Васильев, Квантовая рг в теории критического поведения и стохастической динамики

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Фазовый переход

1го рода - параметр порядка меняется скачком
2го рода - ——— " ——— монотонно (непр. фаз. переход)

T_c - точка ф.п. IIго рода   эксп. набл. — рост восприимчивости $r_c \to \infty$, $T \to T_c$ - корр. радиус

- размерность
- тип взаимодействие
- симметрия

1939 - Ландау, теория среднего поля
- ф. пов - min свободной энергии
- аналитичность св. эн. в окр-ти критич. точки.

магнетики, жидк-пар — ф.п. IIго рода

$\left|\frac{T-T_c}{T_c}\right|^{-1}$ - восприимч.

$\left|\frac{T-T_c}{T_c}\right|^{1/2}$ - намагнич. $\rho_ж - \rho_г$

1942 - точное решение Изинга (Онсагер)
для теплоемкости $\log\left(\frac{T-T_c}{T_c}\right)$
намагниченность $\left|\frac{T-T_c}{T_c}\right|^{1/8}$

не учитывала флуктуации. ~ сильное взаимодействие
⇓
теория случайного поля
~ кв. поле в евклиде

Переход к макромодели - постулат

Ренормгруппа - метод рассчета асимптотики ф-й Грина (либо большие, либо маленькие импульсы)
"–" - $\beta \to 0$ "+" $\beta \to 0$

Предел асимптотич. свободы - рг!
Теория критич. поведения - рг!   Вильсон, нобелевская премия

Обобщенная однородность

$w(e)$ $e = \{e_1 \dots e_n\}$

W - обобщ. одн., если $w(e_1\lambda^{\Delta_1}, e_2\lambda^{\Delta_2} \dots e_n\lambda^{\Delta_n}) = \lambda^{\Delta_w} w(e_1 \dots e_n)$ $\Delta_i \equiv \Delta_{e_i} \equiv \Delta[e_i]$

Если $\Delta_w = 0 \Rightarrow w$ - масштабно-инвариантная

$$\left.\frac{\partial w(e_1\lambda^{\Delta_1},\dots)}{\partial\lambda}\right|_{\lambda=1} = \sum \left(\partial_\lambda e_i\lambda^{\Delta_i}\right)\partial_{e_i} w(e_1\dots e_n)\Big|_{\lambda=1} = \left(\partial_\lambda \lambda^{\Delta_w}\right) w(e_1\dots e_n)\Big|_{\lambda=1}$$

$$\sum \Delta_i \underbrace{e_i\,\partial_{e_i}}_{\mathcal{D}_{e_i}} w(e_1\dots e_n) = \Delta_w w(e_1\dots e_n) \Rightarrow \boxed{\left(\sum \Delta_i \mathcal{D}_{e_i} - \Delta_w\right) w(e) = 0}$$ - ур-е масшт. инв-ти.

величины $\frac{e_i}{|e_n|^{\beta_i}}$ удовл. оди. ур-ю $\beta_i = \frac{\Delta_i}{\Delta_n}$ - инвар. заряды $\beta = \frac{\Delta_w}{\Delta_n}$

$w(e_1 \dots e_n) = e_n^{\beta} f\left(\frac{e_1}{|e_n|^{\beta_1}}, \dots \frac{e_{n-1}}{|e_n|^{\beta_{n-1}}}\right)$ - явно выделенная асимптотика

$\lambda \to \lambda^a \Rightarrow \Delta_i \to a\Delta_i$ принято $\Delta_k = 1$, $\Delta_x = -1$

Гипотеза термодинамического подобия

$Z(e)$ $\Delta_Z = 0$ - статсумма
$e \to 0$ - критич. точка $\tau = \frac{T-T_c}{T_c}$ (или $\tau = T - T_c$)
$V \to \infty$ $(e = e', e''$, $e' \to 0)$

$w = \lim\limits_{V\to\infty} V^{-1} \ln Z_V$ $\Delta_w = d$ - хорошо определенный объект

$W = w^{(s)} + w^{(r)}$ Гипотеза подобия: $w^{(s)}(\lambda e) = \lambda^d w^{(s)}(e)$ в смысле ведущих [illegible]

Более точно:

$$w^{(s)}(\lambda e) = \lambda^d\left[W_0^{(s)}(e) + \lambda^{\omega_1} w_1^{(s)}(e) + \lambda^{\omega_2} w_2^{(s)}(e) + \dots\right] \qquad \lambda e = \vec{\lambda}\vec{e}$$

$$0 < \omega_1 < \omega_2 < \dots$$

$$w_0^{(s)} = \lim_{\lambda \to 0} \frac{w^{(s)}(\lambda e)}{\lambda^d}$$

Флуктуационная теория

$\hat{\varphi}(x)$ - случайное поле

$\rho(\varphi) = e^{S(\varphi)}$ $\quad S(\varphi) = -\beta_c F(\varphi)$ (в. ж.) $\quad \beta_c = \frac{1}{kT_c}$ потому что только вбл-ти фазы точки

$S(\varphi) = S_0(\varphi) + V(\varphi)$

$S_0(\varphi) = -\frac{\varphi K \varphi}{2}$

$Z = \int D\varphi\, e^{S(\varphi)}$

$$\langle Q(\hat{\varphi})\rangle = \frac{\int D\varphi\, Q(\varphi)\, e^{S(\varphi)}}{\int D\varphi\, e^{S(\varphi)}} \qquad G_n(x_1 \dots x_n) = \frac{\int D\varphi\, \varphi(x_1)\dots\varphi(x_n)\, e^{S(\varphi)}}{\int D\varphi\, e^{S(\varphi)}} \qquad G(A) = \frac{\int D\varphi\, e^{S(\varphi)+A\varphi}}{\int D\varphi\, e^{S(\varphi)}}$$

Связные ф.с. $\quad W(A) = \ln(G(A))$

1-неприв. $\quad \Gamma(\alpha) = W(A) - \alpha A \qquad \alpha(x) = \frac{\delta W(A)}{\delta A(x)}$

Пропагатор $\quad D = W_2 = \frac{-1}{\Gamma_2}$

Полиномиальн. действ. 1 вер-ть ренормируемости теории.

Теория φ^4

$$S(\varphi) = \int dx\left(-\frac{(\partial\varphi)^2}{2} - \frac{\tau\varphi^2}{2} - \frac{g\varphi^4}{4!} + h\varphi\right) \qquad \varphi \to -\varphi \text{ при } h=0, \quad \tau = T - T_c$$

(скаляр)

Кубическая симметрия $\quad \frac{g_2}{4!}\sum \varphi_i^4$

Есть ли еще ограничения на действие?

$\Delta_k = 1$

$\int dx \frac{(\partial\varphi)^2}{2} \qquad -d + 2 + 2d_\varphi = 0 \Rightarrow d_\varphi = \frac{d-2}{2}$

$\int d\varphi \frac{\tau\varphi^2}{2} \qquad -d + d_\tau + 2d_\varphi = 0 \Rightarrow d_\tau = 2$

$\int d\varphi \frac{g\varphi^4}{4!} \qquad g\varphi^2 \sim \tau \qquad d_g + d - 2 = 2 \Rightarrow d_g = \frac{4-d}{\cancel{2}}$

$\int d\varphi\, h\varphi \qquad h\varphi = \tau\varphi^2 \qquad d_h = d_\tau + d_\varphi = 2 + \frac{d-2}{2} = \frac{d+2}{2}$

k	x	φ	g	τ	h	g_6	g_n
1	-1	$\frac{d-2}{2}$	4-d	2	$\frac{d}{2}+1$	6-2d	$n-\frac{d(n-2)}{2}$

Канонические размерности.

Критические размерности - это другое

Если что-то не сошлось $\Rightarrow$ не хватает параметров.

Если что-то опред. неоднозначно $\Rightarrow$ какой-то параметр лишний.

$S(\lambda\varphi, \lambda e) = S(\varphi, e)$

$\lambda\varphi = \lambda^{d_\varphi}\varphi(\lambda x)$

$S_0(\varphi) = -\varphi K\varphi/2 \qquad K = -\partial^2 + \tau = p^2 + \tau \qquad \tau = T - T_c \to 0$

$p \to 0$ - интересны большие расст.

Все вклады должны быть одного порядка

как выйдет $gV(\varphi) \qquad g\, e^{-d_g/d_e} \qquad e \to 0 \qquad$ если $d_g > 0$ ($d_\tau > 0$) $\Rightarrow \uparrow$ - ИК-сущест. взаимодействия.

