Lemma 0.1. *Suppose $A \subseteq^\omega \omega$ has a κ-semiscale. Then A is κ-Souslin.*

Proof. Let $(\phi_n \mid n < \omega)$ be a κ-semiscale on A. Let

$$T = \{(x \upharpoonright n, (\phi_0(x), \phi_1(x), \ldots, \phi_{n-1}(x)) \mid x \in A\}.$$

Clearly T is a tree and $A \subseteq p[T]$. On the other hand, if $x \in p[T]$, fix $f \colon \omega \to \kappa$ such that $(x, f) \in T$. For each $n < \omega$ there is now some $a_n \in A$ of length A such that $(x \upharpoonright n, f \upharpoonright n) = (a_n \upharpoonright n, (\phi_0(a_n), \ldots, \phi_{n-1}(a_n))$. We have $\lim_{n<\omega} a_n = x$. Furthermore, for all $m < n < l < \omega$,

$$\phi_m(a_n) = \phi_m(a_l),$$

i.e. $(\phi_m(a_n) \mid n < \omega)$ is eventually constant and thus $x \in A$. $\square$