# CHAPTER 7 INTEGRALS

- **Some Properties of indefinite integrals are as follows:-**
  1. $\int[f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$
  2. For any real number $k$, $\int k\, f(x)dx = k\int f(x)dx$
     More generally, if $f_1, f_2, f_3 \ldots, f_n$ are functions and $k_1, k_2, k_3 \ldots\ldots k_n$ are real number. Then $\int[k_1 f_1(x) + k_2 f_2(x) + .. + k_n f_n(x)]\, dx = k_1 \int f_1(x)\, dx + k_2 \int f_2(x)\, dx + \cdots + k_n \int f_n(x)\, dx$
- **Some standard integrals:**
  1. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C\, , n \neq -1. Particularly, \int dx = x + C$
  2. $\int \cos x\ dx = \sin x + C$
  3. $\int \sin x\ dx = -\cos x + C$
  4. $\int sec^2 x\, dx = \tan x + C$
  5. $\int cosec^2 x\, dx = -cotx\ + \mathrm{C}$
  6. $\int secx\, tanx\, dx = secx + C$
  7. $\int cosecx.cotx\, dx = -cosecx + C$
  8. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
  9. $\int \frac{dx}{\sqrt{1-x^2}} = \cos^{-1} x + C$
  10. $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
  11. $\int \frac{dx}{1+x^2} = \cot^{-1} x + C$
  12. $\int e^x\ dx = e^x + C$
  13. $\int a^x\, dx = \frac{a^x}{\log a} + C$
  14. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$
  15. $\int \frac{dx}{x\sqrt{x^2-1}} = \csc^{-1} x + C$
  16. $\int \frac{1}{x} \mathrm{dx} = \log x + C$
- **Integration by partial fractions:**
  1. $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}, a \neq b$
  2. $\frac{px+q}{(x-a)^2} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
  3. $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
  4. $\frac{px^2+qx+r}{(x-a0)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-c}$
  5. $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

  Where $x^2 + bx + c$ can not be factorized further.

- **Integration by substitution:**

1. $\int \tan x\, dx = \log|x| + C$
2. $\int cotx\, dx = \log|sinx| + C$
3. $\int sex\, dx = \log|secx + tanx| + C$
4. $\int cosecx\, dx = \log|cosecx - cotx| + C$

- **Integrals some special functions:**

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$
2. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$
3. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$
4. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right| + C$
5. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$
6. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + C$

- **Integration by Parts:**

1. For given functions $f_1$ and $f_2$ we have
2. $\int f_1(x). f_2(x)\, dx = f_1(x)\int f_2(x)\, dx - \int\left[\frac{d}{dx}f_1(x).\ \int f_2(x)dx\right]dx$ . i. e. the integral of the product of two functions = first function × integral of the second function – integral of {differential coefficient of the first function × integral of the second function}. Care must be taken in choosing the first function and the second function. Obviously, we must take that function as the second function whose integral is well known to us.

$$\int e^x[f(x) + f'(x)]dx = \int e^x f(x)\ dx + C$$

- **Some special types of integrals:**

1. $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + C$
2. $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2+a^2}\right| + C$
3. $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{b} + C$
4. Integrals of the types $\int \frac{dx}{x^2+bx+c}$ $or$ $\int \frac{dx}{\sqrt{x^2+bx+c}}$ can be transformed into standard from by expressing

$$ax^2 + bx + c = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = a[\left(x + \frac{b}{2a}\right)^2 + (\frac{c}{a} - \frac{b^2}{4a^2})]$$

5. Integrals of the types $\int \frac{px+q\ dx}{ax^2+bx+c}$ $or$ $\int \frac{px+q\ dx}{\sqrt{ax^2+bx+c}}$ can be transformed into standard form by expressing $px + q = A\frac{d}{dx}(ax^2 + bx + c) + B = A(2ax + b) + B,$ where A and B are determined by comparing coefficients on both sides.

- We have define $\int_a^b f(x)\ dx$ as the area of the region bounded by the curve $y = f(x), a \le x \le b,$ the $x - axis$ and the ordinates $x = a$ and $x = b$. Let x be a given point in [a, b]. Then $\int_a^b f(x)\ dx$ represent the **Area function** A(x).
- **First fundamental theorem of integral calculus:**

Let the area function be defined by A(x) = $\int_a^x f(x)\ dx$ for all x≥ $a$, where the $f$ is assumed to be continuous on [a, b]. Then A'(x) = $f(x)$ for all x ∈ [a, b].

- **Second fundamental theorem of integral calculus:**
  Let $f$ be a continuous function of $x$ defined on the closed interval [a, b] and let F be another function such that $\frac{d}{dx}F(x) = f(x)$ for all $x$ in the domain of $f$, then
  $\int_a^b f(x)dx = [F(x) + C]_a^b = F(b) - F(a)$.
  This is called the definite integral of $f$ over the range [a, b], where a and b are called the limits of integration, a being the lower limit and b the upper limit.

