Dynamics of quasi-particle states in a finite one-dimensional repulsive Bose gas

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ITFA Bachelor's thesis

June 26, 2014

Outline

Introduction

- Motivations
- The Lieb-Liniger model
- Excitations of the ground state
- Quasi-particle state

Results

- Density profile: collapse of quasi-particle
- t^{-1} decay behaviour

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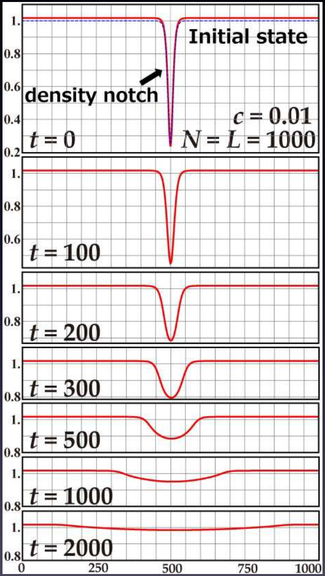
Motivations

J. Sato, E. Kaminishi, and T. Deguchi, Exact quantum dynamics of yrast states in the finite 1D Bose gas, arXiv:1401.4262 [cond-mat.quant-gas]

Decay of quasi-particle



Aim

Take a closer look at decay. Can we find an expression for it?

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N bosons on a ring with contact interaction (a delta peak.)

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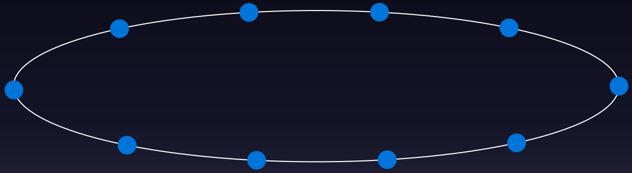

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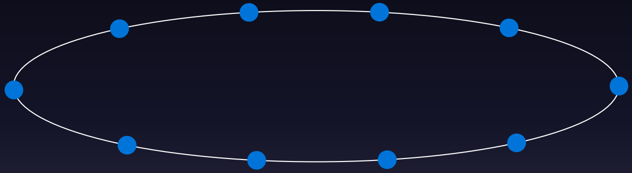

$$H = -\sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + 2c \sum_{i<j} \delta(x_i - x_j)$$

Bethe Ansatz

Solvable through Bethe Ansatz: assume product of plane waves.

$$e^{ikx}$$

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Leads to Bethe equations for bosons' pseudo-momenta:

$$k_j L = 2\pi I_j - 2 \sum_{l=1}^{N} \arctan(\frac{k_j - k_l}{c}) \qquad j = 1, \ldots, N$$

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We label eigenstates by integers I_j.

Ground State

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Ground state for $N = 5$ labeled by:

$$\{I_j\} = \{-2, -1, 0, 1, 2\}$$

Momentum

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$$P = \frac{2\pi}{L} \sum_{j=1}^{N} I_j$$

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Groundstate: I_j's sum to zero:

$$P = 0$$

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Excitations of the ground state

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For hole position m (here $m = 1$):

$$P = \frac{2\pi}{L} m$$

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Quasi-particle state

Sum all one-hole excitations (**momentum eigenstates**) to get a state that is localized in **position**.

$$|\Psi\rangle = \sum_{m=-N}^{N} |P\rangle$$

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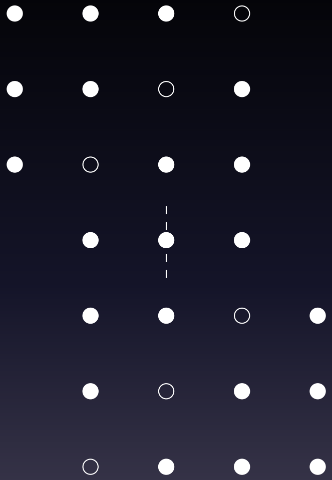
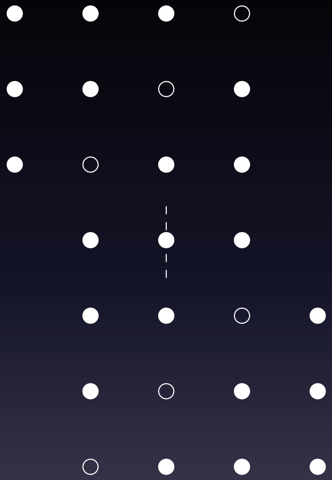
$|P\rangle$ represents the one-hole excitation with momentum $P = \frac{2\pi}{L} m$.





