Finite-χ scaling using CTMRG in the 2D classical Ising model

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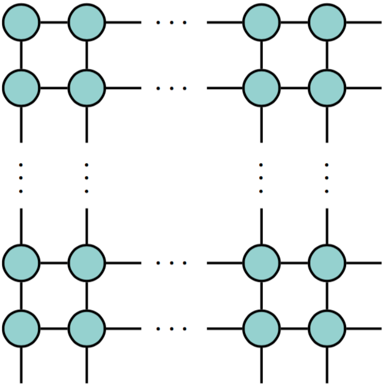
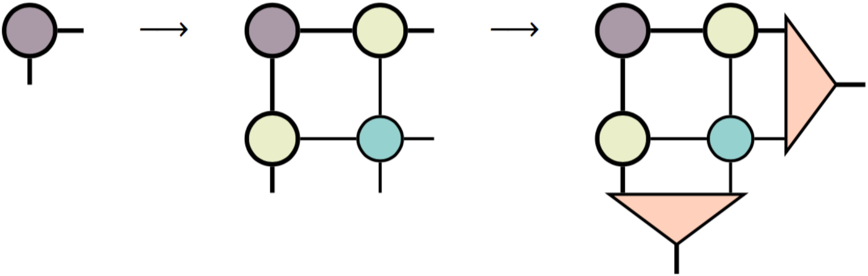
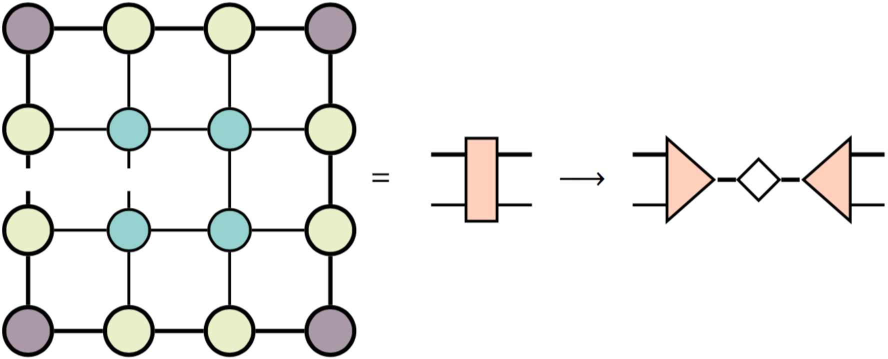
Tomotoshi Nishino and Kouichi Okunishi 1996



$$Z = \mathrm{Tr}\, A^4$$





R. J. Baxter 1982

If one keeps all such eigenvalues and eigenvectors, then the diagonal matrices A_d, B_d, C_d, D_d double in size at each recursion. However, we expect these to tend to infinite-dimensional limits. This is in the sense given in Section 13.4, namely that if their diagonal elements are arranged in numerically decreasing order, then any given such element (e.g. the 6th largest) should tend to a limit.

This suggests a self-consistent truncation of the equations, namely to keep only the larger half of the eigenvalues of $A_tB_tC_tD_t$, and the corresponding right- and left-eigenvectors. This means that we are solving the equations

$$\kappa P_r A_d = A_t Q_r \,, \qquad (13.8.15a)$$

$$\kappa A_d Q_s = P_s A_t \,, \qquad (13.8.15b)$$

$$Q_s Q_r = \mathscr{I} \,, \qquad (13.8.16)$$

together with (13.8.14) and their rotated analogues.

R. Baxter 1978; R. Baxter 1968

Let $\psi(\sigma)$ be the element σ of the vector ψ. Remembering that σ denotes the m spins $\sigma_1, \ldots, \sigma_m$, this can be written more explicitly as $\psi(\sigma_1, \ldots, \sigma_m)$. For reasons that will be given in Section 5, I use the following "trial function" for ψ:

$$\psi(\sigma_1, \ldots, \sigma_m) = \mathrm{Tr}[F(\sigma_1, \sigma_2)F(\sigma_2, \sigma_3)F(\sigma_3, \sigma_4) \cdots F(\sigma_m, \sigma_1)] \qquad (11)$$

