Malaviya National Institute of Technology Jaipur
Department of Mathematics
22MAT101: Mathematics I: Tutorial Sheet 8

Q. 1.	Evaluate $\int\int_S F.\hat{n}\, dS$, where $F = z\hat{\imath} + x\hat{\jmath} + 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ *and* $z = 5$. **Answer: 90**
Q. 2.	If $F = (2x^2 - 3z)\hat{\imath} - 2xy\,\hat{\jmath} - 4x\hat{k}$, evaluate $\int\int\int_V \nabla \times F\, dV$, where V is the region bounded by the coordinate planes and the plane $2x + 2y + z = 4$. **Answer:** $\frac{8}{3}(\hat{\boldsymbol{\jmath}} - \hat{\boldsymbol{k}})$
Q.3.	Evaluate $\int\int_S F.\hat{n}\, dS$ with the help of Gauss theorem for $F = 6z\hat{\imath} + (2x+y)\hat{\jmath} - x\hat{k}$ taken over the region S bounded by the surface of the cylinder $x^2 + z^2 = 9$ included between $x = 0$, $y = 0, z = 0, y = 8$. **Answer:** $\boldsymbol{18\pi}$
Q.4.	Evaluate $\oint_c (xydx + xy^2\, dy)$ by Stoke's theorem, where c is the square in xy-plane with vertices $(1,0), (-1,0), (0,1), (0,-1)$. **Answer:** $\frac{1}{3}$
Q.5.	Verify Stoke's theorem for $F = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ where S is the upper half of the surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.
Q.6.	Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.
Q.7.	Using Green's Theorem, evaluate $\int_c (x^2y\, dx + x^2dy)$, where c is the boundary described counter-clockwise of the triangle with vertices $(0,0), (1,0), (1,1)$. **Answer:** $\frac{5}{12}$
Q.8.	Verify Gauss's divergence theorem for the function $F = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ over the surface S of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
Q.9.	Verify Gauss divergence theorem for the function $\vec{F} = (2x^2y)\hat{\imath} - y^2\hat{\jmath} + (4xz^2)\hat{k}$ taken over the region in the first octant bounded by $z^2 + y^2 = 9, x = 2$.
Q.10.	Verify the Green's Theorem for $\vec{F} = (x^2 + y^2)\,\hat{\imath} + (y + 2x)\hat{\jmath}$ and C is the positively oriented boundary of the region bounded by the curves $y^2 = x$ *and* $x^2 = y$. **Answer:** $\frac{11}{30}$
Q.11.	Use divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}dS$,where $\vec{F} = xy\,\hat{\imath} + \left(y^2 + e^{xz^2}\right)\hat{\jmath} + \sin(xy)\,\hat{k}$ and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes z = 0 , y = 0 and y + z = 2. **Answer:** $\frac{184}{35}$