## CHAPTER - 8 – APPLICATION OF INTEGRALS

- The area of the region bounded by the curve $y = f(x), x - axis$ and the lines $x = a$ and $x = b$ (b > a) is given by the formula: $Area = \int_a^b y\ dx = \int_a^b f(x)dx$.
- The area of the region bounded by the curve $x = \emptyset\ (y), y - axis$ and the lines $y = c$, $y = d$ is given by the formula: $Area = \int_c^d x\ dy = \int_c^d \emptyset\ (y)dy$.
- The area of the region enclosed between two curves $y = f(x), y = g(x)\ and$ the lines $x = a, a = b$ is given by the formula ,
  $Area = \int_b^a [f(x) - g(x)]dx$, where $f(x) \geq g(x) in\ [a, b]$
- If $f(x) \geq g(x) in\ [a, c]$ and $f(x) \leq g(x) in\ [c, b], a < c < b$, then
  $$Area = \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx.$$

## CHAPTER – 9 – DIFFRENTIAL EQUATIONS

- An equation involving derivatives of the dependent variable with respect to independent variable (variables) is known as a differential equation.
- Order of a differential equation is the order of he highest order derivatives occurring in the differential equations.
- Degree of a differential is defined if it is a polynomial equation in its derivatives.
- Degree (when defined) of a differential equation is the highest power (positive integer only) of the highest order derivative in it.
- A function which satisfies the given differential equation is called its solution. The solution which contains as many arbitrary constants as the order of the differential equation is called a general solution and the solution free from arbitrary constants is called particular solution.
- A differential equation which can be expressed in the form $\frac{dy}{dx} = f(x, y) or\ \frac{dy}{dx} = g(x, y)$ where, ○ $f(x, y) and\ g(x, y)$ are homogenous functions of degree zero is called a homogenous differential equation.
- A differential equation of the form $\frac{dy}{dx} + Py = Q$, where P and Q are constants or functions of $x$ only is called a first order linear differential equation.

# CHAPTER – 10 – VECTOR ALGEBRA

- Position vector of a point P(x, y, z) is given as $\overrightarrow{OP}(=\vec{r})= x\hat{\imath}+y\hat{\jmath}+z\hat{k}$, and its magnitude by $\sqrt{x^2+y^2+z^2}$.
- The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
- The magnitude $(r)$, direction ratios $(a, b, c)$ and direction cosines $(l, m, n)$ of any vector are related as: $l=\frac{a}{r},\ m=\frac{b}{r},\ n=\frac{c}{r}$
- The vector sum if the three sides of a triangle taken in order is $\vec{0}$.
- The vector sum of two coinitial vectors is given by the diagonal of the parallelogram whose adjacent sides are the given vectors.
- The multiplication of a given vector by a scalar $\lambda$, changes the magnitude of the vector by the multiple $|\lambda|$, and keeps the direction same according as the value of $\lambda$ is positive.
- For a given vector $\vec{a}$, the vector $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$ gives the unit vector in the direction of $\vec{a}$.
- The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, in the ratio m:n

  (i) internally, is given by $\frac{n\vec{a}+m\vec{b}}{m+n}$ (ii) externally, is given by $\frac{m\vec{b}-n\vec{a}}{m-n}$
- The scalar product of two given vectors $\vec{a}$ and $\vec{b}$ having angle $\theta$ between them is defined as $\vec{a}\,.\,\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

  Also, when $\vec{a}\,.\,\vec{b}$ is given, the angle $'\theta'$ between the vectors $\vec{a}$ and $\vec{b}$ may be determined by $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$
- If $\theta$ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then their cross product is given as

  $$\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\hat{n},$$

  when $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$, such that $\vec{a},\vec{b},\hat{n}$ from right handed system of coordinates axes.
- If we have two vectors $\vec{a}$ and $\vec{b}$, given in component from as $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ and $\lambda$ any scalar then $\vec{a}+\vec{b}=(a_1+b_1)\hat{\imath}+(a_2+b_2)\hat{\jmath}+(a_3+b_3)\hat{k}$

  $$\lambda\vec{a}=(\lambda a_1)\hat{\imath}+(\lambda a_2)\hat{\jmath}+(\lambda a_3)\hat{k}$$

