Malaviya National Institute of Technology Jaipur

Department of Physics

Solutions: Tutorial sheet-3

Solution 1.

Given that $\boldsymbol{B_x} = \mathrm{B_o}(x^2 - y^2)$ and $\boldsymbol{B_z} = 0$

From Maxwell's equation $\nabla.\boldsymbol{B} = 0$

$$[\frac{\partial}{\partial x}B_x + \frac{\partial}{\partial y}B_y + \frac{\partial}{\partial z}B_z] = 0$$

$$[\frac{\partial}{\partial x}B_o(x^2 - y^2) + \frac{\partial}{\partial y}B_y + \frac{\partial}{\partial z}0] = 0$$

$$2B_o x + \frac{\partial}{\partial y}B_y = 0$$

$$\frac{\partial}{\partial y}B_y = -2B_o x$$

$$\boldsymbol{B}_{\boldsymbol{y}} = -\oint 2B_o x\, dy$$

$$\boldsymbol{B_y} = -2B_o xy$$

Solution 2.

Given that

$$\boldsymbol{E} = 100\, sin\,(2000\pi t)\, V/m$$

As we know that displacement current density

$$\boldsymbol{J_d} = \epsilon_o \frac{\partial \boldsymbol{E}}{\partial t}$$

$$J_d = 2000\pi \in_o \times 100\, cos(2000\pi t)\, A/m^2$$

Solution 3.

The magnitude of the emf is:

$$\left|\frac{d\varphi}{dt}\right| = \frac{d}{dt}(6t^2 + 7t) = 12t + 7 = 12(2) + 7 = 31\, mV$$

Solution 4.

$$\rho = \nabla . D = \left[\frac{\partial D}{\partial x}\,\hat{\imath} + \frac{\partial D}{\partial y}\,\hat{\jmath} + \frac{\partial D}{\partial z}\,\hat{k}\right]$$

$$\rho = \frac{\partial}{\partial x}(2y^2 + z) + \frac{\partial}{\partial y}\,(4xy) + \frac{\partial}{\partial z}\,x$$

$$\rho = 4x$$

Volume charge density at (-1,0,3)

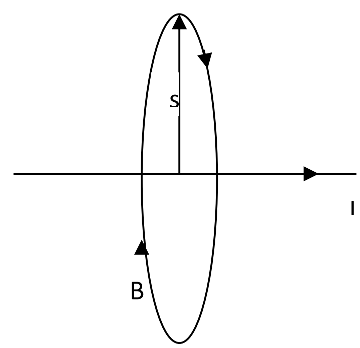
$$\rho = -4\,C/m^3$$

Solution 5.

$$\boldsymbol{E} = -\nabla V = -A\frac{\partial}{\partial r}\left(\frac{e^{-\lambda r}}{r}\right)\hat{r}$$

$$\boldsymbol{E} = Ae^{-\lambda r}(1 + \lambda r)\frac{\hat{r}}{r^2}$$

Solution 6.



From Ampere's law

$\oint \boldsymbol{B}.\boldsymbol{dl} = \mu_o I_{enc}$

$\boldsymbol{B} \oint \boldsymbol{dl} = \mu_o I_{enc}$

$\oint \boldsymbol{dl} = 2\pi s$

$$\boldsymbol{B} = \frac{\mu_o I_{enc}}{2\pi s}\,\hat{\varphi}$$

Solution: 7.

By applying Faraday's law, we get

$$\oint \boldsymbol{E}.\boldsymbol{dl} = E\,(2\pi s) = -\frac{d\varphi}{dt} = -\frac{d(\pi s^2 B(t))}{dt} = -\pi s^2\frac{dB}{dt}$$

$$\boldsymbol{E} = -\frac{s}{2}\frac{dB}{dt}\hat{\varphi}$$

Solution 8.

Given that $\phi = \phi_o(X^2 + Y^2 + Z^2)$

Poisson's equation $\nabla^2\varphi = -\frac{\rho}{\varepsilon_o}$

$$\left[\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}\right] = \frac{\rho}{\varepsilon_o}$$

$$\varphi_o[2 + 2 + 2] = -\frac{\rho}{\varepsilon_o}$$

$$\rho = -6\,\varphi_o\varepsilon_o$$

Solution 9.

$$E = \frac{V}{d} \Rightarrow J_c = \sigma E = \frac{E}{\rho} = \frac{V}{\rho d} \,.\, J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\in E) = \epsilon \frac{\partial}{\partial t}\left[\frac{V_o \cos(2\,\pi\vartheta t)}{d}\right]$$

$$= \frac{\varepsilon V_o}{d}[-2\pi\vartheta \sin(2\pi\vartheta t)]$$

The ratio of the amplitudes is:

$$\frac{J_c}{J_d} = \frac{V_o}{\rho d}\frac{d}{2\pi\vartheta\epsilon V_o} = \frac{1}{2\pi\vartheta\epsilon\rho} = [2\pi(4\times 10^8)(81)(8.85\ \times 10^{-12})(0.23)]^{-1} = 2.41$$

Solution 10.

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & xy^2e^{-t} \end{vmatrix} = \hat{\imath}(2xye^{-t}) + \hat{\jmath}(-y^2\, e^{-t})$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -(\hat{\imath}(2xye^{-t}) + \hat{\jmath}(-y^2\, e^{-t}))$$

$$\boldsymbol{B} = \int -2xy\, e^{-t} dt\, \hat{\imath} + y^2 \int e^{-t}\, dt\ \hat{\jmath}$$

$$\boldsymbol{B} = 2xye^{-t}\,\hat{\imath} - y^2e^{-t}\,\hat{\jmath} + C$$

$$0 = 2xy\,\hat{\imath} - y^2\hat{\jmath} + C \quad \text{[By putting } \mathbf{B} = 0 \text{ at t = 0]}$$

$$\boldsymbol{B} = 2xy\,(-1 + e^{-t})\hat{\imath} - y^2(e^{-t} - 1)\hat{\jmath}$$