1
Results

1.1 Abstract

Cite more

We present finite-size scaling results using the corner transfer matrix renormalization group method on two-dimensional classical square lattices [4]. We compare the results of conventional finite-size scaling in the system size N with scaling in the number of states kept during the renormalization step of the algorithm, denoted by m. Such a comparison was first done in [3]. We highlight the areas in which method excels over the other.

Calculate critical temperature and exponents using information that is directly extractable from the corner transfer matrix.

1.2 Introduction

The first direct comparison of finite-size scaling in the system size N with scaling in the bond dimension of the corner transfer matrix renormalization group method m was done in [3]. In explaining the basic concepts, we largely follow this paper.

The error in the approximation of the partition function (and thus all thermodynamic quantities) in the thermodynamic limit with the corner transfer matrix method depends on two characteristic length scales. The first is the size of the system N. After n steps of the infinite-system algorithm, we have

$$N = 2n + 1. \tag{1.1}$$

The second length scale is related to the finite bond dimension m. Baxter [1], and later Östlund and Rommer [5] (in the context of one-dimensional quantum systems) showed that in the thermodynamic limit, CTMRG and

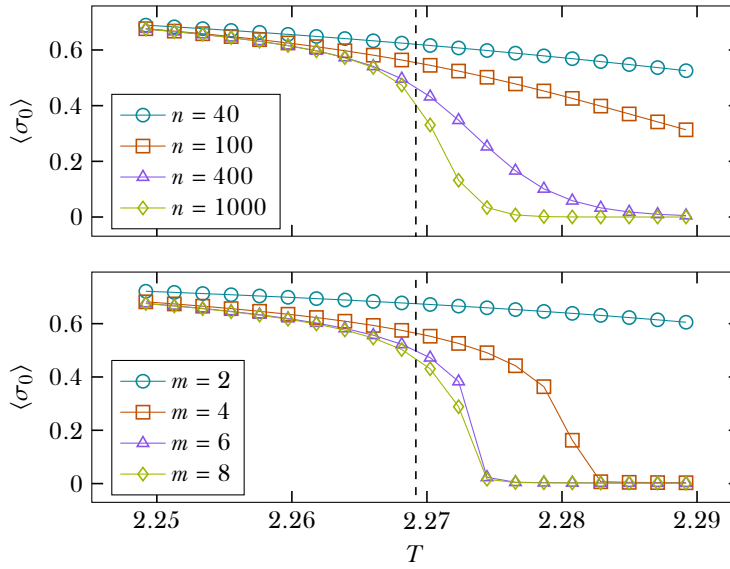

Figure 1.1: Upper panel: expectation value of the central spin $\langle\sigma_0\rangle$ after n CTMRG steps. m is chosen such that the truncation error is smaller than 10^{-6}. Lower panel: $\langle\sigma_0\rangle$ for systems with bond dimension m.

where to explain convergence criteria?

DMRG are variational optimizations in the space of matrix product states.

Can extend this idea a bit.

It is known that an MPS-ansatz with finite bond dimension inherently limits the correlation length of the system to a finite value [8]. Hence, thermodynamic quantities obtained from the CTMRG algorithm with finite m, in the limit $N \to \infty$, cannot diverge and must show finite-size effects similar to those of some effective finite system of size $N_{\text{eff}}(m)$ depending on the bond dimension m.

Figure 1.1 shows the behaviour of the order parameter of the two-dimensional

Ising model for systems of finite-size, where m has been chosen such that the truncation error is smaller than 10^{-6}, and for systems of finite m, where the result is converged in the system size N.

In the thermodynamic limit (corresponding to infinite m and N), we have the following expression for the correlation length of a classical system [2]

$$\xi(T) = \frac{1}{\log\left(\frac{T_0}{T_1}\right)}. \tag{1.2}$$

Here, T_0 and T_1 are the largest and second-largest eigenvalues of the row-to-row transfer matrix T, respectively. With N tending towards infinity and finite m, near the critical point $\xi(T)$ should obey a scaling law of the form

$$\xi(T, m) = N_{\text{eff}}(m)\mathcal{F}\left(\frac{N_{\text{eff}}(m)}{\xi(T)}\right) \tag{1.3}$$

with

$$\mathcal{F}(x) = \begin{cases} \text{const} & \text{if } x \to 0, \\ x^{-1} & \text{if } x \to \infty. \end{cases} \tag{1.4}$$

Hence, the effective length scale corresponding to the finite bond dimension m is proportional to the correlation length of the system at the critical point $t = 0$.

$$N_{\text{eff}}(m) \propto \xi(T = T_c, m). \tag{1.5}$$

To test this assumption, the authors of [3] propose a scaling relation for the order parameter M at the critical temperature of the form

$$M(N, m) = N^{-\beta/\nu}\mathcal{G}(\xi(m)/N) \tag{1.6}$$

with

$$\mathcal{G}(x) = \begin{cases} \text{const} & \text{if } x \to \infty, \\ x^{-\beta/\nu} & \text{if } x \to 0. \end{cases} \tag{1.7}$$

Replicate Nishino's results. Get β/ν in both finite N and m regime.

1.3 Exponent κ and finite-entropy scaling

In the asymptotically large m regime, it is proven that the correlation length for critical systems scales as

$$\xi \propto m^{\kappa}, \tag{1.8}$$

where κ depends only on the central charge of the phase transition [6]. In [7], the authors show that already for modest m, this relation holds approximately.

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