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# How Angle of Attack effects Rocket Stability

## I. Observing effect of Centre of Mass of Fuel Tanks

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**ABSTRACT**

*Context.* To investigate the effect that the level of fuel in a fuel tank aboard a rocket will affect the stability of the system by calculating the moving center of mass of the system

*Aims.* It is shown that in order to maintain complete stability of the rocket, the use of a stability assist system, reaction control systems, engine gimballing, or numerous other sources is a necessity, and that if no such systems were present, the rocket would become incredibly unstable.

*Methods.* A tank is modeled, with a fixed mass and a fixed dry mass (And by extension, a fixed dry centre of mass), and fuel is modelled to be within the tank from an initial filled state, to a "burned out" state. This model is then used to calculate the fuels mass, centre of mass, and then the mass and centre of mass for the whole system. This will show the deviation from the inertial plane, and therefore can find out the stability of the rocket, and what must be done to counter it and cause the system to be stable.

*Results.* The angle of attack of the rocket causes the fuel to develop a tilt, inducing a rotational moment. This needs to be countered by the reaction control system, in order to prevent the rocket from inducing a stall-like flat spin and deviating off course, or becoming entirely destroyed. However, the magnitude of the rotational moment never becomes uncontrollable, even in extreme scenarios.

**Key words.** Centre of Mass – Rocket Stability – Moments

## 1. Introduction

A rocket is a vehicle that's sole purpose is to send cargo from point A to point B very fast. Usually, this involves sending cargo from Earth to some destination in space - be that into orbit, to the ISS, or to an extra-terrestrial body. In order for this to happen smoothly, efficiently and effectively, the rocket must be stable, and able to go in a specified direction. In this respect, a rocket can be treated similarly to a ship. Aboard a ship, there are just 2 main considerations that need to be taken in order to design a stable vessel. These are the centre of mass, and the centre of buoyancy.

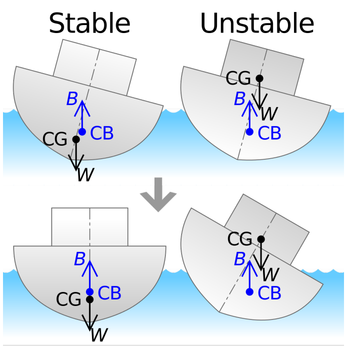


**Fig. 1.** A graphic showing how the stability of a ship is affected by the positions of the centre of mass and the centre of buoyancy. Cmglee (2014)

As seen in figure 1, a ship is stable so long as the centres of mass is below the centre of buoyancy. The system can, in fact, be treated as though it were a pendulum system. In the *unstable* configuration, the system acts as though it were an inverse pendulum, and thus the chaotic nature would mean that any single deviation from perfect stability will cause the system to fail. This would be inconvenient, as ships are designed to float generally in one configuration, and thus any deviation would be catastrophic.

In terms of a rocket this system is very similar, however buoyancy is replaced with thrust. The centre of thrust of the rocket is constantly attempting to spin the rocket upright, and this moment therefore needs to be counteracted through the use of either gimballed engines, or the use of a stability assist system to counteract the moment from the thrusters. Even a slight deviation from the perfect equilibrium will, if left unchecked, eventually cause the rocket to fatally spin and rip itself apart.

In a true rocket flying through space, this is only true at incredibly low speeds. Very quickly, as the rocket develops speed incredibly rapidly, ballistic effects begin to stabilise the rocket, much like a bullet travelling through the air - this is why rockets tend to be aerodynamically shaped to 'cut' through the air. Alongside this, other stabilising features, such as any external fins, gimballing in the engines, stability assist systems, among others, help to stabilise the rocket - through increasing drag in the x and y planes (relative to the rocket), redirecting the thrust to mimic a more stable configuration in figure 1 (aligning the centre of thrust to oppose the rotational moment caused by any movements in the centre of gravity), and by counteracting the rotational moment with a secondary top-down moment. Alongside this, fuel tanks, be they liquid or solid, are usually pressurised or have other systems that are designed to prevent any such negative liquid properties

The model here deals with a rocket in it's simplest form - a cylinder containing fuel, with a massless engine propelling the tank, and the fuel burning with 100% efficiency. As well as that, several further assumptions had to be made. Firstly, all fuel tanks are treated as a liquid fuel, specifically RP-1, which has an average density around 910 $kg/m^3$. When specified that the rocket is using solid rocket fuel, this was presumed to be RDX, which has an average density of 1858 $kg/m^3$. Sloshing, rippling, and any other liquid effects were not considered and ignored - the only 'liquid' feature modelled here is that the surface of the fuel will remain level with gravity - that being that it would be parallel to the horizon. The tank itself is modelled as a cylinder, of height 20m, with a radius of 2m; around half the height and a slightly smaller diameter than the main body of the Ariane V.

