Notes on "Scales in $L(\mathbb{R})$"

Stefan Mesken

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1 Quick Reference

1.1 §0. Introduction

1. Background theory: $\mathrm{ZF} + \mathrm{DC}$.

2. Variables $z, y, x, w \ldots$ range over $\mathbb{R}$.

3. Variables $\alpha, \beta, \gamma, \delta, \ldots$ range over Ord.

4. A pointclass is a class of subsets of $\mathbb{R}$ closed under recursive subsitution.

5. A boldface pointclass is a class of subsets of $\mathbb{R}$ closed under continuous substitution.

6. For a pointclass Γ

    (a)

2 Notes

2.1 §1. The fine structure of $L(\mathbb{R})$

Claim 2.1 (Lemma 1.7). *h is a Skolem function.*

Proof. Let $\emptyset \neq S \in \Sigma_n(J_\alpha(\mathbb{R}), a)$ for some $a \in J_\alpha(\mathbb{R})$. Let $n < \omega$ be such that

$$S = \{s \in J_\alpha(\mathbb{R}) \mid J_\alpha(\mathbb{R}) \models \phi_n[s, a]\}$$

Let ψ be the following Σ_n-formula

$$\psi(p, H) \equiv \exists texists z \in \mathbb{R} \colon p = (t, z) \wedge f_\alpha(H, z >> 2) = (s, a) \wedge \phi_{z(1)}[s, a]$$

Since
$$f_\alpha \colon [\omega\alpha]^{<\omega} \times \mathbb{R} \twoheadrightarrow J_\alpha(\mathbb{R})$$

$\square$