если $d_g < 0 \Rightarrow \downarrow$ - исчез. поправки

$d = d^{**}$ - верхняя критич. разм. $\quad d_g = 0$

$d = d^{*}$ - логарифмич. разм. $\quad V(\varphi) = \int\varphi^n dx \quad d_V = -d + n d_\varphi = 0$

обычно одно и то же

Но! Если $\quad V\varphi = g\tau^2\varphi^4 \qquad d_g = 4 - d - 2d \Rightarrow d^{**} = 4 - 2d$

$S(\varphi) = S_0(\varphi) + gV(\varphi)$

$S_0(\varphi) = -\frac{\varphi K \varphi}{2}$

$K = -\partial^2 + \tau$ критическое поведение $k^2, \tau \to 0$

$\left(g e^{-d_g/d_e}\right)^n$ $d_g > 0$

$d_e > 0$ - ИК-сущ. ос-ть

$e = \{k, \tau\}$, $e \to 0$

$d = d^{**}$: $d_g = 0$ - верх. крит. разм.

$d = d^*$: $V(\varphi) = \int dx\, F(x, \varphi)$ $d_V = d_F - d = 0$ - логарифмическая разм-ть.

$g \frac{\tau^\alpha \varphi^4}{4!}$ $d_\tau = 2$

$d^* = 4$

~~d_g~~ $d_g = 4 - d - 2\alpha \Rightarrow d^{**} = 4 - 2\alpha$

$\alpha_0 : d^{**}(\alpha_0) = 4$ ≷ $d = 3$

$d = 3 \Rightarrow \alpha_0 = 1/2$

$$S(\varphi) = -\int dx \left(\frac{(\partial\varphi)^2}{2} + \overset{\tau = T - T_c}{a_2} \frac{\varphi^2}{2} + a_4 \frac{\varphi^4}{4!} + a_6 \frac{\varphi^6}{6!} \right)$$

$a_2 = 0$ $a_4 > 0$ $a_6 > 0$

$a_4 < 0$ $a_6 > 0$

трикритическая точка

Если $a_4 = g\tau^\alpha$

$\alpha > 0 \Rightarrow$ п/п $\tau \to 0$ $a_4 \to 0$ и может быть

$a_4 \xrightarrow[\tau\to 0]{} 0$

$a_2 \gtrless 0$

$\alpha > 1/2$

$\alpha < 1/2$

$\alpha = 1/2$ - комбинированное состояние

$d = d_c - 2\epsilon$

$\Delta_g = d_g + O(\epsilon)$

<u>Одноосный сегнетоэлектрик</u>

$\varphi_i = \varphi n_i$

$k^2 = k_\perp^2 + k_\parallel^2$

$k_\parallel = (k, n)$

$S_0(\varphi) = \int dk\, \varphi(k) \left[k^2 + \tau + \partial^2 \frac{k_\parallel^2}{k_\perp^2} \right] \varphi(k)$ ← не спрашивайте, откуда это взялось (взято динамически ...)

$V(\varphi) = \frac{g\varphi^4}{24}$

	$k_\parallel$	$k_\perp$	φ	τ	g	∂
$d_F^\parallel$	1	0	$-1/2$	0	$+1$	$-1/2$
$d_F^\perp$	0	1	$\frac{d-1}{2}$	2	$\frac{(3+d)}{2}$	2

$d_F = d_F^\perp + 2d_F^\parallel$ - чтобы соотнош. сохранялось

$k^2 \to 0$

$\tau \to 0$

$\frac{k_\parallel^2}{k^2} \to 0$ при $k_\parallel^2 \ll k_\perp^2$

$k^2 = k_\parallel^2 + k_\perp^2$

$k^2 \sim \frac{k_\parallel^2}{k_\perp^2} \Rightarrow k_\parallel^2 \sim k_\perp^4 \rightarrow \left[k_\perp^2 + \tau + \partial^2 \frac{k_\parallel^2}{k_\perp^2}\right]$

	$k_\parallel$	$k_\perp$	φ	τ	g	∂
$d_F^\parallel$	1	0	$1/2$	0	1	$-1/2$
$d_F^\perp$	0	1	$-\frac{d-1}{2}$	2	$d+3$	2

Если считать в коорд., разм. будут другие

Лучше было в коорд.

$d \to -d$

$\int dk\, \varphi^2 \left(k_\perp^2 + \tau + \partial^2 \frac{k_\parallel^2}{k_\perp^2}\right) + g\varphi^4$

$2d_\varphi + 2 - (d-1) = 0$

$d_\varphi = \frac{d-3}{2}$

$2d_\varphi + 0 + 1 = 0$

$2d_\varphi + 2 + d - 1 = 0$

$2d_\varphi + d + 1 = 0$

$d_\varphi = \frac{-d-1}{2}$

$d_g + d - 1 + d - 1 = 0$

$d_g + 4 \cdot 1/2 - 1 = 0$

$d_g = 1 - 2$

$d_g + 4\frac{d-3}{2} - (d-3) = 0$

$d_g + 2d - 6 - d + 3 = 0$

$d_g = 3 - d$

$2d_\partial - 2 - (d-1) + 4\left(\frac{d-3}{2}\right) = 0$

$2d_\partial - 1 - d + 2d - 6 = 0$

$2d_\partial - 7 + d = 0$

$d_\partial = \frac{7-d}{2}$

	$k_\parallel$	$k_\perp$	φ	τ	g	∂
$d_F^\parallel$	1	0	$1/2$	0	-1	-1
$d_F^\perp$	0	1	$\frac{d-1}{2}$	2	$d+5$	2

$d_g + 2d - 2 + d - 1 = 0$

$2d_\partial - 2 - \overset{-4}{d} + 1 + d - 1 = 0$

Функциональная и диаграммная техника КТП

$$\int D\varphi\, e^{\left[-\frac{\varphi K\varphi}{2} + A\varphi\right]} = \det\left(K/2\pi\right)^{-1/2} e^{\frac{AK^{-1}A}{2}}$$

$$(\varphi K\varphi) =$$

$$(K\varphi)_x = \int dx'\, K(x,x')\, \varphi(x')$$

$$\varphi K\varphi = \int dx\, dx'\, \varphi(x)\, K(x,x')\, \varphi(x')$$

$$K = 1 \to K(x,x') = \delta(x-x')$$

$$K(x,x') = K_x\, \delta(x-x')$$

$$F\left(\delta/\delta A\right) e^{A\varphi}[\ldots] = e^{A\varphi} F\left(A + \delta/\delta A\right)[\ldots]$$

$$F\left(\delta/\delta\varphi\right) e^{A\varphi} = F(A)\, e^{A\varphi}$$

$$e^{A\frac{\delta}{\delta\varphi}} F(\varphi) = F(\varphi + A)$$

$$S = S_0(\varphi) + V(\varphi)$$

$$S_0(\varphi) = -\frac{\varphi K\varphi}{2}$$

$$\Delta = \Delta' = K^{-1}$$

$$P_\varphi = e^{\left[\frac{1}{2}\frac{\delta}{\delta\varphi}\Delta\frac{\delta}{\delta\varphi}\right]} = e^{D_\varphi/2}$$

$$D_\varphi = \iint dx\, dx' \left[\frac{\delta}{\delta\varphi(x)}\Delta(x,x')\frac{\delta}{\delta\varphi(x')}\right]$$

$$P_\psi = c\int D\varphi\; e^{\left[S_0(\varphi) + \varphi\frac{\delta}{\delta\psi}\right]}$$

$$c^{-1} = \int D\varphi\, e^{S_0(\varphi)} = \det(K/2\pi)^{-1/2}$$

$$P_\psi F(\psi) = c\int D\varphi\, F(\varphi + \psi)\, e^{S_0(\varphi)}$$

$$P_\psi F(\psi)\big|_{\psi=0} = c\int d\varphi\, F(\varphi)\, e^{S_0(\varphi)}$$

$$H(\varphi) = P e^{V(\varphi)} \qquad P = e^{D/2}$$

$$H(\varphi) = \sum \frac{H_n \varphi^n}{n!}$$

$$H = \sum_{n=0}^{\infty} PV^n/n! = \sum_{n,m} \frac{D^m V^n}{2^m n!\, m!}$$

$$V_m(x_1, \ldots x_m, \varphi) = \frac{\delta^m V(\varphi)}{\delta\varphi(x_1)\ldots\delta\varphi(x_m)} \qquad + \quad F\left(\frac{\delta}{\delta\varphi}\right)\prod_i F_i(\varphi) = F\left(\sum \frac{\delta}{\delta\varphi_i}\right)\prod_i F_i(\varphi_i)$$

$$PV^n = \left[e^{\left(\sum_i D_{ii}/2 + \sum_{i<k} D_{ik}\right)}\right] V_1 \ldots V_n$$