  $$\vec{a}.\vec{b}=a_1b_1+a_2b_2+a_3b_3$$

  and $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$

- Direction cosines of a line are the cosines of the angels made by the line with the positive directions of the coordinates axes.
- If $l, m, n$ are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.
- Direction cosines of a line joining two points P $(x_1, y_1, z_1)$ and Q $(x_2, y_2, z_2)$ are $\frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ}$
  where $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- Direction ratios of a line are the numbers which are proportional to the direction cosines of a line.
- If $l, m, n$ are the direction cosines and $a, b, c$ are the direction ratios of a line then
  $$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}\ ; m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}\ ;\ n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$
- Skew lines are lines in space which are neither parallel nor intersecting. They lie in different planes.
- Angle between skew lines is the angle between two intersecting lines drawn from any point (preferable through the origin) parallel to each of the skew lines.
- If $l_1, m_1, n_1$ and $l_2, m_2\ n_2$ are the direction cosines of two lines; and $\theta$ is the acute angle between the two lines; then $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$
- If $a_1, b_1, c_1$ and $a_2, b_2, c_2$ are the direction ratios of two lines and $\theta$ is the acute angle between the two line then $\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}\ \sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}\right|$
- Vector equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$
- Equation of a line through a point $(x_1, y_1, z_1)$ and having direction cosines $l, m, n$ is $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$
- If $\theta$ is a acute angle between $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$, then $\cos\theta = \left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}|\,|\overrightarrow{b_2}|}\right|$
- If $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_1}{l_2} = \frac{y-y_1}{m_2} = \frac{z-z_1}{n_2}$
  are the equations of two lines, then the acute angle between the two lines is given by $\cos\theta = \big||l_1 l_2 + m_1 m_2 + n_1 n_2|\big|$.
- Shortest distance between two skew lines is the line segment perpendicular to both the lines.
- Shortest distance between $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is $\left|\frac{(\overrightarrow{b_1}\times\overrightarrow{b_2})(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$
- Shortest distance between the lines : $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_1}{a_2} = \frac{y-y_1}{b_2} = \frac{z-z_1}{c_2}$ is
  $$\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2} + \sqrt{(c_1a_2 - c_2a_1)^2} + \sqrt{(a_1b_2 - a_2b_1)^2}}$$
- Distance between parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \mu\vec{b}$ is $\left|\frac{\vec{b}\times(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\vec{b}|}\right|$

- In the vector form, equation of a plane which is at a distance $d$ from the origin, and $\hat{n}$ is the unit vector normal to the plane through the origin is $\hat{r}.\hat{n} = d$.
- Equation of a plane which is at a distance of $d$ from the origin and the direction cosines of the normal to the plane as $l, m, n \; is \; lx + my + nz = d$.
- The equation of a plane through a point whose position vestor is $\vec{a}$ and perpendicular to the vector $\vec{N}$ is $(\vec{r} - \vec{a}).\vec{N} = 0$.
- Equation of a plane perpendicular to a given line with direction ratios A, B, C and passing through a given point $(x_1, y_1, z_1)$ is $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$
- Equation of a plane passing through three non – collinear points $(x_1, y_1, z_1)$, $(x_2, y_2, z_2)$ and $(x_3, y_3, z_3)$ is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$

- Vector equation of a plane that contains three non collinear points having position vectors $\vec{a}, \vec{b} \; and \; \vec{c}$ is $(\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) \times \overrightarrow{(c - \vec{a})}] = 0$
- Equation of a plane that cuts the coordinates axes at $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ is
$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$
- Vector equation of a plane that passes through the intersection of planes $\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ is $\vec{r}.(\overrightarrow{n_1} + \lambda\overrightarrow{n_2}) = d_1 + \lambda d_2$ where $\lambda$ is any nonzero constant.
- Cartesian equation of a plane that passes through the intersection of two given planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is
$$(A_1x + B_1y + C_1z + D_1 + \lambda A_2x + B_2y + C_2z + D_2) = 0$$
- Two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ are coplanar if $(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0$
- In the Cartesian form two lines= $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \; and \; \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$
- In the vector form, if $\theta$ is the angle between the two planes $\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$, then $\theta = \cos^{-1}\frac{|\overrightarrow{n_1}.\overrightarrow{n_2}.|}{|\overrightarrow{n_1}|.|\overrightarrow{n_2}|}$
- The angle $\phi$ between the lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and the plane $\vec{r}\,\hat{n} = d$ is $\sin\phi = \left|\frac{\vec{b}\,\hat{n}}{|\vec{b}|\,|\hat{n}|}\right|$
- The angle $\theta$ between the planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by $\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1{}^2 + B_1{}^2 + C_1{}^2}\,\sqrt{A_2{}^2 + B_2{}^2 + C_2{}^2}}\right|$
- The distance of a point whose position vectors is a $\vec{a}$ from the plane $\vec{r}\,\hat{n} = d$ is $|d - \vec{a}\,\hat{n}|$
- The distance from a point $(x_1, \; y_1, z_1)$ to the plane $Ax + By + Cz + D = 0$ is $\left|\frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}}\right|$

# CHAPTER – 12 – LINEAR PROGRAMMING

- **Linear Programming Problems –** Problems which concern with finding the minimum or maximum value of a linear function Z (called objective function) of several variables (say x and y), subject to certain conditions that the variables are non-negative and satisfy a set of linear inequalities (called linear constraints) are known as linear programming problems.
- **Feasible Region –** The common region determined by all the constraints including non–negative constraints $x, y \geq 0$ of linear programming problem is known as feasible region (or solution region) If we shade the region according to the given constraints, then the shaded areas is the feasible region which is the common area of the regions drawn under the given constraints.