## 2. Moments and Integrals of Fuel Tank

In order to begin analysing how the fuel level effects the stability of the rocket, the following information is crucial:

- Centre of Mass,
- Mass of the Fuel and Tank,
- Angle of attack of the Rocket.

In the model used in this paper, the angle of attack is assumed to be constant - stability is assumed to be possible, regardless of whether this is realistically possible or not. The tanks were modeled as a vector with magnitude $\overline{v}$ starting at a co-ordinate $p_0$ and ending at a co-ordinate $p_1$, with a cylinder modelled around this vector of radius $R$. With this information, one can begin to model the tank of fuel:

| | |
|---|---|
| $R$ | radius of the tank |
| $\overline{v}$ | vector of tank |
| $\theta$ | angle of attack of the rocket (with respect to z-axis) |
| $\rho_t$ | density of tank material |
| $\rho_f$ | density of fuel |
| $l_f$ | level of fuel within the tank |
| $V$ | volume of tank |
| $A$ | area of plane |

We can calculate the volume of the tank

$$V = \pi R^2 |\overline{v}| \tag{1}$$

and the mass of the tank itself, with the assumption made that the radius is the radius of the volume within the tank, so any thickness is extraneous; and calculating the fuel mass is also simple, from (1):

$$m_t = \pi |\overline{v}| \left( (R+t)^2 - R^2 \right) \rho_t \tag{2}$$

$$m_f = V \cdot \rho_f \cdot l_f \tag{3}$$

The next step is to calculate the integral for the centre of mass of the fuel within the tank. For the purposes of the simulation, any true liquid properties are ignored such as sloshing and any internal reverberations and "echoing". If the tank is treated as a cylinder, then by nature of the shape, if the tank is tilted in the x direction, then it is symmetrical in the y-z plane, and thus we need only to calculate the centre of mass in y and in z, as in x the CoM will always be at 0. This is how the simulation was modelled. In order to calculate the positions of the centre of mass in y and z, the area of the plane to be integrated:

$$A = \int_{-R}^{R} |\overline{v}| l_f + y \tan\theta \; dy \tag{4}$$

Then, using equation 4, we can calculate the centres of mass in x and in y with the following:

$$CoM_y = \frac{1}{A} \int_{-R}^{R} y \left( y \tan\theta + |\overline{v}| \, l_f \right) dy \tag{5}$$

$$CoM_z = \frac{1}{A} \int_{-R}^{R} \frac{1}{2} \left( y \tan\theta + |\overline{v}| \, l_f \right)^2 dy \, ; \tag{6}$$

And finally, the centre of mass of the tank as a whole can be calculated using the fuel and tank components

$$CoM_{tot} = \frac{CoM_f m_f + CoM_t m_t}{m_f + m_t} \tag{7}$$

The moments of the tank are not calculated here, due to them depending strongly on external factors such as the thrust of the engines, any aerodynamic and/or ballistic effects of the rocket, the exact locations of the stability assist systems, as well as random variables such as wind gusts, air pressure, temperature, etc. Due to this, the model used here is an incredibly basic generalisation that will only somewhat mimic real life. However, a lot of other factors will assist in the stability of the rocket rather than hinder it, and as such the calculations here are akin to a worst case scenario.

## 3. Results of Simulations

With the equations, we can begin calculating centres of mass of the fuel tank, given various tilts. Due to the nature of the model, it falls apart at both extremes - when the tank is near full, and when the tank is near empty. Between the extremes, however, the model works at values between the extremes. As such, the tank will be running from a value of 95% full, to 5% full. With this taken into effect, we can begin calculating the Centre of Mass of a cylindrical tank, using equations (4), (5), (6), and (7).

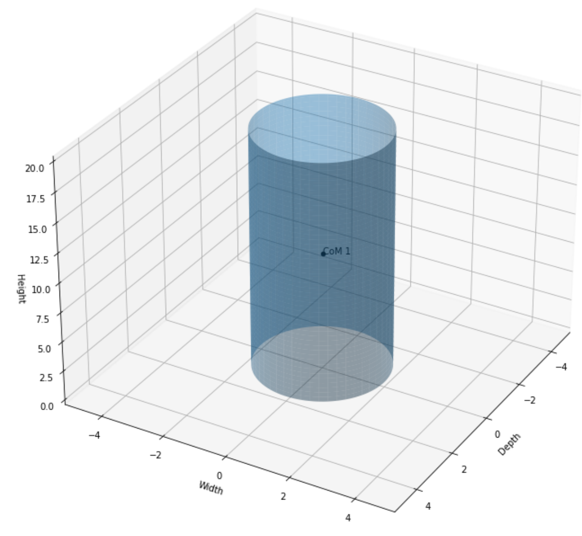


**Fig. 2.** The modelled cylinder, treated as a full tank of fuel, with the centre of mass of the entire tank and fuel labeled CoM