$$V_i = V(\varphi_i)$$

$$[\;\;] = \sum_i \prod_i \left[(D_{ii}/2)^{\pi_{ii}}/\pi_{ii}!\right] \cdot \prod_{i<k}\left[(D_{ik})^{\pi_{ik}}/\pi_{ik}!\right]$$

$$\frac{n!}{S} \frac{1}{2^{\sum \pi_{ii}} \prod(\pi_{ii}!) \prod(\pi_{ik}!)} = \frac{1}{n!}$$

$$\frac{c_2}{c_1} = \epsilon \frac{N(D_1 \to D_2)}{N(D_2 \to D_1)} \qquad \epsilon = \begin{matrix} 1 - \text{не петля} \\ 1/2 - \text{петля} \end{matrix}$$

$$G = c\int D\varphi\, e^{\sum \frac{\vartheta_n \varphi^n}{n!}} = P\, e^{\sum_{n\geq 2} \frac{\vartheta_n \varphi^n}{n!}}\Big|_{\varphi=0}$$

$$c^{-1} = \int D\varphi\, e^{\vartheta_2 \varphi^2/2}$$

$$C = \frac{1}{3 \cdot 2^{\,n+\pi} \prod \pi_{ii}! \prod_{i<k} \pi_{ik}!} \qquad \Delta = -\frac{1}{\vartheta_2}$$

$\vartheta_n \quad n\geq 2$

полн. ф. Г. неnорм.

$$G(A) = e^{W(A)} = c\int D\varphi\, e^{S(\varphi) + A\varphi}$$

$$\text{вакуум} \quad c^{-1} = \int D\varphi\, e^{S_0(\varphi)} = \det\left(K/2\pi\right)^{-1/2}$$

Норм. – нормировка на $e^{S(\varphi)}$

$$G_0 = G(0) = \frac{Z}{Z_0}$$

$$W_0 = W(0) = \ln\frac{Z}{Z_0}$$

$$G(A) = P e^{V(\varphi) + A\varphi}\Big|_{\varphi=0}$$

$$F\left(\frac{\delta}{\delta\varphi}\right) e^{A\varphi}[\ldots] = e^{A\varphi} F\left(A + \frac{\delta}{\delta\varphi}\right)[\ldots]$$

$$G(A) = \left(e^{[A\Delta A/2 + A\Delta\frac{\delta}{\delta A}]}\right) \underbrace{P e^{V(\varphi)}}_{H}\Big|_{\varphi=0}$$

$$e^{A\frac{\delta}{\delta\varphi}} F(\varphi) = F(\varphi + A)$$

$$G(A) = H(\Delta A)\, e^{A\Delta A/2}$$

$$\Delta A = \int dx'\, \Delta(x, x')\, A(x')$$

$$G(A) = P e^{V(\varphi) + A\varphi}\Big|_{\varphi=0}$$

$$\tilde{\vartheta}_1 = A + V_1\big|_{\varphi=0} = A + h$$

$$\tilde{\vartheta}_n = \frac{\delta^n}{\delta\varphi \ldots \delta\varphi} V(\varphi)\Big|_{\varphi=0} \qquad V(\varphi) = \sum \frac{\vartheta_n \varphi^n}{n!}$$

$$S(\varphi) = \int dx \left(-\frac{\partial\varphi^2}{2} - \tau\frac{\varphi^2}{2} - g\frac{\varphi^4}{4!} + h\varphi\right) \qquad \Delta = \frac{1}{k^2 + \tau}$$

$$V_1 = h - g\frac{\varphi^3(x)}{3!}$$

$$V_2 = -g\varphi^2(x_1)\,\delta(x_1 - x_2)/2$$

$$V_3 = -g\varphi(x_1)\,\delta(x_1 - x_2)\,\delta(x_2 - x_3)$$

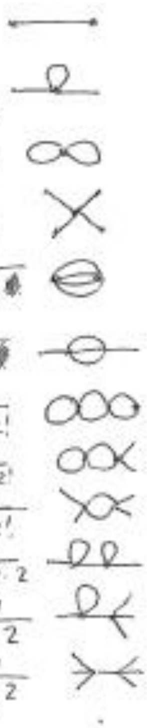
$$V_4 = -g\,\delta(x_1 - x_2)\,\delta(x_1 - x_3)\,\delta(x_1 - x_4)$$

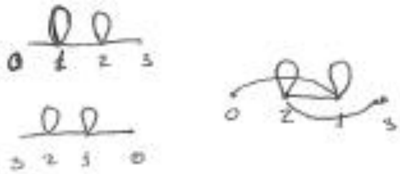
$$V_n = 0.$$

$$\Rightarrow \vartheta: \quad \vartheta_1 = A(x) + h(x), \quad \vartheta_4(x_1 \ldots x_4) = -g\,\delta_{12}\delta_{13}\delta_{14}$$

$$W(A) = \ln G(A) =$$

$= \frac{1}{2}$, $\frac{1}{2\cdot 2}$, $\frac{1}{2^3\cdot 3!}$, $\frac{1}{4!}$, $\frac{1}{2\cdot 4!}$, $\frac{1}{2\cdot 3!}$, $\frac{1}{2\cdot 2^3\cdot 2!}$, $\frac{1}{2\cdot 2\cdot 2!}$, $\frac{1}{2\cdot 4\cdot 2!}$, $\frac{1}{2\cdot 2\cdot 2}$, $\frac{1}{3!\, 2}$, $\frac{1}{3!\,3!\,2}$

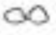


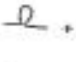


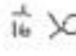
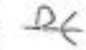
$$= -g\int dx\, dx'\, dx''\, \vartheta_1(x)\, \Delta(x, x')\, \Delta(x', x')\, \Delta(x', x'')\, \vartheta_1(x'')$$

$$W(A) = \sum \frac{W_n(x_1 \ldots x_n) A^n}{n!}$$

$$W_n(\ldots) = \frac{\delta^n W(A)}{\delta A \ldots \delta A}\Big|$$

$W_0 = \frac{1}{8}$  $+ \frac{1}{48}$  $+ \dots$

$W_2 = 2! \Big[\frac{1}{2}$ $+ \frac{1}{4}$ $+ \frac{1}{12}$  $+ \dots \Big]$

$W_4 = 4! \Big[\frac{1}{4!}$ $+ \frac{1}{16}$  $+ \frac{1}{12}$  $+ \dots \Big]$

 $= (-g)^2 \, \mathrm{Sym} \int dx\, dx' \, \Delta(x_1 x)\, \Delta(x_2, x)\, \Delta^2(x, x')\, \Delta(x' x_3)\, \Delta(x' x_4)$

$W(A,h) = W(A_1 + h, 0)$

$\Gamma(\alpha) = W(A) - \alpha A$

$W_1 = \alpha$

$\Gamma_1 = -A$

$\Gamma_2 W_2 = -1$

$W_{n+1} = D_A^{n+1} W = D_A W_n \qquad \Gamma_n = D_\alpha^n \Gamma(\alpha)$

$D_A = (D_A \alpha) D_\alpha = (D_A W_1) D_\alpha = \left(-\frac{1}{\Gamma_2} D_\alpha\right)$

$D_A = \frac{\delta}{\delta A} \qquad D_x M^{-1} = -M^{-1} [D_x M] M^{-1}$

$W_3 = D_A W_2 = -\frac{1}{\Gamma_2} D_\alpha \underset{-\frac{1}{\Gamma_2}}{\underset{\shortparallel}{(W_2)}} = \left(-\frac{1}{\Gamma_2}\right)^3 D_\alpha(\Gamma_2) = \left(-\frac{1}{\Gamma_2}\right)^3 \Gamma_3 \qquad W_3 =$ 

$W_4 = D_A W_3 = -\frac{1}{\Gamma_2} D_\alpha(W_3) = -\frac{1}{\Gamma_2} D_\alpha\left[\left(-\frac{1}{\Gamma_2}\right)^3 \Gamma_3\right] = -\frac{1}{\Gamma_2}\left(+3\left(-\frac{1}{\Gamma_2}\right)^4 \underset{\Gamma_3}{(D_\alpha \Gamma_2)} \Gamma_3 + \left(-\frac{1}{\Gamma_2}\right)^3 D_\alpha \Gamma_3\right) =$

$= 3\left(-\frac{1}{\Gamma_2}\right)^5 \Gamma_3^2 + \left(-\frac{1}{\Gamma_2}\right)^4 \Gamma_4$ 

