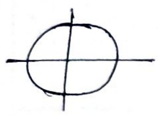
## Vertical Circular Motion :



If Rod is attached

$\boxed{V_{bottom} = \sqrt{4gl}}$

$\boxed{V_{Lowest} = \sqrt{5rg}}$ $\boxed{a_L = 5g}$

$\boxed{V_{High} = \sqrt{rg}}$ $\boxed{a_H = g}$

$\boxed{V_{mid} = \sqrt{3rg}}$ $\boxed{a_{mid} = 3g}$

$\boxed{V_L^2 - V_H^2 = 4rg}$

$\boxed{V_m^2 - V_H^2 = 2rg}$

$\boxed{V_{any\ point} = \sqrt{rg(3+2\cos\theta)}}$

$\boxed{f_{cp} = \frac{mv^2}{r} = mr\omega^2 = mv\omega}$

## Kinematical Equation

| | |
|---|---|
| $v = u + at$ | $\omega_f = \omega_i + \alpha t$ |
| $s = ut + \frac{1}{2}at^2$ | $\theta = \omega_i t + \frac{1}{2}\alpha t^2$ |
| $v^2 = u^2 + 2as$ | $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ |
| $s = \left(\frac{u+v}{2}\right)t$ | $\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$ |

## Law of Conservation of Momentum

$\vec{\tau} = \vec{r} \times \vec{p}$ $\boxed{L = mvr}$

$\boxed{\frac{dL}{dt} = \vec{\tau}}$ $\boxed{\frac{d\vec{p}}{dt} = \vec{F}}$ $\boxed{\tau = \vec{r} \times \vec{F}}$

$\boxed{\tau = I\alpha}$

If $\vec{\tau} = 0$

$\vec{L} = \text{Constant}$ $\boxed{L = I\omega}$

$\boxed{I_1\omega_1 = I_2\omega_2}$

| Linear | Angular |
|---|---|
| $K.E = \frac{1}{2}mv^2$ | $K.E = \frac{1}{2}I\omega^2$ |
| $f = ma$ | $\tau = I\alpha$ |
| $P = mv$ | $L = I\omega$ |
| $W = F.S$ | $W = \tau.\theta$ |
| $P = \vec{F}\vec{v}$ | $P = \vec{\tau}\vec{\omega}$ |

Agar ⇒ disc, Ring, S.S, H.S rolled which will come first

$V \propto \frac{1}{\sqrt{\left(1+\frac{k^2}{R^2}\right)}}$ $\boxed{V = \sqrt{\frac{2gh}{\left(1+\frac{k^2}{R^2}\right)}}}$

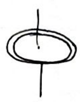
∴ Jiska $\frac{k^2}{R^2} \uparrow$ ⇒ uska $v \downarrow$

| | | |
|---|---|---|
| Ring | 1 | ← last |
| Disc | 1/2 | ← 2nd |
| Solid-S | 2/5 | ← 1st |
| Hollow-S | 2/3 | ← 3rd |

## Moment of inertia

$\boxed{I = \frac{ML^2}{12}}$

$\boxed{I = \frac{ML^2}{3}}$



$I = MR^2$

$I = \frac{MR^2}{2}$

S.S $I = \frac{2}{5}MR^2$

H.S $I = \frac{2}{3}MR^2$

$\vec{S} = \vec{\theta} \times \vec{r}$

$\vec{V} = \vec{\omega} \times \vec{r}$

$\vec{a}_T = \vec{\alpha} \times \vec{r}$

## Conical pendulum :

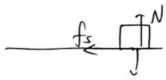
$\boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$ $\boxed{\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L\cos\theta}}}$ $\boxed{T = mg\sec\theta}$

$\boxed{V = \sqrt{rg\tan\theta}}$

$\boxed{C.F = mg\tan\theta}$

## Application of UCM :

① Vehicle Along a Horizontal circular track:



$\boxed{V_{max} = \sqrt{\mu_s r g}}$

② Wall of Death:

$\boxed{V_{min} = \sqrt{\frac{gr}{\mu_s}}}$

③ Vechicle on a Banked Road :

optimum speed : $V = \sqrt{rg\tan\theta}$

Banking angle : $\theta = \tan^{-1}\left(\frac{V^2}{rg}\right)$

④ Lower Speed Limit:

$\boxed{V_{min} = \sqrt{rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s\tan\theta}\right)}}$

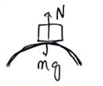
Upper speed limit :

$\boxed{V_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right)}}$

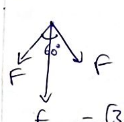
Parallel axis Theorem : $\boxed{I_0 = I_{COM} + Mh^2}$

Prd axis Theorem : $\boxed{I_z = I_x + I_y}$

⑤ Vechicle at the Top of Convex over Bridge.



$\boxed{V_{max} = \sqrt{rg}}$



$f_{rest} = \sqrt{3}F$

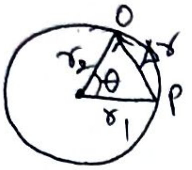
No. of revolution $= \frac{\theta}{2\pi}$

## Vector angular velocity ($\vec{\omega}$)

$V = \omega r \sin\theta$

$$\boxed{\vec{V} = \vec{\omega} \times \vec{r}}$$

## Change in position (Linear Disp.)



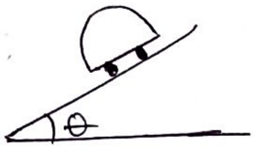

$r_1 = r_2 = r$

$$\boxed{\Delta r = 2r \sin\frac{\theta}{2}}$$

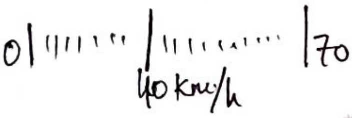
## Change in velocity:

$$\boxed{\Delta V = 2V \sin\frac{\theta}{2}}$$

## Banked Road :




$$V_{max} = \sqrt{rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)}$$




if $\mu_s = 0$ then $V = V_{optimum}$

$$\boxed{V_{opt} = \sqrt{rg\tan\theta}}$$

## Vehicle on convex Bridge.


$$\boxed{V_{max} = \sqrt{gr}}$$

if $\theta = 0^\circ$

$$\boxed{N = mg - \frac{mv^2}{r}}$$

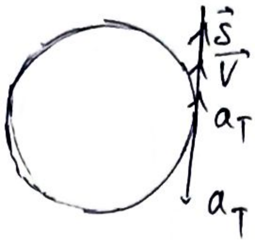
$$\boxed{N = mg\cos\theta - \frac{mv^2}{r}}$$

$S = r\theta$
$V = r\omega$
$a = r\alpha$

scalar form

$\vec{\delta S} = \vec{\delta\theta} \times \vec{r}$
$\vec{V_T} = \vec{\omega} \times \vec{r}$
$\vec{a_T} = \vec{\alpha} \times \vec{r}$

vector form




$\alpha \to$ with $\omega \to \omega\uparrow$

$\alpha \to$ opp $\omega \to$ if $\omega\downarrow$

$\alpha = 0$ if $\omega = 0$

acceleration (a)

↳ change in velocity → speed → $a_t$
→ velocity direction → $a_N$ or $a_c$

$a_t$ : changes speed only

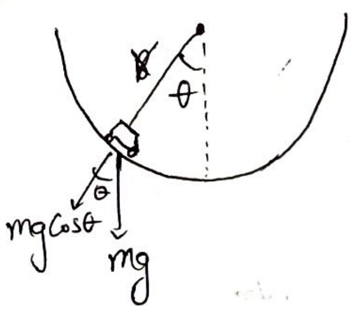
$a_N / a_c$ : changes direction only.


