

COMPARISON OF MILP SOLVERS WITH USING RELATED-KEY DIFFERENTIAL ATTACK

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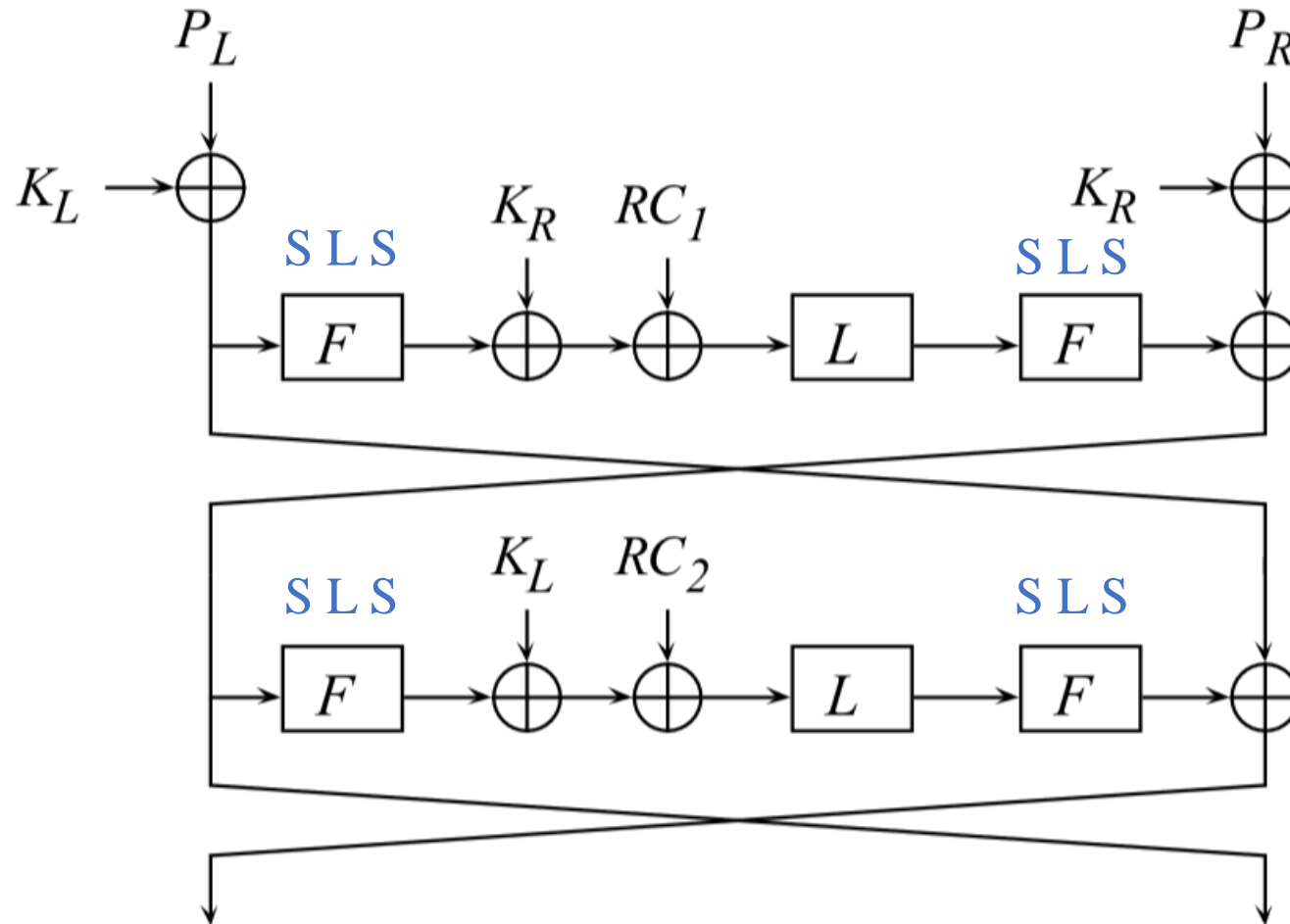
TDD / Kripto Analiz Laboratuvarı

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2024



1. ITUbee Block Cipher
2. MILP Model of Related-Key Differential Attack
3. GLPK Solver
4. HiGHS Solver
5. Gurobi Solver
6. CPLEX Solver
7. Comparison



The encryption operation proceeds as follows:

1. $X_1 \leftarrow P_L \oplus K_L$ and $X_0 \leftarrow P_R \oplus K_R$.
2. for $i = 1 \dots 20$ do
 - (a) if $i \in \{1, 3, \dots, 19\}$
 - i. $RK \leftarrow K_R$.
 - (b) else
 - i. $RK \leftarrow K_L$.
 - (c) $X_{i+1} \leftarrow X_{i-1} \oplus F(L(RK \oplus RC_i \oplus F(X_i)))$. Note that 16-bit round constant is added to the rightmost 16 bits.
3. $C_L \leftarrow X_{20} \oplus K_R$ and $C_R \leftarrow X_{21} \oplus K_L$.