$W_5 = -\frac{1}{\Gamma_2} D_\alpha \left(+3\left(-\frac{1}{\Gamma_2}\right)^5 \Gamma_3^2 + \left(-\frac{1}{\Gamma_2}\right)^4 \Gamma_4\right) = \left(-\frac{1}{\Gamma_2}\right)\left[(-3)5(-)\left(-\frac{1}{\Gamma_2}\right)^6 \Gamma_3^3 + 6\left(-\frac{1}{\Gamma_2}\right)^5 \Gamma_3 \Gamma_4 + 4\left(-\frac{1}{\Gamma_2}\right)^5 \Gamma_4 \Gamma_3 + \left(-\frac{1}{\Gamma_2}\right)^4 \Gamma_5\right]$

$-\frac{1}{\Gamma_2} D_\alpha \left[\left(3\left(-\frac{1}{\Gamma_2}\right)^4 \Gamma_3 \left(-\frac{1}{\Gamma_2}\right)\Gamma_3\right) + \left(-\frac{1}{\Gamma_2}\right)^4 \Gamma_4\right)$

$\left(-\frac{1}{\Gamma_2}\right) \cdot 12 \left(-\frac{1}{\Gamma_2}\right)^4 \Gamma_3 \left(-\frac{1}{\Gamma_2}\right) \Gamma_3 \left(-\frac{1}{\Gamma_2}\right) \Gamma_3$

$3\left(-\frac{1}{\Gamma_2}\right)\left(-\frac{1}{\Gamma_2}\right)^4 \Gamma_4 \left(-\frac{1}{\Gamma_2}\right) \Gamma_3$

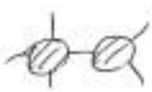
$3\left(-\frac{1}{\Gamma_2}\right)^5 \Gamma_3 \left(-\frac{1}{\Gamma_2}\right) \Gamma_3 \left(-\frac{1}{\Gamma_2}\right) \Gamma_3$

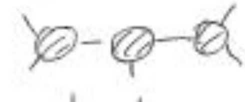
$3\left(-\frac{1}{\Gamma_2}\right)^5 \Gamma_3 \left(-\frac{1}{\Gamma_2}\right) \Gamma_4$

$4\left(-\frac{1}{\Gamma_2}\right)^5 \Gamma_3 \left(-\frac{1}{\Gamma_2}\right) \Gamma_4$

$\left(-\frac{1}{\Gamma_2}\right)^5 \Gamma_5$

12 

3 

3 

3 

4 

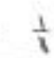
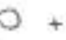
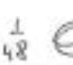
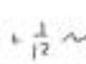
1

$w(A) = \frac{A\Delta A}{2} + \tilde{w}(A)$

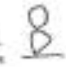
$\Gamma(A) = -\frac{\alpha\Delta^{-1}\alpha}{2} + \tilde{\Gamma}(\alpha)$

$\tilde{\Gamma}(\alpha)$ = 1-неп. часть $\tilde{w}(A = \Delta^{-1}\alpha)$

$\Delta A \to \alpha$ 

$\Gamma(\alpha) = \underbrace{-\frac{\alpha\Delta^{-1}\alpha}{2} + \frac{1}{4}}$  $+ \frac{1}{8}$  $+ \frac{1}{24}$  $+ \frac{1}{48}$  $+ \frac{1}{12}$  $+ \ldots$

$\tilde{\Gamma}(\alpha)$

$\Gamma_2 = -\Delta^{-1} + \frac{1}{2}$  $+ \frac{1}{4}$  $+ \ldots$

$\Gamma_4 = X + \frac{3}{2}$  $+ \ldots$

$c = \frac{n!\, c_n}{\prod_a n_a!}$

<u>Фурье</u>

$F(x,x') = (2\pi)^{-d}\int dk\, F(k)\, e^{ik(x-x')}$

$F(k) = \int d(x-x')\, F(x,x')\, e^{ik(x'-x)}$

$F(p_1 \ldots p_n) = \int dx_1 \ldots dx_n\, F_n(x_1 \ldots x_n)\, e^{-i\sum p_s x_s}$

$\tilde{F}_n(p_1 \ldots p_n) = (2\pi)^d\, \delta(\sum p_s)\, F_{np}(p_1 \ldots p_n)$

$F_{np} = \iint dx_1 \hat{dx_i} \ldots dx_n\, F_n(x_1 \ldots x_n)\, e^{i\sum p_s x_s}$
по всем
без одного

$\vartheta_4(x \ldots x_4) \quad \Delta(x_i)$

$\vartheta_{4\varphi} = -g$

$d[\tilde{F}_n(p_i)] = d[F_{pn}(p_i)] - d = d[F_n(x_i)] - nd$

Подграф – набор вершин и линий (не пустой и не полный)
2 варианта: 1) Коллинз – что угодно
2) Боголюбов – со всеми вн. рёбрами

$d(\Gamma_{np}) = d - nd_\varphi$

$\gamma_{nv} = (-g)^v I_{nv}$

$\omega = d[I_{nv}] = d[\Gamma_n] - v d_g$ – формальный инд. расх. диагр. $\omega \geq 0$ – расх
$\omega < 0$ – непонятно

Пов-но расх подграф – 1-неп. подгр. с полным, о реальн. инд. расх. ≥ 0

Кон. ч. пов. расх – суперреном.
Беск., но с кон. числом нов – реном.
–″– , – беск. # типов – нереном.

$\int^{\Lambda} \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+\tau} = \ldots = \frac{i\sqrt{\tau}}{2}\left(\ln\frac{\Lambda + i\sqrt{\tau}}{\Lambda - i\sqrt{\tau}} - \ln\frac{i\sqrt{\tau}}{i\sqrt{\tau}}\right).$

$= \frac{1}{16\pi^2}\left(\Lambda^2 - \tau\ln\left(1+\frac{\Lambda^2}{\tau}\right)\right)$

$\Lambda \to \infty \quad \tau\ln\frac{\Lambda^2}{\tau} = \tau\ln\frac{\Lambda^2}{\mu^2} + \ln\frac{\mu^2}{\tau}$ расх. часть – полином по τ – примитивные расх-ти

φ^4

$\Gamma_0 = c\Lambda^4 + c\Lambda^2\tau + c\tau^2$

$\Gamma_2 = c\Lambda^2 + c\tau + cp^2$

$\Gamma_4 = c$

$\Gamma_{2n} = 0$

$$\int d^4k \frac{1}{k^2+\tau}\cdot\frac{1}{(p-k)^2+\tau} = \int d^4k \frac{1}{(k^2+\tau)^2} = \int_0^\Lambda \frac{\Omega k^3 dk}{(2\pi)^4 (k^2+\tau)^2}$$

$$= \frac{\Omega}{(2\pi)^4} \qquad \int_0^{\Lambda} \frac{\Omega\, r\, dr}{2(r+\tau)^2} = \qquad -\partial_\tau \Omega \cdot \Big|_{p=0}$$

$$\begin{cases} S(\varphi, \Theta_0) = -\frac{(\partial\varphi)^2}{2} - \tau_0 \frac{\varphi^2}{2} - g_0 \frac{\varphi^4}{4!} \\ Z_\varphi = 1 + \sum_{n=1}^{\infty} g^n \lambda_n \\ \tau_0 = \tau + \sum_{n=1}^{\infty} g^n \alpha_n \\ g_0 = g\left[1 + \sum_{1}^{\infty} g^n \beta_n\right] \\ \Gamma_{2R} = Z_\varphi^2 \Gamma_2 = Z_\varphi^2\left(-p^2 - \tau_0 + \frac{1}{2}\,\Omega + \frac{1}{4}\,\mathcal{8} + \frac{1}{6}\,\ominus + \ldots\right] \\ \Gamma_{4R} = Z_\varphi^4 \Gamma_4 = Z_\varphi^4\left(-g_0 + \frac{3}{2} \ldots + \frac{3}{4} \ldots + 3 \ldots + \frac{3}{2} \ldots\right] \end{cases}$$

$\frac{1}{4\pi^2}(\Lambda^2 - \tau \ln \frac{\Lambda^2}{\tau})$

$-\ln\left(1+\frac{\Lambda^2}{\tau}\right) + \frac{\tau}{\tau}\frac{1}{\tau+\Lambda^2} \quad -\ln\left(1+\frac{\Lambda^2}{\tau}\right) + \frac{1}{\tau+\Lambda^2}$

$\ln\frac{\Lambda^2}{p^2}$

$\Gamma_{2R}^{(1)} = g\{-2\lambda_1(p^2+\tau_R) - \alpha_1 - \frac{1}{2}I\,\Omega\}$ $\quad c\Lambda^2 + c\tau \quad \to \lambda_1 = 0$

$\Gamma_{4R}^{(1)} = g^2\{-4\lambda_1 - \beta_1 + \frac{3}{2}I[\ldots]\}$ $\quad \alpha_1 = ?$, $\beta_1 = ?$

