Vectors

① Distance of point P from origin $O(0,0,0)$

$l(OP) = \sqrt{x^2+y^2+z^2}$

② Dist. betⁿ two points $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$

$l(AB) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

③ Distance of Point P

i) from X-axis

→ $d(PA) = \sqrt{y^2+z^2}$

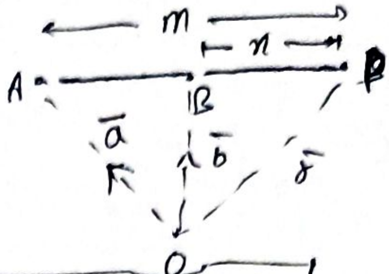
ii) from Y-axis

→ $d(PA) = \sqrt{z^2+x^2}$

iii) from Z-axis

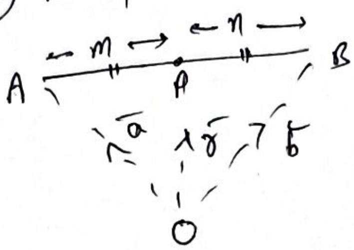
→ $d(PA) = \sqrt{y^2+x^2}$

② External (−)



$$\boxed{\bar{r} = \frac{m\bar{b} - n\bar{a}}{m-n}}$$

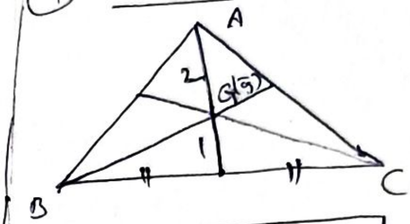
③ Midpoint formula



$\because m = n$ so, $m : n = 1:1$

$$\boxed{\therefore \bar{r} = \frac{\bar{a}+\bar{b}}{2}}$$

④ Centroid formula

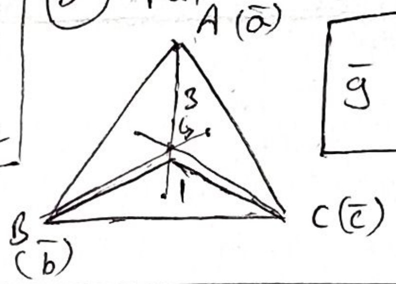


$$\boxed{\bar{g} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}}$$

Note: If vectors $x\hat{i}+y\hat{j}+z\hat{k}$ & $x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ are ∥/collinear then cofficients i, j, k are in ratio.

$$\boxed{\frac{x}{x_1} = \frac{y}{y_1} = \frac{z}{z_1}}$$

⑤ For tetrahedron

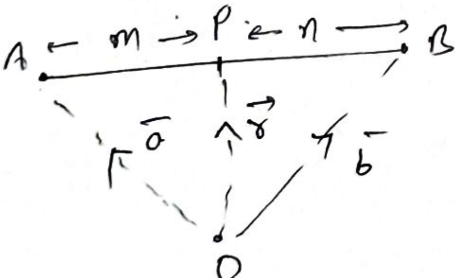


$$\boxed{\bar{g} = \frac{\bar{a}+\bar{b}+\bar{c}+\bar{d}}{4}}$$

Note :- median → midpoint join

altitude → 90° join

① Internal division (+)



$$\boxed{\vec{r} = \frac{m\bar{b} + n\bar{a}}{m+n}}$$

Note :- perpendicular = orthogonal

i.e $\theta = \frac{\pi}{2}$

$\hat{i}\cdot\hat{i} = 1$

$\hat{j}\cdot\hat{j} = 1$

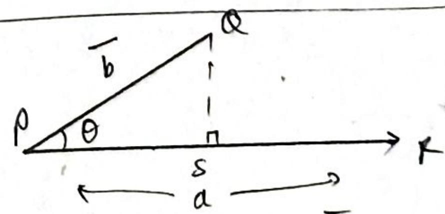
$\hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = 0$

