Assignment 4 Part I

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$$
\begin{aligned}
E_{agg}(x) &= E\left[\left\{\frac{1}{M}\sum_{i=1}^{M}\epsilon_i(x)\right\}^2\right] \\
&= \frac{1}{M^2}E\left[\left\{\sum_{i=1}^{M}\epsilon_i(x)\right\}^2\right] \\
&= \frac{1}{M^2}E\left[\sum_{i=1}^{M}\epsilon_i^2(x) + \sum_{i=1}^{M}\sum_{j=1}^{M}\epsilon_i(x)\epsilon_j(x)\right] \qquad // \ \forall i \neq j, E[\epsilon_i(x)\epsilon_j(x)] = 0 \\
&= \frac{1}{M^2}E\left[\sum_{i=1}^{M}\epsilon_i^2(x)\right] \\
&= \frac{1}{M^2}\sum_{i=1}^{M}E[\epsilon_i^2(x)] \qquad // \ E_{avg} = \frac{1}{M}\sum_{i=1}^{M}E[\epsilon_i^2(x)] \\
&= \frac{1}{M}E_{avg}
\end{aligned}
$$

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Since all *convex* function have following inequality:

$$f\left(\sum_{i=1}^{M}\lambda_i x_i\right) \leq \sum_{i=1}^{M}\lambda_i f(x_i)$$

Given that $f : x \to x^2$ is a convex function.

$$\begin{aligned}
E_{agg}(x) &= E\left[\left\{\frac{1}{M}\sum_{i=1}^{M}\epsilon_i(x)\right\}^2\right] \\
&= \frac{1}{M^2}E\left[\left\{\sum_{i=1}^{M}\epsilon_i(x)\right\}^2\right] \\
&= \frac{1}{M^2}E\left[f\left\{\sum_{i=1}^{M}\epsilon_i(x)\right\}\right] \\
&\leq \frac{1}{M^2}E\left[\sum_{i=1}^{M}f\big(\epsilon_i(x)\big)\right] \\
&= \frac{1}{M}E_{avg}
\end{aligned}$$