MATH 128 Quiz # 6 (Practice Version)

Bubble your answer(s) for each question 1 - 6 on the last page of the exam.

[1] **1.** Select all the improper integrals.

(a) $\displaystyle\int_1^\infty \frac{1}{x^2+1}\,dx.$

(b) $\displaystyle\int_0^{\pi/2} \tan(x)\,dx.$

(c) $\displaystyle\int_0^5 \frac{1}{(x-1)^2}\,dx.$

(d) $\displaystyle\int_1^3 \frac{e^x}{x}\,dx.$

[1] **2.** Select all the improper integrals.

(a) $\displaystyle\int_0^\infty \frac{2x}{\sqrt{x^2+1}}\,dx.$

(b) $\displaystyle\int_0^1 \frac{2x}{\sqrt{x^2+1}}\,dx.$

(c) $\displaystyle\int_1^2 \sqrt{x^2-1}\,dx.$

(d) $\displaystyle\int_0^1 xe^{\frac{1}{x}}\,dx.$

(e) $\displaystyle\int_2^4 \ln(|x^2-4|)\,dx.$

[1] **3.** Is the following calculation correct?

$$\begin{aligned}
\int_{-\infty}^{\infty} x^3\,dx &= \lim_{t\to\infty}\int_{-t}^{t} x^3\,dx && (1)\\
&= \lim_{t\to\infty} \frac{1}{4}x^4\Big|_{-t}^{t} && (2)\\
&= \lim_{t\to\infty}\left(\frac{1}{4}t^4 - \frac{1}{4}(-t)^4\right)\\
&= \lim_{t\to\infty}\left(\frac{1}{4}t^4 - \frac{1}{4}t^4\right) && (3)\\
&= \lim_{t\to\infty} 0\\
&= 0
\end{aligned}$$

(a) Yes, it is correct.

(b) No, the first line with a mistake is equation (1).

(c) No, the first line with a mistake is equation (2).

(d) No, the first line with a mistake is equation (3).

Bubble your answer(s) for each question 1 - 6 on the last page of the exam.

[1] **4.** Consider the following calculation.

$$\begin{aligned}
\lim_{t\to 3^-}\int_0^t \frac{1}{x^2-9}\,dx &= \lim_{t\to 3^-}\int_0^t \left(\frac{-\frac{1}{6}}{x+3}+\frac{\frac{1}{6}}{x-3}\right)\,dx \\
&= \lim_{t\to 3^-}\left[-\frac{1}{6}\ln(|x+3|)+\frac{1}{6}\ln(|x-3|)\right]_0^t && (1)\\
&= \lim_{t\to 3^-}\frac{1}{6}\left[\ln\left(\left|\frac{x-3}{x+3}\right|\right)\right]_0^t && (2)\\
&= \frac{1}{6}\lim_{t\to 3^-}\left(\ln\left(\left|\frac{t-3}{t+3}\right|\right)-\ln|(-1)|\right) && (3)\\
&= -\infty
\end{aligned}$$

(a) Yes, it is correct.

(b) No, the first line with a mistake is equation (1).

(c) No, the first line with a mistake is equation (2).

(d) No, the first line with a mistake is equation (3).

[1] **5.** What is the Taylor series centered at $a = 0$ for xe^{x^2}?

(a) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n!}x^{2n}$

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n}$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n!}x^{2n+1}$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n+1}$

[1] **6.** Is the following calculation correct?

$$\begin{aligned}
\int_{-\infty}^{\infty}\frac{2x}{1+x^2}\,dx &= \lim_{t\to-\infty}\int_t^0\frac{2x}{1+x^2}\,dx+\lim_{s\to\infty}\int_0^s\frac{2x}{1+x^2}\,dx \\
&= \lim_{t\to-\infty}\ln(1+x^2)\Big|_t^0+\lim_{s\to\infty}\ln(1+x^2)\Big|_0^s && (1)\\
&= \lim_{t\to-\infty}\big(\ln(1)-\ln(1+t^2)\big)+\lim_{s\to\infty}\big(\ln(1+s^2)-\ln(1)\big) && (2)\\
&= -\infty+\infty\\
&= 0 && (3)
\end{aligned}$$

(a) Yes, it is correct.

(b) No, the first line with a mistake is equation (1).

(c) No, the first line with a mistake is equation (2).

(d) No, the first line with a mistake is equation (3).

[3] **7.** Determine whether $\displaystyle\int_{\pi/4}^{\pi/2} \frac{\sec^2(x)}{\tan^3(x)}\,dx$ converges or diverges.

[3] **8.** Determine whether $\displaystyle\int_0^\infty \frac{\ln(x)}{x^2}\,dx$ converges or diverges.

9. Determine whether the series converges or diverges.

[1] (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5^{n+1}}$

[2] (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n3^n}{n!}$.

[4] **10.** Determine whether $\displaystyle\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$ converges or diverges.

[4] **11.** Determine whether $\displaystyle\int_0^{\infty} \frac{1}{(x-2)(x+2)}\,dx$ converges or diverges.