$\hat{j}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{i} = 0$

Projection

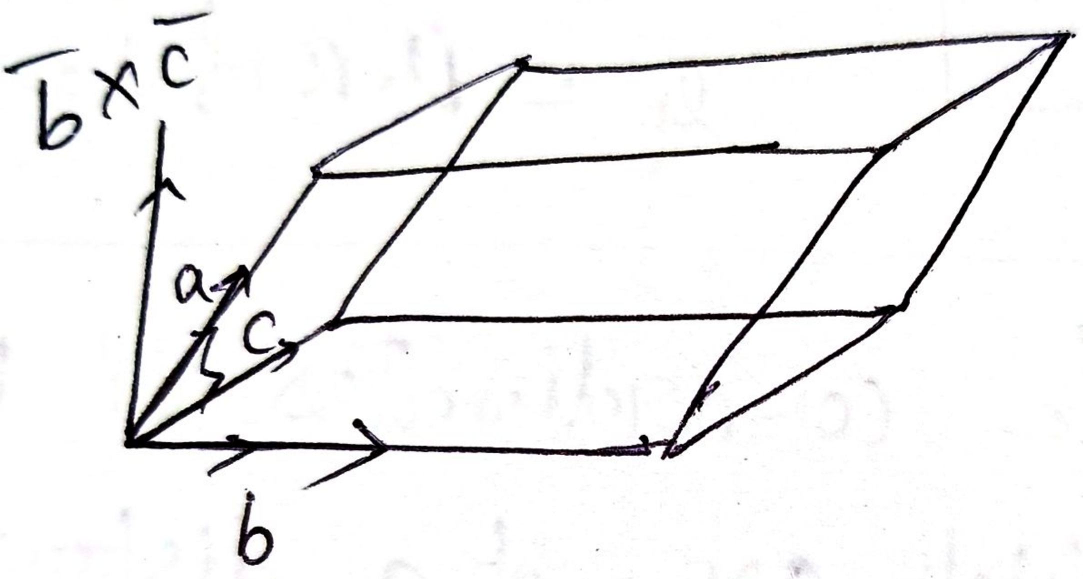


Scalar projection of $\bar{b}$ on $\bar{a}$

$= \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|}$

$$\boxed{S.P \text{ of } \bar{a} \text{ on } \bar{b} = \frac{\bar{a}\cdot\bar{b}}{|\bar{b}|}}$$

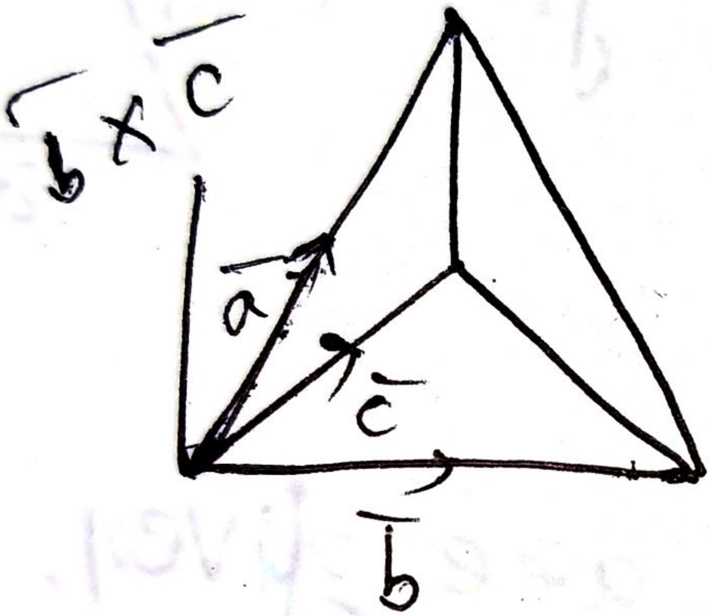
Volume of parallelopiped



$V = [\bar{a}\ \bar{b}\ \bar{c}]$

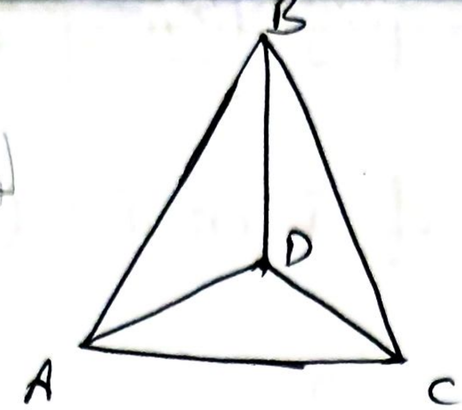
$V = \bar{a} \cdot (\bar{b} \times \bar{c})$ or $(\bar{a} \times \bar{b}) \cdot \bar{c}$

