Dynamics of quasi-particle states in a finite one-dimensional Bose gas

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ITFA Bachelor's thesis

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Outline

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Motivations

J. Sato, E. Kaminishi, and T. Deguchi, Exact quantum dynamics of yrast states in the finite 1D Bose gas, arXiv:1401.4262 [cond-mat.quant-gas]

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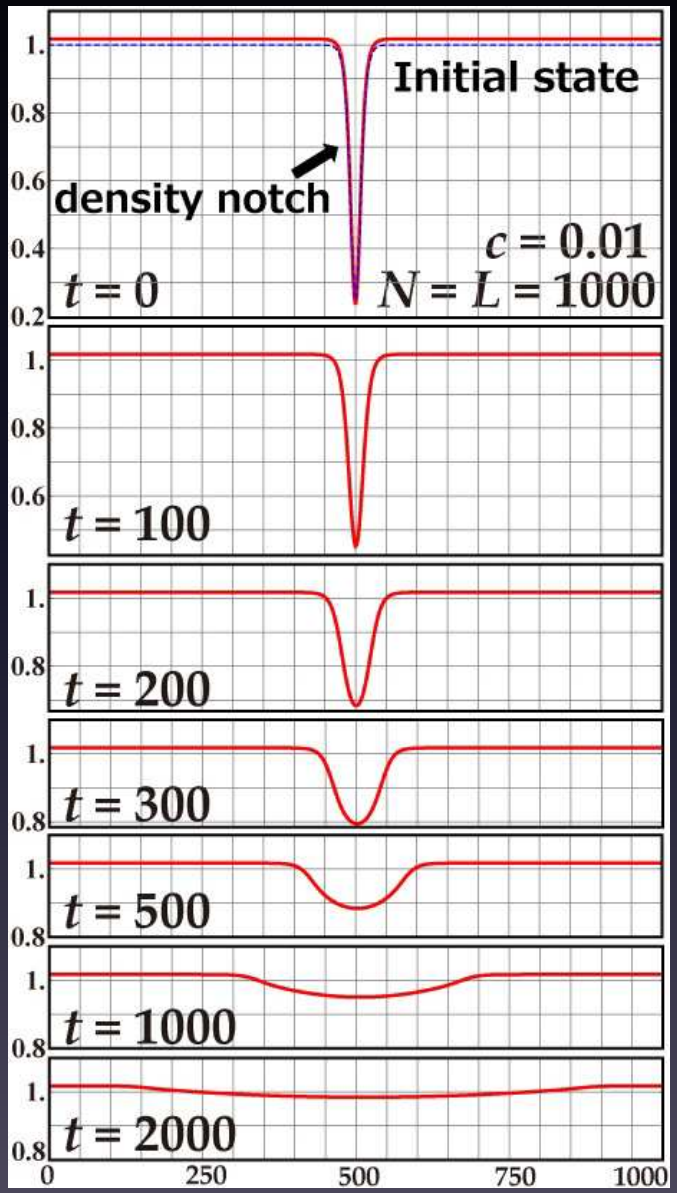
"Japanese guys" demonstrate dynamics of a quasi-particle state in 1D Bose gas

Decay of quasi-particle



Aim

Take a closer look at decay. Can we find an expression for it?

Outline

The Lieb-Liniger model

N bosons on a ring with contact interaction (a delta peak.)

