Kapil Khanal,

Neelima Pandey, N'dri Diby



DEPARTMENT OF MATHEMATICS AND STATISTICS

Winona State University

#### Introduction

We analyze how the spatial distribution, dispersal rates, and carrying capacities of caves affect equilibrium populations of Mexican free-tailed bat (Tadarida brasiliensis mexicana; Tbm) demographics throughout their year long life cycle. We derive equations which track the demographics of the Tbm during dispersal stages in and around the caves.

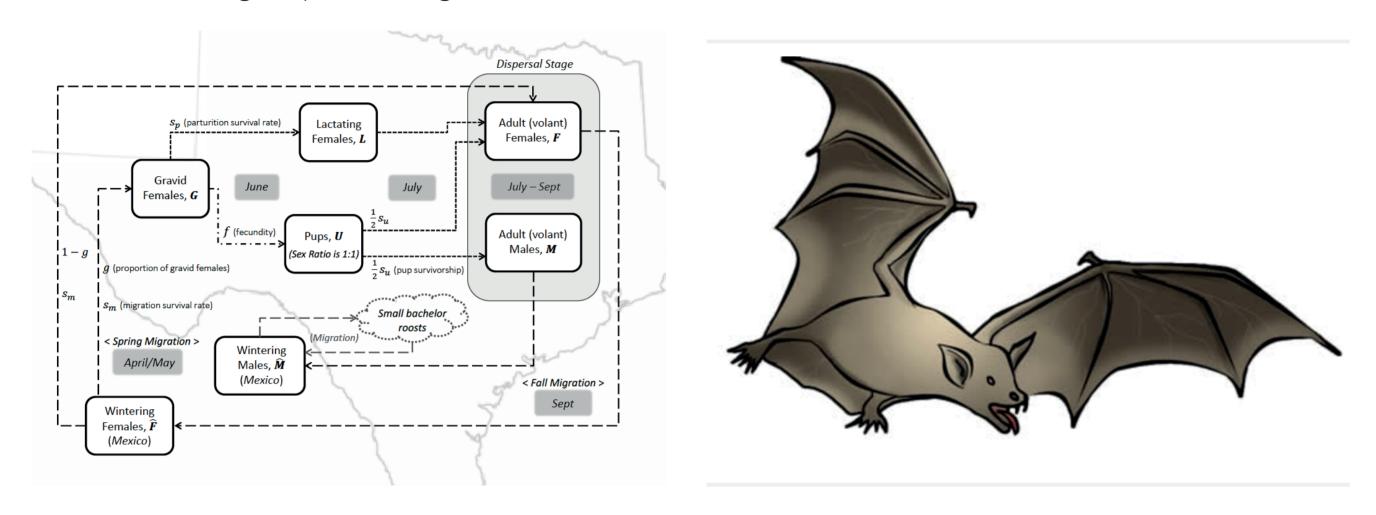


Figure 1: Mexican free tailed Tbm lifecycle.

# Integrodifference Equation Model

We construct a model of a metapopulation in a point-patch habitat (caves in Texas) in which bats use olfactory gradients to find new patches (caves) upon dispersing. Population demographics are modeled by a logistic growth model for bats in maternity caves i = 1, 2, ..., N on day n throughout the dispersal stage where population growth rates  $r_i$  and carrying capacity  $C_i$  are climate dependent.

$$P_i(n+1) = P_i(n) + r_i P_i(n) \left(1 - \frac{P_i(n)}{C_i}\right)$$
 (1)

 We assume that the birth rate and carrying capacity of a cave (and immediately surrounding area) increases with local annual mean temperature as well as with annual rainfall.

