Notes on "PFA implies $\mathrm{AD}^{L(\mathbb{R})}$"

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January 12, 2019

1 Quick Reference §1

1. κ is a singular, strong limit cardinal such that $\square_\kappa$ fails.
2. $A_0 \subseteq \kappa$ codes V_κ.
3. $\lambda = \kappa^{+\mathrm{Lp}(A_0)}$.
4. $\mathrm{cf}(\lambda) < \mu < \kappa$ and $\mu^\omega = \mu$.
5. g is a $\mathrm{Col}(\omega, \mu)$-generic filter.
6. Given $U \subseteq \mathbb{R}^g$ and $k < \omega$ a coarse (k, U)-Woodin mouse (witnessed by $S, T, \Sigma, \delta_0, \ldots, \delta_k$) is a countable transitive model of ZFC such that
   (a) $N \models \delta_0 < \ldots < \delta_k$ are Woodin cardinals ,
   (b) $N \models S, T$ are trees which project to complements after the collapse of δ_k to be countable and
   (c) there is a $\omega_1 + 1$-iteration strategy Σ for N such that whenever $i \colon N \to P$ is an iteration map by Σ and P is countable, then $p[i(S)] \subseteq U$ and $p[i(T)] \subseteq \mathbb{R}^g \setminus U$.
7. For a scale $\vec{\phi}$ we let $\vec{\phi}^*$ be the associated sequence of prewellorderes.
8. (W^*_α) Let $U \subseteq \mathbb{R}^g$ and suppose there are scales $\vec{\phi}, \vec{\psi}$ on U and $\mathbb{R}^g \setminus U$ such that $\vec{\phi}^*, \vec{\psi}^* \in J_\alpha(\mathbb{R}^g)$. Then for all $k < \omega$ and all $x \in \mathbb{R}^g$ there are N, Σ such that
   (a) $x \in N$ and N is a coarse (k, U)-Woodin mouse as witnessed by Σ and
   (b) $\Sigma \restriction \mathrm{HC}^{V[g]} \in J_\alpha(\mathbb{R}^g)$.