① ⓐ The electric field is in y-direction and wave is propagating in z-direction, hence magnetic field will be in x-direction

From maxwell curl equation,

$$\nabla \times H = \frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t} \qquad (\because J=0)$$

$$= -\hat{j}\,\varepsilon\, 10 \sin(10^8 t + 20z) . 10^8$$

or $-\hat{j}\left(\cancel{\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}}\right)$

$$-\hat{j}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) = -\hat{j}\,\varepsilon\, 10^9 \sin(10^8 t + 20z)$$

since $H_z = 0$

$$\boxed{\frac{\partial H_x}{\partial z} = 10^9 \varepsilon \sin(10^8 t + 20z)}$$

Integrating $H_x = \dfrac{-10^9 \varepsilon \cos(10^8 t + 20z)}{20} = \dfrac{-10^8}{2} \varepsilon \cos(10^8 t + 20z)$ Amp/m

$$\boxed{\text{or } H = -\hat{i}\,\frac{10^8}{2} \varepsilon \cos(10^8 + 20z) \text{ Amp/m}}$$

(b) phase velocity $v = \dfrac{\omega}{\beta} = \dfrac{10^8}{20} = 0.5 \times 10^7$ m/s

(c) $v = \dfrac{1}{\sqrt{\mu\varepsilon}} \Rightarrow \varepsilon = \dfrac{1}{v^2 \mu} = \left(\dfrac{1}{0.5 \times 10^7}\right)^2 \times \dfrac{1}{4\pi \times 10^{-7}}$

$$= \frac{4 \times 10^{-14}}{4\pi \times 10^{-7}}$$

$\therefore$ dielectric constant $\Rightarrow \varepsilon_r = \dfrac{\varepsilon}{\varepsilon_0} = \dfrac{4 \times 10^{-14} \times 36\pi \times 10^9}{4\pi \times 10^{-7}}$

$$\boxed{\varepsilon_r = 3600}$$

(2) Given $H_z = 0.4\omega\varepsilon_0 \cos(\omega t - 50x)$

From Maxwell's curl equation

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

in free space, $\vec{J} = 0$

$$\therefore \quad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_x \end{vmatrix} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\hat{i}\left[\frac{\partial H_z}{\partial y} - 0\right] + \hat{j}\left[0 - \frac{\partial H_z}{\partial x}\right] + \hat{k}\left[0-0\right] = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$\therefore$ But $\frac{\partial H_z}{\partial y} = 0$ since $\vec{H}$ is not function of y.

$$\therefore \quad -\frac{\partial H_z}{\partial x}\hat{j} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

or $$\frac{\partial \vec{E}}{\partial t} = -\hat{j}\frac{1}{\varepsilon_0}\frac{\partial}{\partial x}\left[0.4\omega\varepsilon_0 \cos(\omega t - 50x)\right]$$

$$= 0.4\omega \sin(\omega t - 50x) \times (-50)\hat{j}$$

$$\Rightarrow \frac{\partial \vec{E}}{\partial t} = -20\omega \sin(\omega t - 50x)\hat{j}$$

$$\Rightarrow E = 20\cos(\omega t - 50x)\hat{j} \text{ V/m}$$

The displacement current density

$$J_d = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$J_d = \varepsilon_0 \frac{\partial}{\partial t}\left[20\cos(\omega t - 50x)\hat{j}\right]$$

$$J_d = -20\varepsilon_0\omega \sin(\omega t - 50x)\hat{j} \text{ A/m}^2$$

(3) Solar energy recieved by earth per unit area per unit time, i.e. Poyanting vector,

$$\vec{P} = \vec{E} \times \vec{H}$$

$$P = EH = 2 \text{ cal/min/} \cancel{\text{sec}} \text{ cm}^2$$

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\times3.14\times10^{7}}{8.85\times10^{-12}}} = 377\ \Omega$$

$$\therefore\ E = \sqrt{1400\times377} = 7265\ \text{V/m}$$

$$H = \frac{E}{377} = \frac{726.5}{377} = 1.927\ \text{A/m}$$

(4) Given, $E_x = \frac{1}{\sqrt{2}} \cos\left[4\pi\times10^{7}\left(t - \frac{z}{c}\right)\right]$

Comparing with standard electric field equation

$$E_x = E_0 \cos\omega\,(t - z/c)$$

$$\omega = 2\pi\nu = \frac{2\pi c}{\lambda} = 4\pi\times10^{7} \quad (\because c = \nu\lambda)$$

$$\therefore\ \lambda = \frac{2\pi\times3\times10^{8}}{4\pi\times10^{7}} = 15\text{m}.$$

As the electric vector $\vec{E} = E_x\hat{i}$ is vibrating along x-direction, hence it will be plane-polarized along x-direction.

(b) $\vec{B} = \frac{1}{c}\hat{k}\times\vec{E} = \frac{E}{c}(\hat{k}\times\hat{i}) = \frac{E}{c}\hat{j}$

or $\vec{B} = \frac{1}{\sqrt{2}\times3\times10^{8}} \cos\left[4\pi\times10^{7}\,(t - z/c)\right]$ wb/m2

in y-direction.