From figure 2, we can see that with a full tank, and an Angle of Attack (AoA) of 0 degrees, the centre of mass is in fact located at the exact centre of the cylinder. Plotting with various fuel levels ranging from 95% to 5% full, the following graph is plotted detailing the location of the centre of mass in the Y and Z plane (labeled Width and Height - A note that we are assuming the tank to be symmetrical in the x-plane, labeled as depth).

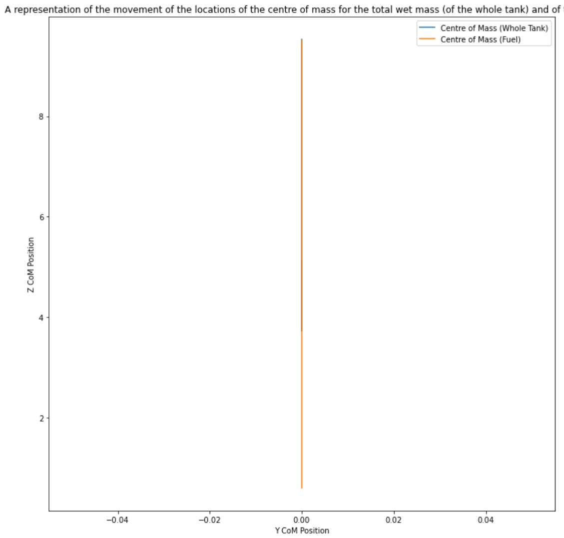


**Fig. 3.** A graph showing the position of the centre of mass of a fuel tank at 0 degrees AoA, showing only a variation in Z and no variation in Y.

Shown in figure 3, there is no deviation in the Y plane with regards to the position of the centre of mass - it only moves up and down from the frame of reference of the tank. This is logical, as the surface of the fuel is rectangular in shape, and therefore symmetrical about the z axis, so only deviates in the z axis. A more interesting question is how any deviation in angle of attack (and the fuel level) will effect the stability of the rocket if at all, and thus if a rotational moment will be induced. Therefore the following must be done:

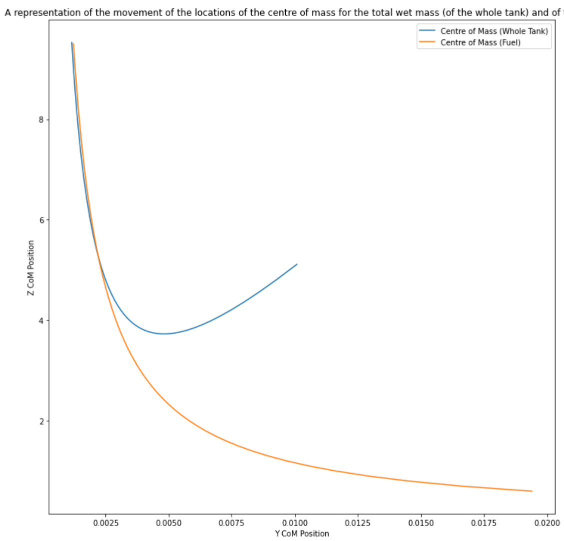


**Fig. 4.** A graph showing the position of the centre of mass of a fuel tank at 1 degrees AoA, showing variation in both the Y and Z direction.

1. Plot the deviation of centre of mass (CoM) with respect to the angle of attack (AoA)
2. Plot this over several Angles of Attack and compare if there is any difference between them

Part 1 of this plan is done by inducing a tilt of 1 degree to the system, and interesting results are obtained.

Given figure 4, a much better picture is developed as to what is occurring with respect to the movement of the centre of mass within the tank. We can see that there is now an induced deviation in the centre of mass in the Y direction - although this is a tiny deviation (in the order of a couple of centimetres), it is still significant enough to cause a rotational moment - if no steps are taken to correct this deviation, the rotational moment *will* cause the rocket to flip.

A plot can now be made from various AoAs, and this was done ranging from 0 to 10 degrees in the case of this simulation. The results, shown in figure 5, paint a clear picture.

