COMPUTER PROJECT 2

DIGITAL IMAGE PROCESSING

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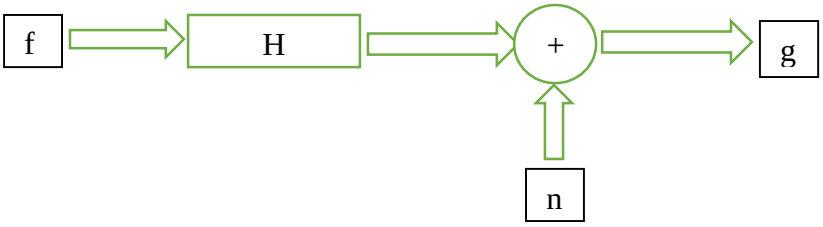
OBJECTIVE:

The objective of the project is to design a MATLAB code and simulate the code along with the SNR produced after performing the following restoration techniques on a 256 x 256 image.

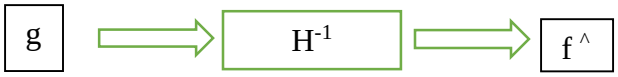
1. Inverse Filter
2. Least Square Estimation
3. Constrained Least Square Estimation
   a. Constrained Least Square Pseudo Inverse
   b. Laplacian Constrained Least Square
   c. Parametric Wiener Filter
4. Minimum Mean Square Estimation Wiener Filter
5. Maximum Entropy Pseudo Inverse

METHODOLOGY:

Degradation:



Restoration:



Terminology:

$F^{\wedge}(K_1,K_2)$ -Restored image pixel at (K_1,K_2) in frequency domain

$H(K_1,K_2)$ –Channel transfer function at (K_1,K_2) in frequency domain

$G(K_1,K_2)$-Degraded image pixel at (K1,K2) in frequency domain

$S_n(K_1,K_2)$-Power spectral density of noise at (K_1,K_2) in frequency domain

$S_f(K_1,K_2)$-Power spectral density of image at (K_1,K_2) in frequency domain

1. **Inverse Filter:**

Inverse filter restores a blurred image perfectly from an output of a noiseless linear system. The simplest approach to restoration is direct inverse filtering.The equation is given as:

$$F^{\wedge}(K_1, K_2) = \frac{1}{H(K_1, K_2)}$$

2. **Least Square Estimation:**

Least mean squares (LMS) algorithms are a class of adaptive filter used to mimic a desired filter by finding the filter coefficients that relate to producing the least mean squares of the error signal (difference between the desired and the actual signal). Consider g=Hf+n and let g˜=HBTf˜ Let us assume H characterizes time invariant system then

$$f^{\wedge}_{LS} = \text{argmin} \|g - g\tilde{}\|^{\wedge}2$$

$$f^{\wedge}_{LS} = \text{argmin} \|g - H_{BT} f\tilde{}\|^{\wedge}2$$

$$f^{\wedge}_{LS} = (H_{BT}^T H_{BT})^{\wedge}-1 H_{BT}^T g$$

$$f^{\wedge}_{LS} = H_{BT}^T g$$

3. **Constrained Least Square Estimation:**

The equation for constrained least square estimation is given by

$$F^{\wedge}(K_1, K_2) = \frac{H^*(K_1, K_2) * G(K_1, K_2)}{|H(K_1, K_2)|^2 + \lambda^{-1}|Q(K_1, K_2)|^2}$$

Now depending on various estimations are possible.

a. **Constrained Least Square Pseudo Inverse:**

For this method Q is considered as Identity and the following equation is obtained

$$F^{\wedge}(K_1, K_2) = \frac{H^*(K_1, K_2) * G(K_1, K_2)}{|H(K_1, K_2)|^2 + \lambda^{-1}}$$

b. **Laplacian Constrained Least square :**

In this method Q is considered such that it can find the rate of rate of change. This can be denoted as a laplacian which is an operator for calculating the rate of rate of change.