Volume of tetrahedron

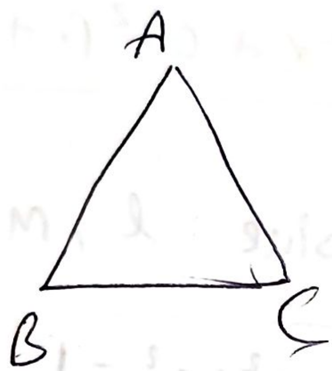


$$\boxed{V = \frac{1}{6}[\bar{a}\ \bar{b}\ \bar{c}]}$$

$$\boxed{V = \frac{1}{6}\bar{a}\cdot(\bar{b}\times\bar{c})}$$



Volume $= \frac{1}{6}[\overline{AB}\ \overline{AC}\ \overline{AD}]$



Area Vector of ΔABC $= \frac{1}{2}[\vec{AB} \times \vec{AC}]$

Note :- if $[a\ b\ c]$ is given

if a, b, c are are mutually $\perp^{rd}$ to each other. then

$\boxed{|a||b||c|}$

#. $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

Vector projection of $\bar{a}$ on $\bar{b} = (\bar{a}\cdot\bar{b})\dfrac{\bar{b}}{|\bar{b}|^2}$

V.P of $\bar{b}$ on $\bar{a} = (\bar{a}\cdot\bar{b})\dfrac{\bar{a}}{|\bar{a}|^2}$

Note: if $l_1 \parallel l_2$ then direction ratio of l_1 = D.R of l_2

Three - 3-D

direction - angle : α, β, γ

direction cosine : $\cos\alpha, \cos\beta, \cos\gamma$

Note: $\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$

also,

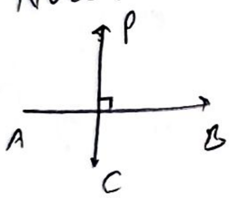
direction cosine : l, m, n

Note : $\boxed{l^2 + m^2 + n^2 = 1}$

Direction ratio $\longrightarrow$ ~~d~~ to D.C

$$\boxed{\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \lambda}$$

Note :



D.r of line PC be (a_1, b_1, c_1) and of line AB be (a_2, b_2, c_2)

if perpendicular

$$\boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$$

Note :- $\boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}$$

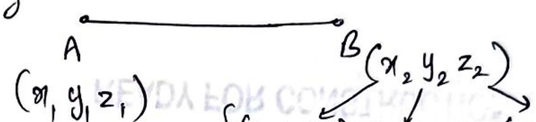
$$n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

Note

The co-ordinates of the points which are at a distance of 'd' units from the point (x_1, y_1, z_1) are given by

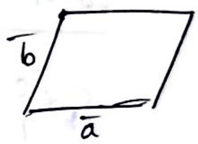
$$\{(x_1 \pm ld), (y_1 \pm md), (z_1 \pm nd)\}$$

eg



$$\{(x_1 \pm ld), (y_1 \pm md), (z_1 \pm nd)\}$$

Area of parallelogram



$$\boxed{A = |\bar{a}\times\bar{b}|}$$

if diagonals are given,

$$\boxed{A = \frac{1}{2}|\bar{a}\times\bar{b}|}$$

$ax^2 + bx + c = 0$

suppose $x = 2$ $\quad x = -3$

$\quad\quad\quad\;\;\downarrow\alpha \quad\quad \downarrow\beta$

then, $\boxed{\alpha\cdot\beta = \frac{c}{a}}$ $\quad \boxed{\alpha + \beta = -\frac{b}{a}}$

Product Scalar Triple

$\bar{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\bar{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$\bar{c} = a_3\hat{i} + b_3\hat{j} + c_3\hat{k}$

$$[\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$[\bar{a}\ \bar{b}\ \bar{c}]$ is zero

if 1) any vector is zero vector

2) any two vector are colliner

3) any three vector are coplaner