qpu

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1 Initial Setup

1.1 Imports

[1]:
```
import math
import re
from IPython.display import Latex
from qiskit import QuantumCircuit, transpile
from qiskit_aer import AerSimulator
from qiskit.circuit import Qubit
from qiskit.providers.fake_provider import FakeCairo
```

1.2 Global Variables

[2]:
```
# Number of Qubits
N = 15

# IBMQ Mock Backend (https://qiskit.org/documentation/stable/0.42/apidoc/
↪providers_fake_provider.html#fake-v1-backends)
backend = { 'device': FakeCairo() }
backend['name'] = re.sub(r'(_|fake|v\d)', ' ', backend['device'].backend_name.
↪lower()).title()
backend['num_qubits'] = backend['device'].configuration().num_qubits
backend['simulator'] = AerSimulator.from_backend(backend['device'])

# Total Runs
shots = 1024

# linear GHZ container
linear = {
  'circuit': { 'draw': None, 'execute': None},
  'transpiled': None,
  'job': None,
  'result': None,
  'counts': { '0': None, '1': None },
  'time': None,
  'bias': { '0': None, '1': None },
```

```
  'error': None
}

# logarithmic GHZ container
log = {
  'circuit': { 'draw': None, 'execute': None},
  'transpiled': None,
  'job': None,
  'result': None,
  'counts': { '0': None, '1': None },
  'time': None,
  'bias': { '0': None, '1': None },
  'error': None
}
```

---

2 Generate $|\mathrm{GHZ}_N\rangle$ Circuits

2.1 Generate Linear Time Complexity Circuits for $|\mathrm{GHZ}_N\rangle$ [1]

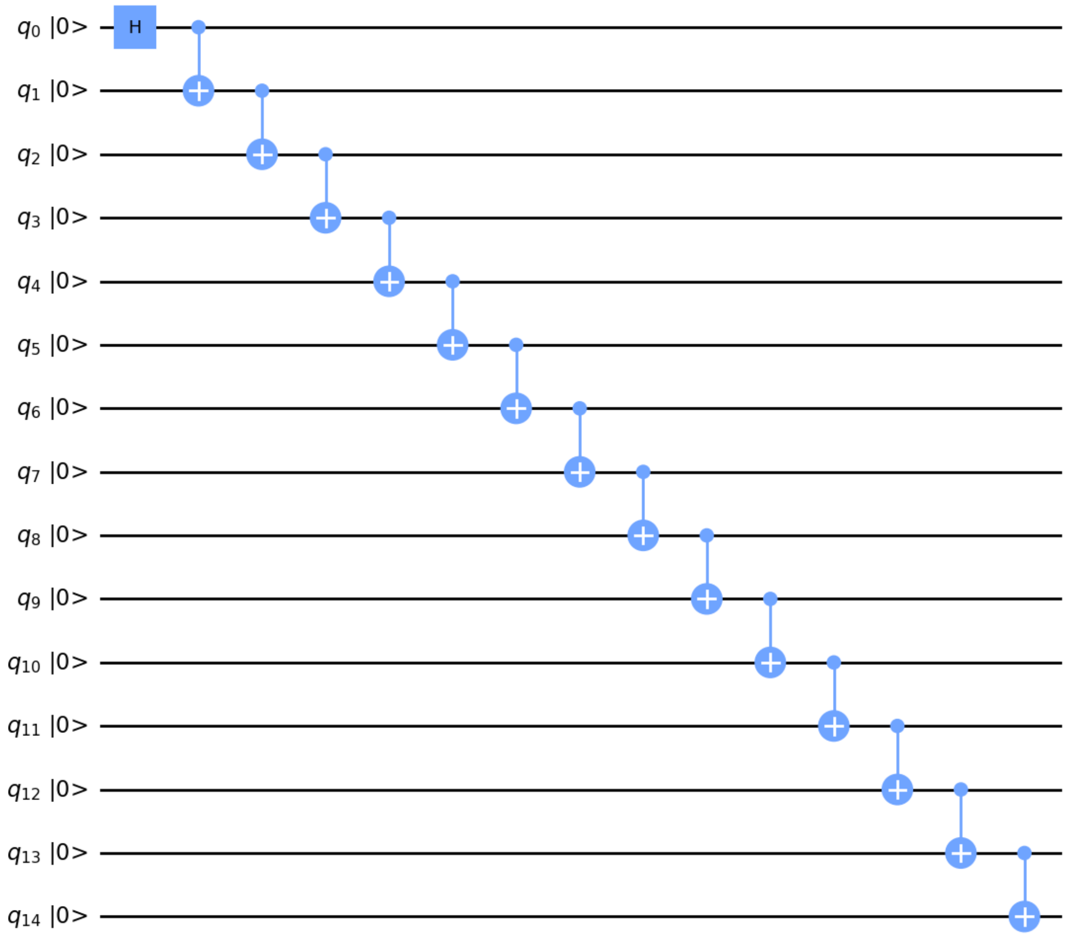
[3]:
```
def linear_complexity_GHZ(N: int, measure: bool = True) -> QuantumCircuit:
  if not isinstance(N, int):
    raise TypeError("Only integer arguments accepted.")
  if N < 1:
    raise ValueError("There must be one or more qubits.")

  c = QuantumCircuit(N)
  c.h(0)
  for i in range(1, N):
    c.cx(i-1, i)
  if measure:
    c.measure_active()

  return c
```