$$\text{Q matrix} = \begin{bmatrix} -2 & 1 & 0 & \cdots & 1 \\ : & & \ddots & & \vdots \\ 1 & 0 & & \cdots & -2 \end{bmatrix}.$$

c. Parametric wiener filter:

In this method Q is considered as $R_f^{-1/2}R_n^{1/2}$ where R_f covariance matrix of image f and R_n covariance matrix of noise .The equation for this method is as follows:

$$F^{\wedge}(K_1, K_2) = \frac{H^*(K_1, K_2) * G(K_1, K_2)}{|H(K_1, K_2)|^2 + \lambda^{-1}\frac{S_n(K_1, K_2)}{S_f(K_1, K_2)}}$$

4. Minimum Mean Square Estimation Wiener Filter:

The Wiener filter is a filter used to produce an estimate of a desired or target random process by linear time-invariant filtering an observed noisy process. The Wiener filter minimizes the mean square error between the estimated random process and the desired process. It removes the additive noise and inverts the blurring simultaneously. The Wiener filtering is optimal in terms of the mean square error. The equation is given by:

$$F^{\wedge}(K_1, K_2) = \frac{H^*(K_1, K_2) * G(K_1, K_2)}{|H(K_1, K_2)|^2 + \frac{S_n(K_1, K_2)}{S_f(K_1, K_2)}}$$

5. Maximum Entropy Pseudo Inverse:

In this we consider the image as a probability density function and we maximize the entropy of this function. We obtain the equation as:

$$F^{\wedge}(K_1, K_2) = \frac{H^*(K_1, K_2) * G(K_1, K_2)}{|H(K_1, K_2)|^2 + \lambda^{-1}/2}$$

RESULTS AND CONCLUSION:

In this project, we implemented the restoration of a linear degradation of 256x256 image in MATLAB. A few of the features of a good introductory image processing program have been seen. There are many more complex modifications you can make to the images. For example, you can apply a variety of filters to the image. The filters use mathematical algorithms to modify the image. The SNR'S produced are:

(a) Inverse Filter = 4.8486e+16
(b) Least-Squares Filter = 1.9844e+5
(c) Maximum Entropy Pseudo-Inverse Filter = 144.9312
(d) Norm-Constrained Least-Squares Pseudo-Inverse Filter = 6.48e+4
(e) Laplacian-Constrained Least-Squares Filter = 1.5184e+5
(f) Covariance-Constrained Least-Squares Parametric Wiener Filter = 144.9191
(g) Linear Minimum Mean-Square Error Wiener Filter = 0.9947

1. Inverse Filter

SNR:

2. Least Square Estimation

SNR:

3. Constrained Least Square Estimation

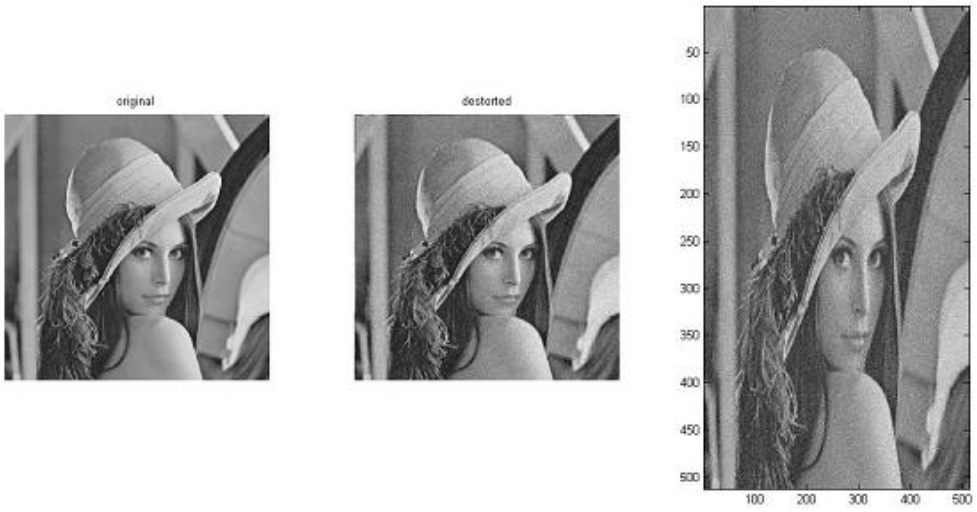
a. Constrained Least Square Pseudo Inverse