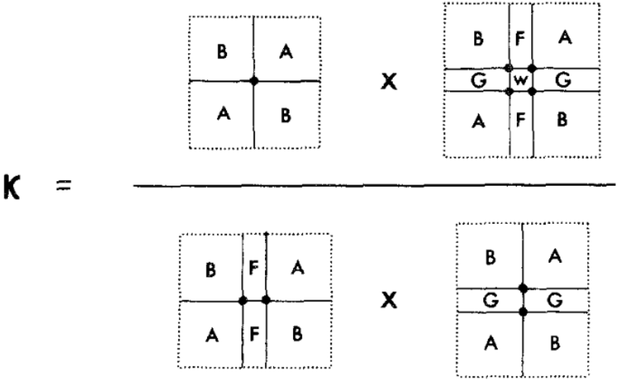
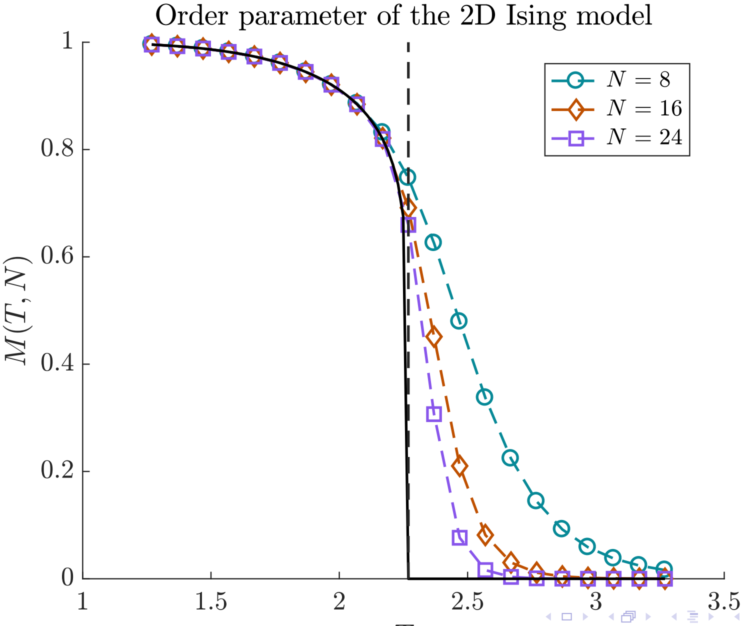


Fig. 5. Graphical representation of the variational expression (31) for κ.

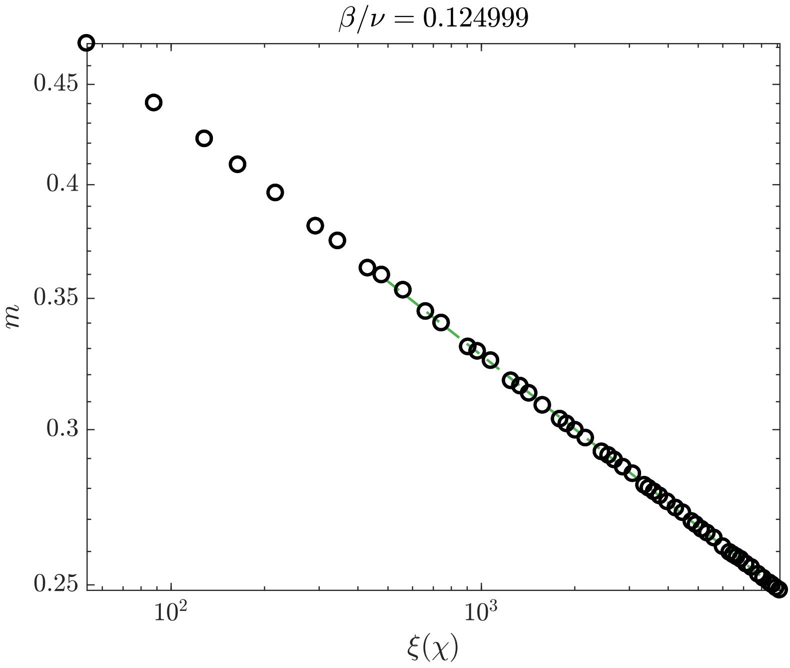
Finite systems distort critical behaviour