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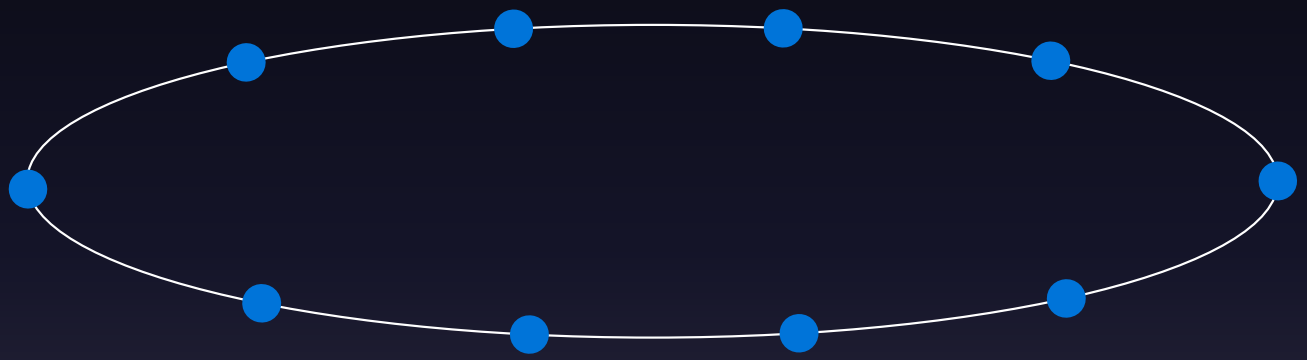

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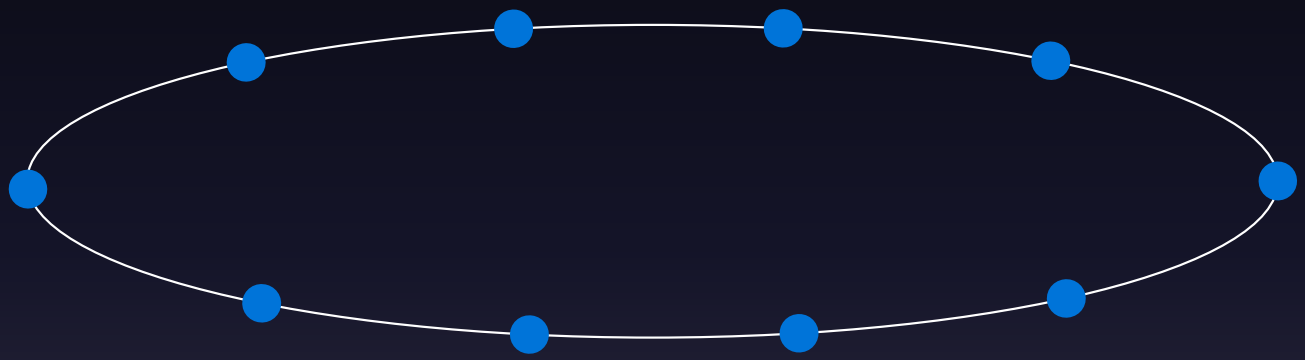

$$H = -\sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} + 2c \sum_{i<j} \delta(x_i - x_j)$$

Bethe Ansatz

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Leads to Bethe equations for bosons' pseudo-momenta:

$$k_j L = 2\pi I_j - 2\sum_{l=1}^{N} \arctan\left(\frac{k_j - k_l}{c}\right) \qquad j = 1, \ldots, N$$

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We label eigenstates by integers I_j.

Ground State

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Ground state for $N = 5$ labeled by:

$$\{I_j\} = \{-2, -1, 0, 1, 2\}$$

Momentum

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Groundstate: I_j's sum to zero:

$$P = 0$$

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Excitations of the ground state

One-hole excitations: create a hole somewhere.

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For hole position m (here $m = 1$):

$$P = \frac{2\pi}{L} m$$

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Quasi-particle state

Sum all one-hole excitations (**momentum eigenstates**) to get a state that is localized in **position**.