SNR:

b. Laplacian Constrained Least Square

SNR:

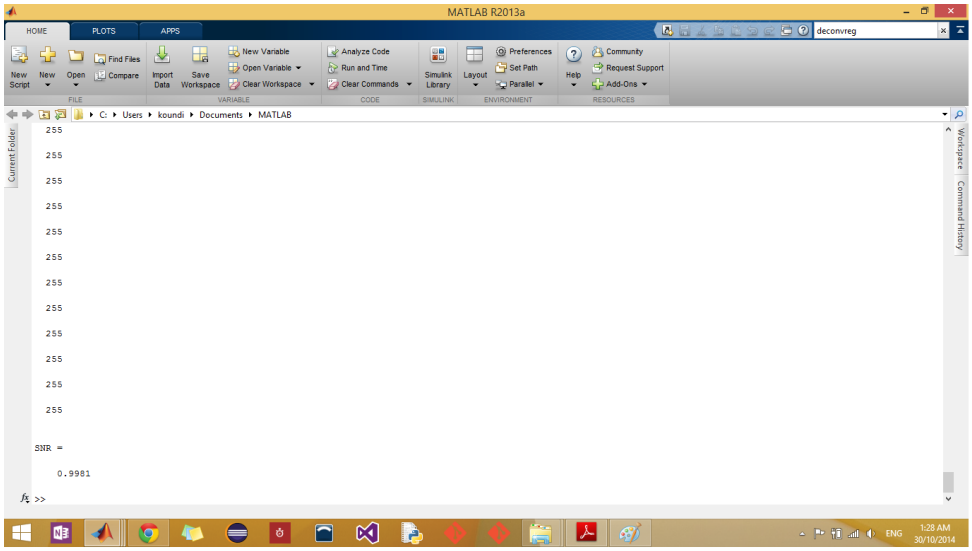
c. Parametric Wiener Filter



SNR:

4. Minimum Mean Square Estimation Wiener Filter

SNR:



5. Maximum Entropy Pseudo Inverse

SNR:

APPENDIX

Inverse Filter

```
clc;
close all;
clear all;

%input
img=imread('C:\Users\koundi\Desktop\lena.jpg'); %input image in spatial
domain
img=rgb2gray(img);
F= fft2(img);   %input image in freq domain
M=size(img,1);  %rows of input image
N=size(img,2);  %columns of the input image


%channel
h=ones(4,4)./8; %channel in spatial domain
H=fft2(h,M,N); %channel in freq domain


%Degradation
G= F.*H ;   %degraded image in spatial domain
g = (ifft2(G)); %degraded image in freq domain
g=mat2gray(g);


%Restoration
Hin=1./H;
Fcap = G.*Hin;
fcap = abs(ifft2(Fcap)); %restored image in freq domain
fcap= mat2gray(fcap);

%SNR calculation
SNR = norm(F,2)/norm(Fcap-G,2)


%Plotting of Results
figure;
subplot(1,3,1)
imshow(img);
title('original');
subplot(1,3,2)
imshow(g);
title('degraded');
subplot(1,3,3);
imshow(fcap);
title('Restored');
```

Least Square Estimation

```
clc;
close all;
clear all;

%input
img=imread('C:\Users\koundi\Desktop\lena.jpg'); %input image in spatial
domain
img=rgb2gray(img);
F= fft2(img);   %input image in freq domain
M=size(img,1);  %rows of input image
N=size(img,2);  %columns of the input image


%channel
h=ones(M,N); %channel in spatial domain
H=fft2(h,M,N); %channel in freq domain

%Noise
n=randn(M,N); %Noise in spatial domain
Ns=fft2(n,M,N); %noise in freq domain

%Degradation
G= F.*H +Ns;   %degraded image in spatial domain
g = abs(ifft2(G)); %degraded image in freq domain
g=mat2gray(g);

%Restoration
Fcap = G./H;
fcap = abs(ifft2(Fcap)); %restored image in freq domain
fcap= mat2gray(fcap);

%SNR calculation
SNR = norm(Fcap,2)/norm(Ns,2)


%Plotting of Results
figure;
subplot(1,3,1)
imshow(img);
title('original');
subplot(1,3,2)
imshow(g);
title('degraded');
subplot(1,3,3);
imshow(fcap),title('Restored');
```