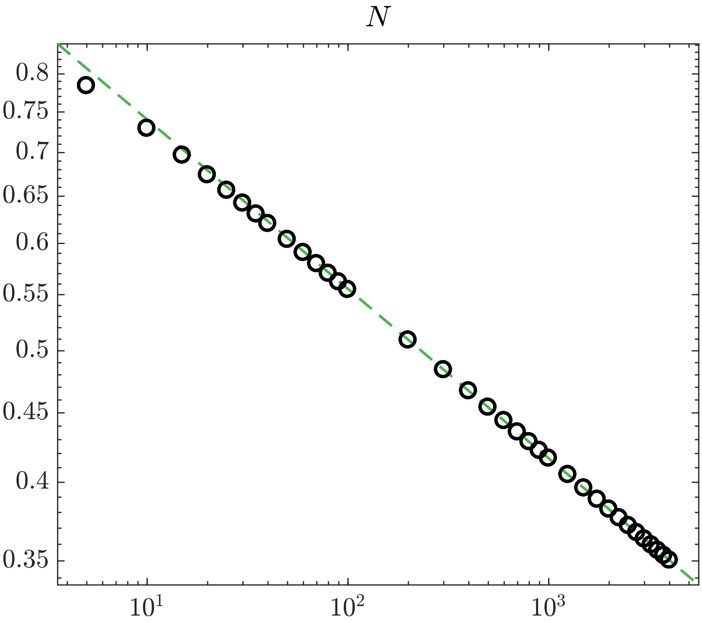
Finite-χ scaling

- Östlund and Rommer 1995: DMRG becomes MPS-ansatz in thermodynamic limit, MPS has inherently finite correlation length.
- T Nishino, K Okunishi, and Kikuchi 1996 numerical evidence: effective length scale given by

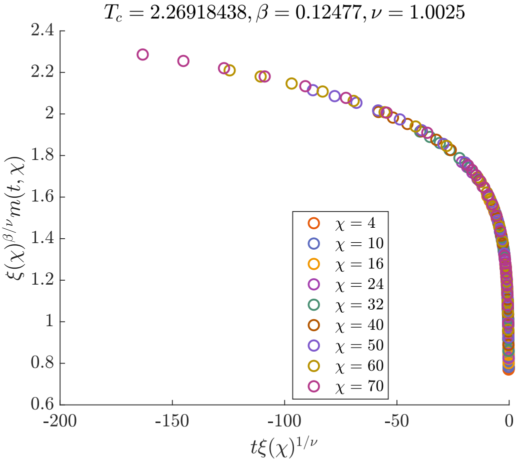
$$\xi(T = T_c, N \to \infty, \chi) \propto N_{\text{eff}}$$



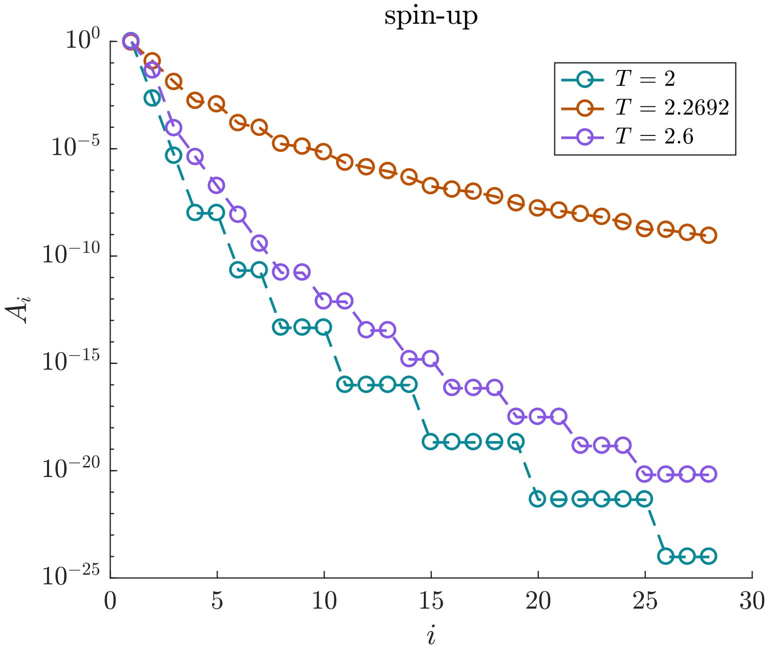
$\beta/\nu = 0.124999$
0.45
0.4
0.35
0.3
0.25
m
10^2
10^3
$\xi(\chi)$



