MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

DEPARTMENT OF MATHEMATICS

Tutorial Sheet III (Answer Key)

Subject: 22MAT102 Mathematics II

Topics: Formation of first and second-order partial differential equations. Solution of first-order partial differential equations: Lagrange`s equation, Charpit's method, Linear partial differential equations with constant coefficients

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Notations used: $p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y},\ D = \frac{\partial}{\partial x},\ D' = \frac{\partial}{\partial y}.$

Q.1. Form a partial differential equation, by eliminating arbitrary constants from the following equations.

(i) $z = a(x+y) + b(x-y) + abt + c,$ **Ans.** $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = 4\frac{\partial z}{\partial t}.$

(ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$ **Ans.** $zxr + xp^2 = zp$ **OR** $zyt + yq^2 = zq.$

Q.2 Form a partial differential equation, by eliminating arbitrary functions from the following equations

(i) $x + y + z = f(x^2 + y^2 + z^2),$ **Ans.** $(y-z)\mathrm{p} + (z-x)\mathrm{q} = x + y$.

(ii) $y = f(x - at) + g(x + at),$ **Ans.** $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}.$

(iii) $z = f(x^2 - y) + g(x^2 + y)$, **Ans.** $p + 4\,x^3 t = rx$.

(iv) $f(x + yz, x^2 + y^2 + z^2) = 0,$ **Ans.** $p(y^2 - z^2) + q(z - xy) = xz - y.$

(v) $f(x^2 + y^2,\ z - xy) = 0,$ **Ans.** $xq - yp = x^2 - y^2.$

Q.3. Solve the following partial differential equation

(i) $\frac{y-z}{yz}p + \frac{z-x}{zy}q = \frac{x-y}{xy},$ **Ans.** $\phi\{x + y + z\ ,\ xyz\} = 0.$

(ii) $(x^3 + 3xy)p + (y^3 + 3\,x^2 y)q = 2(x^2 + y^2)z,$ **Ans.** $\phi\left\{(x-y)^{-2} - (x+y)^{-2},\ \frac{xy}{z^2}\right\} = 0.$

(iii) $pz - qz = z^2 + (x+y)^2,$ **Ans.** $\phi\left(x + y,\ \frac{e^{2x}}{(x^2+y^2+z^2+2xy)}\right) = 0.$

(iv) $(x + 2z)p + (4zx - y)q = 2x^2 + y,$ **Ans.** $\phi(xy - z^2,\ x^2 - y - z) = 0.$

(v) $x^2(y-z)p + y^2(z-x)q = z^2(x-y),$ **Ans.** $f\left(xyz,\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0.$

Q.4. Solve the following partial differential equation

(i) $p^2 + q^2 = 1,$ **Ans.** $z = ax \pm \left\{\sqrt{(1-a^2)}\right\}y + C.$

(ii) $x^2p^2 + y^2q^2 = z^2,$ **Ans.** $\log z = A\log x \pm \left\{\sqrt{(1-A^2)}\right\}\log y + C.$

(iii) $p^3 + q^3 = 27z$, **Ans.** $(1 + a^3)z^2 = 8(ax + y + b)^3.$

(iv) $p^2 + q^2 = x + y,$ **Ans.** $\frac{2}{3}\left\{(x+a)^{3/2} + (y-a)^{3/2}\right\} + b.$

(v) $q = xyp^2,$ **Ans.** $(2z - ay^2 - 2b)^2 = 16ax.$

(vi) $(y - px)(p - 1) = p$, **Ans.** $y = cx + c/(c-1)$.

(vii) $x^2y^2p^2q = z^3$, **Ans.** $z = ax + \sqrt{1-a^2}y + c$.

(viii) $p(x+p) + q(y+q) = z$, **Ans.** $z = ax + by + a^2 + b^2$.

Q.5. Using Charpit's method, find the complete integral of the following equations.

(i) $2xz - px^2 - 2qxy + pq = 0$, **Ans.** $z = ay + b(x^2 - a)$.

(ii) $pxy + pq + qy - yz = 0$, **Ans.** $(z - ax)(y + a)^a = be^y$.

(iii) $z = p^2x + q^2y$, **Ans.** $\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + c$.

(iv) $pqxy - z^2 = 0$, **Ans.** $z^{\sqrt{a}} = bx^a y$.

(v) $p = (qy + z^2)$, **Ans.** $yz = ax + 2\sqrt{ay} + c$.

(vi) $px + qy = pq$, **Ans.** $az = \frac{1}{2}(ax + y)^2 + b$.

(vii) $p(1+q) = qz$, **Ans.** $az = 1 + be^{x+ay}$.

Q.6 Solve the following linear homogeneous partial differential equations:

(i) $(D^2 - 2DD' + D'^2)z = 12xy$, **Ans.** $z = f_1(y+x) + xf_2(y+x) + 2x^3y + x^4$.

(ii) $(D^3 - 3D^2D' + 4D'^3)z = e^{x+2y}$, **Ans.** $z = f_1(y-x) + f_2(y+2x) + xf_3(y+2x) + \frac{1}{27}e^{x+2y}$.

(iii) $(D^2 + D'^2)z = \cos mx \cos ny$, **Ans.** $z = f_1(y+ix) + f_2(y-ix) + \frac{1}{(m^2+n^2)}\cos mx \cos ny$.

(iv) $(D^2 - DD' - 2D'^2)z = (y-1)e^x$, **Ans.** $z = f_1(y-x) + f_2(y+2x) + ye^x$.

(v) $(D^2 - 4D'^2)z = \frac{4x}{y^2} - \frac{y}{x^2}$, **Ans.** $z = f_1(y-2x) + f_2(y+2x) + x\log y + y\log x + 3x$.

(vi) $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$, **Ans.** $z = x\,\phi_1(y-x) + \phi_2(y-x) + x\sin y$.