[4]:
```
linear['circuit']['execute'] = linear_complexity_GHZ(N)
linear['circuit']['draw'] = linear_complexity_GHZ(N, False)
linear['circuit']['draw'].draw(output='mpl', fold=-1, initial_state=True)
```

[4]:



2.2 Generate Logaritmic Complexity Circuits for $|\mathrm{GHZ}_{2^m}\rangle$ [1]

[5]:
```
def _log_complexity_GHZ(m: int) -> QuantumCircuit:
  if not isinstance(m, int):
    raise TypeError("Only integer arguments accepted.")
  if m < 0:
    raise ValueError("`m` must be at least 0 (evaluated 2^m).")

  if m == 0:
    c = QuantumCircuit([Qubit()])
    c.h(0)
    c.barrier()
  else:
    c = _log_complexity_GHZ(m - 1)
    for i in range(c.num_qubits):
      c.add_bits([Qubit()])
```

```
        new_qubit_index = c.num_qubits - 1
        c.cx(i, new_qubit_index)
      c.barrier()

    return c
```

2.3 Generate Logaritmic Complexity Circuits for $|\mathrm{GHZ}_N\rangle$ [1]

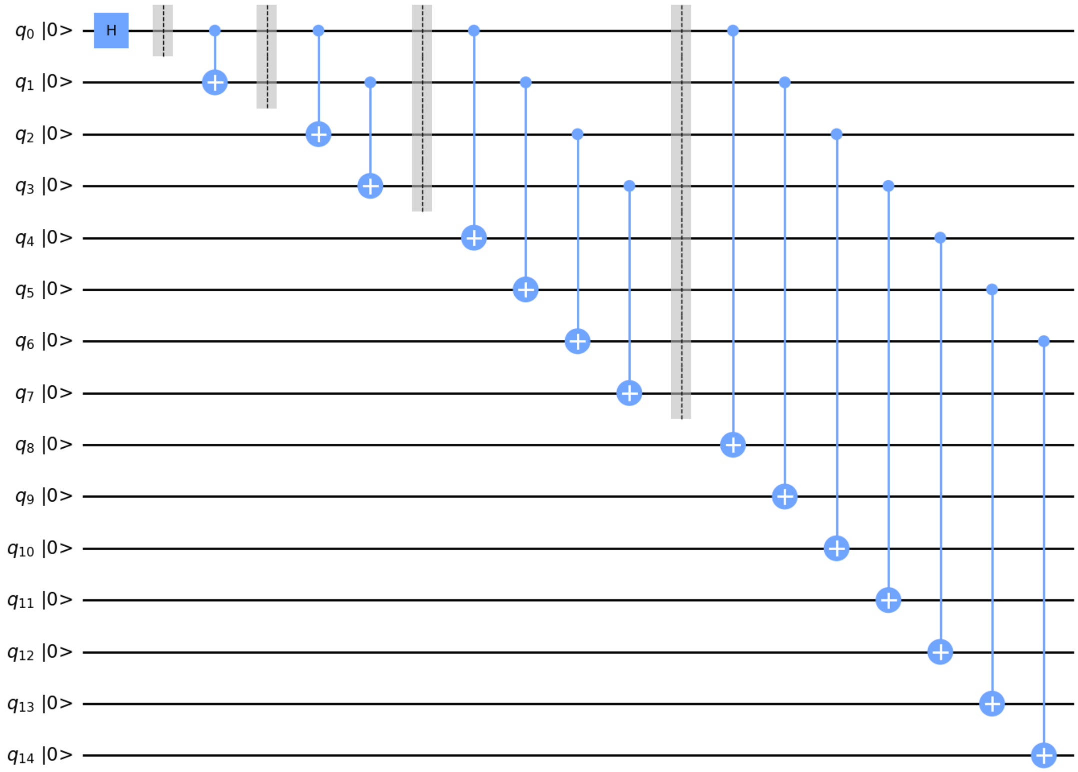
[6]:
```
def log_complexity_GHZ(N: int, measure: bool = True) -> QuantumCircuit:
    if not isinstance(N, int):
      raise TypeError("Only an integer argument is accepted.")
    if N < 1:
      raise ValueError("There must be one or more qubits.")

    m = math.ceil(math.log2(N))
    num_qubits_to_erase = 2**m - N
    old_circuit = _log_complexity_GHZ(m=m)
    new_num_qubits = old_circuit.num_qubits - num_qubits_to_erase
    new_circuit = QuantumCircuit(new_num_qubits)
    for gate in old_circuit.data:
      qubits_affected = gate.qubits