At dusk, bats forage and disperse (on a 1-dimensional landscape) according to a probability distribution  $K_i(x)$ , called the **dispersal kernel**, with parameters that depend on climate conditions near each cave. We use Laplace distribution as dispersal Kernel.

$$K_i(x) = \frac{1}{2d_i} e^{\frac{-|x|}{d_i}} \tag{2}$$

where  $d_i$  is the climate-dependent mean dispersal distance. To obtain the total density of thm that dispersed from all caves, denoted by B(x), we add the densities from all caves,

$$B(x) = P_1(n)K_1(x - x_1) + P_2(n)K_2(x - x_2).... + P_N(n)K_N(x - x_N) :$$
(3)

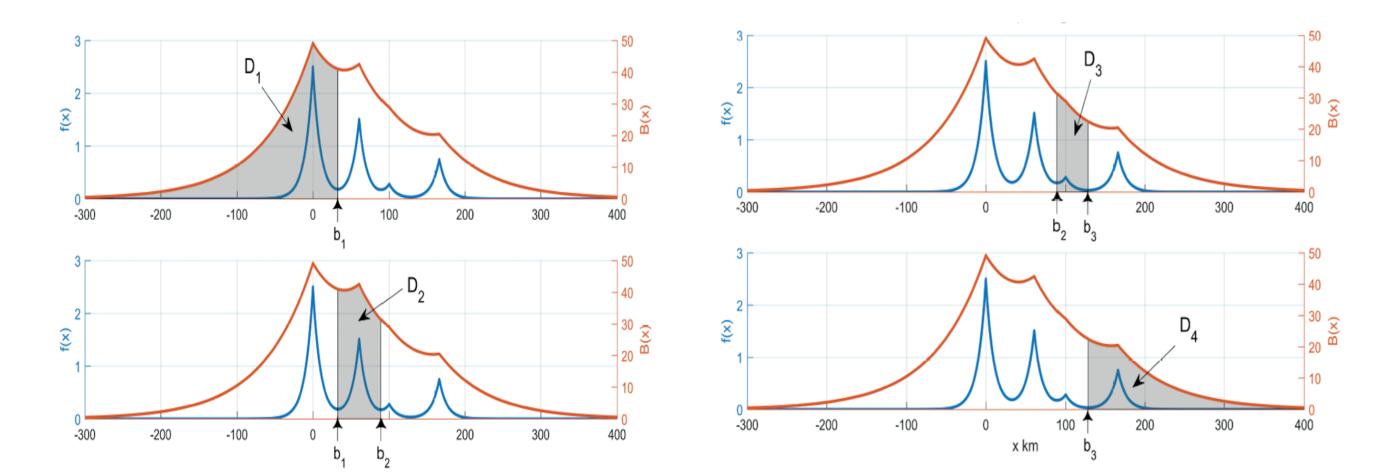


Figure 2: Minimums of f are denoted by  $b_i$ , each of which forms the boundary between the region in which olfactory cue concentration gradients(order cue function) lead the to cave i and the region in which they lead the to cave i+1. The total number of tbm attracted to cave i on day n, denoted by  $D_i(n)$ , is the area under the bat density curve B(x)

# Extension to 2-Dimensional Landscape

Consider a set of N caves located at points  $(x_1, y_1), ...(x_N, y_N)$  on the xy-plane. Using a bivariate Laplace distribution as dispersal kernel,