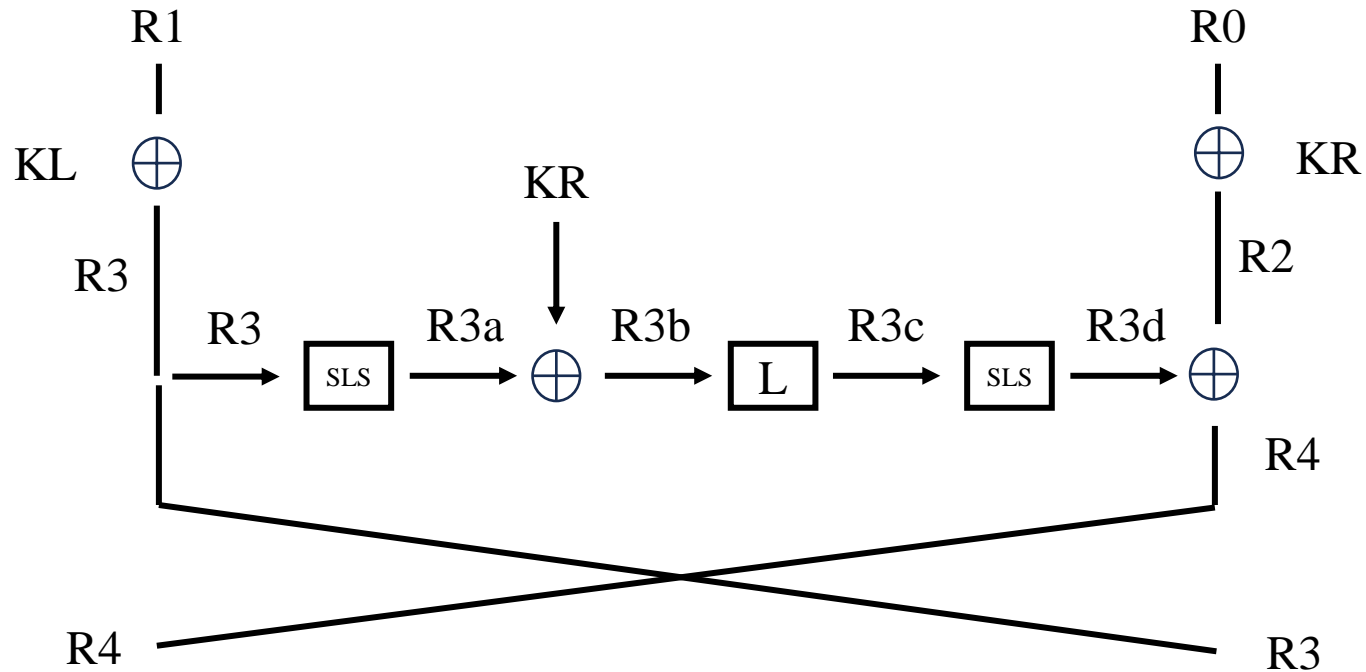
- $F(X) = S(L(S(X)))$.
- $S(a\|b\|c\|d\|e) = s[a]\|s[b]\|s[c]\|s[d]\|s[e]$ where a, b, c, d, e are 8-bit values and s is the S-box used in AES [10].
- $L(a\|b\|c\|d\|e) = (e \oplus a \oplus b)\|(a \oplus b \oplus c)\|(b \oplus c \oplus d)\|(c \oplus d \oplus e)\|(d \oplus e \oplus a)$.

ITUBEE offers 80-bit security. Also, the cipher provides the same security level against attacks in related key models as well as in single key attack models. In addition, the cipher has no weak keys.