      if all(old_circuit.find_bit(qubit).index < new_num_qubits for qubit in␣
   ↪qubits_affected):
        new_circuit.append(gate[0], [old_circuit.find_bit(qubit).index for qubit␣
   ↪in qubits_affected])
    if measure:
      new_circuit.measure_active()

    return new_circuit
```

[7]:
```
log['circuit']['execute'] = log_complexity_GHZ(N)
log['circuit']['draw'] = log_complexity_GHZ(N, False)
log['circuit']['draw'].draw(output='mpl', fold=-1, initial_state=True)
```

[7]:



---

3 Computational Cost Analysis

3.1 Cost of Hadamard (H) Gate [2]

[8]:
```
H_cost = 2
```

3.2 Cost of CNOT (CX) Gate [2]

[9]:
```
CX_cost = 5
```

3.3 Program Cost

[10]:
```
T_cost = ((N - 1) * CX_cost) + H_cost
Latex(f"""\\begin{{equation*}}{T_cost}\\end{{equation*}}""")
```

[10]:

3.4 Program Sections (s)

[11]:
```
def program_sections() -> int:
  init = [ 0, 1, 2 ]
  i = len(init)
  k = 1
  while len(init) <= N:
    init += [i] * 2**k
    i += 1
    k += 1
  return init[N]
```

[12]:
```
s = program_sections()
Latex(f"""\\begin{{equation*}}{s}\\end{{equation*}}""")
```

[12]:

$$5$$

3.5 Cost per Section

[13]:
```
def cost_per_section() -> list:
  init = [ H_cost ]
  _N = N - 1
  k = 0
  while len(init) < (s - 1):
    init.append(CX_cost * 2**k)
    _N -= 2**k
    k += 1
  if _N > 0:
    init.append(CX_cost * _N)
  return init
```

[14]:
```
section_cost_list = cost_per_section()
Latex(f"""\\begin{{equation*}}{section_cost_list}\\end{{equation*}}""")
```