$$\boxed{a_{net} = \sqrt{(a_c)^2 + (a_t)^2}}$$

$$\boxed{|\vec{a_c}| = \frac{V^2}{r} = r\omega^2 = V\omega}, \quad \boxed{a_t = \frac{dV}{dt}}$$

$$\boxed{\vec{a_c} = -\frac{V^2}{r}\hat{r_0} = -\vec{r}\omega^2 = \vec{\omega} \times \vec{V}}$$

## Vehicle on concave Bridge:




$$\boxed{N = mg\cos\theta + \frac{mv^2}{r}}$$

$$\boxed{\text{No. of revolutions} = \frac{\theta}{2\pi} = \left(\frac{f_i + f_f}{2}\right).t = \frac{\omega t}{2\pi} = nt}$$

→ The distance travelled by the particle performing uniform circular motion in $t$ seconds is given by the formula, $\boxed{d = \frac{2\pi r}{T} t}$

→ The angle made by the resultant acceleration with the radius,

$$\boxed{\alpha = \tan^{-1}\left(\frac{a_t}{a_{rk}}\right)}$$

| | |
|---|---|
| $\pi = 3.14$ | $5\pi \doteq 15.70$ |
| $2\pi = 6.28$ | $7\pi = 21.99$ |
| $3\pi = 9.42$ | $9\pi = 28.27$ |

| Angular Displacement ($\theta$) [rad] | Angular velocity ($\omega$) [rad/s] | Angular acceleration ($\alpha$) [rad/s²] |
|---|---|---|
| $s = r\theta$ | $\omega = \frac{d\theta}{dt}$ | $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$ |

Uniform C.M

($\omega$ = const)

$\alpha = 0$

$|\vec{v}|$ = const

$a_t = 0$

$a_t = R\alpha$

Non-Uniform C.M.

$\omega \neq$ const

$\alpha \neq 0$

$\alpha$ = constant

* U.C.M $\boxed{W = F \cdot S = 0}$ $\vec{F} \perp \vec{V}$

S.H.M $\boxed{W = F \cdot S = 0}$

↳ Here displa = 0

→ $v = u + at$

→ $s = ut + \frac{1}{2}at^2$

→ $v^2 = u^2 + 2as$

→ $s = \left(\frac{u+v}{2}\right)t$

→ $s = vt - \frac{1}{2}at^2$

(4) Relation betⁿ linear & angular Q

| | | |
|---|---|---|
| displacement | $\vec{s}$ | $\vec{\theta}$ |
| velocity | $\vec{v}$ | $\vec{\omega}$ |
| accⁿ | $\vec{a}_T$ | $\vec{\alpha}$ |

| Vector Relations: | Scalar Relⁿ |
|---|---|
| $\vec{S} = \vec{\theta} \times \vec{r}$ | $S = r\theta$ |
| $\vec{v} = \vec{\omega} \times \vec{r}$ | $v = r\omega$ |
| $\vec{a}_T = \vec{\alpha} \times \vec{r}$ | $a = r\alpha$ |

# All linear quantities above are tangential vectors.

# All angular quantities above are axial vectors

# for UCM $a_T = 0$ & $\alpha = 0$

for UCM n → frequency

$\boxed{n = \frac{1}{T}}$ $\boxed{\omega = 2\pi n = \frac{2\pi}{T}}$

Radial / centripetal Accⁿ

$\vec{a}_c = -\vec{r}\omega^2 = \vec{\omega} \times \vec{v} = -\frac{v^2}{r}\hat{r}_0$ } vector form

# Radial / cent. accⁿ is must for UCM

# Radial / centr. accⁿ cannot perform any work

# Always directed towards centre of circle.

$P = \tau\omega$, $W = \tau\theta$

$\tau = I\alpha$

# Applications of UCM

(1) CAR moving on horizontal curved road No skidding condition

$V_{max} = \sqrt{\mu_s R g}$

(2) ON Banked road:

$V_{max} = \sqrt{Rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)}$

$V_{min} = \sqrt{Rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s \tan\theta}\right)}$

Optimum or most safe speed

$\boxed{V_{opt} = \sqrt{Rg\tan\theta}}$

(3) Well of Death

$V_{min} = \sqrt{\frac{rg}{\mu_s}}$

(4) Concial pendulum

Time period

$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$


⇒ $V = \sqrt{rg\tan\theta}$

⇒ Centripetal force = $mg\tan\theta$

⇒ Tension in string = $mg\sec\theta$

$= \frac{mgL}{\sqrt{L^2 - r^2}}$

# $h = l\cos\theta$

⇒ $\Delta U = mgL(1 - \cos\theta)$

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \left(\frac{u+v}{2}\right)t$

$\omega_f = \omega_i + \alpha t$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$

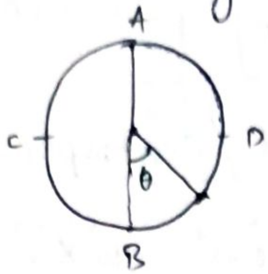
## Vertical Circular Motion

i) Stone tied to a string

$V_L = \sqrt{5gr}$

$V_m = \sqrt{3gr}$

$V_H = \sqrt{gr}$ → min. velocity




$$\boxed{T_L - T_H = 6mg}$$

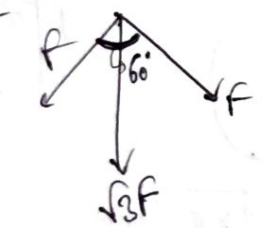
$$\boxed{T = \frac{mv^2}{r} + mg\cos\theta}$$ → Tension at any point θ

Note Mass attached to a rod.

$$\boxed{V_{bottom} = \sqrt{4gl}}$$

$$\boxed{\text{no. of revolutions} = \frac{\theta}{2\pi}}$$

#Note :-




VCM: Total Energy at any point

$$E = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$$

VCM: Mass tied to RoD:

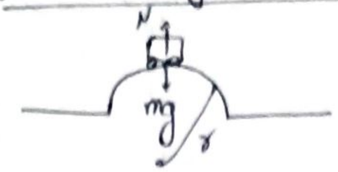
1) $T_L - T_H = 6mg$

2) $V_H = 0$

3) $V_M = \sqrt{2rg}$

4) $V_L = \sqrt{4rg}$

## Car moving on over bridge




$$\frac{mv^2}{r} = mg - N$$

$$N = mg - \frac{mv^2}{r}$$

$$\boxed{V_{max} = \sqrt{rg}}$$

## Sphere of Death

$$N + mg = \frac{mv^2}{r}$$

$$\boxed{V_{min} = \sqrt{gR}}$$ → at top


# 1rps = 60rpm

## Conservation of Angular Momentum

$\vec{\tau}_{ext} = 0$ then $\vec{L} = \text{const}$

i.e $I_1\omega_1 = I_2\omega_2$

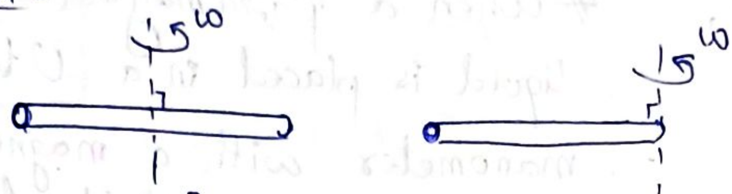
| $K^2/R^2$ for | Ring | Disc | Hollow spher | solid sphere |
|---|---|---|---|---|
| $K^2/R^2$ | 1 | $1/2$ 0.5 | $2/3$ 0.66 | $2/5$ 0.4 |