All one-hole excitations for $N = 3$.

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Expectation value of particle density operator

$$\rho(x,t) = \langle \Psi(t)|\rho(x)|\Psi(t)\rangle = \sum_{m,m'=-N}^{N} e^{i(P-P')x - i(E_m - E_{m'})t} \langle P|\rho(0)|P'\rangle$$

Expectation value of particle density operator

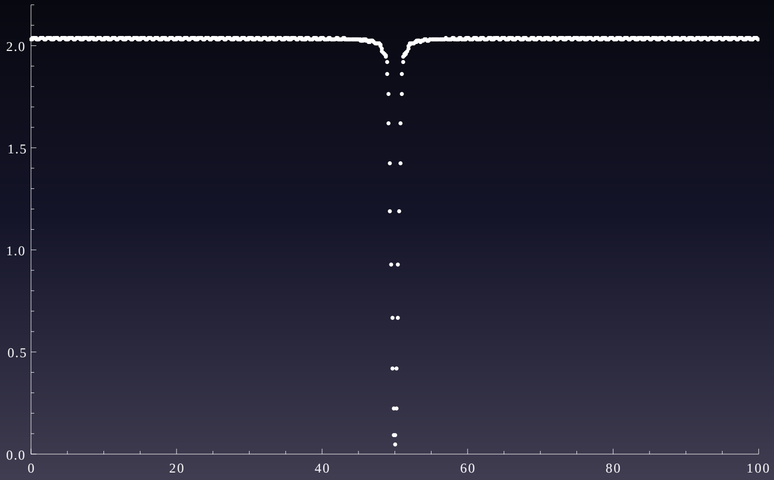
$$\rho(x,t) = \langle \Psi(t) | \rho(x) | \Psi(t) \rangle = \sum_{m,m'=-N}^{N} e^{i(P-P')x - i(E_m - E_{m'})t} \langle P | \rho(0) | P' \rangle$$

$\langle P | \rho(0) | P' \rangle$ and E calculated numerically.

