

# Malaviya National Institute of Technology, Jaipur

## Department of Mathematics

### MAT 101 Tutorial 1- Rank of a matrix, Solution of linear simultaneous equations.

1. Find the rank of the matrix  $A = \begin{bmatrix} 2 & 1 & -2 \\ -1 & -1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$ .

2. Reduce the following matrices to row echelon form and hence find their ranks.

(i)  $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$  (ii)  $\begin{bmatrix} 2 & 0 & -1 & 0 \\ 4 & 1 & 0 & 5 \\ 0 & 1 & 3 & 6 \\ 6 & 1 & -2 & 6 \end{bmatrix}$ .

3. Reduce the following matrices to column echelon form and hence find their ranks.

(i)  $\begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$  (ii)  $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$ .

4. For what value of  $k$  the matrix  $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & k \end{bmatrix}$  has rank 3.

5. Check if the following system of equations is consistent or inconsistent:

$$x + y + z = 1, \quad x + 2y + 4z = 3, \quad x + 4y + 10z = 9.$$

6. Test for consistency and find the solution to the equation

$$x + y + z = 6, \quad x - y + 2z = 5, \quad 3x + y + z = 8, \quad 2x - 2y + 3z = 7.$$

7. For what values of  $\lambda$  the equations

$$x + y + z = 1, \quad 2x + y + 4z = \lambda, \quad 4x + y + 10z = \lambda^2$$

have a solution and solve them completely in each case.

8. Investigate the values of  $\lambda$  and  $\mu$  so that the equations

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu$$

have (i) no solution, (ii) a unique solution, and (iii) an infinite number of solutions.

9. Examine whether the following equations are consistent and solve them if they are consistent:

$$2x + 6y + 11 = 0, \quad 6x + 20y - 6z + 3 = 0, \quad 6y - 18z + 1 = 0.$$

10. Determine the value of  $\lambda$  for which the following set of equations may possess a nontrivial solution:

$$2x + y + 2z = 0, \quad x + y + 3z = 0, \quad 4x + 3y + \lambda z = 0.$$

Also, find the solution.

11. Determine the conditions for which the following system

$$\begin{aligned} x + y + z &= 1, \\ x + 2y - z &= b, \\ 5x + 7y + az &= b^2 \end{aligned}$$

admits (i) unique solution, (ii) no solution, (iii) infinite solutions.