Схема вычитаний на нулевых импульсах:

$\Gamma_{2R}|_{p=0} = -\tau \qquad \frac{\partial \Gamma_{2R}}{\partial p^2}\Big|_{p=0} = -1$

$\Gamma_{4R}|_{p=0} = -g$

$\alpha_1 = -\frac{1}{2}I[\Omega]\big|_{p=0}$

$\beta_1 = \frac{3}{2}I[\ldots]\big|_{p=0}$

$\lambda_1 = 0$

$g_0 = g(1+\beta_1 g + \ldots)$

$\Delta_0(k) = \frac{1}{k^2+\tau_0} = \frac{1}{k^2+\tau+g\alpha_1+\ldots}$

$\text{—} \to \text{—} - g\alpha_1 \text{—}$

$\Gamma_{2R} = (1+g\lambda_1+g^2\lambda_2)^2(-p^2-\tau-g\alpha_1-g^2\alpha_2-\ldots+\frac{g}{2}I(\Omega)+\frac{g^2}{4}\mathcal{8}+\frac{g^2}{6}\ominus\ldots)$

$\Gamma_{4R} = (1+g\lambda_1+g^2\lambda_2)^4(-(g+g^2\beta_1+g^3\beta_2+\frac{3}{2}I(\ldots)+\frac{3}{4}I(\ldots)+3\ldots+\frac{3}{2}\ldots))$

$\Gamma_{2R} = (1+g^2\lambda_1^2+2g^2\lambda_2+2g\lambda_1)(-p^2-\tau-g\alpha_1-g^2\alpha_2+gI(\Omega)+g^2I_2)$

$\Gamma_{4R} = (1+4g^2\lambda_1^2+2g^2\lambda_1^2+4g^3\lambda_2+4g\lambda_1)(-g-g^2\beta_1-g^3\beta_2+g^2\tilde{I}_1+g^3\tilde{I}_2)$ $\quad +4g^3\lambda_1\lambda_2$

$\Gamma_{2R} = (-p^2-\tau-g\alpha_1-g^2\alpha_2+gI_1+g^2I_2-g^2\lambda_1^2p^2-g^2\lambda_1^2\tau-2g^2\lambda_2p^2-2g^2\lambda_2\tau-2g\lambda_1p^2-$
$-2g\lambda_1\tau-2g^2\lambda_1\alpha_1+2g^2\lambda_1 I_1)$

$\Gamma_{4R} = (-g-g^2\beta_1-g^3\beta_2+g^2\tilde{I}_1+g^3\tilde{I}_2-\cancel{4g^2\lambda_1}-4g^3\lambda_1^2-2g^3\lambda_1^2-4g^3\lambda_2-4g^2\lambda_1-4g^3\lambda_1\beta_1+4g^3\tilde{I}_1)$

$$\Gamma_{2R} = -p^2-\tau+g(-\alpha_1+I_1-\lambda_1p^2-2\lambda_1\tau)+g^2(-\alpha_2+I_2-\lambda_1^2p^2-\lambda_1^2\tau-2\lambda_2p^2-2\lambda_2\tau-2\lambda_1\alpha_1+2\lambda_1 I_1)$$

$$\Gamma_{4R} = -g+g^2(-\beta_1+\tilde{I}_1-4\lambda_1)+g^3(-\beta_2+\tilde{I}_2-4\lambda_1^2-2\lambda_1^2-4\lambda_2-4\lambda_1\beta_1+4\tilde{I}_1)$$

$$\begin{cases}\Gamma_{2R} = -p^2-\tau+g(-\alpha_1+I_1)+g^2(-\alpha_2+I_2-2\lambda_2p^2-2\lambda_2\tau) \\ \Gamma_{4R} = -g+g^2(-\beta_1+\tilde{I}_1)+g^3(-\beta_2+\tilde{I}_2-4\lambda_2+4\tilde{I}_1)\end{cases}$$

R-операция

$S(\varphi, e_0)$

$S_R(\varphi, e) = \cancel{S(\ldots)} = S(\overbrace{Z_\varphi \varphi}^{\varphi_0}, e_0)$

$Z_\varphi = Z_\varphi(e)$

$e_0 = e_0(e)$

$e_0 = Z_e e$

$\downarrow$

$e_B = e$

$S_B(\varphi, e_B) = S(\varphi, e_{0}(e))$

$e_B(\varphi) = e_0(e)$ в низш. прибл.

$S_R = S \neq S_B$

$S_R(\varphi, e) = S_B(\varphi, e) + \underbrace{\Delta S}_{\text{контрчлены}}$

L, R, R' операции.

$L\gamma$ - контрчлен графа

$L\gamma \neq 0 \Rightarrow \gamma$ - сущ.

$L\gamma = 0 \Rightarrow \gamma$ - несущ.

$L(1\text{-прив}) = 0$

$L(a\gamma) = aL(\gamma)$ $\quad a$ - число

R' - неполная R-операция

$R' = 1 - \sum_\gamma L_\gamma + \sum_{\substack{\gamma\gamma' \\ \gamma\cap\gamma' = \varnothing}} L_\gamma L_{\gamma'} - \sum_{\gamma\cap\gamma' = \varnothing} L_\gamma L_{\gamma'} L_{\gamma''} + \ldots$

$L_\gamma \cdot [\ldots] = \ldots$

$L\gamma = \mu^{\varepsilon} K \mu^{-\varepsilon} R'\gamma$

K - все пов. расх. коэф. сущ. и любой ...

- полином по импульсам и содержит всю расх. часть.

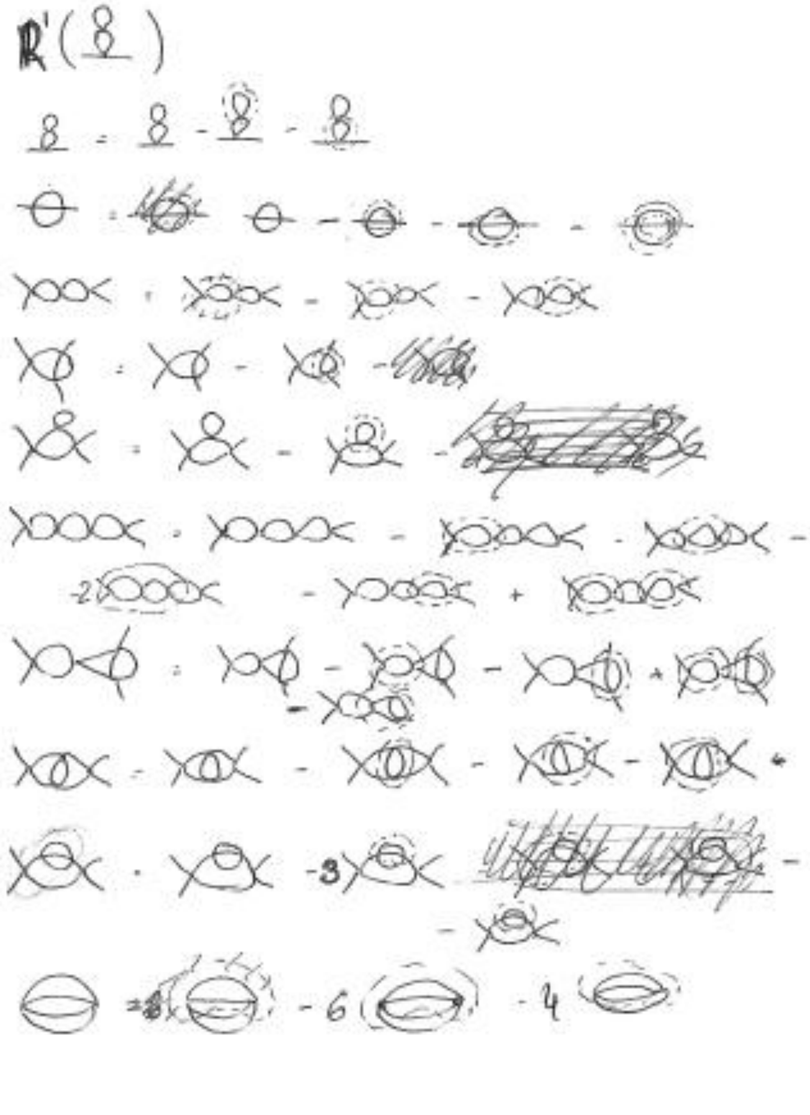
$$S = -\frac{(\partial\varphi)^2}{2} - \tau_0 \frac{\varphi^2}{2} - \frac{g_0 \varphi^4}{4!}$$

$$S_R = -Z_1 \frac{(\partial\varphi)^2}{2} - \tau Z_2 \frac{\varphi^2}{2} - \frac{g Z_3 \varphi^4 \mu^{2\varepsilon}}{4!}$$

$$S_B = -\frac{(\partial\varphi)^2}{2} - \tau \frac{\varphi^2}{2} - \frac{g \mu^{2\varepsilon} \varphi^4}{4!}$$

$$\Delta S = (Z_1 - 1)\frac{(\partial\varphi)^2}{2} - (Z_2 - 1)\tau\frac{\varphi^2}{2} - g\mu^{2\varepsilon}(Z_3 - 1)\frac{\varphi^4}{4!}$$