Density Profile

Notch Up Close

Notch depth



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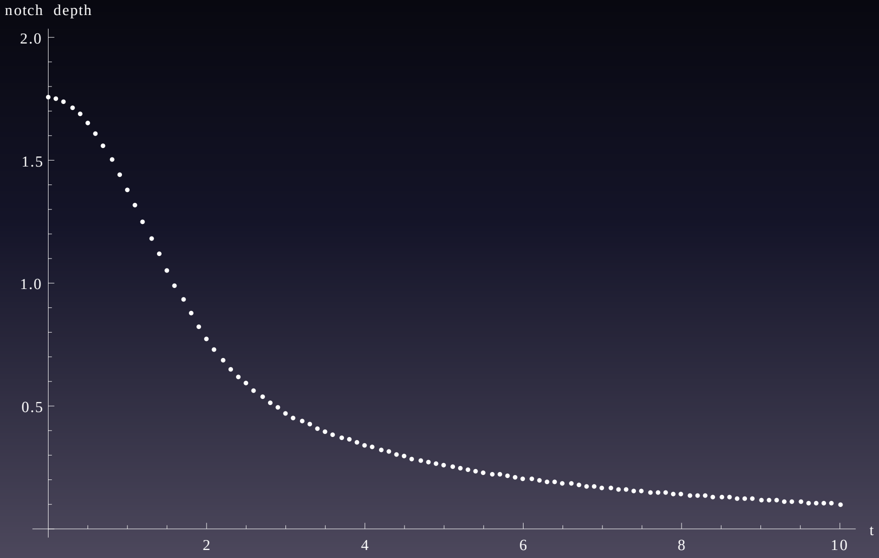
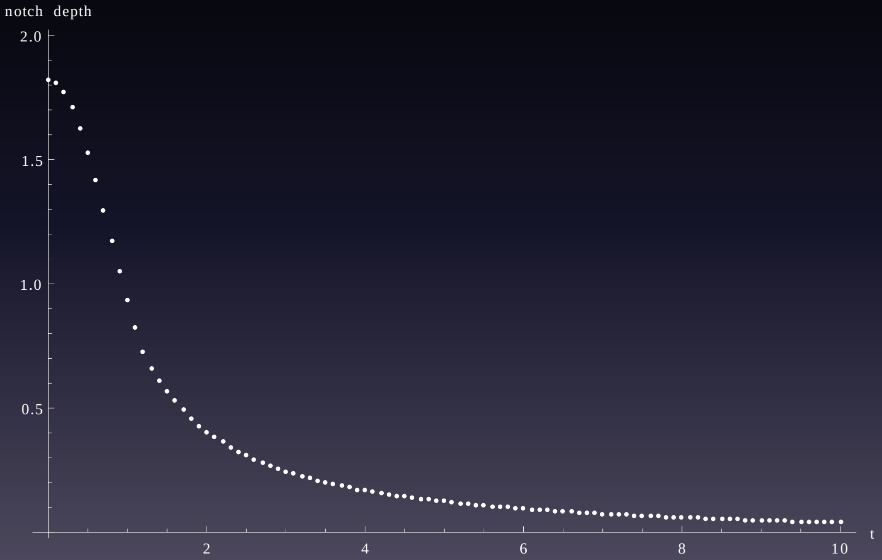
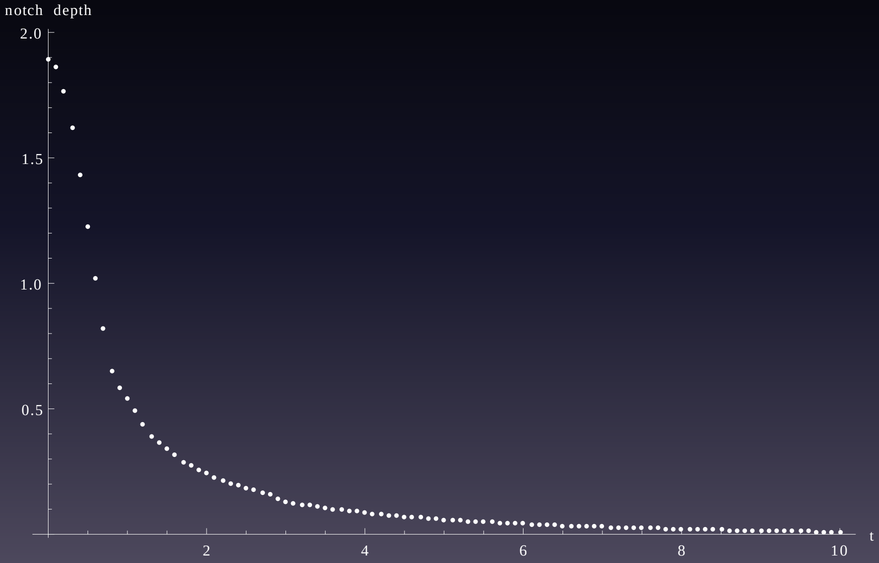
Decay Behaviour $c = 1$