- Points within and on the boundary of the feasible region represent feasible solutions of the constraints.
  Any point outside the feasible region is an **infeasible solution.**
- Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an **optimal solution.**
- **Theorem 1 –** Let R be the feasible region (convex polygon) for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, the optimal value must occur at a corner point of the feasible region.
- **Theorem 2 –** Let R be the feasible region for a linear programming problem, and let Z = ax + by be the objective function. If R is bounded them the objective function Z has both maximum and minimum value on R and each of these occurs at a corner point of R.
- **Corner point method** for solving a linear programming problem. The method comprises of the following steps:-
  (i) Find the feasible region of the linear programming problem and determine its corner points (vertices).
  (ii) Evaluate the objective function $Z = ax + by$ at each corner point. Let M and $m$ respectively be the largest and smallest values at these points.
  (iii) If the feasible region is bounded, M and $m$ respectively are the maximum and minimum values of the objective function.
  If the feasible region is unbounded, then
  (i) M is the maximum value of the objective function, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, the objective function has no maximum value.
  (ii) $m$ is the maximum value of the objective function, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, the objective function has no minimum value.

# CHAPTER – 13 – PROBABILITY

- The conditional probability of an event E, given the occurrence of the event F is given by $P(E|F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$
- $0 \leq P(E|F) \leq 1, P(E'|F) = 1 - P(E|F)$
- $P\big((E \cup F)\big|G\big) = P(E|G) + (F|G) - P\big((E \cap F)\big|G\big)$
- $P(E \cap F) = P(E)P(F|E), P(E) \neq 0;\ P(E \cap F) = P(F)P(E|F), P(F) \neq 0$
- If E and F are independent ,then $P(E \cap F) = P(E)\,P(F)$
- $P(E|F) = P(E), P(F) \neq 0;\ P(F|E) = P(F), P(E) \neq 0$
- **Theorem of total probability**
- Let $(E_1, E_2 \dots, E_n)$ be a partition of a sample space and suppose that each of $E_1, E_2 \dots, E_n$ has non zero probability. Let A be any event associated with S, then
  P(A) = P($E_1$) $P(A|E_1)$ + P($E_2$) $P(A|E_2)$ + ………… + P($E_n$) $P(A|E_n)$

- **Bayes' theorem** If $E_1, E_2, \dots\dots\dots E_3$ are events which constitute a partition of sample space S, i.e. $E_1, E_2, \dots\dots E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \dots\dots \cup E_n = S$ and A be any event with nonzero probability, then $P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A|E_j)}$
- A random variable is a real valued function whose domain is the sample space of a random experiment.
- The probability distribution of a random variable X is the system of numbers

| | | | | |
|---|---|---|---|---|
| X | : $x_1$ | $x_2$ | ……. | $x_n$ |
| P(X) | : $p_1$ | $p_2$ | ……. | $p_n$ |

  Where, $p_i > 0, \sum_{i=1}^{n} p_i = 1\ \ i = 1, 2, \dots, n$
- Let X be a random variable whose possible values $x_1, x_2, x_3, \dots, x_n$ occur with probabilities $p_1, p_2, p_3, \dots, p_n$ respectively. The mean of X, denoted by $\mu$, is the number $\sum_{i=1}^{n} x_i\,p_i$
  The mean of random variable X is also called the expectation of X, denoted by E(X).
- Let X be a random variable whose possible values $x_1, x_2, \dots, x_n$ occur with probabilities $p(x_1), p(x_2) \dots\dots p(x_n)$
  Let $\mu = E(X)$ be the mean of X. The variance of X, denoted by Var (X) or ${\sigma_x}^2$, is defined as ${\sigma_x}^2 = Var\ (X) = \sum_{i=1}^{n} (x_i - \mu)^2\ p(x_i)$ or equivalently number ${\sigma_x}^2 = E\ (X - \mu)^2$
  The non-negative number $\sigma_x = \sqrt{Var\ (X)} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2\ p(x_i)}$ is called the standard deviation of the random variable X.
- Var (X) = E($X^2$) $- [E(X)]^2$
- For binomial distribution B(n, p), P(X = x) = ${}^{n}C_r\ q^{n-x} p^x, x = 0, 1, \dots\dots, n$

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