Dynamics of quasi-particle states in a finite one-dimensional Bose gas

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ITFA Bachelor's thesis

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Outline

Introduction

- Motivations
- The Lieb-Liniger model
- Excitations of the ground state
- Quasi-particle state

Results

- Density profile: collapse of quasi-particle
- t^{-1} decay behaviour and lifetime parameter τ

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Motivations

J. Sato, E. Kaminishi, and T. Deguchi, Exact quantum dynamics of yrast states in the finite 1D Bose gas, arXiv:1401.4262 [cond-mat.quant-gas]

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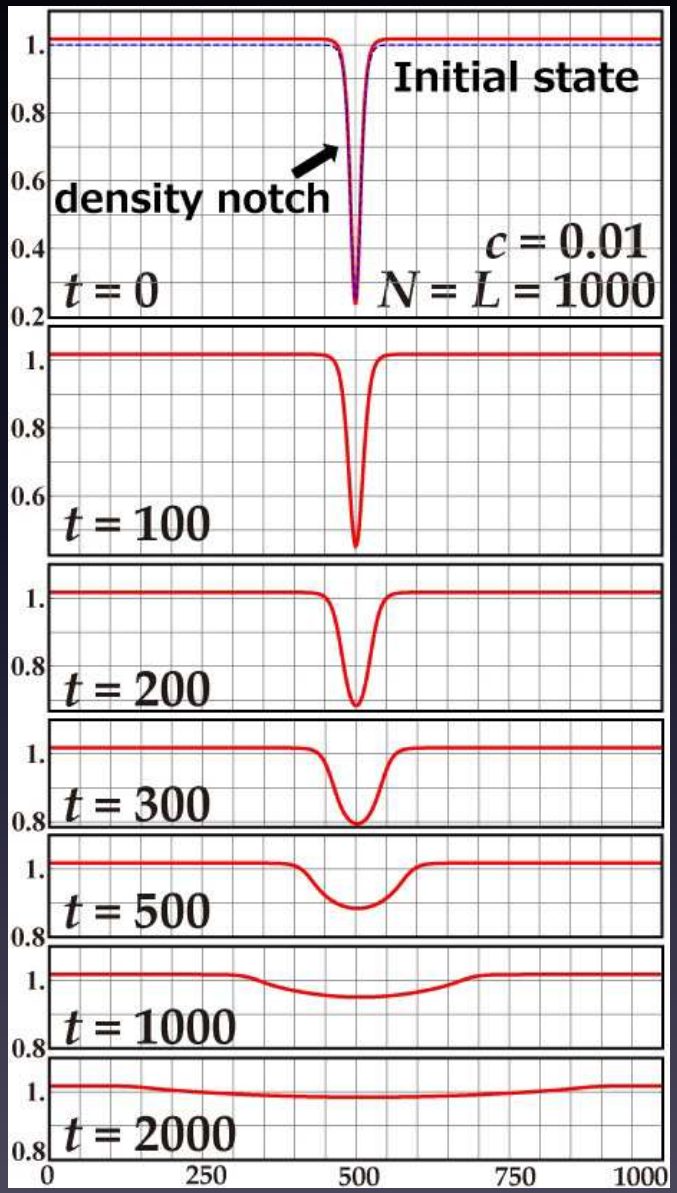
J. Sato, E. Kaminishi, and T. Deguchi, Exact quantum dynamics of yrast states in the finite 1D Bose gas, arXiv:1401.4262 [cond-mat.quant-gas]

"Japanese guys" demonstrate dynamics of a quasi-particle state in 1D Bose gas

Decay of quasi-particle



Aim

Take a closer look at decay. Can we find an expression for it?

