   [Ans.: $\theta_1 = 13° 53'$; N = 8]

May '19

1. Write the formula for dispersive power of the grating. Explain how it can be increased. **[3]**
2. Explain Fraunhofer's double slit diffraction experiment and obtain expression for resultant intensity of light on the screen and derive the formula for missing orders in the double slit diffraction pattern. **[8]**
3. Interference fringes are produced by monochromatic light falling normally on a wedge shaped film of refractive index 1.4. The angle of wedge is 20 seconds of an arc and the distance between successive bright fringes is 0.25 cm. Calculate wavelength of the light used. **[5]**

   [Ans.: λ = 6787 A°]

Dec. '18

1. What is antireflection coating? What should be the refractive index and minimum thickness of the coating? **[3]**
2. A parallel beam of light of wavelength 5890 Á is incident on a glass plate having refractive index μ = 1.5 such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the glass plate which will appear dark by reflected light. **[3]**

   [Ans.: t_{min} = 3920 A°]

3. With the help of a proper diagram and necessary expression, explain how Newton's ring experiment is useful to determine the radius of curvature of a plano convex lens. In a Newton ring's experiment the diameter of 5th dark ring is 0.336 cm and the diameter of 15th dark ring is 0.590 cm. Find the radius of curvature of a plano convex lens, if the wavelength of light used is 5890 Á. **[8]**

   [Ans.: R = 99.88 cm]

4. What is diffraction grating and grating element? Explain the experimental method to determine the wavelength of the spectral line using diffraction grating. **[7]**
5. Derive the condition for absent spectra in grating. **[5]**
6. A Newton's rings set up is used with a source emitting two wavelengths λ_1 = 6000 Á and λ_2 = 4500 Á. It is found that nth dark ring due to 6000Á coincides with (n + 2)th dark ring due to 4500Á. If the radius of curvature of the lens is 90 cm. find the diameter of nth dark ring for 6000Á. **[5]**

   [Ans.: Radius of nth dark ring = 0.504 cm]

May '18

1. Explain how interference in wedge shaped film is used to test optical flatness of given glass plate. **[3]**

2. What is diffraction grating? What is the advantage of increasing the number of lines in the grating? [3]
3. In Newton's rings pattern what will be the order of the dark ring which will have double the diameter of the 40th dark ring. [3]

[Ans.: n′ = 160]

4. Derive the conditions for maxima and minima due to interference of light reflected from thin film of uniform thickness. [8]
5. Discuss the Fraunhofer diffraction at single slit and obtain the condition for minima. In plane transmission grating the angle of diffraction for second order principal maxima for wavelength 5×10^{-5} cm is 35°. Calculate number of lines /cm on diffraction grating. [8]

[Ans.: 5747]

6. A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between consecutive bright fringes is 0.12 cm. Calculate wavelength of light used. [5]

[Ans.: 4654A°]

7. Explain Newton's rings experiment and show that diameters of n^{th} dark rings are proportional to square root of natural numbers. [5]

Dec. '17

1. Why the Newton's rings are circular and centre of interference pattern (reflected) is dark? [3]
2. What is Rayleigh's criterion of resolution? Define resolving power of a grating? [3]
3. What will be the fringe pattern if wedge shaped air film is illuminated with white light? [3]
4. Obtain the condition for maxima and minima of the light reflected from a thin transparent film of uniform thickness. Why is the visibility of the fringe much higher in the reflected system than in the transmitted system? [8]
5. Explain the experimental method to determine the wavelength of spectral line using diffraction grating. [8]
   A diffraction grating has 5000 lines /cm and the total ruled width is 5cm. Calculate dispersion for a wavelength of 5000A° in the second order.

[Ans.: 11547]

6. A wedge shaped air film having angle of 40 seconds is illuminated by monochromatic light. Fringes are observed vertically through a microscope. The distance between 10 consecutive dark fringes is 1.2cm. Find the wavelength of monochromatic light used. [5]

[Ans.: λ = 4654 A°]

7. Describe in detail the concept of anti-reflecting film with a proper ray diagram. [5]

May '17

1. Why the Newton's rings are circular and fringes in wedge shaped film are straight? [3]
2. What is grating and grating element? [3]
3. What do you mean by thin film? Comment on the colors in thin film in sun light. [3]
4. Derive the conditions for maxima and minima due to interference of light transmitted from thin film of uniform thickness. [8]

5. Explain the experimental method to determine the wavelength of spectral line using diffraction grating. **[8]**

What is the highest order spectrum which can be seen with monochromatic light of wavelength 6000 Á by means of a diffraction grating with 5000 lines /cm?

[**Ans.:** maximum order of spectrum = 3]

6. Two optically plane glass strips of length 10cm are placed one over the other. A thin foil of thickness 0.01 mm is introduced between them at one end to form an air film. If the light used has wavelength 5900 A°. Find the separation between consecutive bright fringes. **[5]**

[**Ans.:** β = 2.95 mm]

7. With Newton's ring experiment explain how to determine the refractive index of liquid? **[5]**

❑❑❑❑❑