Учет контрчленов для R-операции



1. $= \frac{1}{\epsilon}$

2. $= G(1,1) \cdot G(1,1)\, G(1,\epsilon) = \frac{1}{\epsilon}\left(-\frac{1}{4}\right) = -\frac{1}{4\epsilon}$

3. $= - 2(KR' \ldots) \ldots = -\frac{1}{\epsilon^2}$

4. $= -\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}$

5. $= \frac{1}{6\epsilon^2} + \frac{4}{3\epsilon}$

6. $= \frac{1}{\epsilon^3}$

7. $= \frac{1}{2\epsilon^3} - \frac{1}{2\epsilon^2}$

8. $= -\frac{1}{12\epsilon^2} - \frac{3}{8\epsilon}$

9. $= \frac{1}{3\epsilon^3} - \frac{1}{3\epsilon^2} - \frac{1}{3\epsilon}$

10.

11. $= \frac{1}{6\epsilon^3} - \frac{1}{2\epsilon^2} + \frac{2}{3\epsilon}$

12. $= 6\zeta(3)\cdot\frac{1}{3\epsilon} \cdot \frac{2\zeta(3)}{\epsilon}$

13. $= \frac{1}{3\epsilon^3} - \frac{2}{3\epsilon^2} + \frac{1}{3\epsilon}$

$= - KR'(\ldots) - 2\, KR'(\ldots)$

$= \frac{1}{\epsilon}\tau^{-2\epsilon}\left(\frac{1}{3\epsilon^2} - \frac{5}{3\epsilon} + \frac{1}{6}\pi^2 + \frac{7}{3}\right)$

$-\frac{1}{\epsilon}\tau^{-2\epsilon}\left(\frac{1}{\epsilon^2} + 4 + \frac{\pi^4}{36}\epsilon^2 - \frac{4}{\epsilon} + \frac{\pi^2}{3} - \frac{2}{3}\pi^2\right)$

$-2\tau^{-\epsilon}\left(\frac{1}{\epsilon} - 2 + \frac{1}{6}\pi^2\epsilon\right)\left(-\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\right) =$

$= \frac{1}{3\epsilon^3} - \frac{5}{3\epsilon^2} + \left(\frac{1}{6}\pi^2 + \frac{7}{3}\right)\frac{1}{\epsilon}$

$-\frac{1}{\epsilon^3} - \frac{4}{\epsilon} + \frac{4}{\epsilon^2} + \frac{\pi^2}{36} - \frac{2\pi^2}{36}$

$+\frac{1}{2\epsilon^3} - \frac{1}{2\epsilon^2} + \frac{1}{\epsilon^2} + \frac{1}{2\epsilon} + \frac{\pi^2}{6\epsilon}$

$= \frac{1}{3\epsilon^3} - \frac{2}{3\epsilon^2} + \frac{1}{3\epsilon}$

Правила

α $= \frac{1}{(k^2)^\alpha}$

$\alpha \;\; \beta$ $=$ $\alpha\beta$

$= \alpha+\beta-\frac{d}{2}\;\; \frac{1}{(4\pi)^{d/2}}\, H(\alpha,\beta,d-\alpha-\beta) =$

$= \alpha+\beta-\frac{d}{2}\;\; \frac{1}{(4\pi)^{d/2}}\, G(\alpha,\beta)$

$H(\alpha) = \frac{\Gamma(d/2-\alpha)}{\Gamma(\alpha)}$

Прочее

$g\left(\frac{p^2}{\mu^2}\right)^{-\epsilon}$

$\tilde{g} = g(1+O(\epsilon))$

$u \equiv G(1,1) \equiv \frac{1}{\epsilon}$

$u = \frac{g}{(4\pi)^{d/2}\, G(1,1)\,\epsilon}$ — расписываем по этому заряду

$\frac{1}{16\pi^2}(1+O(\epsilon)) \quad G(1,1)\,\epsilon = 1 + O(\epsilon)$

$G(1,\epsilon) = -\frac{1}{4} + \frac{5}{8}\epsilon + O(\epsilon^2)$

$G(1,1+\epsilon) = \frac{1}{2\epsilon} + \frac{1}{2} + \frac{3}{2}\epsilon + O(\epsilon^2)$

$G(1,2\epsilon) = -\frac{1}{3} - \frac{4}{3}\epsilon + O(\epsilon^2)$

$G(1,1+2\epsilon) = \frac{1}{3\epsilon} + \frac{2}{3} + \frac{8}{3}\epsilon + O(\epsilon^2)$

$G(2,1+\epsilon) = -\frac{1}{\epsilon} + 2 + 6\zeta(3)\epsilon^2 + O(\epsilon^3)$

$G(2,1) = -\frac{1}{\epsilon} + 2$

$G(1,1) = \frac{1}{\epsilon}$

4. $= - KR'(\ldots) \cdot - G(1,1) \cdot \epsilon = -\frac{1}{\epsilon}\cdot\frac{1}{\epsilon}(p^2)^{-\epsilon}$

$= G(1,1) - \frac{1}{\epsilon^2}(p^2)^{-\epsilon} = G(1,1)\, G(1,1+\epsilon)\,(p^2)^{-2\epsilon}$

$= \frac{1}{\epsilon}\left(\frac{1}{2\epsilon} + \frac{1}{2} + \frac{3}{2}\epsilon\right)(p^2)^{-2\epsilon} - \frac{1}{\epsilon^2}(p^2)^{-\epsilon} = -\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}$

5. $= - 2\, KR'(\ldots) \cdot = \frac{1}{\epsilon^2}\left(-\frac{1}{4} + \frac{5}{8}\epsilon\right) + \frac{1}{2\epsilon^2}$

$= \frac{1}{6\epsilon^2} + \frac{4}{3\epsilon}$

6. $= - 2KR'(\ldots) - KR'(\ldots) - 2KR'(\ldots) + KR'(\ldots)\, KR'(\ldots) =$

$= \frac{1}{\epsilon^3} - \frac{2}{\epsilon}\cdot\left(+\frac{1}{\epsilon^2}\right) - 2\left(-\frac{1}{\epsilon^2}\right)\cdot\frac{1}{\epsilon} + \frac{1}{\epsilon}\cdot\frac{1}{\epsilon}\cdot\frac{1}{\epsilon} = \frac{1}{\epsilon^3}$

$= -\frac{G(1,1)}{\epsilon}\left(G(2,1) - G(1+\epsilon,2)\right)\frac{1}{(p^2)^{\ldots}}$

$= \tau^{-3\epsilon}\left(\frac{1}{3\epsilon^2} - \frac{5}{3\epsilon} + \frac{1}{6}\pi^2 + \frac{7}{3} + O(\epsilon)\right)$

$\frac{1}{(2\pi)^d}\int d^d k\, \frac{1}{(k^2+\tau)^2} = \tau^{-\epsilon}\left(\frac{1}{\epsilon} - 2 + \frac{1}{6}\pi^2\epsilon + O(\epsilon^2)\right)$

7. $= - KR'(\ldots) - KR'(\ldots) + KR'(\ldots)\, KR'(\ldots)$

$- KR'(\ldots) = \frac{1}{\epsilon^2} - \frac{1}{\epsilon}\left(-\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\right) - \frac{1}{\epsilon}\left(-\frac{1}{\epsilon^2}\right) + \frac{1}{\epsilon^3} - \left(-\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\right)\cdot\frac{1}{\epsilon}$

$= \frac{1}{\epsilon^2}\left(+\frac{1}{2\epsilon} + \frac{1}{2} + \frac{3}{2}\epsilon\right) - \frac{2}{\epsilon}\left(-\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon} + \frac{3}{2}\right) + \frac{2}{\epsilon^3} = \frac{1}{2\epsilon^3} + \frac{1}{2\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^2} - \frac{3}{\epsilon} + \frac{2}{\epsilon^3} = \frac{7}{2\epsilon^3} - \frac{1}{2\epsilon^2} - \frac{3}{2\epsilon}$

$\frac{1}{\epsilon}\left(-\frac{1}{2\epsilon^2} + \frac{4}{2\epsilon} + \frac{3}{2}\right)$

