VC Dimension Examples

If X = R (the set of real numbers).

Class 0 | Class 1

Hypothesis H1:
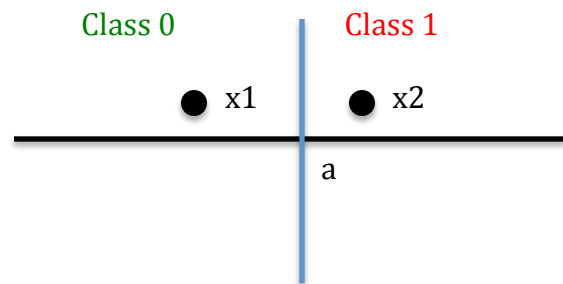   If x > a then y=1 else y=0

a

One point → find dichotomy
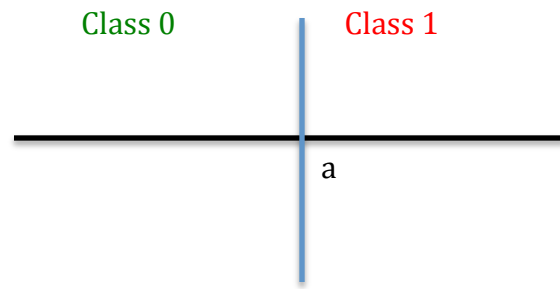
        The point may be class 1 or 0. We can represent it by changing value of a

Two points → find dichotomy (assume x2 > x1 on the axis)

Class of x1	Class of x2	Can be represented
0	0	Yes (a should be greater than both)
0	1	Yes (a can be 0.5)
1	0	No (can not be represented)
1	1	Yes (a can be 0)

Therefore, VC=1

Class 0   Class 1

● x1   ● x2

a

Now change hypothesis as:

If x > a then y = 1 else y = 0 OR
If x > a then y = 0 else y = 1

It can represent VC=2.

Can it represent 1 0 1 (i.e 3 points with in increasing order with classes 1 0 1)?

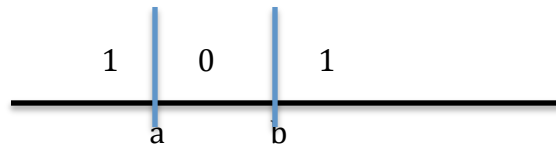What about hypothesis

H3: if $a < x < b$ then $y = 1$ else $y = 0$

Can it represent points 1 0 1

Remember the hypothesis is like a closed rectangle, any point within the rectangle is class 1.

What about –

H4: if $a < x < b$ then $y = 1$ else $y = 0$
or, if $a < x < b$ then $y = 0$ else $y = 1$

Second part of the above hypothesis can represent 1 0 1 as inside the box, it can be 0 and outside 1.

$$
\begin{array}{c c c}
1 & 0 & 1 \\
& a \quad b &
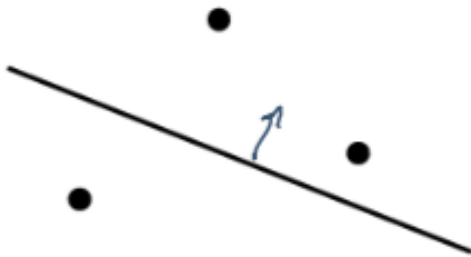\end{array}
$$

What about lines on a plane?

VC = 3 (it can shatter any combination)
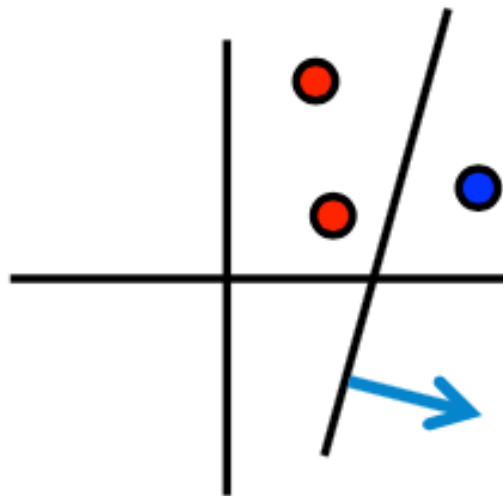
VC Dimension of a straight line classifier

What is VC dimension of lines in a plane?

$H_2 = \{ ((w_0 + w_1x_1 + w_2x_2) > 0 \rightarrow \quad y=1) \}$

Ask yourself, can it handle any dichotomy of 3 points?

Yes.

What about 4 points i.e. VC dim = 4? No