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Methods

1.1 Abstract

We describe the technical details of the algorithms used to compute quantities of interest. We report the convergence behaviour of the algorithms and discuss validity and sources of error.

1.2 Technical details of the CTMRG algorithm

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1.3 Stopping criterion for the corner transfer matrix renormalization group algorithm

1.3.1 Simulations with finite bond dimension

The convergence of the CTMRG algorithm with fixed bond dimension m (the infinite system algorithm) can be defined in multiple ways (*cite*). In this thesis, the convergence after step i of the algorithm is defined as

$$c_i = \sum_{\alpha=1}^{m} |s_\alpha^{(i)} - s_\alpha^{(i-1)}|, \tag{1.1}$$

where s_α are the singular values of the corner transfer matrix A. If the convergence falls below some threshold ϵ, the algorithm terminates.

The assumption is that once the singular values stop changing to some precision, the optimal projection is sufficiently close to its fixed point and the

Figure 1.1: hallootjes

transfer matrices A and T represent an environment only limited by the length scale given by m, i.e.

$$\xi(m) \ll N \tag{1.2}$$

is satisfied.

The convergence of the order parameter of the Ising model is shown in Figure 1.1.

Cross check with correlation length, report on boundary conditions

1.3.2 Simulations with finite system size

In the finite-system algorithm, we require

$$N \ll \xi(m), \tag{1.3}$$

so the question becomes at which m the results are sufficiently converged in m. Throughout this work, we have used the residual probability (also called truncation error)

$$P(m)^{(i)} = \frac{\sum_{\alpha=m+1}^{dm} (s_\alpha^{(i)})^2}{\sum_{\alpha=1}^{dm} (s_\alpha^{(i)})^2}, \tag{1.4}$$

which quantifies the fraction of the spectrum of the corner transfer matrix that is thrown away, as a measure of how accurate the transfer matrices represent the finite system of size N. Here, d is the dimension of the local tensors ($d = 2$ for the Ising model).

If, for a given m, we have

$$P(m) < P_{\max} \tag{1.5}$$

we deem the result accurate enough.

In the limit $m \to d^n$, with

$$n = \frac{N-1}{2} \tag{1.6}$$

the number of algorithm steps, we obtain the exact result for the transfer matrices and hence the partition function, i.e. $P(m) \to 0$.

To justify that for small enough $P_{\max}$, we obtain good results for a wide range of N, figure bla bla.