$$K_i(x,y) = \frac{1}{2\pi d_i^2} e^{-\frac{\sqrt{x^2 + y^2}}{d_i}}$$

we modify (3) as

$$B(x,y) = P_1(n)K_1(x - x_1, y - y_1)....P_N(n)K_N(x - x_n, y - y_n)$$

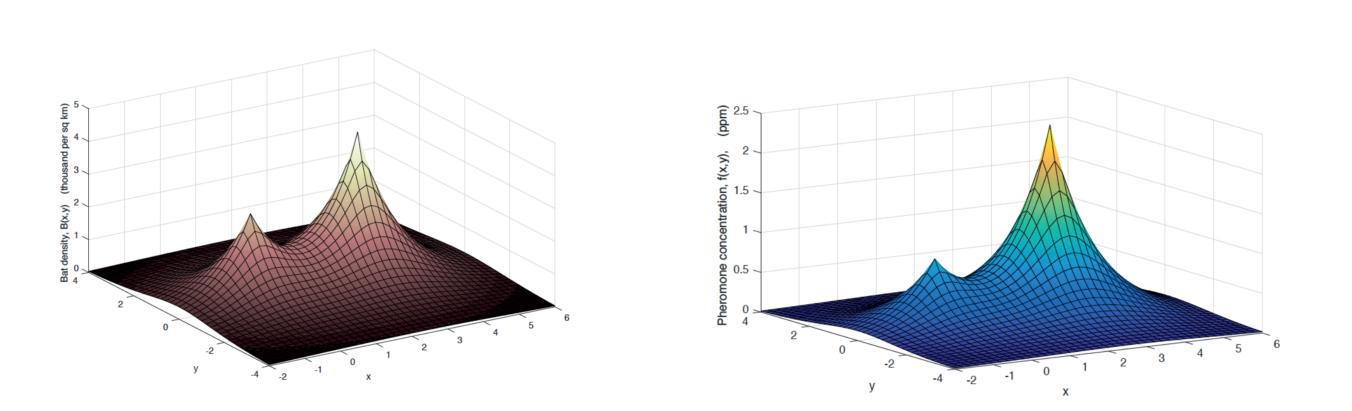


Figure 3: B(x,y) across 2-dimension, Odor concentration function f(x,y) across 2-dimension

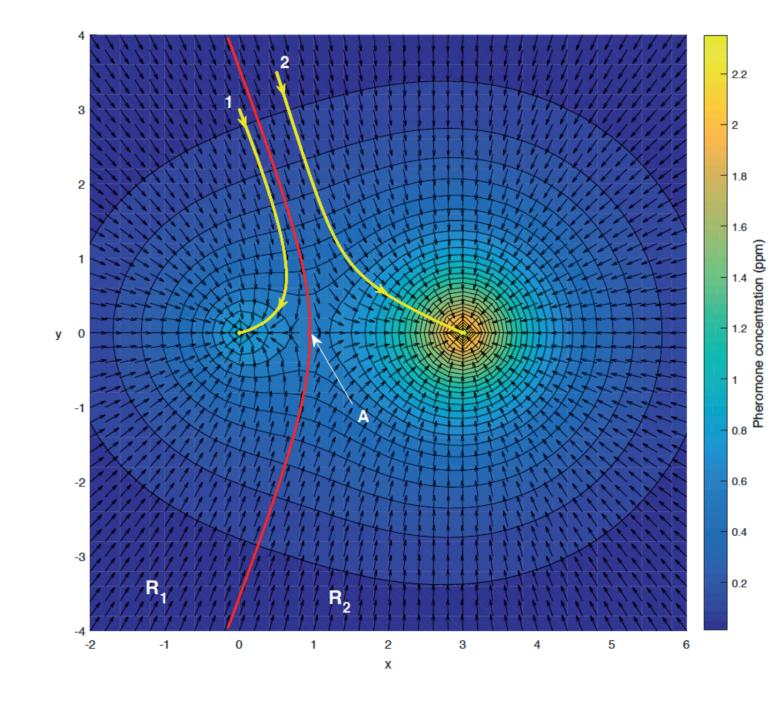


Figure 4: Gradient vector field of f(x,y), for caves with two example bat trajectories (yellow curves) in cave finding after foraging. The boundary separating the region,  $R_1$ , in which thm return to cave 1 from the region,  $R_2$ , in which tbm return to cave 2 is called a separatrix (red curve)

With an arbitrary number of caves (N), the gradient field will partition the plane into N regions,  $R_1, R_2, ..., R_N$ , in which thm return to caves 1, 2, ..., N respectively. The total number of thm attracted to cave i after foraging on day n is determined by the total density of thm in region  $R_i$ ,

$$D_i(n) = \int \int_{R_i} B(x, y) dA \tag{4}$$

### Model Parameterization, Simulation, and Prediction

#### **4-Cave Model Simulation**

We plan to parameterize the model with data gathered by remote sensors that track daily populations emerging from caves. This will be done in collaboration with bat researcher Laura Kloepper. We also investigate the effect of climate conditions on equilibrium populations. We provide a brief expository analysis of the 4-Cave model in 1-dimension here.