$$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{m=-N}^{N} |P\rangle$$

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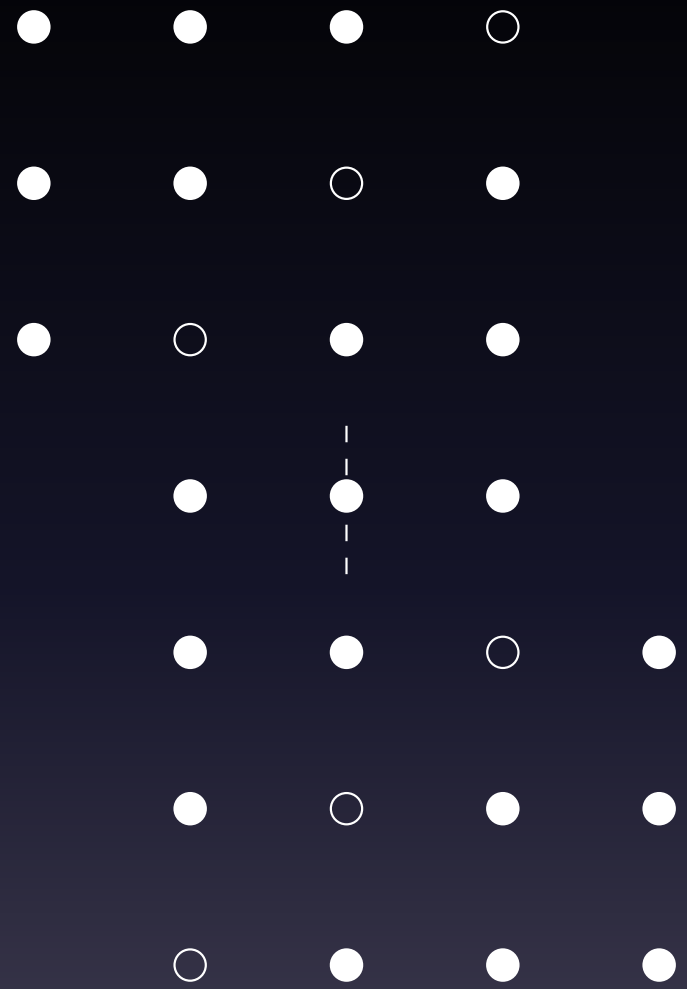
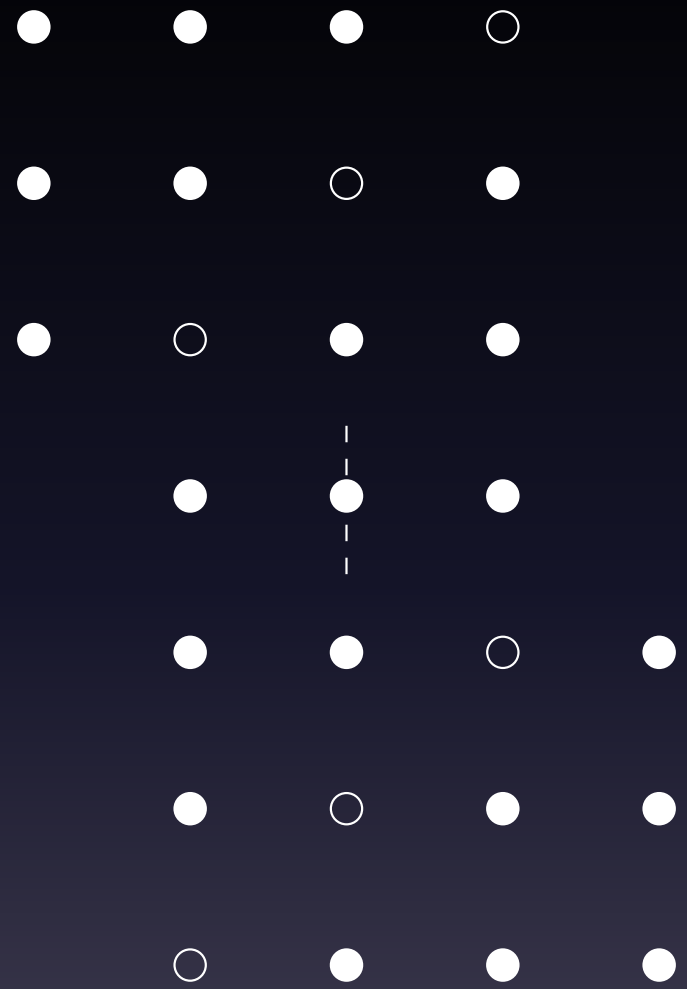
$|P\rangle$ represents the one-hole excitation with momentum $P = \frac{2\pi}{L} m$.





All one-hole excitations for $N = 3$.

