

Assignment 4 Part I

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$$\begin{aligned} E_{agg}(x) &= E \left[\left\{ \frac{1}{M} \sum_{i=1}^M \epsilon_i(x) \right\}^2 \right] \\ &= \frac{1}{M^2} E \left[\left\{ \sum_{i=1}^M \epsilon_i(x) \right\}^2 \right] \\ &= \frac{1}{M^2} E \left[\sum_{i=1}^M \epsilon_i^2(x) + \sum_{i=1}^M \sum_{j=1}^M \epsilon_i(x) \epsilon_j(x) \right] \quad // \forall i \neq j, E[\epsilon_i(x) \epsilon_j(x)] = 0 \\ &= \frac{1}{M^2} E \left[\sum_{i=1}^M \epsilon_i^2(x) \right] \\ &= \frac{1}{M^2} \sum_{i=1}^M E[\epsilon_i^2(x)] \quad // E_{avg} = \frac{1}{M} \sum_{i=1}^M E[\epsilon_i^2(x)] \\ &= \frac{1}{M} E_{avg} \end{aligned}$$

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Since all *convex* function have following inequality:

$$f\left(\sum_{i=1}^M \lambda_i x_i\right) \leq \sum_{i=1}^M \lambda_i f(x_i)$$

Given that $f : x \rightarrow x^2$ is a convex function.

$$\begin{aligned} E_{agg}(x) &= E \left[\left\{ \frac{1}{M} \sum_{i=1}^M \epsilon_i(x) \right\}^2 \right] \\ &= E \left[\left\{ \sum_{i=1}^M \frac{1}{M} \epsilon_i(x) \right\}^2 \right] \\ &\leq E \left[\sum_{i=1}^M \frac{1}{M} \left\{ \epsilon_i(x) \right\}^2 \right] \\ &= \frac{1}{M} E \left[\sum_{i=1}^M \left\{ \epsilon_i(x) \right\}^2 \right] \\ &= E_{avg} \end{aligned}$$