[14]:

$$[2, 5, 10, 20, 35]$$

---

4 Parallel v. Sequential Analysis in Logarithmic Circuit

4.1 Gates Per Section

[15]:
```
def gates_per_section() -> list:
  if s == 1:
    return [1]
  elif s == 2:
```

```
    return [1, 1]
  else:
    init = [1, 1]
    for i in range(len(init), s):
      init.append(int(section_cost_list[i] / CX_cost))
    return init
```

[16]:
```
num_gates_list = gates_per_section()
Latex(f"""\\begin{{equation*}}{num_gates_list}\\end{{equation*}}""")
```

[16]:

$$[1, 1, 2, 4, 7]$$

4.2 Percent Sequential

[17]:
```
def add_sequential_portions() -> float:
  seq = 0
  for cost, gates in zip(section_cost_list, num_gates_list):
    seq += (1 / gates) * (cost / T_cost)
  return seq
```

[18]:
```
sequential_portion = add_sequential_portions()
Latex(f"""\\begin{{equation*}}{sequential_portion * 100}\%\\end{{equation*}}""")
```

[18]:

$$30.555555555555557\%$$

4.3 Percent Parallel

[19]:
```
def add_parallel_portions() -> float:
  par = 0
  for cost, gates in zip(section_cost_list, num_gates_list):
    par += ( (gates - 1) / gates ) * (cost / T_cost)
  return par
```

[20]:
```
parallel_portion = add_parallel_portions()
Latex(f"""\\begin{{equation*}}{parallel_portion * 100}\%\\end{{equation*}}""")
```

[20]:

$$69.44444444444444\%$$

---

5 Quantum Simulation

5.1 Device

[21]:
```
Latex(f"""\\begin{{equation*}}
         \\text{{{backend['name']}} ({backend['num_qubits']} qubits)}}
         \\end{{equation*}}""")
```

[21]:

$$\text{Cairo (27 qubits)}$$

5.2 Transpile Circuits [3]

[22]:
```
linear['transpiled'] = transpile(linear['circuit']['execute'],
                                 backend['simulator'],
                                 scheduling_method="asap",
                                 optimization_level=0)
```

[23]:
```
log['transpiled'] = transpile(log['circuit']['execute'],
                              backend['simulator'],
                              scheduling_method="asap",
                              optimization_level=0)
```

5.3 Run Simulations

[24]:
```
linear['job'] = backend['device'].run(linear['transpiled'])
```

[25]:
```
log['job'] = backend['device'].run(log['transpiled'])
```

5.4 Block for Results

[26]:
```
linear['result'] = linear['job'].result()
```

[27]:
```
log['result'] = log['job'].result()
```

---

6 Speed-Up Analysis

6.1 Run-Times

6.1.1 Linear

[28]:
```
linear['time'] = linear['result'].time_taken
Latex(f"""\\begin{{equation*}}{linear['time']}\space\\text{{seconds}}\\end{{equation*}}""")
```

[28]:

$$20.897703170776367\text{seconds}$$

6.1.2 Logarithmic

[29]:
```
log['time'] = log['result'].time_taken
Latex(f"""\\begin{{equation*}}{log['time']}\space\\text{{seconds}}\\end{{equation*}}""")
```

[29]:

$$7.604283332824707\text{seconds}$$

6.2 Theoretical Max Speed-Up (Amdahl's Law [4])

$$S_{\text{latency}} = \lim_{F \to \infty} \frac{1}{S_{\text{eq}} + \frac{P}{F}} = \frac{1}{S_{\text{eq}}}$$

- S_{eq} represents the portions of the program running sequentially
- P represents the portions of the program running in parallel
- F represents the level of concurrency (i.e. number of cores in classical computing)

[30]:
```
Latex(f"""\\begin{{equation*}}{1 / sequential_portion}\\end{{equation*}}""")
```

[30]:

$$3.2727272727272725$$

6.3 Observed Speed-Up Factor (S_{latency})

[31]:
```
S_latency = linear['time'] / log['time']
Latex(f"""\\begin{{equation*}}{S_latency}\\end{{equation*}}""")
```