Outline

Expectation value of particle density operator

$$\langle \Psi | \rho(x) | \Psi \rangle = \sum_{m,m'=-N}^{N} e^{i(P-P')x - i(E_m - E_{m'})t} \langle P | \rho(0) | P' \rangle$$

Density Profile

Notch Up Close

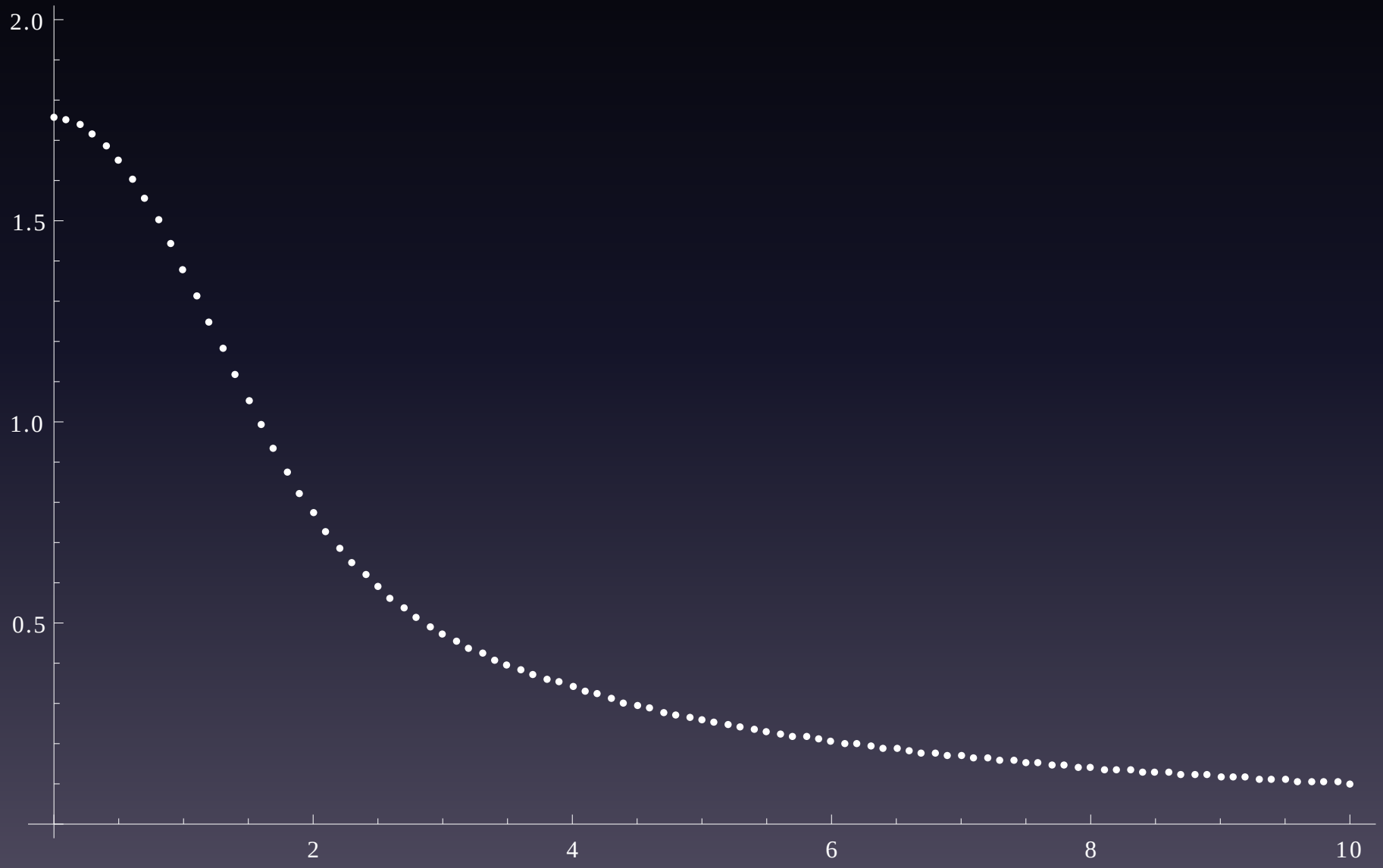
Notch depth

TODO:

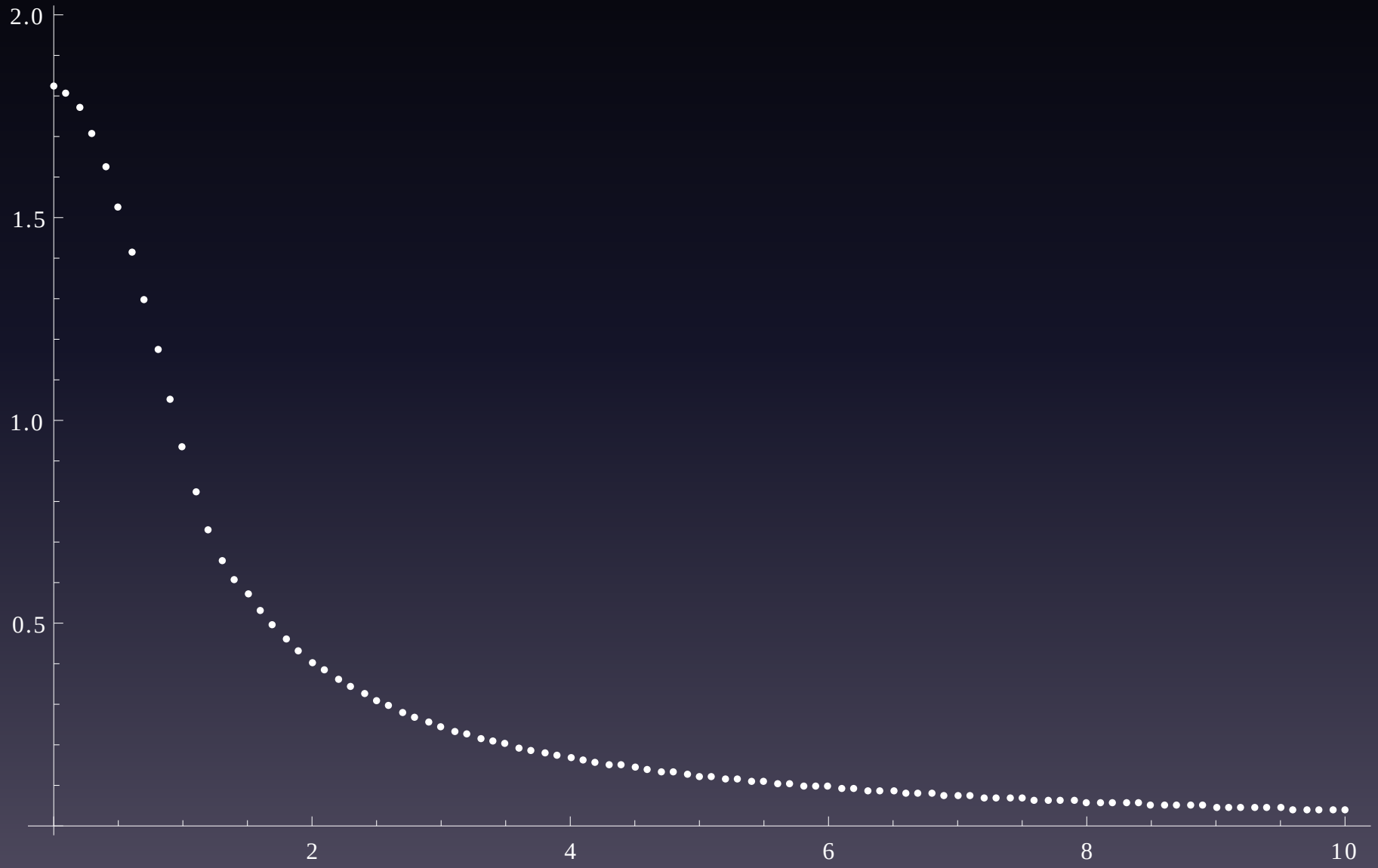
- betere gif ($0 < t < 10$)
- sowieso mooiere plaatjes met gnuplot
- Plaatje notch depth
- afleiding stationary phase approx?
- expectation value density operator erin