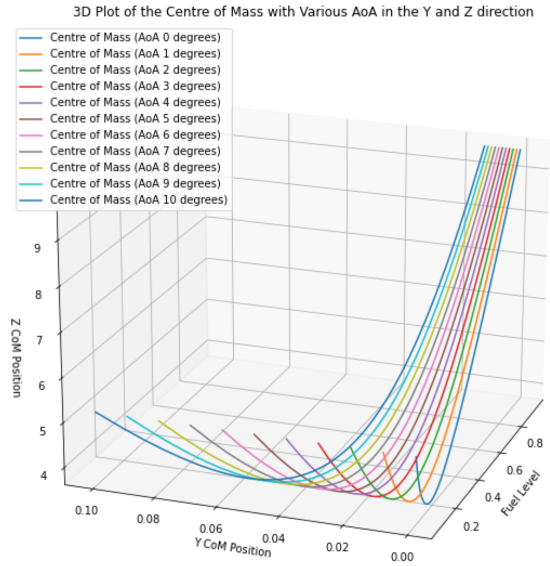


**Fig. 5.** A 3d Plot showing the deviation of AoA and how this effects the CoM location in the Y and Z plane. A clear trend is seen, suggesting that a larger angle of attack produces a greater deviation in the Y direction in terms of the CoM. Different colours denote different angles of attack.

Figure 5 shows that the deviation in angle of attack does in fact cause a greater shift in deviation of the centre of mass. Figure 5 shows a very clear trend of an increasing deviation in terms of the CoM in y with an increasing AoA. Interestingly enough, it seems to show no deviation in the shift in the Z co-ordinate between the angles, which can be confirmed in Figure 6 by changing the viewpoint, to be side on in the Y axis.

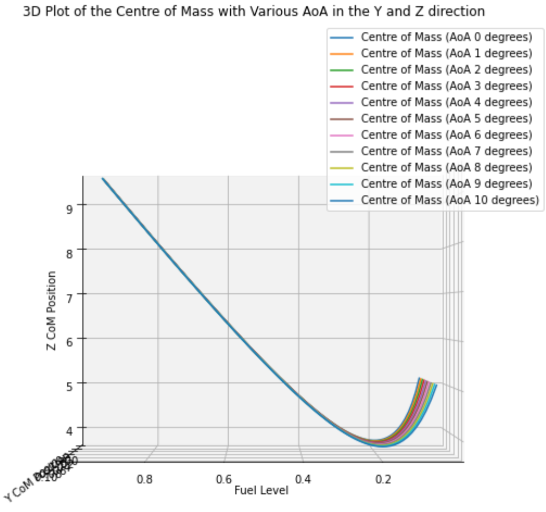


**Fig. 6.** As in Fig. 5, but with the viewpoint changed to observe the difference (or, lack thereof) between the deviations in the Z centre of mass of various angle of attacks; any apparent differences are merely due to parallex from viewing a 3d plot side-on. Different colours denote different angles of attack.

Knowing that the only deviations that are apparent are due to the parallax in Z, and the only true deviations are in the Y axis, it would be more constructive to plot a 2 dimensional graph of the fuel level against deviation in y, which is done in figure 7.

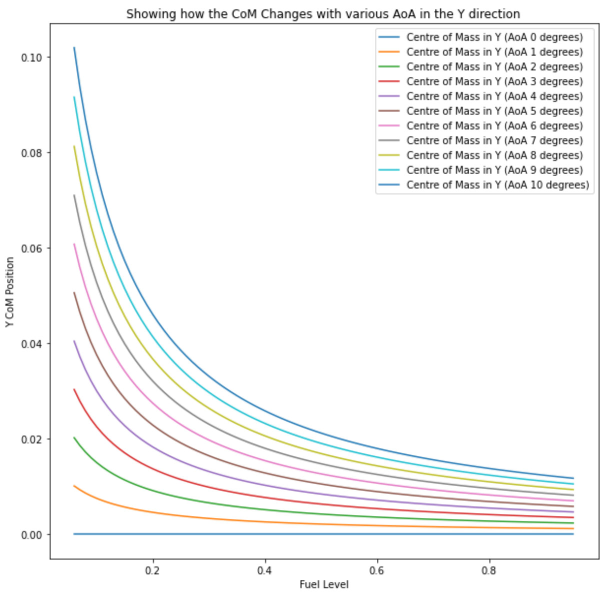


**Fig. 7.** Plotting the deviations in the y axis of various angles of attack, with regards to the fuel level. Different colours denote different angles of attack.

Figure 7 shows that there is actually something of a direct scalable relationship between the AoA and the deviation in the CoM - that is, a deviation of $2x$ degrees will be twice as effective at shifting the CoM in the Y direction as a deviation of $x$ degrees.

