Solution of Heat transfer equation using finite element method - project for Differential and Difference Equations Course at AGH UST. Program gets positive number n (number of elements we will use in finite element method) and plots the function we want to find.
Equation:

$$\begin{gathered}
-k(x)\frac{d^2u(x)}{dx^2} = 100x \\
u(2) = 2 \\
\frac{du(0)}{dx} + u(0) = 20 \\
k(x) = \begin{cases} x+1, & x \in [0,1] \\ 2x, & x \in (1,2] \end{cases}
\end{gathered} \tag{1}$$

Where u is the function we want to find:

$$[0,2] \ni x \to u(x) \in \mathbb{R} \tag{2}$$

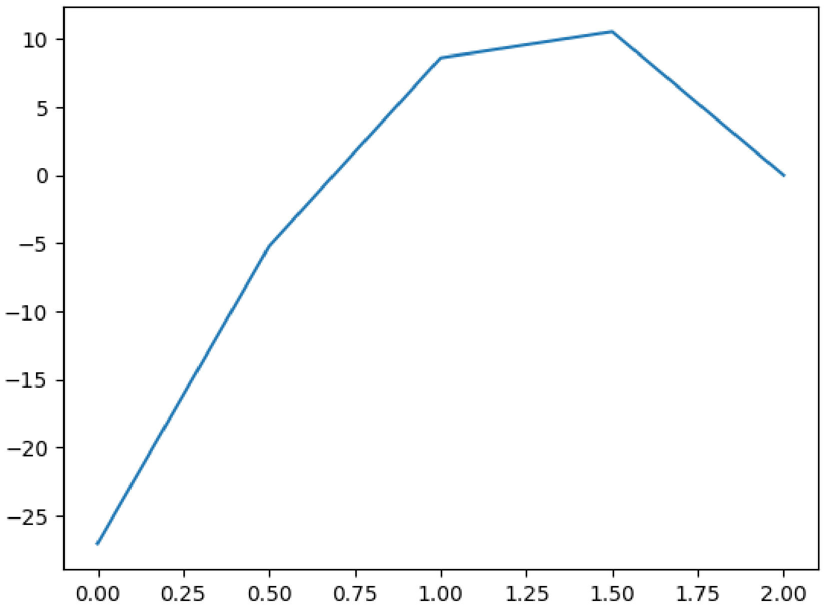
Solution of the equation:

$$\begin{gathered}
-k(x)\frac{d^2u(x)}{dx^2} = 100x \qquad \Omega = [0,2] \\
u(2) = 0 \\
u'(0) + u(0) = 20 \quad \Rightarrow \quad u'(0) = 20 - u(0) \\
k(x) = \begin{cases} x+1, & x \in [0,1] \\ 2x, & x \in (1,2] \end{cases} \\
-k(x)u''(x) = 100x \quad / : k(x), \quad k(x) \neq 0 \ \ \forall x \in \Omega \\
-u''(x) = \frac{100x}{k(x)}
\end{gathered} \tag{3}$$

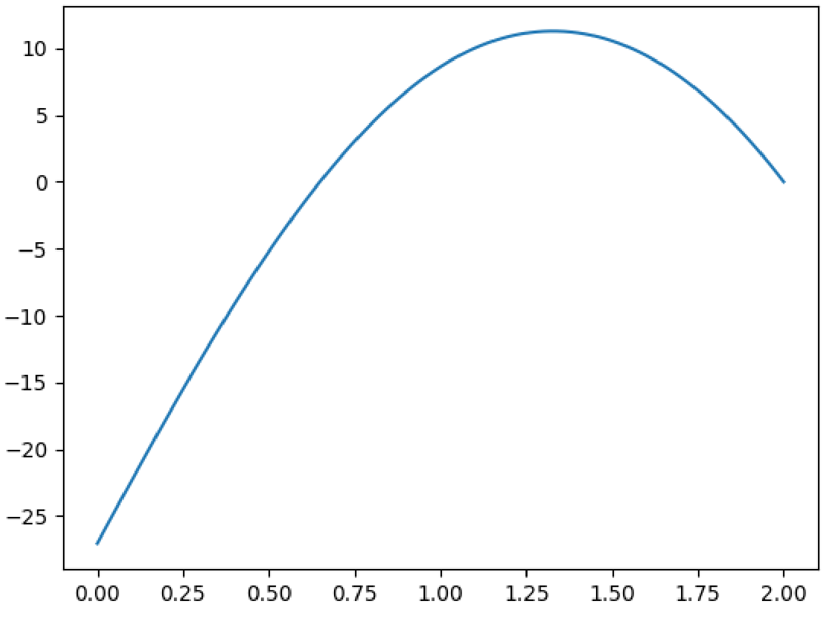
Multiply by test function v with Dirichlet boundary condition: $v(2) = 0$

$$\begin{gathered}
-u''(x)v(x) = \frac{100x}{k(x)}v(x) \\
\int_\Omega -u''(x)v(x)\ dx = \int_\Omega \frac{100x}{k(x)}v(x)\ dx \\
-\int_0^2 u''(x)v(x)\ dx = -[u'(x)v(x)]_0^2 + \int_0^2 v'(x)u'(x)\ dx = -u'(2)v(2) + u'(0)v(0) + \int_0^2 v'(x)u'(x)\ dx \\
-\int_0^2 u''(x)v(x)\ dx = v(0)(20 - u(0)) + \int_0^2 u'(x)v'(x)\ dx = 20v(0) - u(0)v(0) + \int_0^2 u'(x)v'(x)\ dx \\
-u(0)v(0) + \int_0^2 v'(x)u'(x)\ dx = \int_0^1 \frac{100x}{x+1}v(x)\ dx + \int_1^2 50v(x)\ dx - 20v(0) \\
B(u,v) \ = \ L(v)
\end{gathered} \tag{4}$$

Example output of the program for n=4 and n=100:



n=4



n=100