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# 1
# Results clock model

## 1.1 Abstract

We present results of scaling in bond dimension and system size with the CTMRG algorithm for the five- and six-state clock model.

## 1.2 Introduction

In the field of phase transitions and critical phenomena, the two-dimensional topological phase transition discovered by Kosterlitz and Thouless [1, 2] received much attention. This phase transition is characterized not by an order parameter which indicates a breaking of symmetry, but by the proliferation of topological defects.

The XY model consists of planar rotors on the square lattice. It exhibits the Kosterlitz-Thouless (BK) phase transition and by the Mermin-Wagner-Hohenberg theorem the symmetry of the ground state is broken for all temperatures, due to the $O(2)$ (planar rotational) symmetry of the potential [3, 4].

The $q$-state clock model possesses the discrete $\mathbb{Z}_q$ symmetry and is an interpolation between the Ising model, which corresponds to $q = 2$, and the XY model, which corresponds to $q \to \infty$. Its energy function is given by

$$H_q = -\sum_{\langle ij \rangle} \cos(\theta_i - \theta_j). \tag{1.1}$$

It has been proven that for high enough $q$, this model indeed exhibits a Kosterlitz-Thouless transition [5].

In the Villain formulation of the potential [6], it has been proven that the model has two phase transitions for $q \geq 5$: a symmetry broken phase for

$T < T_1$, an intermediate phase with power law decay of the correlation function, and a high-temperature phase with exponential decay of the correlation function for $T > T_2$. The transition at $T_2$ is a BK-transition, and for a broad range of temperatures, the thermodynamic behaviour becomes identical to the XY model for high enough $q$ [7].

## 1.3 Previous work

## 1.4 Results

# Bibliography


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