$\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\cdot\frac{1}{\epsilon} - \frac{1}{\epsilon}\cdot\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\cdot\frac{1}{\epsilon}\cdot\frac{1}{\epsilon} - \left(-\frac{1}{\epsilon} + \frac{1}{2\epsilon}\right)\frac{1}{2} = \frac{1}{2\epsilon^2} - \frac{1}{2\epsilon^2} \quad \left(-\frac{1}{4} + \frac{5}{8}\epsilon\right)\left(\frac{1}{3\epsilon} + \frac{2}{3} + \frac{8}{3}\epsilon\right)$

8. $= - 3KR'(\ldots) - KR'(\ldots) + \frac{1}{\epsilon} - \left(-\frac{1}{4\epsilon}\right) \cdot \frac{1}{\epsilon}(\ldots) + \frac{1}{4\epsilon}\left(-\frac{1}{\epsilon} + 2\right)$

$= \frac{1}{\epsilon}\left(\frac{1}{2\epsilon} + \frac{1}{2} + \frac{3}{2}\epsilon\right)\left(-\frac{1}{3} - \frac{4}{3}\epsilon\right) + \frac{1}{6\epsilon^2} + \frac{1}{2\epsilon} = -\frac{1}{6\epsilon^2} - \frac{5}{3\epsilon} - \frac{1}{6\epsilon} - \frac{1}{4\epsilon^2} + \frac{1}{2\epsilon} = -\frac{5}{12\epsilon^2} - \frac{1}{3\epsilon}$

$\frac{1}{\epsilon}\left(-\frac{1}{4} - \frac{5}{8}\epsilon\right)\left(\frac{1}{3\epsilon} + \frac{2}{3} + \frac{8}{3}\epsilon\right) - \frac{1}{8\epsilon} = -\frac{1}{12\epsilon^2} - \frac{1}{6\epsilon} - \frac{5}{24\epsilon} + \frac{1}{4\epsilon^2} = -\frac{1}{12\epsilon^2} - \frac{3}{8\epsilon}$

9. $= KR'(\ldots) - KR'(\ldots) - KR'(\ldots) = \frac{1}{\epsilon^2} - 2\frac{1}{\epsilon}\cdot\frac{1}{\epsilon}\left(\frac{1}{2\epsilon} + \frac{1}{2} + \frac{3}{2}\epsilon\right) - \frac{1}{\epsilon}\left(-\frac{1}{\epsilon^2}\right) =$

$= \frac{1}{\epsilon^2}\left(\frac{1}{3\epsilon} + \frac{2}{3} + \frac{8}{3}\epsilon\right) - \frac{2}{\epsilon^2}\left(\frac{1}{2\epsilon} + \frac{1}{2} + \frac{3}{2}\epsilon\right) + \frac{1}{\epsilon^3} = \frac{1}{3\epsilon^3} + \frac{2}{3\epsilon^2} + \frac{8}{3\epsilon} - \frac{1}{\epsilon^3} - \frac{1}{\epsilon^2} - \frac{3}{\epsilon} + \frac{1}{\epsilon^3} = \frac{1}{3\epsilon^3} - \frac{1}{3\epsilon^2} - \frac{1}{3\epsilon}$

11. $= - KR'(\ldots) - KR'(\ldots) = \frac{1}{\epsilon} - \frac{1}{\epsilon^2} - \frac{1}{\epsilon}\left(-\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\right) =$

$= \frac{1}{\epsilon}\left(\frac{1}{2\epsilon} + \frac{1}{2} + \frac{3}{2}\epsilon\right)\left(+\frac{1}{3\epsilon} + \frac{2}{3} + \frac{8}{3}\epsilon\right) - \frac{1}{\epsilon^2}\left(\frac{1}{2\epsilon} + \frac{1}{2} + \frac{3}{2}\epsilon\right) - \frac{1}{\epsilon}\left(-\frac{1}{2\epsilon^2} + \frac{1}{2\epsilon}\right) = \ldots \frac{13}{6\epsilon}$

$-\frac{7}{6\epsilon^2}$

$= \frac{1}{6\epsilon^3} + \frac{1}{3\epsilon^2} + \frac{4}{3\epsilon} + \frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \ldots$ [illegible]

Уравнения РГ

$$\{e_0, \varphi\} \xrightarrow{\Lambda_{\text{рен}}} \{e_0', \varphi\} \longrightarrow \{e, \underset{\varphi_R}{Z_\varphi^{-1}\varphi}\}$$

$$G_{nR}(\dots, e) = Z_\varphi^{-n} G_n(\dots, e_0')$$

$Z_\varphi(e, \mu)$ $\quad \underline{e_0' = e_0'(e,\mu)}$ $\qquad g_0 = g\mu^{2\epsilon} Z_g$

не меняются при изм. рг-масштаба

ренорм-инварианти не зависят от μ

$$\tilde{\mathcal{D}}_\mu F = 0$$

$$\tilde{\mathcal{D}}_\mu = \mu\partial_\mu\big|_{e_0,\Lambda}$$

$$0 = \tilde{\mathcal{D}}_\mu G_n = \tilde{\mathcal{D}}_\mu Z_\varphi^n G_{nR}$$ - уравнение РГ в неприличном виде

$$\tilde{\mathcal{D}}_\mu = \underbrace{\mathcal{D}_\mu + \sum_{e_i}(\tilde{\mathcal{D}}_\mu e_i)\frac{\partial}{\partial e_i}}_{\mathcal{D}_{\text{РГ}}}$$

$$\tilde{\mathcal{D}}_\mu Z_\varphi^n G_{nR} = Z_\varphi^n\Big(\mathcal{D}_\mu + \sum_{e_i}(\mathcal{D}_\mu e_i)\frac{\partial}{\partial e_i}\Big) G_{nR} + Z_\varphi^{n-1}(n\tilde{\mathcal{D}}_\mu Z_\varphi)\, G_{nR}$$

$$\Big(\mathcal{D}_\mu + \sum_{e_i}(\mathcal{D}_\mu e_i)\frac{\partial}{\partial e_i}\Big) G_{nR} + \underbrace{\tilde{\mathcal{D}}_\mu \ln Z_\varphi}_{\gamma_\varphi} n\, G_{nR} = 0$$

$$\Big(\mathcal{D}_\mu + \sum_{e_i}(\mathcal{D}_\mu e_i)\frac{\partial}{\partial e_i} + n\gamma_\varphi\Big) G_{nR} = 0 \quad - \boxed{\text{ур-е РГ}}$$

$e = \tau, g$

$\varphi = Z_\varphi \varphi_R \quad g_0 = g\mu^{2\epsilon} Z_g \quad \tau_0' = \tau Z_\tau \quad Z = Z(g, \lambda, \epsilon) \quad \lambda = \frac{\tau}{\mu^2}$ $\qquad \boxed{MS: Z = Z(g,\epsilon)}$

нет завис-ти от μ, но иногда бывает!

$$\beta(g) = \tilde{\mathcal{D}}_\mu(g)$$

~~$\gamma(a)$~~ $\gamma_a = \tilde{\mathcal{D}}_\mu \ln Z_a \quad a = g, \tau, \varphi$ - аном. разм-ти

$$\beta(g) = \tilde{\mathcal{D}}_\mu g$$

$$\tilde{\mathcal{D}}_\mu g_0 = 0 = \tilde{\mathcal{D}}_\mu(g\mu^{2\epsilon} Z_g) = \beta(g)\cdot \cancel{\mu^{2\epsilon}}\cancel{Z_g} + g\, 2\epsilon \cancel{\mu^{2\epsilon}}\cancel{Z_g} + g\cancel{\mu^{2\epsilon}}\underbrace{\tilde{\mathcal{D}}_\mu Z_g}_{\gamma_g}$$

$$\beta(g) + g\,2\epsilon + g\gamma_g = 0 \qquad \beta(g) = -g(2\epsilon + \gamma_g)$$

$$\tilde{\mathcal{D}}_\mu F(g) = \beta(g)\partial_g F(g) = -(2\epsilon + \gamma_g)\mathcal{D}_g F(g)$$

$$\gamma_g = \tilde{\mathcal{D}}_\mu \ln Z_g = \beta\partial_g \ln Z_g = \underline{-(2\epsilon+\gamma_g)\, g\,\partial_g \ln Z_g}$$

$$\left[\begin{aligned} &\gamma_g = \frac{-2\epsilon\,\mathcal{D}_g \ln Z_g}{1 + \mathcal{D}_g \ln Z_g} \\ &\beta(g) = -g(2\epsilon + \gamma_g) = \frac{-2\epsilon g}{1 + \mathcal{D}_g \ln Z_g} \\ &\gamma_a = -(2\epsilon + \gamma_g)\mathcal{D}_g \ln Z_a \end{aligned}\right.$$