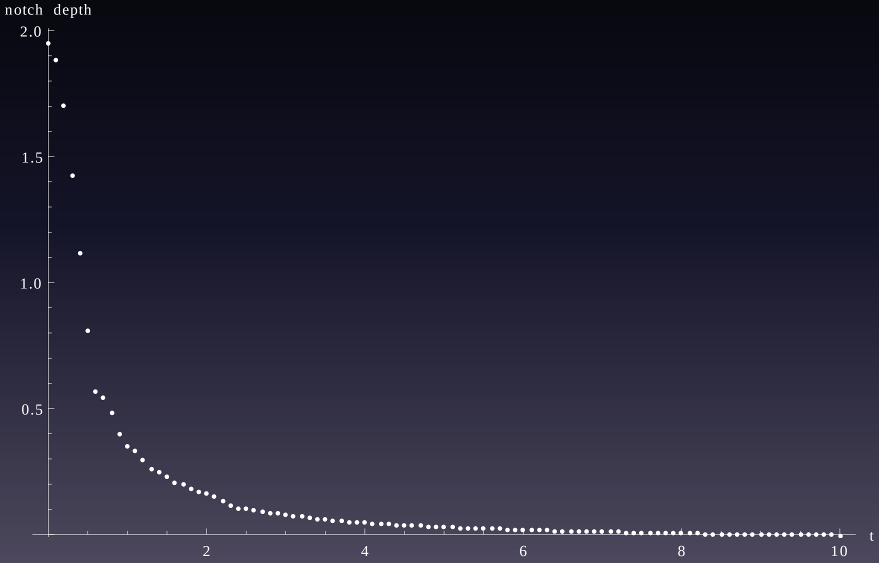
Decay Behaviour $c = 2$



Decay Behaviour $c = 4$



Decay Behaviour $c = 8$



Fit t^{-1} model

$$\frac{A}{t+B}$$

Find values for A and B.

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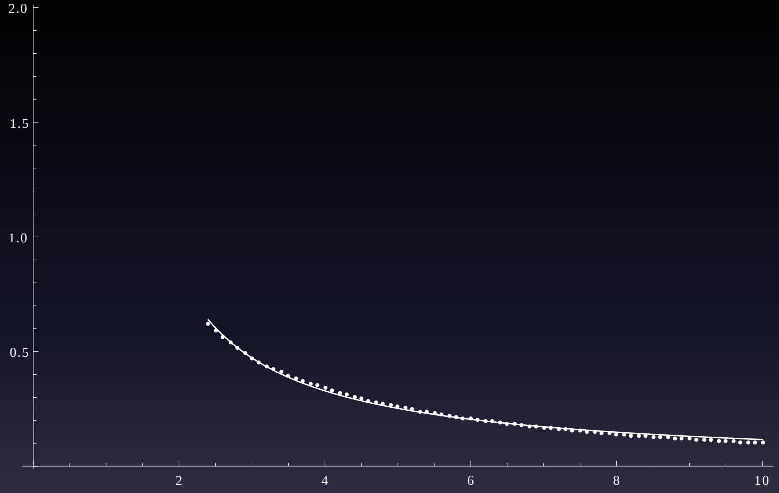
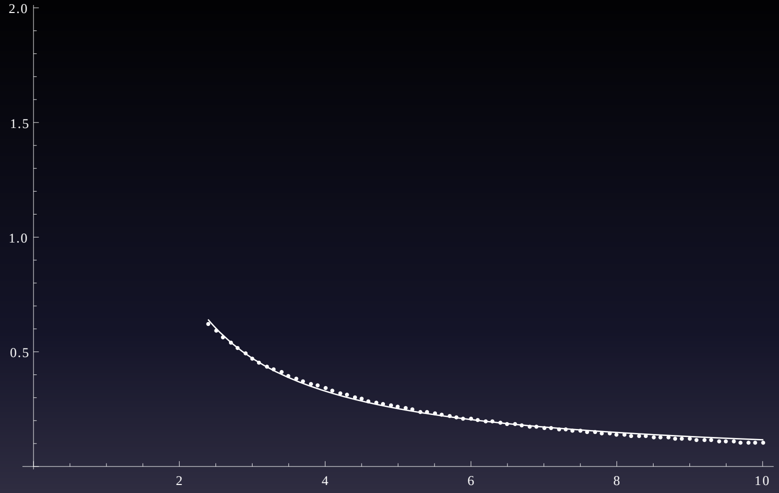
$$\frac{A}{t+B}$$

Find values for A and B.

For large t (for which notch depth is smaller than $\frac{1}{e}$ of initial depth.)


2.0
1.5
1.0
0.5
2
4
6
8
10



For $c = 1$, $N = L = 100$:

$$A = 1.18083, B = 0.517246$$

Derivation of t^{-1} Behaviour

$$\rho(x,t) = \sum_{m,m'=-N}^{N} e^{i(P-P')x - i(E_m - E_{m'})t} \langle P | \rho(0) | P' \rangle$$

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$$\approx \int_{\mathbb{R}^2} dm dm' A_m A_{m'} e^{-i[E(m)-E(m')]t}$$

Stationary Phase Approximation

$$\int_{\mathbb{R}} dm A_m e^{iE(p)t} \approx \frac{\#}{\sqrt{t}}$$

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Double integral gives $\frac{1}{t}$ relation.

Summary

Reproduced results of Sato et al.

Found a relation for decay process in large t limit with simple argument.