(c) Energy flux, $= \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{1}{\mu_0}(E_x\hat{i} + B_y\hat{j}) = \frac{E_x B_y}{\mu_0}\hat{k}$

$$= \frac{1}{\sqrt{2}\times\sqrt{2}\times3\times10^{8}\times4\pi\times10^{7}} = 13.32\times10^{-4}\ \text{W/m}^2$$

(5) $\vec{E} = E_0 \cos(kz - \omega t)\hat{i} + E_0 \cos(kz + \omega t)\hat{i}$

$$B = \int (\nabla \times \vec{E}) dt$$

$$= -\int \frac{\partial E_0}{\partial z} (\hat{j})$$

$$= -\int -k E_0 \sin(kz - \omega t)\, dt\, \hat{j} + \int -k E_0 \sin(kx + \omega t)\, dt\, \hat{j}$$

$$= \frac{k}{\omega} E_0 \cos(kz - \omega t)\hat{j} - \frac{k_0}{\omega} E_0 \cos(kz + \omega t)\hat{j}$$

$$\therefore \frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_0}{\partial t^2} \quad \text{or} \quad -k^2 E = -\frac{\omega^2}{c^2} E$$

$$\Rightarrow \frac{\omega}{k} = c$$

$$\therefore \vec{B} = \frac{E_0}{c} \cos(kz - \omega t)\hat{j} - \frac{E_0}{c} \cos(kz + \omega t)\hat{j}$$

$$\therefore \vec{E} = E_0[\cos(kz)\cos\omega t - \sin(kz)\sin(\omega t)\hat{i}$$
$$+ E_0[\cos kz \cos(\omega t) - \sin(kz)\sin(\omega t)\hat{i}$$

$$E_0 = 2E_0 \cos(kz)\cos\omega t)\hat{i}$$

$$\therefore B = \frac{2E_0}{c} \sin(kz)\sin(\omega t)\hat{j}$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{4E_0^2}{c} \sin(kz)\cos(kz)\cos(\omega t)\sin(\omega t)\hat{k}$$

$$\vec{S} = \frac{E_0^2}{c} \sin(2kz)\cos(2\omega t)\hat{k}$$

$$\langle \vec{S} \rangle = 0 \quad \text{as} \quad \frac{1}{T}\int_0^T \cos(2\omega t)\, dt = 0$$

This represents a standing wave so does not transport power

(6) $\vec{E_z}(y,t) = E_0 e^{i(ky-\omega t)}$

$\therefore \quad B_x = \frac{E_0}{c} e^{i(ky-\omega t)}$

$\therefore \quad H_x = \frac{E_0}{c\mu_0} e^{i(ky-\omega t)}$

$\therefore \quad H_0 = \frac{E_0}{c\mu_0} = \frac{5}{3\times10^8 \times 4\pi\times10^{-7}} = \frac{5}{377}$

$\therefore \quad H_0 = 13.26 \times 10^{-3}$ A/m

$S_y = E_z H_x = 5\times 13.3\times10^{-3} \cos^2(ky-\omega t)$

$\langle S_y \rangle = \frac{1}{2}\times 5 \times 13.3\times10^{-3} = 33.16\times10^{-3}$ W/m²

(7) $H_z = \frac{B}{\mu_0} = A\sin(\alpha y)\cos(\omega t)$

$\nabla \times H = A\alpha \cos\alpha y \cos\omega t \, \hat{i}$

as, From Maxwell's equation

$\nabla\times\vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

$E = \frac{A\alpha}{\epsilon_0 \omega} \cos\alpha y \sin\omega t \, \hat{i}$

So for poyanting vector,

$S = \vec{E}\times\vec{H}$

$= \frac{A^2\alpha}{4\epsilon_0\omega} \sin(2\alpha y)\sin 2\omega t \, \hat{j}$

$\langle S \rangle = \frac{1}{T}\int_0^T S\, dT = 0.$

(8) Energy flux is given by

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$S = \frac{EB}{\mu_0} = \frac{E_{rms}^2}{C\mu_0}$$

But $E_{rms} = \frac{E_0}{\sqrt{2}}$

$$\therefore \quad S = \frac{E_0^2}{2\mu c} = \quad E_0 = \sqrt{2\mu_0 c S}$$

$$\Rightarrow E_0 = \sqrt{2 \times 4 \times 3.14 \times 10^{-7} \times 3 \times 10^8 \times 1380}$$

$$\boxed{E_0 = 1.02\ KV/m}$$

and $\quad B_0 = \frac{E_0}{c} = \frac{1.02 \times 10^3}{3 \times 10^8}$

$$\boxed{B_0 = 3.4 \times 10^{-6}\ Wb/m^2}$$

(9) In free space, Maxwell's curl equation

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \hat{j}$$

$$\Rightarrow \quad -\frac{\partial H_z}{\partial x} \hat{j} = \epsilon_0 \frac{\partial E_y}{\partial t} \hat{j}$$

$$\frac{2}{\mu_0} \sin x \cos t = 2\epsilon_0 \sin x \cos t$$

$$\Rightarrow \quad \mu_0 \epsilon_0 = 1$$

Which is not correct, Hence both the fields do not satisfy Maxwell's equations

(10) Given $f = 60\,Hz$

$\sigma = 5.8 \times 10^{8}$ S/m

$a = 1 \times 10^{-3}$ m

$I_c = 1$ A

The electric field $E = E_0 \sin\omega t$

, $\omega = 2\pi \times 60$ Rad/s

$I_c = AJ_c = \sigma E = A\sigma E_0 \sin\omega t$

$$I_d = A\frac{\partial \vec{D}}{\partial t} = A\varepsilon\frac{\partial \vec{E}}{\partial t} = A\varepsilon\omega_0 E_0 \sin\omega t$$

The ratio of displacement current to conduction current is given by

$$\left|\frac{I_d}{I_c}\right| = \frac{\omega\varepsilon}{\sigma} = \frac{2\pi \times 60 \times 8.85 \times 10^{-12}}{5.8 \times 10^{8}} = 5.75 \times 10^{-18}$$

$\therefore$ Displacement current $I_d = 5.75 \times 10^{-18}$ A.