4.3 Related Key Differential Attacks

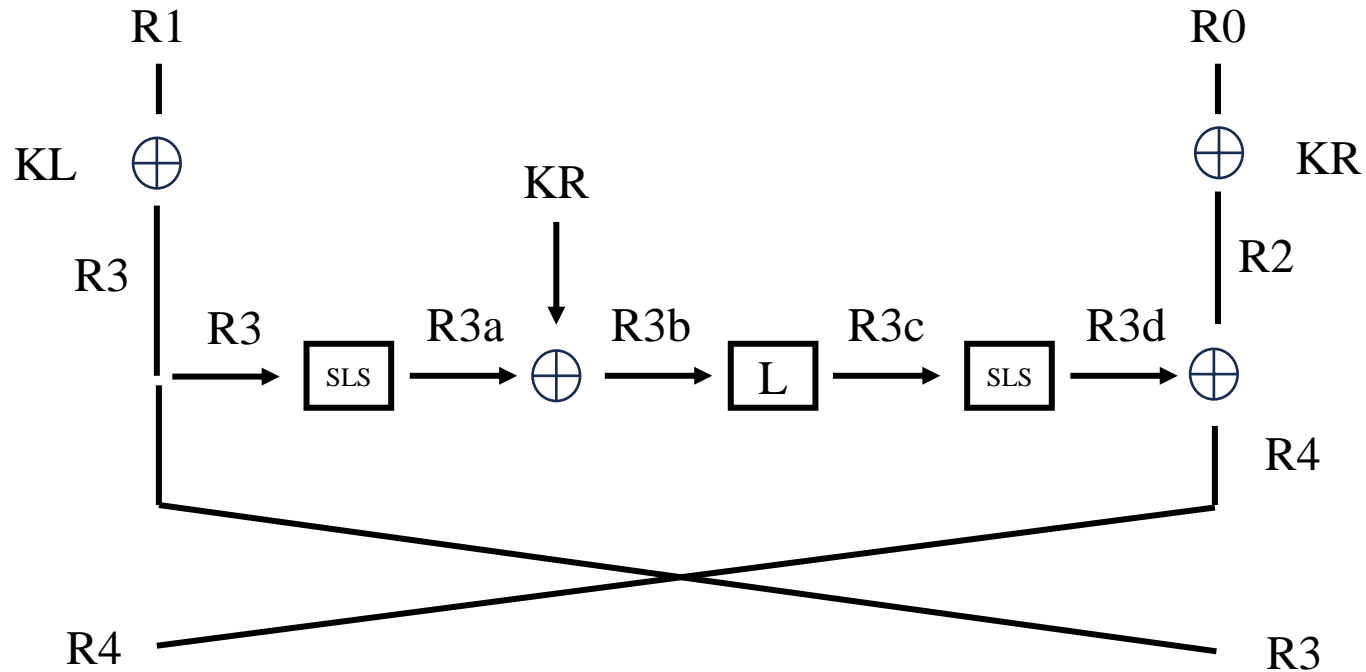
In the related key attack model, the adversary is able to collect plaintext and ciphertext pairs under related keys. In our algorithm, we divide the master key into two parts K_L and K_R and we use these parts between F functions in the rounds. It is trivial to see that when there is a difference in the key used between F functions, then at least one of the F functions will be active. In the best case for the adversary, the difference will be only one part of the key K_L or K_R . As a result, in two consecutive rounds there exists at least one active F function in the case of the related key attack. The probabilities of differentials for the F function is less than 2^{-17} as given in the Section 4.1. Thus, for 10 consecutive rounds the probabilities will be less than $(2^{-17})^5 = 2^{-85}$ which is not usable in an attack.

MILP Model of Related-Key Differential Attack



Minimize: $R3 + R3a + R3c + R3d$

MILP Model of Related-Key Differential Attack



Add Constraint : $KL + KR \geq 1$

XOR:

$$x_{in_1}^{\oplus} + x_{in_2}^{\oplus} + x_{out}^{\oplus} \geq 2d^{\oplus} ,$$

$$d^{\oplus} \geq x_{in_1}^{\oplus} ,$$

$$d^{\oplus} \geq x_{in_2}^{\oplus} ,$$

$$d^{\oplus} \geq x_{out}^{\oplus} .$$

```
def xor(p,in1,in2,out,dummy):  
    for i in range(len(in1)):  
        p.add_constraint(in1[i]+in2[i]+out[i]-2*dummy[i] >=0)  
        p.add_constraint(dummy[i]- in1[i] >= 0)  
        p.add_constraint(dummy[i]- in2[i] >= 0)  
        p.add_constraint(dummy[i]- out[i] >= 0)
```