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The Lieb-Liniger model

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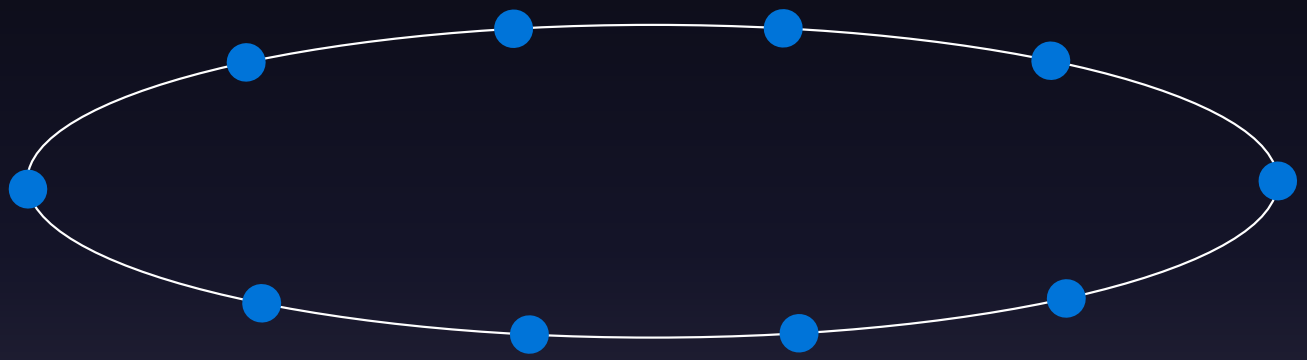

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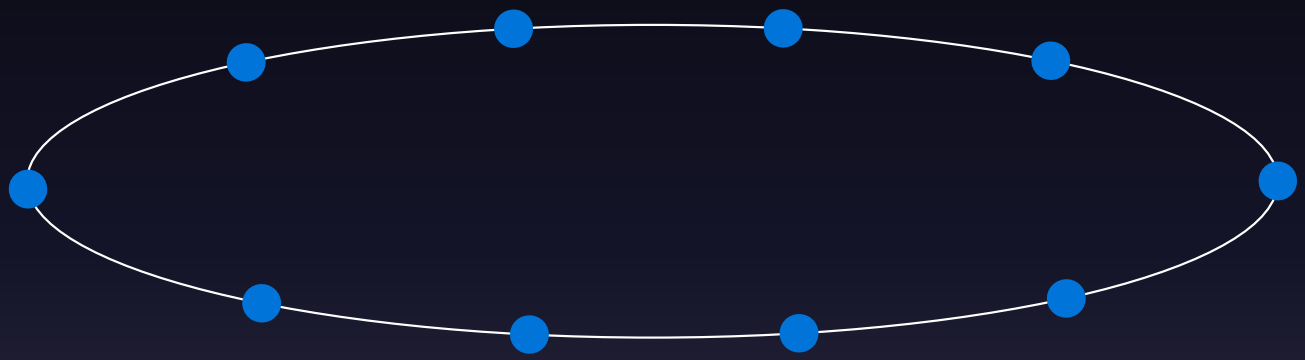

$$H = -\sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + 2c \sum_{i<j} \delta(x_i - x_j)$$

Bethe Ansatz

Solvable through Bethe Ansatz: assume product of plane waves.

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Leads to Bethe equations for bosons' pseudo-momenta:

$$k_j L = 2\pi I_j - 2\sum_{l=1}^{N} \arctan\left(\frac{k_j - k_l}{c}\right) \qquad j = 1, \ldots, N$$

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We label eigenstates by integers I_j.

Ground State

Ground State

Ground state for $N = 5$ labeled by:

$$\{I_j\} = \{-2, -1, 0, 1, 2\}$$

Momentum

Momentum given by:

$$P = \frac{2\pi}{L} \sum_{j=1}^{N} I_j$$

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Groundstate: I_j's sum to zero:

$$P = 0$$

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Excitations of the ground state

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For hole position m (here $m = 1$):

$$P = \frac{2\pi}{L} m$$

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Quasi-particle state

Sum all one-hole excitations (**momentum eigenstates**) to get a state that is localized in **position**.