$$\Big(\mathcal{D}_\mu + \beta(g)\frac{\partial}{\partial g} - \gamma_\tau \mathcal{D}_\tau\Big) G_{nR} = 0$$

$$\Big(\mathcal{D}_\mu + \sum_{e_i}(\mathcal{D}_\mu e_i)\frac{\partial}{\partial e_i} + n\gamma_\varphi\Big) G_{nR} = 0$$

$$Z_g = 1 + \frac{C_{11}}{\epsilon}g + g^2\Big(\frac{C_{21}}{\epsilon} + \frac{C_{22}}{\epsilon^2}\Big) + O(g^3)$$

$$\gamma_g = -2C_{11}g - g^2\Big(4C_{21} + 4\frac{C_{22}}{\epsilon} \cancel{\ } - C_{11}^2\Big)$$

$$\mathcal{D}_g \ln Z_g \sim \frac{\mathcal{D}_g Z_g}{Z_g} \qquad O(g^3)$$

~~γ_g ///2ϵ~~ $\quad \mathcal{D}_g Z_g = \Big(\frac{C_{11}}{\epsilon}g + g^2\Big(\frac{2C_{21}}{\epsilon} + \frac{2C_{22}}{\epsilon^2}\Big)\Big)$

$$\frac{\mathcal{D}_g Z_g}{Z_g} = \Big(\frac{C_{11}}{\epsilon}g + g^2\Big(\frac{2C_{21}}{\epsilon} + \frac{2C_{22}}{\epsilon^2}\Big)\Big)\Big(1 - \frac{C_{11}}{\epsilon}g\Big) =$$

$$= \frac{C_{11}}{\epsilon}g + g^2\Big(\frac{2C_{21}}{\epsilon} + \frac{2C_{22}}{\epsilon^2} - \frac{C_{11}^2}{\epsilon^2}\Big)$$

$$\frac{-2C_{11}g - g^2\Big(\frac{4C_{21}}{} + \frac{4C_{22}}{\epsilon} - \frac{2C_{11}^2}{\epsilon}\Big)}{1 + \frac{C_{11}}{\epsilon}g + g^2\Big(\frac{2C_{21}}{\epsilon} + \frac{2C_{22}}{\epsilon^2}\Big)} =$$

$$= \Big[-2C_{11}g - g^2\Big(4C_{21} + 4\frac{C_{22}}{\epsilon} - \frac{2C_{11}^2}{\epsilon}\Big)\Big]\Big(1 - \frac{C_{11}}{\epsilon}g\Big) =$$

$$= -2C_{11}g - g^2\Big(4C_{21} + 4\frac{C_{22}}{\epsilon} - \cancel{\frac{2C_{11}^2}{\epsilon}} + \cancel{\frac{2C_{11}^2}{\epsilon}}\Big)$$

$$\gamma_a = \beta(g)\,\partial_g \ln z_a$$

$$\ln z_a = \int \frac{\gamma_a(x)}{\beta(x)}\,dx$$

$$z_a = \exp\left\{\int_0^g \frac{\gamma_a(x)}{\beta(x)}\,dx\right\}$$

$$z_a(g=0) = 1$$

$$z_g = e^{\left\{-\int_0^g \frac{2\epsilon}{\beta(x)} : \frac{1}{x}\,dx\right\}}$$

Я не знаю, как раскрасить вторую страницу

$$\left(D_\mu + \beta(g)\frac{\partial}{\partial g} - \gamma_\tau D_\tau\right) G_{nR}(g,\tau,\mu)$$

$$L F(u) = \gamma(u)$$

$$L = -D_s + \sum_{i=2}^{n} Q_i(u)\frac{\partial}{\partial e_i}$$

$s = u_1$

$e_i = u_i \quad i \geq 2 \quad Q_1 = -s \qquad L\bar{e}_i(s,e) = 0$

инв. переменные $\qquad \bar{e}_i(1,e) = e_i \qquad s = p/\mu$

$$LF(u) = 0 \iff F(\bar{u}(t)) = \mathrm{const}$$

$$D_t \bar{\bar{s}}(t) = -\bar{\bar{s}}(t) \qquad \bar{\bar{s}}\big|_{t=1} = s$$

$$D_t \bar{\bar{e}}_i(t) = Q_i(\bar{\bar{s}}, \bar{\bar{e}}) \quad \bar{\bar{e}}_i\big|_{t=1} = e_i$$

$$LF(u) = \gamma(u)$$

$$D_t F(\bar{u}(t)) = \gamma(\bar{u}(t))$$

$$F(u) = F(\bar{u}(t)) - \int_1^t \frac{dt'}{t'}\gamma(\bar{u}(t'))$$

$$\bar{\bar{e}}_i(t,s,e)\big|_{t=s} = \bar{e}_i(s,e)$$

1. $LF = 0 \quad F(s,e) \Rightarrow F(s,e) = F(1,\bar{e}(s,e))$
2. $F(s,e) : LF(s,e) = \gamma(s,e) \Rightarrow F(s,e) = F(1,\bar{e}(s,e)) - \int_1^s \frac{dt'}{t'}\gamma(\bar{\bar{s}}(t',s), \bar{\bar{e}}(t',s))$
3. Q_i не зав. от S

$$D_s \bar{e}_i(s,e) = Q_i(\bar{e}(s,e)), \quad \bar{e}\big|_{s=1} = e_i$$

$$F(s,e) = F(1,\bar{e}(s,e)) - \int_1^s \frac{ds'}{s'}\gamma(\bar{e}(s',e))$$

полная ф-ия (ф-я от g)

$$\left(-D_s + \beta(g)\partial_g - \sum_a \Delta_a D_a + \gamma(g)\right)\Phi(s,g,a) = 0$$

$\Phi = e^F$ - подстановка

$$Q_g = \beta(g) \qquad Q_a = -a\Delta_a(g)$$

$$\bar{g}(s,g) \quad D_s\bar{g} = \beta(\bar{g}) \quad \bar{g}\big|_{s=1} = g \qquad \ln s = \int_g^{\bar{g}} \frac{dx}{\beta(x)}$$

$$D_s\bar{a} = -\bar{a}\Delta_a(\bar{g}) \qquad \bar{a}\big|_{s=1} = a$$

$$D_s = \beta(\bar{g})\frac{d}{d\bar{g}} \Rightarrow \beta(\bar{g})\frac{d\ln\bar{a}}{dg} = -\Delta_a$$

$$\bar{a} = a\,e^{-\int_g^{\bar{g}} dx \frac{\Delta_a(x)}{\beta(x)}}$$

$$\Phi(s,g,a) = \Phi(1,\bar{g},\bar{a})\,e^{\int_g^{\bar{g}} dx \frac{\gamma(x)}{\beta(x)}}$$

Парный коррелятор

$G_2 \equiv D$

$$D_R(p, g, \tau, \mu) = p^{-2}\Phi(s, g, z) \qquad s = \frac{p}{\mu} \quad z = \frac{\tau}{\mu^2}$$

$$\left[D_{RG} + 2\gamma_\varphi\right] D_R = 0$$

$$\left[-D_s + \beta(g)\partial_g - (2+\gamma_\tau)D_z + 2\gamma_\varphi(g)\right]\Phi(s, g, z) = 0$$

$$\Phi_{sgz} = \Phi(1, \bar{g}, \bar{z})\, e^{2\int_g^{\bar{g}} dx\, \frac{\gamma_\varphi(x)}{\beta(x)}}$$

$$\bar{z}(s, g, z) = z s^{-2} e^{-\int_g^{\bar{g}} dx\, \frac{\gamma_\tau(x)}{\beta(x)}}$$

$$\ln s = \int_g^{\bar{g}} \frac{dx}{\beta(x)}$$

$\beta(g^*) = 0$ – фиксированная точка

$g = 0$ всегда есть

$g^* \sim \varepsilon$ (обычно)

Что происх. с $\beta(g)$ при $g \sim 1$?

$\beta(g) \simeq \omega(g - g^*)^n$

$$\ln s = \int_g^{\bar{g}} \frac{dx}{\beta(x)}$$

$$\ln s(\bar{g}) \to \omega^{-1} \ln(\bar{g} - g^*)$$

$$\omega > 1 \to \frac{1}{\omega(1-n)}(\bar{g} - g_*)^{1-n}$$

$$D_R(g, p, \tau, \mu) = p^{-2}\left(\frac{p}{\mu}\right)^{2\gamma_\varphi^*}\left(a_0 + a_1\left(\frac{p}{\mu}\right)^{\omega} + \dots\right)$$