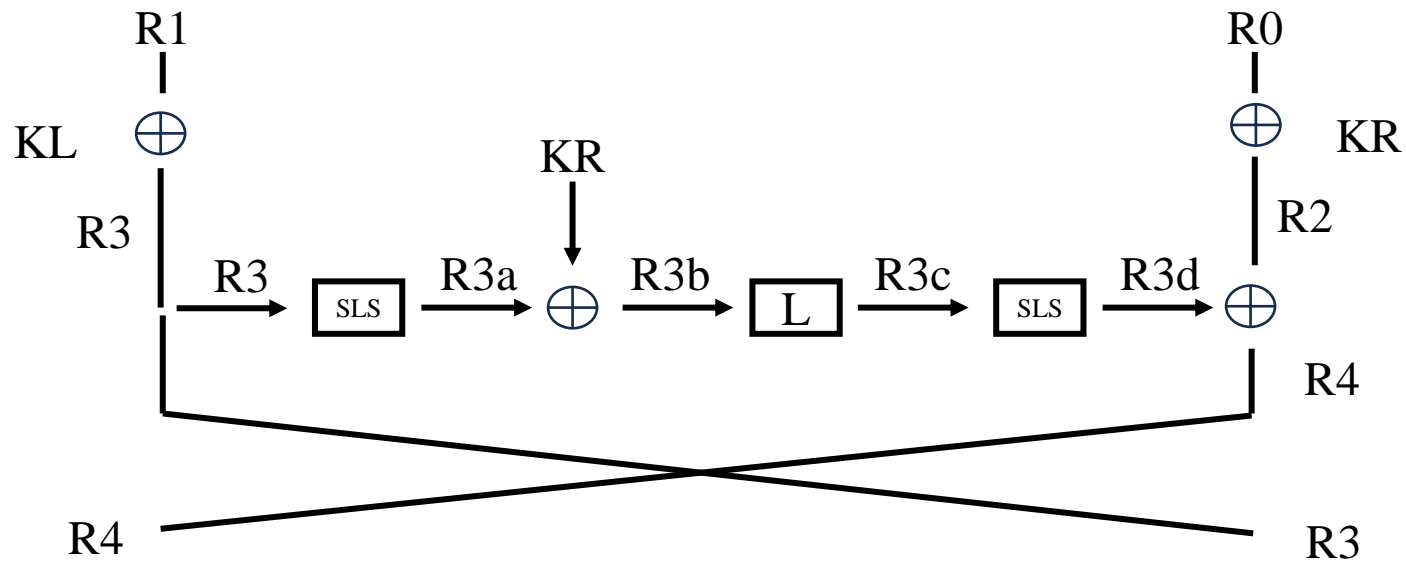
MILP Model of Related-Key Differential Attack

L Function:

```
pp = Polyhedron(vertices=[  
  
  [ 0,0,0,0,0,0,0,0,0,0 ],  
  [ 0,0,0,0,1,1,0,0,1,1 ],  
  [ 0,0,0,1,0,0,0,1,1,1 ],  
  [ 0,0,0,1,1,1,0,1,0,0 ],  
  [ 0,0,0,1,1,1,0,1,1,1 ],  
  [ 0,0,1,0,0,0,1,1,1,0 ],  
  [ 0,0,1,0,1,1,1,1,0,1 ],  
  [ 0,0,1,0,1,1,1,1,1,1 ],  
  [ 0,0,1,1,0,0,1,0,0,1 ],  
  [ 0,0,1,1,1,1,1,0,1,0 ],  
  [ 0,0,1,1,1,1,1,0,1,1 ],  
  [ 0,0,1,1,0,0,1,1,1,1 ],  
  [ 0,0,1,1,1,1,1,1,1,1 ],  
  [ 0,0,1,1,1,1,1,1,1,0 ],
```

MILP Model of Related-Key Differential Attack

Toplam aktif s-kutusu:	16.0
KL-KR	{0: 0.0, 1: 1.0, 2: 0.0, 3: 1.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R1-R0	{0: 0.0, 1: 1.0, 2: 0.0, 3: 1.0, 4: 0.0} {0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0}
R3-R2	{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0}
R3a-R3b-R3c-R3d	{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R4-R3	{0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R4a-R4b-R4c-R4d	{0: 0.0, 1: 1.0, 2: 0.0, 3: 1.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R5-R4	{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0}
R5a-R5b-R5c-R5d	{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R6-R5	{0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R6a-R6b-R6c-R6d	{0: 0.0, 1: 1.0, 2: 0.0, 3: 1.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R7-R6	{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0}
R7a-R7b-R7c-R7d	{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R8-R7	{0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R8a-R8b-R8c-R8d	{0: 0.0, 1: 1.0, 2: 0.0, 3: 1.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R9-R8	{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0}
R9a-R9b-R9c-R9d	{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R10-R9	{0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R10a-R10b-R10c-R10d	{0: 0.0, 1: 1.0, 2: 0.0, 3: 1.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}
R11-R10	{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0}