[31]:

$$2.748148938713157$$

6.4 Approximated Level of Concurrency (F)

$$F = \frac{P \cdot S_{\text{latency}}}{1 - S_{\text{eq}} \cdot S_{\text{latency}}}$$

[32]:
```
F = (parallel_portion * S_latency) / (1 - (sequential_portion * S_latency))
Latex(f"""\\begin{{equation*}}{F}\\end{{equation*}}""")
```

[32]:

$$11.906311484000096$$

---

7 Error & Bias Analysis

7.1 Get Counts

[33]:
```
try:
  linear['counts']['0'] = linear['result'].get_counts()['0' * N]
except KeyError:
  linear['counts']['0'] = 0
try:
  linear['counts']['1'] = linear['result'].get_counts()['1' * N]
except KeyError:
  linear['counts']['1'] = 0
try:
  log['counts']['0'] = log['result'].get_counts()['0' * N]
except KeyError:
  log['counts']['0'] = 0
try:
  log['counts']['1'] = log['result'].get_counts()['1' * N]
except KeyError:
  log['counts']['1'] = 0
```

7.2 Error

7.2.1 Linear Error Percent

[34]:
```
linear['error'] = (shots - (linear['counts']['0'] + linear['counts']['1'])) /␣
 ↪shots
Latex(f"""\\begin{{equation*}}{linear['error'] * 100}\%\\end{{equation*}}""")
```

[34]:

$$77.1484375\%$$

7.2.2 Logarithmic Error Percent

[35]:
```
log['error'] = (shots - (log['counts']['0'] + log['counts']['1'])) / shots
Latex(f"""\\begin{{equation*}}{log['error'] * 100}\%\\end{{equation*}}""")
```

[35]:

$$75.390625\%$$

7.3 Bias

7.3.1 Linear Bias Percent

State $|0\rangle$

[36]:
```
linear['bias']['0'] = linear['counts']['0'] / (linear['counts']['0'] +␣
 ↪linear['counts']['1'])
Latex(f"""\\begin{{equation*}}{linear['bias']['0'] *␣
 ↪100}\%\\end{{equation*}}""")
```

[36]:

$$82.90598290598291\%$$

State $|1\rangle$

[37]:
```
linear['bias']['1'] = linear['counts']['1'] / (linear['counts']['0'] +␣
↪linear['counts']['1'])
Latex(f"""\\begin{{equation*}}{linear['bias']['1'] *␣
↪100}\%\\end{{equation*}}""")
```

[37]:

$$17.094017094017094\%$$

7.3.2 Logarithmic Bias Percent

State $|0\rangle$

[38]:
```
log['bias']['0'] = log['counts']['0'] / (log['counts']['0'] +␣
↪log['counts']['1'])
Latex(f"""\\begin{{equation*}}{log['bias']['0'] * 100}\%\\end{{equation*}}""")
```

[38]:

$$82.14285714285714\%$$

State $|1\rangle$

[39]:
```
log['bias']['1'] = log['counts']['1'] / (log['counts']['0'] +␣
↪log['counts']['1'])
Latex(f"""\\begin{{equation*}}{log['bias']['1'] * 100}\%\\end{{equation*}}""")
```

[39]:

$$17.857142857142858\%$$

---

8 References

1. Cruz, Diogo, Romain Fournier, Fabien Gremion, Alix Jeannerot, Kenichi Komagata, Tara Tosic, Jarla Thiesbrummel, et al. (2018). Efficient Quantum Algorithms for *GHZ* and *W* States, and Implementation on the IBM Quantum Computer. ArXiv. 1-2.

2. Lee, Soonchil & Lee, Seong-Joo & Kim, Taegon & Lee, Jae-Seung & Biamonte, Jacob & Perkowski, Marek. (2006). The cost of quantum gate primitives. Journal of Multiple-Valued Logic and Soft Computing. 12. 571.

3. Scheduling Methods

4. Amdahl's Law Definition