Therefore, assuming the rocket is stable when still, or when in a completely upright position, when a rocket is firing upwards, this off centred centre of mass, much like in figure 1, will cause a rotational moment to be induced, causing the rocket to flip. Therefore, in the presence of no ballistic effects to stabilise the craft, a rocket engine that can change the direction of thrust, to correct any deviations, or stability assist systems above the centre of mass to counteract any rotational moments induced by the rocket propelling the craft, are needed to prevent this from happening.

## 4. Conclusions

From the models and graphs given, we can gain useful information about a spacecraft.

1. An unpressurised fuel tank, with no mechanisms to prevent fuel from flowing freely, will experience a shift in the centre of mass due to a tilted level of fuel. Due to fuel taking up such a large portion of the mass of a rocket, this shift in the centre of mass is noticeable and will potentially have an effect.
2. This shift in the centre of mass is dependant entirely on the angle that the fuel is levelling to - a larger 'tilt' in fuel level will induce a greater shift in the centre of mass.
3. An off-central axis centre of mass will cause a rotational moment to be experienced by the rocket as a whole due to the upthrust of the (centrally mounted) rocket, assuming the rocket does not tilt.
4. This induced moment will, if left unchecked, cause the rocket to spin, much like an inverted pendulum, although as the rocket is constantly propelling the craft, this will mean a spin is induced instead, much akin to a flat spin stall in a standard aircraft.

It is therefore safe and logical to assume that the presence of any stability assist systems, a pressurised container preventing fuel flowing freely, a rocket engine able to be pivoted, or some combination of any of the above, are a necessity on all spacecraft, and that due to the real world not being a perfect system, any slight deviation off of pure alignment of the z-axis, the centre of thrust, and the centre of mass would cause total failure.

*Acknowledgements.* Part of this work was supported by many contributors on Stack Overflow, as well as numerous GitHub repositories and sites containing useful information on the Centre of Mass, inertia, and moments about a point.

## 5. Appendix: Notes on techniques and other Works

This paper focuses on the Centres of Mass of a body, and the applications therein with regards to a rocket maintaining stable flight. Centres of Mass of a volume typically involve a triple integral over a small volume element, dV, and are typically computationally incredibly intensive, especially with regards to an ever changing volume. In terms of this program, the technique used is the np.quad function within the numpy library, which using a mixture of techniques, and compares the results to see if a required set accuracy is achieved (or in this case, the default accuracy). I am led to believe it uses a mixture of a 7-point Gauss Rule, and 15-point Kronrod rule, within the range specified (from a to b) and compares the results. If they are not to a satisfactory level of accuracy, then the interval is bisected and the process is repeated, until it's accurate enough.

Integration is used incredibly widely through academia and industry. Within academia, I read several interesting papers including integrals, including Espinosa (1999), where integrals are used in topological analysis of hydrogen bonds, specifically to analyse the effects of hydrogen bonding within various organic compounds to propose a new, more accurate model for electron density at critical points. Another use is in the Kirkwood (1935) paper exploring chemical potentials in gases and in liquids, where molecular pair distribution functions are used to calculate properties of said fluids. Additionally, they are used in the Helm (1973) paper to evaluate synchrotron integrals, to calculate the motion of electrons within a synchrotron for use in experimental and theoretical physics.

Outside of science, there are many uses. In this paper Bruno (2022) they are used to calculate the kinematics of moving limbs, to better learn to assist parkinson's patients in recovery or in treatments. In this paper M. Wilmer (2021) integrals are used to best calculate how to limit emissions such as carbon particulates from a conventional diesel engine through analysing the number of particle emissions. And finally, integrals have been used in to study and model airflow within a room from an air conditioner to optimise the design and efficiency.

## References

Wikipedia, https://en.wikipedia.org/wiki/Ship_stability

E. Espinosa, M. Souhassou, H. Lachekar and C. Lecomte in Acta Crystallographica Section B

J. Kirkwood, The Journal of Chemical Physics, Volume 3, Issue 5, p.300-313

RH Helm, MJ Lee, PL Morton, M Sands, IEEE Trans. Nucl. Sci, 1973

A Study on Lower Limb Asymmetries in Parkinson's Disease during Gait Assessed through Kinematic-Derived Parameters, Bruno Leban, Bioengineering, 2022

Solid particulate mass and number from ducted fuel injection in an optically accessible diesel engine in skip-fired operation, Brady M Wilmer, Christopher W Nilsen, Drummond E Biles, Charles J Mueller, William F Northrop, , 2021

Comparative Study of Positioning of Air Conditioner in a Room Using CFD, Madhu Kalyan Reddy Pulagam, C.V Raman Journal, 2020