Toplam aktif s-kutusu: 16.0

KL-KR {0: 0.0, 1: 1.0, 2: 0.0, 3: 1.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}

R1-R0 {0: 0.0, 1: 1.0, 2: 0.0, 3: 1.0, 4: 0.0} {0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0}

R3-R2 {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0}

R3a-R3b-R3c-R3d {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}

{0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}

R4-R3 {0: 1.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 1.0} {0: 0.0, 1: 0.0, 2: 0.0, 3: 0.0, 4: 0.0}

GLPK (GNU Linear Programming Kit)



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Introduction to GLPK

The GLPK (GNU Linear Programming Kit) package is intended for solving large-scale linear programming the form of a callable library.

GLPK supports the *GNU MathProg modeling language*, which is a subset of the AMPL language.

The GLPK package includes the following main components:

- primal and dual simplex methods
- primal-dual interior-point method
- branch-and-cut method
- translator for GNU MathProg
- application program interface (API)
- stand-alone LP/MIP solver

HiGHS - high performance software for linear optimization

Open source serial and parallel solvers for large-scale
sparse linear programming (LP),
mixed-integer programming (MIP), and quadratic programming (QP) models

Solvers

LP

HiGHS has implementations of the three main solution techniques for LP. HiGHS will choose the most appropriate technique for a given problem, but this can be over-ridden by setting the option **solver**.

Simplex

HiGHS has efficient implementations of both the primal and dual simplex methods, although the dual simplex solver is likely to be faster and is more robust, so is used by default. The novel features of the dual simplex solver are described in

HiGHS - high performance software for linear optimization

Open source serial and parallel solvers for large-scale
sparse linear programming (LP),
mixed-integer programming (MIP), and quadratic programming (QP) models

Interior point

HiGHS has one interior point (IPM) solver based on the preconditioned conjugate gradient method, as discussed in *Implementation of an interior point method with basis preconditioning*, Mathematical Programming Computation, 12, 603-635, 2020. DOI: [10.1007/s12532-020-00181-8](https://doi.org/10.1007/s12532-020-00181-8).

This solver is serial. An interior point solver based on direct factorization is being developed.

Setting the option **solver** to "ipm" forces the IPM solver to be used

Primal-dual hybrid gradient method

HiGHS includes the **cuPDLP-C** primal-dual hybrid gradient method for LP (PDLP). Currently this only runs on CPU, so it is unlikely to be competitive with the HiGHS interior point or simplex solvers. Enabling HiGHS to run PDLP on a GPU is work in progress.

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PROBLEM	SOLVER	TIME(sec)	SPEED
Related-Key Differential Cryptanalysis of ITUbee algoirthm (8 round) 48649 constraints 320 variables	GLPK	257	x (base)
	HiGHS	231	1.1x
	Gurobi	37	6.9x
	CPLEX	16	16x