Outline

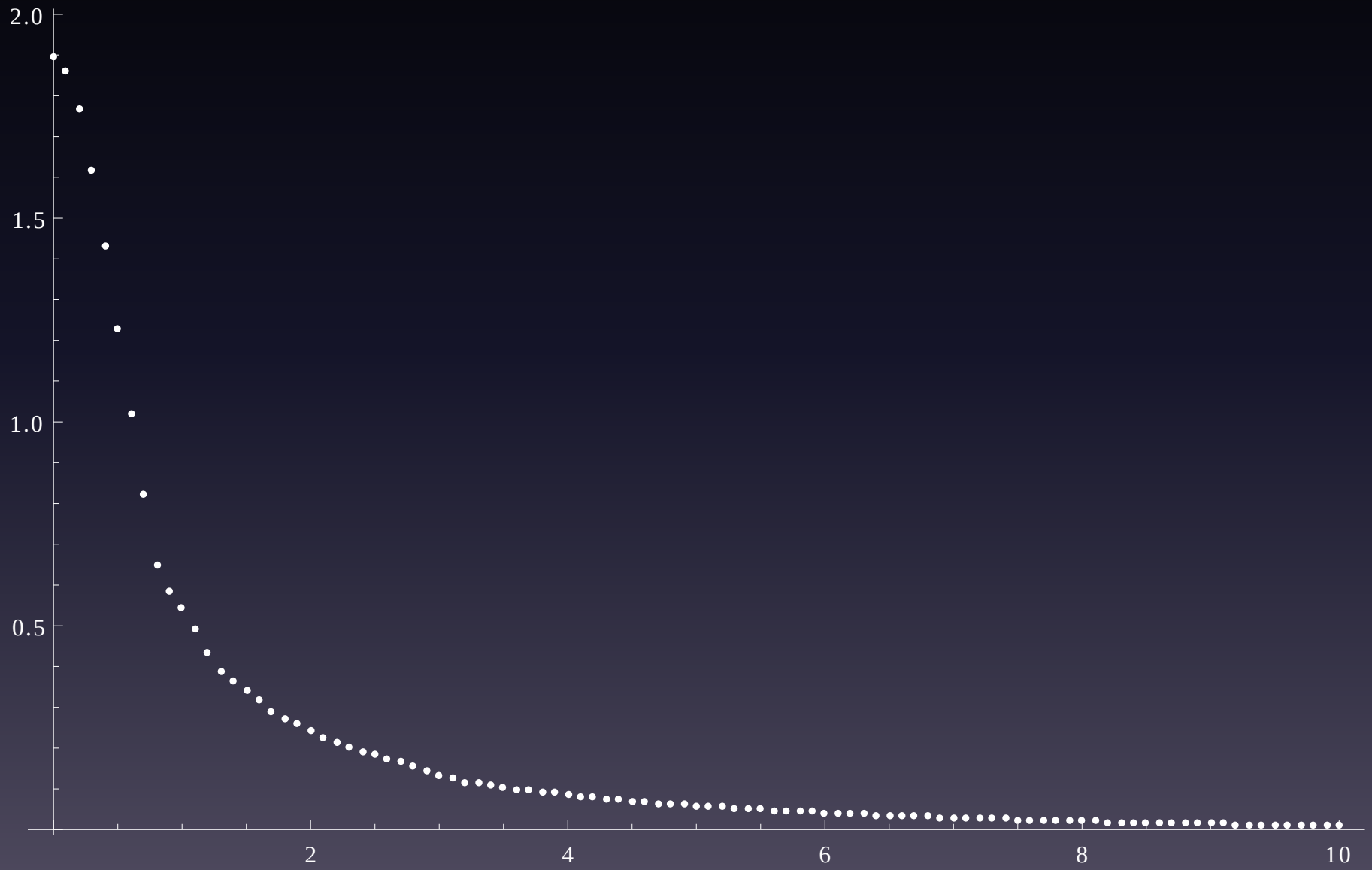
Decay Behaviour $c = 1$



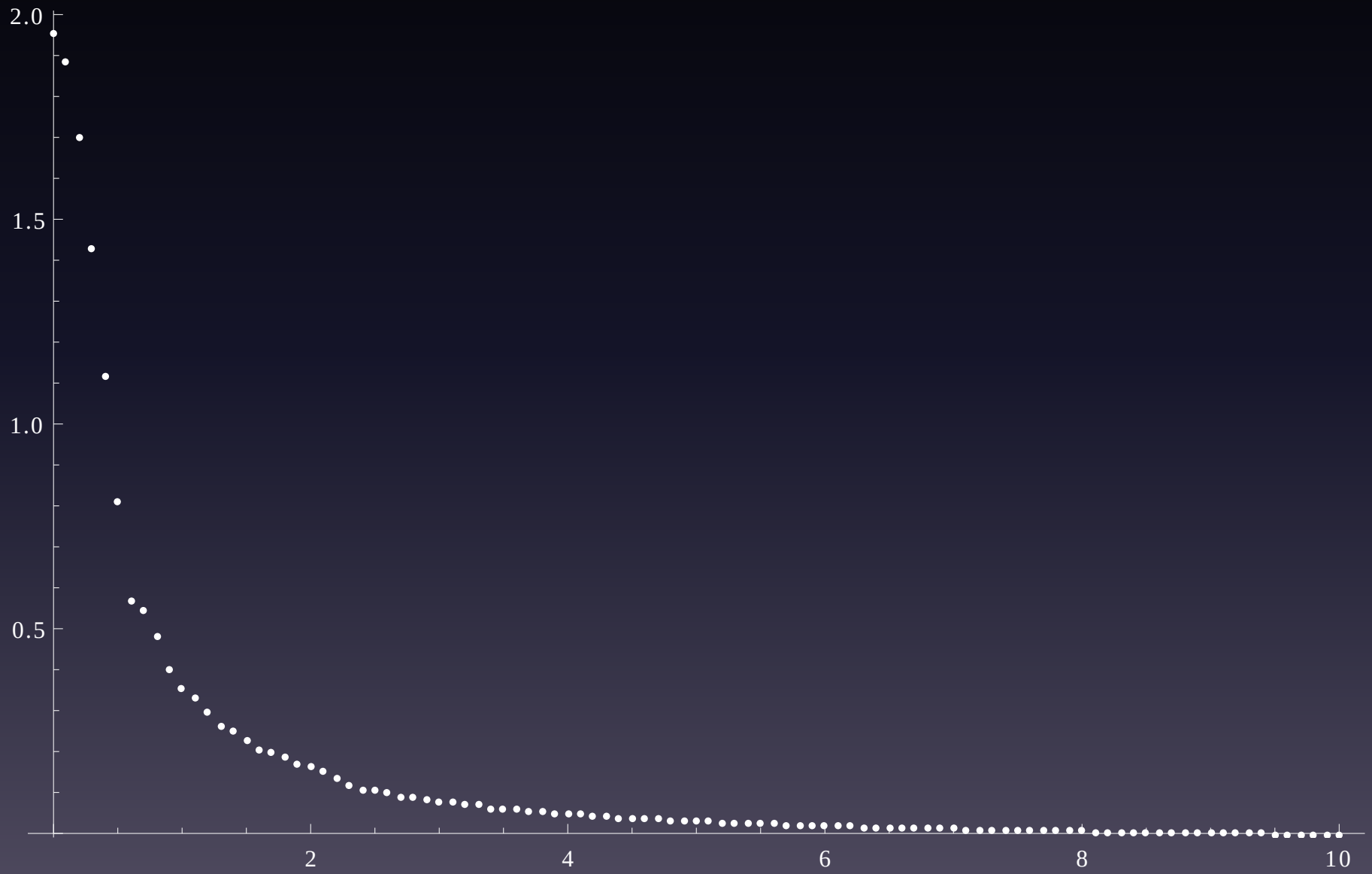
Decay Behaviour $c = 2$



Decay Behaviour $c = 4$



Decay Behaviour $c = 8$



Fit t^{-1} model

$$\frac{A}{t + B}$$

Find values for A and B.