km/hr → se m/s → ko ⊗ By $\frac{5}{18}$

$$K.E = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{L^2}{I}$$

In VCM: Total Energy is constant $= \frac{5mgr}{2}$

**Rod:**

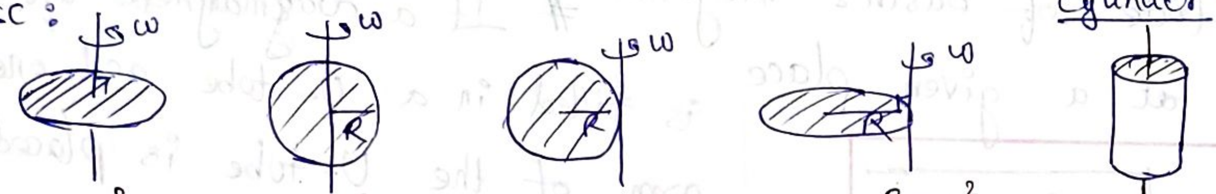


$\frac{ML^2}{12}$ $\frac{ML^2}{3}$

**Ring:** **Cylinder (Hollow):**

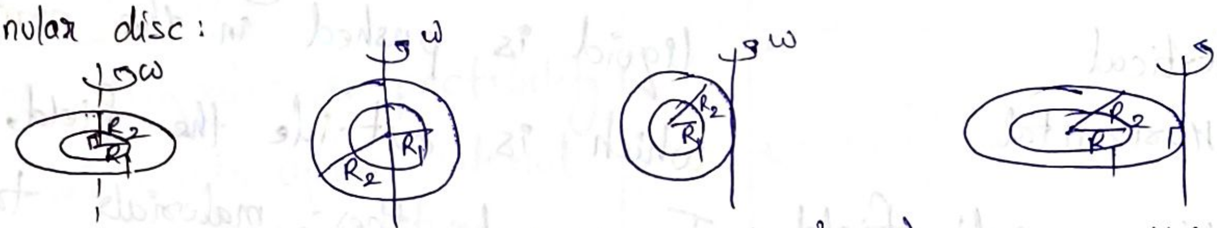
$MR^2$ $\frac{MR^2}{2}$ $\frac{3}{2}MR^2$ $2MR^2$ $MR^2$

**Disc:** **Cylinder (solid)**



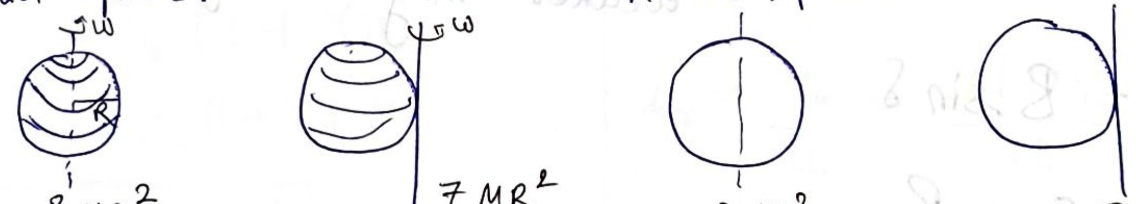
$\frac{MR^2}{2}$ $\frac{MR^2}{4}$ $\frac{5MR^2}{4}$ $\frac{3}{2}MR^2$ $\frac{MR^2}{2}$

**Annular disc:**



$\frac{M}{2}(R_1^2+R_2^2)$ $\frac{M}{4}(R_1^2+R_2^2)$ $\frac{M}{4}(5R_1^2+R_2^2)$ $\frac{M}{2}(3R_1^2+R_2^2)$

**Solid sphere:** **Hollow sphere:**



$\frac{2}{5}MR^2$ $\frac{7}{5}MR^2$ $\frac{2}{3}MR^2$ $\frac{5}{3}MR^2$

$\frac{(2,7)}{5}$ $\frac{(2,5)}{3}$

**Cone:**

(Hollow) (solid)

$\frac{MR^2}{2}$ $\frac{3}{10}MR^2$