Constrained Least Square Pseudo Inverse

```
clc;
close all;
clear all;

%input
img=imread('C:\Users\koundi\Desktop\lena.jpg'); %input image in spatial
domain
img=rgb2gray(img);
F= fft2(img);   %input image in freq domain
M=size(img,1);  %rows of input image
N=size(img,2);  %columns of the input image


%channel
h = fspecial('gaussian', [3 3], 0.5); %channel in spatial domain
H= fft2(h,M,N); %channel in freq domain

%Noise
noise = input('Enter the magnitude for the noise signal : ');
n = fspecial('gaussian', [3 3], noise); %Noise in spatial domain
%n=noise*rand(M,N);
Ns=fft2(n,M,N); %noise in freq domain

%Degradation
G= F.*H +Ns;   %degraded image in spatial domain
g = abs(ifft2(G)); %degraded image in freq domain
g=mat2gray(g);

%Restoration
lambda=1;
Fcap = zeros(M,N);
for k1=1:M
for k2=1:N
Fcap(k1,k2) = ( conj( H(k1,k2) ) / ( pow2( abs( H(k1,k2) ) )+ lambda^-1 ) )*
G(k1,k2); % restored image in spatial domain
end
end
fcap = abs(ifft2(Fcap)); %restored image in freq domain
fcap= mat2gray(fcap);

%SNR calculation
SNR = norm(Fcap,2)/norm(Ns,2)

%Plotting of Results
figure;
subplot(1,3,1)
imshow(img);
title('original');
subplot(1,3,2)
imshow(g);
title('degraded');
subplot(1,3,3);
imshow(fcap),title('Restored');
```

Laplacian Constrained Least Square

```
clc;
close all;
clear all;

%input
img=imread('C:\Users\koundi\Desktop\lena.jpg'); %input image in spatial
domain
img=rgb2gray(img);
F= fft2(img);   %input image in freq domain
M=size(img,1);  %rows of input image
N=size(img,2);  %columns of the input image


%channel
h = fspecial('gaussian', [3 3], 0.5); %channel in spatial domain
H= fft2(h,M,N); %channel in freq domain

%Noise
noise = input('Enter the magnitude for the noise signal : ');
n = fspecial('gaussian', [3 3], noise); %Noise in spatial domain
Ns=fft2(n,M,N); %noise in freq domain

%Degradation
G= F.*H +Ns;   %degraded image in spatial domain
g = abs(ifft2(G)); %degraded image in freq domain
g=mat2gray(g);

%Restoration
q=fspecial('laplacian');
Q=fft2(q,M,N);

lambda=1;
Fcap = zeros(M,N);
for k1=1:M
for k2=1:N
Fcap(k1,k2) = ( conj( H(k1,k2) ) / ( pow2( abs( H(k1,k2) ) )+ lambda^-1*pow2(
abs( Q(k1,k2) ) ) ) )* G(k1,k2); % restored image in spatial domain
end
end
fcap = abs(ifft2(Fcap)); %restored image in freq domain
fcap= mat2gray(fcap);

%SNR calculation
SNR = norm(Fcap,2)/norm(Ns,2)


%Plotting of Results
figure;
subplot(1,3,1)
imshow(img);
title('original');
```

```
subplot(1,3,2)
imshow(g);
title('degraded');
subplot(1,3,3);
imshow(fcap),title('Restored');
```

