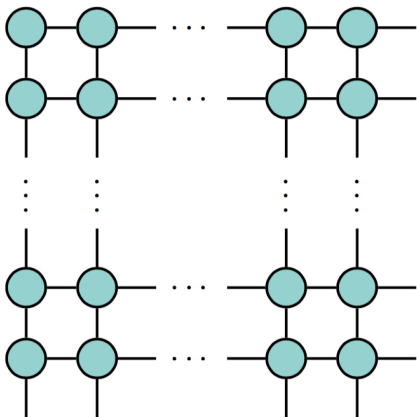


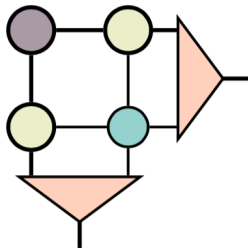
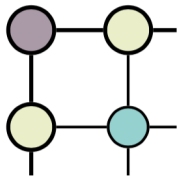
# Finite- $\chi$ scaling using CTMRG in the 2D classical Ising model

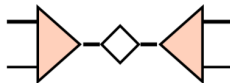
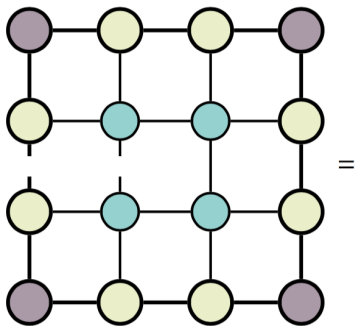
Geert Kapteijns

May 29, 2017



$$Z = \text{Tr} A^4$$





If one keeps all such eigenvalues and eigenvectors, then the diagonal matrices  $A_d, B_d, C_d, D_d$  double in size at each recursion. However, we expect these to tend to infinite-dimensional limits. This is in the sense given in Section 13.4, namely that if their diagonal elements are arranged in numerically decreasing order, then any given such element (e.g. the 6th largest) should tend to a limit.

This suggests a self-consistent truncation of the equations, namely to keep only the larger half of the eigenvalues of  $A_t B_t C_t D_t$ , and the corresponding right- and left-eigenvectors. This means that we are solving the equations

$$\kappa P_r A_d = A_t Q_r, \quad (13.8.15a)$$

$$\kappa A_d Q_s = P_s A_t, \quad (13.8.15b)$$

$$Q_s Q_r = \mathcal{I}, \quad (13.8.16)$$

together with (13.8.14) and their rotated analogues.

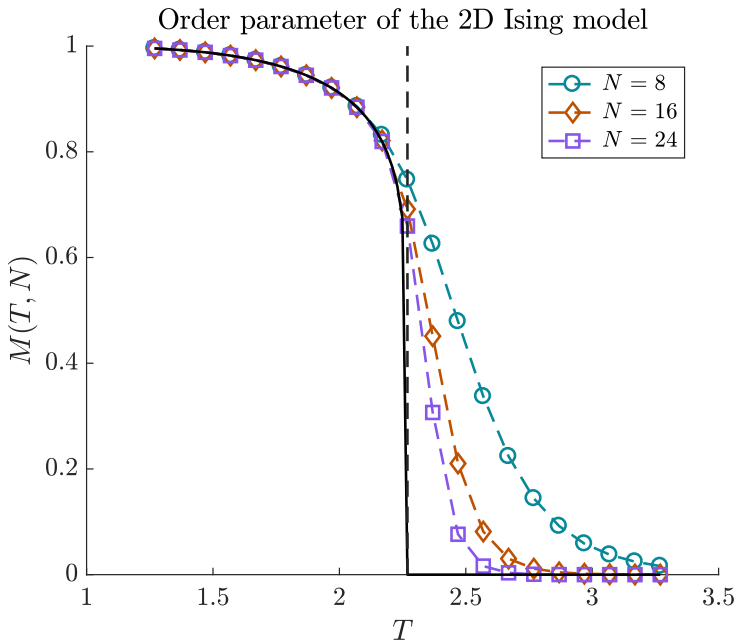
Let  $\psi(\sigma)$  be the element  $\sigma$  of the vector  $\psi$ . Remembering that  $\sigma$  denotes the  $m$  spins  $\sigma_1, \dots, \sigma_m$ , this can be written more explicitly as  $\psi(\sigma_1, \dots, \sigma_m)$ . For reasons that will be given in Section 5, I use the following “trial function” for  $\psi$ :

$$\psi(\sigma_1, \dots, \sigma_m) = \text{Tr}[F(\sigma_1, \sigma_2)F(\sigma_2, \sigma_3)F(\sigma_3, \sigma_4) \cdots F(\sigma_m, \sigma_1)] \quad (11)$$

$$\mathbf{K} = \frac{
 \begin{array}{|c|c|} \hline B & A \\ \hline A & B \\ \hline \end{array}
 \quad \times \quad
 \begin{array}{|c|c|c|} \hline B & F & A \\ \hline G & w & G \\ \hline A & F & B \\ \hline \end{array}
 }{
 \begin{array}{|c|c|c|} \hline B & F & A \\ \hline A & F & B \\ \hline \end{array}
 \quad \times \quad
 \begin{array}{|c|c|} \hline B & A \\ \hline G & G \\ \hline A & B \\ \hline \end{array}
 }$$

Fig. 5. Graphical representation of the variational expression (31) for  $\kappa$ .

# Finite systems distort critical behaviour







Baxter, RJ (1978). “Variational approximations for square lattice models in statistical mechanics”. In: *Journal of Statistical Physics* 19.5, pp. 461–478.



– (1968). “Dimers on a rectangular lattice”. In: *Journal of Mathematical Physics* 9.4, pp. 650–654.



Baxter, Rodney J (1982). *Exactly solved models in statistical mechanics*. Elsevier. Chap. 13.