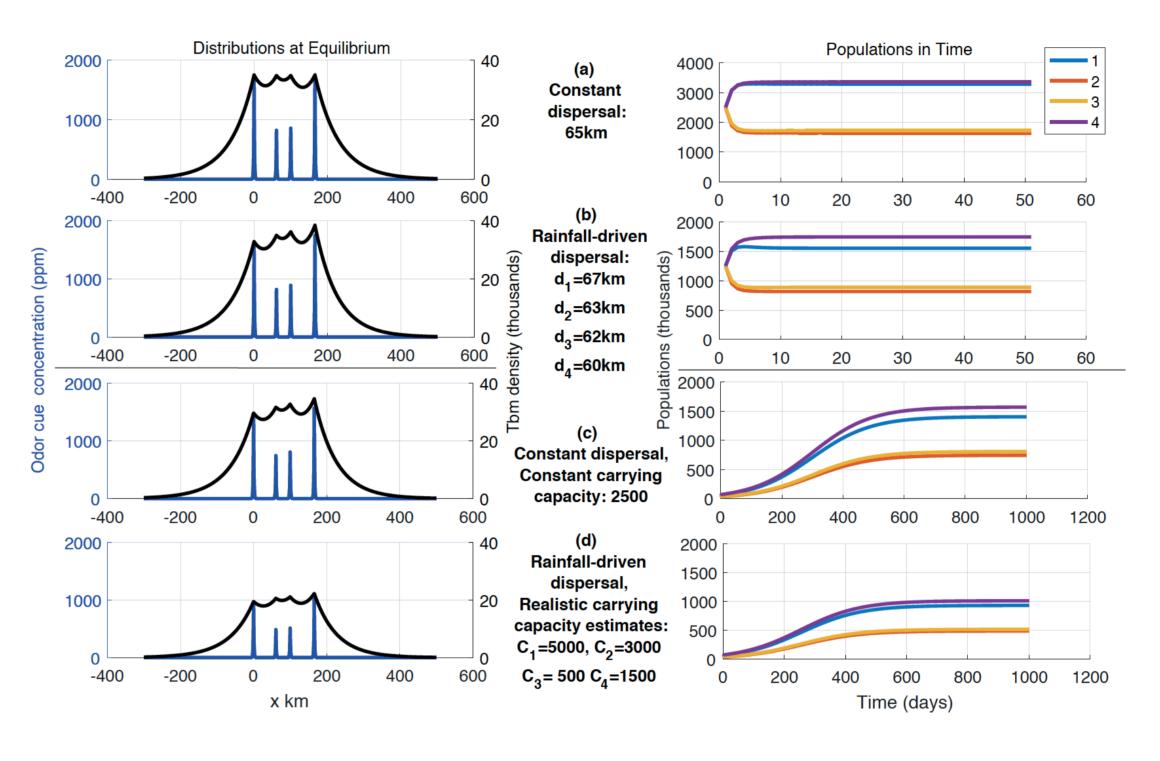


Figure 5: Simulation of the model with constant dispersal rate (65 km) and carrying capacity (2.5 million) for closely situated caves (a), and caves significantly far apart (b). Dispersal is more of a factor than carrying capacity in determining equilibrium populations when caves are close together. If caves are far apart, the system tends more toward the carrying capacities than the intrinsic dispersal-driven equilibrium populations.

# **Equilibrium Analysis**

Simulations give good qualitative insight into the spatiotemporal dynamics of bat populations in point-patch habitats. However, our long-term goal is to derive predictions of equilibrium populations in terms of spatial distribution of caves, as well as climate-dependant dispersal rates, birth rates, and carrying capacities. We can then use such models to analyze the effect of climate change on long term (equilibrium) populations.

#### Acknowledgements

We would like to thank Jacob Duncan(Winona State University, Mathematics and Statistics) and Laura Kloepper (Saint Mary's College - Notre Dame Biology) for advising us in this research project. Bat clipart from http://clipartandscrap.com/bat-clipart<sub>1</sub>9333/