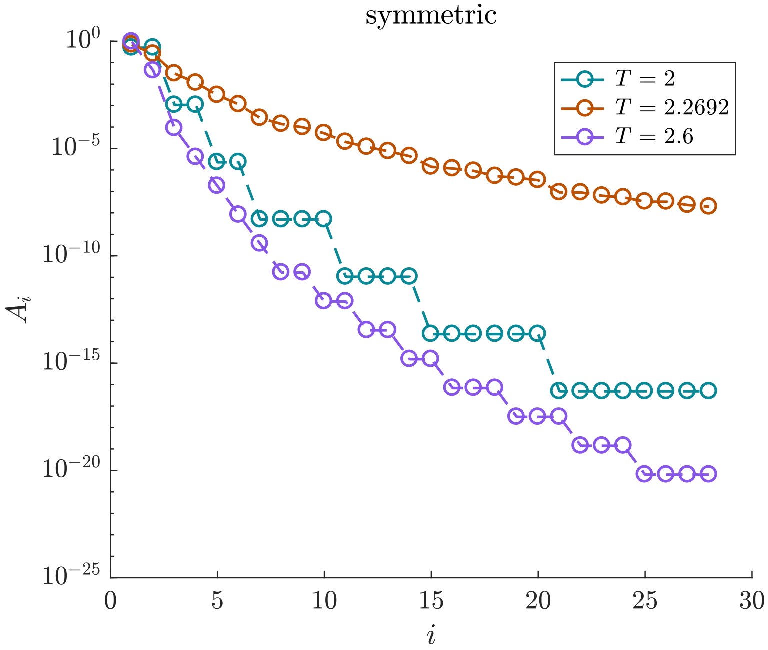
N
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
10^1
10^2
10^3



Fitting a data collapse: Bhattacharjee and Seno 2001



spin-up
$T = 2$
$T = 2.2692$
$T = 2.6$
A_i
i
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
0
5
10
15
20
25
30


symmetric
$T = 2$
$T = 2.2692$
$T = 2.6$
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
A_i
0
5
10
15
20
25
30
i

Postulated exponent $\xi(\chi) \propto \chi^\kappa$

- Andersson, Boman, and Östlund 1999 found

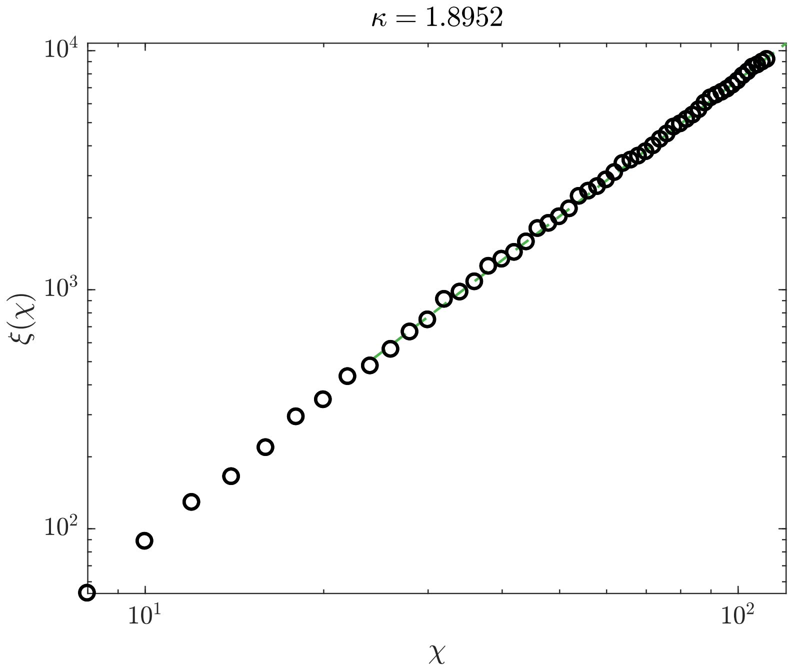
$$\xi(\chi) \propto \chi^{1.3} \tag{1}$$

  for gapless system of free fermions in DMRG calculations.
- Tagliacozzo et al. 2008 numerical evidence for:

$$\kappa_{\text{Ising}} \approx 2, \tag{2}$$
$$\kappa_{\text{Heisenberg}} \approx 1.37. \tag{3}$$