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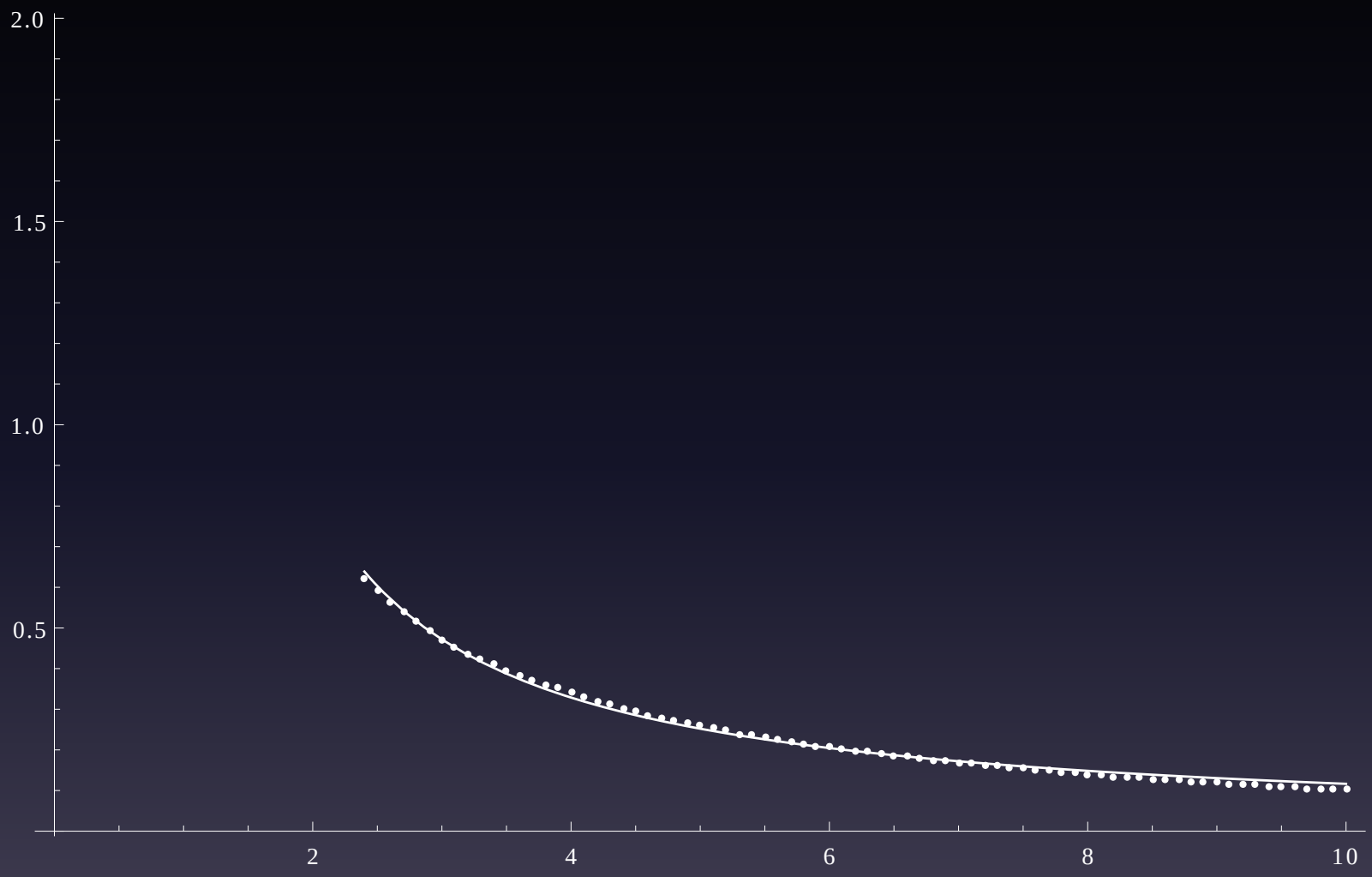
For large t (for which notch depth is smaller than $\frac{1}{e}$ of initial depth.)


2.0
1.5
1.0
0.5
2
4
6
8
10