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Maximum Likelihood Estimate

1.

Example 1: Suppose that X is a discrete random variable with the following probability mass function: where $0 \le \theta \le 1$ is a parameter. The following 10 independent observations

X	0	1	2	3
$P(X)$	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

were taken from such a distribution: (3,0,2,1,3,2,1,0,2,1). What is the maximum likelihood estimate of θ.

Solution: Since the sample is (3,0,2,1,3,2,1,0,2,1), the likelihood is

$$\begin{aligned} L(\theta) = \quad & P(X=3)P(X=0)P(X=2)P(X=1)P(X=3) \\ \times \quad & P(X=2)P(X=1)P(X=0)P(X=2)P(X=1) \end{aligned} \tag{2}$$

Substituting from the probability distribution given above, we have

$$L(\theta) = \prod_{i=1}^{n} P(X_i|\theta) = \left(\frac{2\theta}{3}\right)^2 \left(\frac{\theta}{3}\right)^3 \left(\frac{2(1-\theta)}{3}\right)^3 \left(\frac{1-\theta}{3}\right)^2$$

L(θ) is not easy to maximize. So take the log of L and find the parameter $\hat{\theta}$ that maximizes L.

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2.

Example 2: Suppose $X_1, X_2, \cdots, X_n$ are i.i.d. random variables with density function $f(x|\sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|x|}{\sigma}\right)$, please find the maximum likelihood estimate of σ.

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3.
Take n coin flips, x_1, x_2, …, x_n. The number of heads and tails are n_0 and n_1, respectively. θ is the probability of getting heads, and thus the probability of tails is 1 – θ.

Find the MLE estimate of θ assuming that the coin flips are independent.

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4. The Pareto distribution is given as below:

$$f(x|x_0, \theta) = \theta x_0^{\theta} x^{-\theta-1}, \quad x \geq x_0, \quad \theta > 1$$

Find the value of the parameter $\theta = \hat{\theta}$, that maximizes the likelihood of the estimate.

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5.

Exercise 1: Let $X_1, \cdots, X_n$ be an i.i.d. sample from a Poisson distribution with parameter λ, i.e.,

$$P(X = x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}.$$

Please find the MLE of the parameter λ.

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6.

Let $X_1, X_2, X_3 \ldots\ldots X_n$ be a random sample from the exponential distribution with p.d.f.

$$f(x;\theta) = \frac{1}{\theta} e^{\frac{-x}{\theta}} 0 < x < \infty, \theta \in \Omega = \{\theta | 0 < \theta < \infty\}$$

Find the MLE of θ, assuming independence of X_1, X_2, X_3, … , X_n