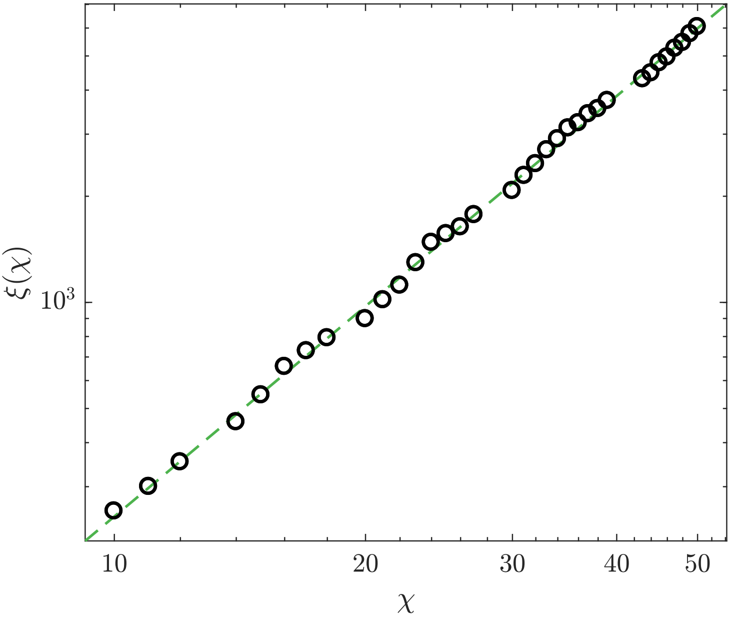
  using iTEBD

Theoretic framework predicting κ by Pollmann et al. 2009

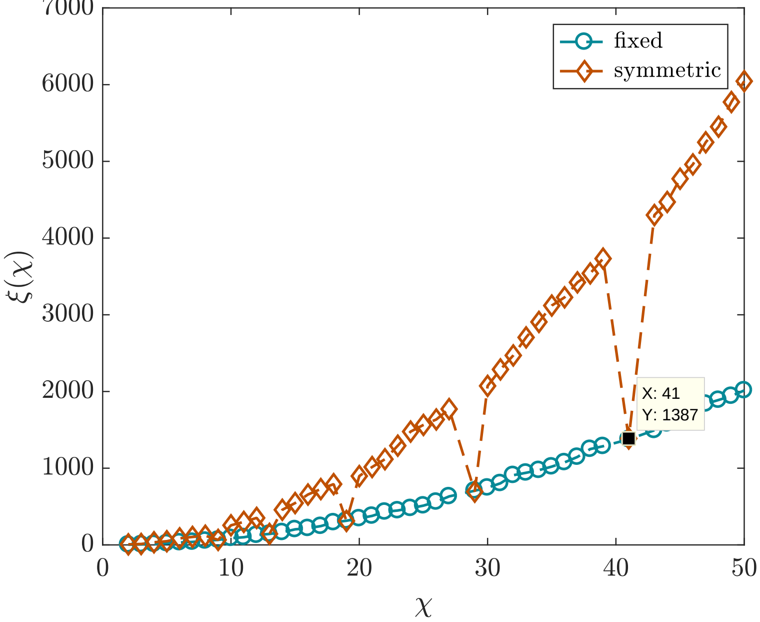


$\kappa = 1.8952$
10^4
10^3
10^2
$\xi(\chi)$
10^1
10^2
χ

Ising model at T_c. Symmetric boundary conditions: $\kappa = 1.9811$.



Ising model. Correlation length at T_c.



References I

Nishino, Tomotoshi and Kouichi Okunishi (1996). "Corner transfer matrix renormalization group method". In: *Journal of the Physical Society of Japan* 65.4, pp. 891–894.

Baxter, Rodney J (1982). *Exactly solved models in statistical mechanics*. Elsevier. Chap. 13.

Baxter, RJ (1978). "Variational approximations for square lattice models in statistical mechanics". In: *Journal of Statistical Physics* 19.5, pp. 461–478.

– (1968). "Dimers on a rectangular lattice". In: *Journal of Mathematical Physics* 9.4, pp. 650–654.

Östlund, Stellan and Stefan Rommer (1995). "Thermodynamic limit of density matrix renormalization". In: *Physical review letters* 75.19, p. 3537.

References II

Nishino, T, K Okunishi, and M Kikuchi (1996). "Numerical renormalization group at criticality". In: *Physics Letters A* 213.1-2, pp. 69–72.

Bhattacharjee, Somendra M and Flavio Seno (2001). "A measure of data collapse for scaling". In: *Journal of Physics A: Mathematical and General* 34.33, p. 6375.

Andersson, Martin, Magnus Boman, and Stellan Östlund (1999). "Density-matrix renormalization group for a gapless system of free fermions". In: *Physical Review B* 59.16, p. 10493.

Tagliacozzo, L et al. (2008). "Scaling of entanglement support for matrix product states". In: *Physical review b* 78.2, p. 024410.

Pollmann, Frank et al. (2009). "Theory of finite-entanglement scaling at one-dimensional quantum critical points". In: *Physical review letters* 102.25, p. 255701.