(PHT-101) Tutorial Problem Set-7

March 27, 2025

1. Why Do we need wavepackets for a fitting description of material wave? Can a single wave represent a particle?
2. Prove that wave nature of particles are mutually exclusive to particulate nature.
3. Show that the Heisenberg's uncertainty principle can be motivated (argued for) by the use of group velocity in quantum mechanics!
4. What do you mean by orthogonality of wavefunctions?
5. Show that the Shroedinger wavefunctions is continuous even if the potential is discontinuous (finite numbers of finite discontinuity)
6. Write down the expression for Probability Density & Probability Current Density in quantum Mechanics. Show that these two densiticies obey the continuity equation for steady state cases, e.g. single partcile in an 1 D box.
7. Let $\hat{A}$ be any dynamical variable of a system in which $\hat{H}$ being the Hamiltonian, Derive Heisenberg's equation of motion

$$\frac{\mathrm{d}\hat{\mathrm{A}}}{\mathrm{dt}} = \frac{\partial\hat{\mathrm{A}}}{\partial\mathrm{t}} + \frac{\mathrm{i}}{\hbar}\left[\hat{\mathrm{H}},\hat{\mathrm{A}}\right]$$

8. Express quantuam mechanical expection value of 1D momentum in terms of 1D probability current density.
9. Show that quantum mechanical operators obey laws of Newtonian mechanics. Specifically

$$\langle\hat{p}_x\rangle = m\frac{\mathrm{d}}{\mathrm{dt}}\langle\mathrm{x}\rangle$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\langle\hat{p}_x\rangle = \left\langle -\frac{dV}{dx}\right\rangle$$

10. Given a list of operators & their corresponding eigenfunctions, find out their respective eigenvalues.

$$\hat{\mathrm{A}} \equiv -\frac{\mathrm{d}^2}{\mathrm{dx}^2}; \quad \psi_A = \sin 2x$$

$$\hat{\mathrm{A}} \equiv -\frac{\hbar^2}{2\mathrm{m}}\frac{\mathrm{d}^2}{\mathrm{dx}^2} + \frac{1}{2}\mathrm{kx}^2; \quad \psi_A = \exp\left(-\frac{\alpha^2 x^2}{2}\right)$$