CPU: 11th Gen Intel(R) Core(TM) i7-1165G7 @ 2.80GHz

RAM: 32,0 GB

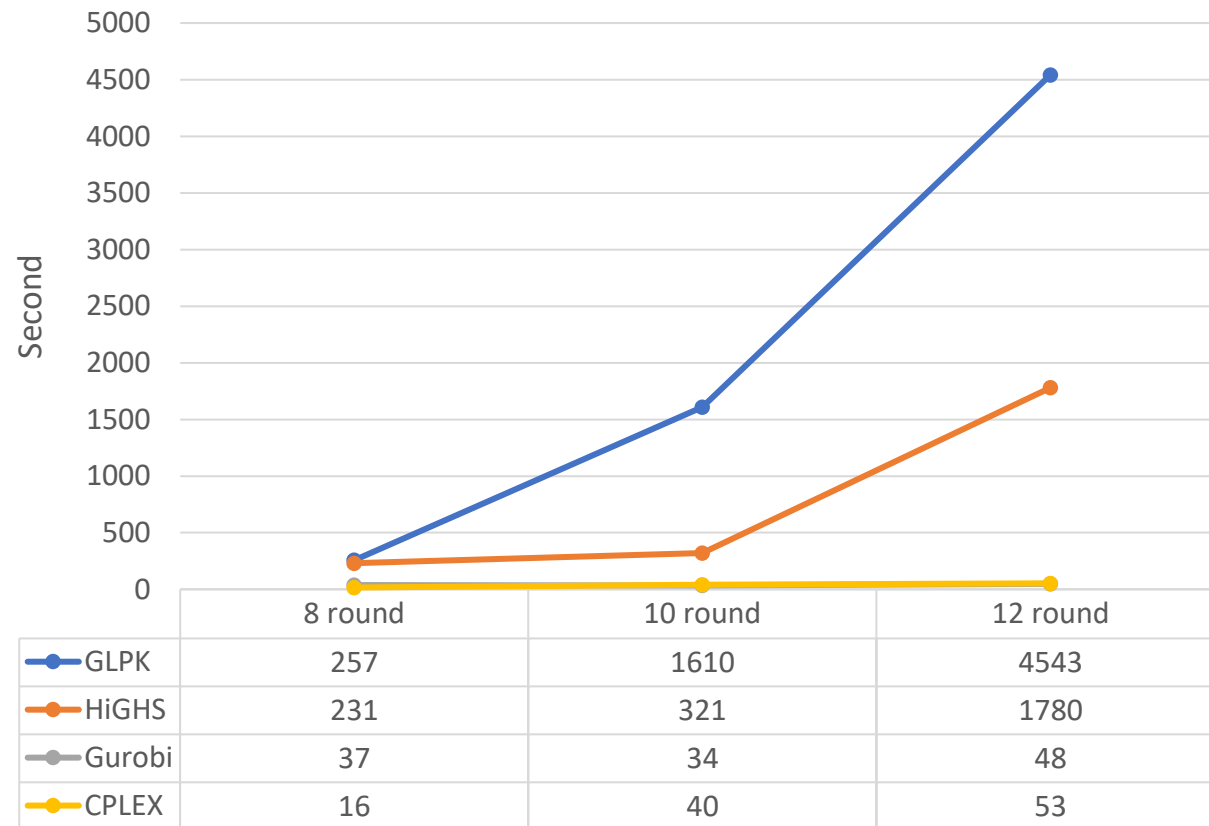
PROBLEM	SOLVER	TIME(sec)	SPEED
Related-Key Differential Cryptanalysis of ITUbee algorithm (10 round) 60801 constraints 390 variables	GLPK	1610	x (base)
	HiGHS	321	5x
	Gurobi	34	47x
	CPLEX	40	40x

CPU: 11th Gen Intel(R) Core(TM) i7-1165G7 @ 2.80GHz

RAM: 32,0 GB

PROBLEM	SOLVER	TIME(sec)	SPEED
Related-Key Differential Cryptanalysis of ITUbee algoirthm (12 round) 72953 constraints 460 variables	GLPK	4543	x (base)
	HiGHS	1780	2.5x
	Gurobi	48	94x
	CPLEX	53	85x

CPU: 11th Gen Intel(R) Core(TM) i7-1165G7 @ 2.80GHz
 RAM: 32,0 GB



CPU: 11th Gen Intel(R) Core(TM) i7-1165G7 @ 2.80GHz
RAM: 32,0 GB

- Karakoç, F., Demirci, H., & Harmancı, A. E. (2013). ITUbee: a software oriented lightweight block cipher. In *Lightweight Cryptography for Security and Privacy: Second International Workshop, LightSec 2013, Gebze, Turkey, May 6-7, 2013, Revised Selected Papers 2* (pp. 16-27). Springer Berlin Heidelberg.
- Mouha, N., Wang, Q., Gu, D., & Preneel, B. (2012). Differential and linear cryptanalysis using mixed-integer linear programming. In *Information Security and Cryptology: 7th International Conference, Inscrypt 2011, Beijing, China, November 30–December 3, 2011. Revised Selected Papers 7* (pp. 57-76). Springer Berlin Heidelberg.
- Developers, S. (2020). SageMath. *the Sage Mathematics Software System (Version 9.0)*. Retrieved July, 21, 2022.
- Gurobi, I. (2013). Gurobi Optimizer 5.0. *Gurobi*.
- Manual, C. U. S. (1987). Ibm ilog cplex optimization studio. *Version, 12*(1987-2018), 1.