$$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{m=-N}^{N} |P\rangle$$

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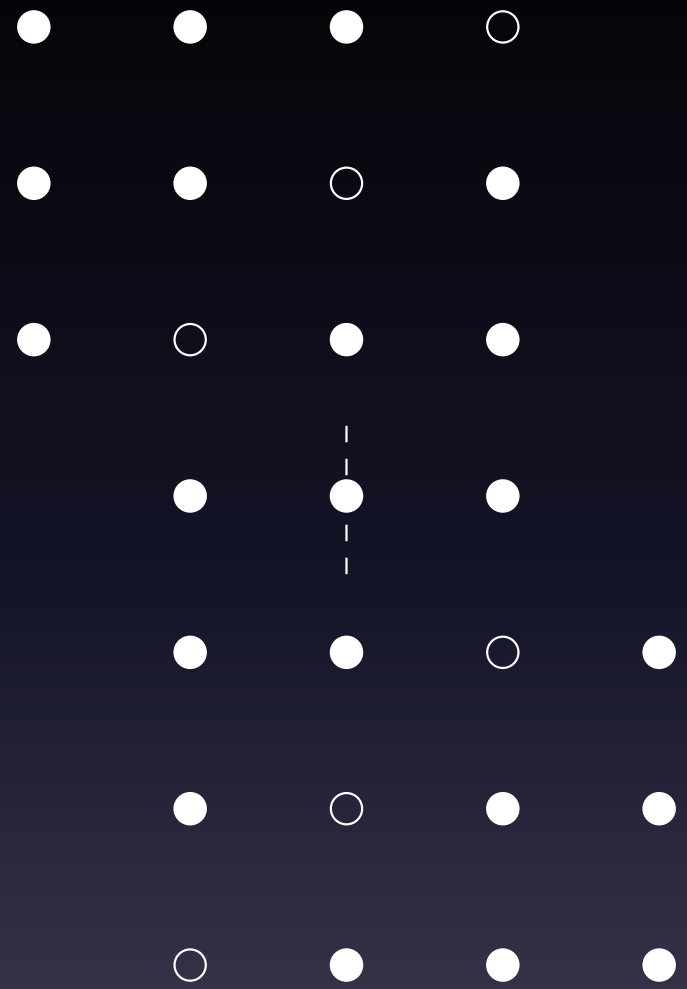
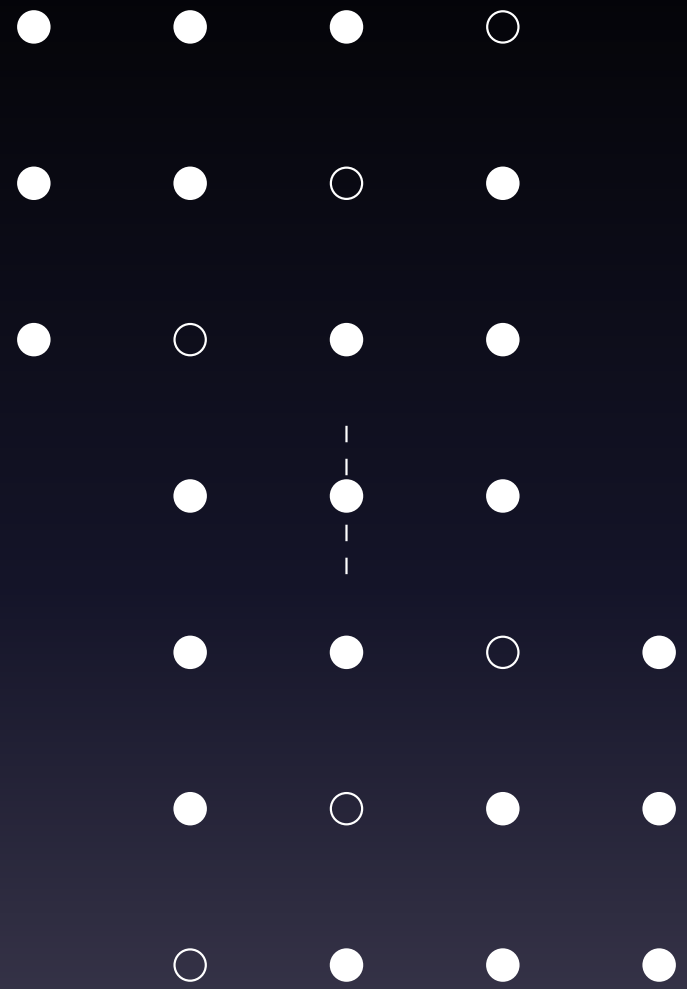
$|P\rangle$ represents the one-hole excitation with momentum $\frac{2\pi}{L} m$.





All one-hole excitations for $N = 3$.

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Density Profile

Make Titles Informative.

Theorem

In left column.

Make Titles Informative.

Theorem

In left column.

Corollary

In right column.
New line

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Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - What we have not done yet.
  - Even more stuff.

For Further Reading I

A. Author.
Handbook of Everything.
Some Press, 1990.

S. Someone.
On this and that.
Journal on This and That. 2(1):50–100, 2000.