Parametric Wiener Filter

```
clc;
close all;
clear all;

%input
img=imread('C:\Users\koundi\Desktop\lena.jpg'); %input image in spatial
domain
noise = input('Enter the magnitude for the noise signal : ');
img=rgb2gray(img);
F= fft2(img); %input image in freq domain
M=size(img,1); %rows of input image
N=size(img,2);  %columns of the input image

%channel
h = fspecial('gaussian', [3 3], 0.5); %channel in spatial domain
H= fft2(h,M,N); %channel in freq domain

%Noise
n=noise*randn(M,N); %Noise in spatial domain
Ns=fft2(n,M,N); %noise in freq domain

%Degradation
G= F.*H +Ns;  %degraded image in spatial domain
g = abs(ifft2(G)); %degraded image in freq domain


%Restoration
q=fspecial('laplacian');
Q=fft2(q,M,N);
lamda=1;
Fcls=zeros(M,N);
for k1=1:M
for k2=1:N
Fcls(k1,k2)=(conj(H(k1,k2))*G(k1,k2))/(abs(H(k1,k2)))^2+lamda^-
1*(abs(Q(k1,k2))^2); %restored image in freq domain
end;
end;
f=ifft2(Fcls);

%SNR calculation
SNR=norm(F,2)/norm(Ns,2)

%Plotting of Results
subplot(1,3,1)
```

```
imshow(img);
title('original')
subplot(1,3,2)
g=mat2gray(g);
imshow(g);
title('destorted')
subplot(1,3,3)
imagesc(f);
colormap(gray)
```

Minimum Mean Square Estimation Wiener Filter

```
clc;
clear all;
close all;

%input
RGB=imread('C:\Users\koundi\Desktop\lena.jpg'); %input image in spatial
domain
org = rgb2gray(RGB);

%Degradation
degr=imnoise(org,'gaussian',0,0.005);

%In built Wiener Functions
rest10=wiener2(degr,[10 10]);
rest6=wiener2(degr,[6 6]);
p=double(0);
disp(p);
for x=1:1:256
disp(p);
for y=1:1:256
q=double((org(x,y)).^2)/((org(x,y)-rest6(x,y)).^2);
p=p+q;
end;
end;

%SNR calculation
SNR=norm(double(rest10))/norm(double(degr))

%Pltoting the Results
figure
subplot(2,2,1)
imshow(org),title('Original');
subplot(2,2,2)
imshow(degr),title('Degraded');
subplot(2,2,3)
imshow(rest10),title('Restoration10');
subplot(2,2,4)
imshow(rest6),title('Restoration6');
```

Maximum Entropy Pseudo Inverse

```
clc;
close all;
clear all;

%input
img=imread('C:\Users\koundi\Desktop\lena.jpg'); %input image in spatial
domain
img=rgb2gray(img);
F= fft2(img);   %input image in freq domain
M=size(img,1);  %rows of input image
N=size(img,2);  %columns of the input image


%channel
h = fspecial('gaussian', [3 3], 0.5); %channel in spatial domain
H= fft2(h,M,N); %channel in freq domain

%Noise
noise = input('Enter the magnitude for the noise signal : ');
n =noise*randn(M,N);
%n =fspecial('gaussian', [3 3], noise); %Noise in spatial domain
Ns=fft2(n,M,N); %noise in freq domain

%Degradation
G= F.*H +Ns;   %degraded image in spatial domain
g = abs(ifft2(G)); %degraded image in freq domain
g=mat2gray(g);

%Restoration


lambda=1;
Fcap = zeros(M,N);
for k1=1:M
for k2=1:N
Fcap(k1,k2) = ( conj( H(k1,k2) ) / ( pow2( abs( H(k1,k2) ) )+ 0.5*lambda^-
1*pow2( abs( H(k1,k2) ) ) ) )* G(k1,k2); % restored image in spatial domain
end
end
fcap = abs(ifft2(Fcap)); %restored image in freq domain
fcap= mat2gray(fcap);

%SNR calculation
SNR = norm(F,2)/norm(Ns,2)


%Plotting of Results
figure;
subplot(1,3,1)
imshow(img);
title('original');
subplot(1,3,2)
```

```
imshow(g);
title('degraded');
subplot(1,3,3);
imshow(fcap),title('Restored');
```