Date
Page

Gaurav Amarnani
D7A/67.

1)

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$0+0 \mod 6 = 0$
$0+1 \mod 6 = 1$
$0+2 \mod 6 = 2$
$0+3 \mod 6 = 3$
$0+4 \mod 6 = 4$
$0+5 \mod 6 = 5$

All the entries in the composition table are element of the set G. Hence G is closed with respect to addition module 6 (+6)
- composition +6 is associative. If a, b, c are any three element of G. then

$$a +_6 (b +_6 c) = a +_6 b +_6 c$$

let $a = 1$, $b = 2$, $c = 3$

$$1 + (2 +_6 3) = (1 +_6 2) +_6 3$$

$$1 +_6 5 = 3 +_6 3$$

$$0 = 0$$

Hence $+_6$ is an associative operation since it is satisfying for a, (a, b), c ∈ G from composition table we also see the left inverse of 0, 1, 2, 3, 4, 5 are 0, 5, 4, 3, 2, 1 respectively since

$0 +_6 0 = 0 \quad 1 +_6 5 = 0 \quad 2 +_6 4 = 0$
$3 +_6 3 = 0 \quad 4 +_6 2 = 0 \quad 5 +_6 1 = 0$

- composition is commutative as the corresponding rows & columns in the position are identical number of elements in set $h = 6$ $(G, +_6)$ is a finite Abelian group of order 6.

2.

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	7	2
4	4	5	6	1.	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5

$2 \times 1 \bmod 7 = 2$
$2 \times 2 \bmod 7 = 4$
$2 \times 3 \bmod 7 = 6$
$2 \times 4 \bmod 7 = 1$
$2 \times 5 \bmod 7 = 3$
$2 \times 6 \bmod 7 = 5$

- All the entries in the composition table are element of G Hence h is closed multiplication module 7.

- Composition of $\times$ is associate if a, b, c are any three elements of G then $a \times_7 (b \times_7 c) = (a \times_7 b) \times c$.

Date
Page

let a = 1, b = 2, c = 3
$1 \times_7 (2 \times_7 3) = (1 \times_7 2) \times_7 3$
$1 \times_7 6 = 2 \times_7 3$
$6 = 6$

let a = 4, b = 5, c = 6
$4 \times_7 (5 \times_7 6) = (4 \times_7 5) \times_7 6$
$4 \times_7 2 = 6 \times_7 6$
$1 = 1$

Hence $\times_7$ is associative operation

- We have $1 \in G$. If a is any element of G then from the composition table $1 \times_7 a = a = a \times_7 1$.
$\therefore$ 1 is an identity element.

- from composition table we can see that left invests of 1, 2, 3, 4, 5, 6 are 1, 4, 5, 2, 3, 6 respectively

composition $\times_7$ is commutative or the corresponding rows and columns in the table are identical.
number of element in set G = 6
$(G, \times_7)$ is a finite abelian group of order 6.

Q3 A {1, 2, 3, 4, 5, 6} multiplication module 7

multiplication module 7 table

a.

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	7	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5

b. $2 \times_7 4 = 4 \times_7 2 = 1$

$3 \times_7 5 = 5 \times_7 3 = 1$

$6 \times_7 6 = 6 \times_7 6 = 1$

Inverse of 2 is 4 Inverse of 5 is 3

Inverse of 3 is 5 Inverse of 6 is 6

We have: $2^1 = 2$

$2^2 = 4$

$2^3 = 2^2 \times_7 2 = 4 \times_7 2 = 1$

$2^4 = 2^3 \times_7 2 = 1 \times_7 2 = 2$

Hence $|2| = 3$ $\therefore$ 2 is not generator of group.

Now, $3 = 3$

$3^2 = 3 \times_7 3 = 2$

$3^3 = 3^2 \times_7 3 = 2 \times_7 3 = 6$

$3^4 = 3^3 \times_7 3 = 6 \times_7 3 = 4$

$3^5 = 3^4 \times_7 3 = 4 \times_7 3 = 5$

$3^6 = 3^6 \times_7 3 = 5 \times_7 3 = 1$

4. $\langle \{3, 4\} \rangle$

Inverse of 3 is 5 & 4 is 2

$\therefore$ 3, 4, 5, 2 $\in \langle \{3, 4\} \rangle$

$3 \times_7 4 = 5$

$3 \times_7 3 = 2$

$3 \times_7 5 = 1$

$4 \times_7 4 = 2$

$3 \times_7 2 = 6$

$3 \times_7 5 = 4$

$5 \times_7 6 = 2$

$5 \times_7 4 = 6$

$6 \times_7 6 = 1$

$5 \times_7 1 = 5$

$1 \times_7 1 = 1$

$5 \times_7 2 = 3$

$3 \times_7 6 = 4$

$2 \times_7 2 = 4$

$\langle \{3, 4\} \rangle = \langle 1, 2, 3, 4, 5, 6 \rangle$

Inverse of 2 is 4 and 3 is 5

$\therefore$ 2, 3, 4, 5 $\in \langle \{2, 3\} \rangle$

$2 \times_7 3 = 6$

$4 \times_7 4 = 2$

$2 \times_7 4 = 1$

$4 \times_7 1 = 4$

$2 \times_7 5 = 3$

$4 \times_7 5 = 6$

$2 \times_7 6 = 5$

$6 \times_7 6 = 1$

$3 \times_7 4 = 5$

$5 \times_7 5 = 4$

$3 \times_7 5 = 1$

$5 \times_7 6 = 2$

$3 \times_7 6 = 4$

$5 \times_7 1 = 5$

$3 \times_7 1 = 3$

$3 \times_7 3 = 2$

$\therefore \langle \{2, 3\} \rangle = \langle 1, 2, 3, 4, 5, 6 \rangle$

$\therefore$ subgroup generated by $\langle \{2, 3\} \rangle$ set A and is of order 6.