Александр Андрианов

<u>Физика Стандартной Модели элементарных частиц</u>

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Ренормгруппа Стандартной Модели I

Метод фонового поля = background field method

$$\Gamma(\phi_c) = -\mathcal{W}(j) - (\phi_c, j); \quad \phi_c(x) = -\frac{\delta \mathcal{W}(j)}{\delta j(x)}; \quad (\phi_c, j) \equiv \int d^4x \phi_c(x) j(x).$$

Depending on the goal of an investigation one can treat as an independent variable either ϕ_c or j . Let us check the duality relation,

$$\frac{\delta \Gamma(\phi_c)}{\delta \phi_c(x)} = -\int d^4y \Big(\frac{\delta j(y)}{\delta \phi_c(x)} \frac{\delta \mathcal{W}(j)}{\delta j(x)} + \frac{\delta j(y)}{\delta \phi_c(x)} \phi_c(x) \Big) - j(x) = -j(x),$$

in virtue of the definition for "classical field" ϕ_c. Thus this Legendre transformation between ϕ_c and $j(x)$ is invertible and the effective actions $\Gamma(\phi_c)$ and $\mathcal{W}(j)$ are dual each to other.

Background formalism and 1-loop correction of classical action

Let's continue with semiclassical approach to calculation of the QFT path integral. for this purpose introduce the *background formalism*. First change variables $\boxed{\phi(x) = \eta(x) + \phi_c.}$ As

shift does not make any influence on the path integration. Now let us expand around ϕ_c adjusting(!) the external source $j(x)$ from the classical equation $j(x) = -\delta S(\phi_c)/\delta \phi_c(x)$,

$$S(\phi) + \int d^4x j(x)\phi(x) \simeq S(\phi_c) + \int d^4x j(x)\phi_c(x) + \frac{1}{2} \int d^4x d^4y \eta(x) \frac{\delta^2 S(\phi)}{\delta\phi(x)\delta\phi(y)}\Big|_{\phi=\phi_{cl}} \eta(y) + \mathcal{O}(\eta^3)$$

Denote

$$\frac{\delta^2 S(\phi)}{\delta\phi(x)\delta\phi(y)}\Big|_{\phi=\phi_{cl}} \equiv \langle x|\hat{S}''|y\rangle = \Big(-\partial_\mu^2 - m^2 - \frac{\lambda}{2}\phi_{cl}^2 \Big) \delta^{(4)}(x-y);$$

$$\int d^4x d^4y \eta(x) \frac{\delta^2 S(\phi)}{\delta\phi(x)\delta\phi(y)}\Big|_{\phi=\phi_{cl}} \eta(y) \equiv \Big(\eta, \hat{S}''\eta\Big).$$

The generating functional,

$$\mathcal{Z}(j) \simeq \exp\Big\{ \frac{i}{\hbar}[S(\phi_{cl}) + \int d^4x j(x)\phi_{cl}(x)] \Big\} \int \mathcal{D}\eta(x) \exp\Big\{ \frac{i}{2\hbar}\Big(\eta, \hat{S}''\eta\Big) \Big\},$$

Наикратчайший способ сосчитать однопетлевое эффективное действие (Larry Abbot, 1982)

$$\int \mathcal{D}\eta(x)\exp\left\{\frac{i}{2\hbar}\Big(\eta,\hat{S}''\eta\Big)\right\} = C(\det(\hat{S}''))^{-1/2} = \tilde{C}\det\Big(\frac{\hat{S}''}{\hat{S}''_0}\Big)^{-1/2}.$$

Thus the essential part of the first quantum correction to the classical result is,

$$= \exp\Big\{-\frac{1}{2}\mathrm{Tr}\log\Big(\frac{\hat{S}''}{\hat{S}''_0}\Big)\Big\} = \exp\Big\{-\frac{1}{2}\int d^4x\langle x|\log\Big(1-\frac{\lambda}{2}\frac{1}{-\partial_\mu^2-m^2+i\epsilon}\phi_{cl}^2\Big)|x\rangle\Big\}.$$

$$-\frac{1}{2}\int d^4x\langle x|\log\Big(1-\frac{\lambda}{2}\frac{1}{-\partial_\mu^2-m^2+i\epsilon}\phi_{cl}^2\Big)|x\rangle$$
$$=\frac{\lambda}{4}D(0)\int d^4x\phi_{cl}^2(x)+\frac{\lambda^2}{16}\int d^4x\int d^4y\phi_{cl}^2(x)\big(D(x-y)\big)^2\phi_{cl}^2(y).$$

Квантовая электродинамика
производящий функционал

$$\mathcal{Z}_{QED}(j_\mu,\eta,\bar{\eta}) = \int\frac{1}{C_{QED}}\prod_x\prod_{\mu=0}^{3}\mathcal{D}A_\mu\mathcal{D}\psi(x)\mathcal{D}\bar{\psi}(x)\mathcal{D}c(x)\mathcal{D}\bar{c}(x)$$

$$\times\exp\Big\{i\int d^4x\Big(-\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2\xi e^2}(\partial_\mu A^\mu)^2+\bar{\psi}(i\not\partial-\not A-m)\psi+\partial_\mu\bar{c}\partial^\mu c$$
$$+\frac{1}{e}A_\mu j^\mu+\bar{\eta}\psi+\bar{\psi}\eta\Big)\Big\}$$

Ф.-П. духи обеспечивают правильный счет физических степеней фотона в статсумме при ненулевых температурах.

Background approach to one-loop action

Let's now calculate the second variation around the quasiclassical solution. Define,

$$A^\mu(x)=A^\mu_{cl}(x)+a^\mu(x);\quad \psi(x)=\psi_{cl}(x)+q(x);\quad \bar{\psi}(x)=\bar{\psi}_{cl}(x)+\bar{q}(x).$$

The part of the Lagrangian density bilinear in fluctuation fields $a^\mu(x),q(x),\bar{q}(x)$,

$$\mathcal{L}^{(2)}=-\frac{1}{4e^2}(\partial_\mu a_\nu-\partial_\nu a^\mu)^2-\frac{1}{2\xi e^2}(\partial_\mu a^\mu)^2+\bar{q}(i\not\partial-\not A_{cl}-m)q-\bar{\psi}_{cl}\not a q-\bar{q}\not a\psi_{cl}.$$

Integration over fermion fluctuations $q,\bar{q}$ in the generating functional,

$$\mathcal{Z}^{(2)}=\int\prod_x\prod_{\mu=0}^{3}\mathcal{D}a_\mu\mathcal{D}q(x)\mathcal{D}\bar{q}(x)\exp\Big\{i\int d^4x\mathcal{L}^{(2)}\Big\}=\exp\Big\{\mathrm{Tr}\log\Big(1-\frac{1}{i\not\partial-m+i\epsilon}\not A_{cl}\Big)\Big\}$$

$$\times\int\prod_x\prod_{\mu=0}^{3}\mathcal{D}a_\mu\exp\Big\{i\int d^4x\Big(-\frac{1}{4e^2}(\partial_\mu a_\nu-\partial_\nu a^\mu)^2-\frac{1}{2\xi e^2}(\partial_\mu a^\mu)^2\Big)$$
$$-i\int d^4xd^4y\bar{\psi}_{cl}(x)\not a(x)\langle x|\frac{1}{i\not\partial-\not A_{cl}-m+i\epsilon}|y\rangle\not a(y)\psi_{cl}(y)\Big\}\equiv\mathcal{Z}^{(2)}_\gamma\cdot\mathcal{Z}^{(2)}_f.$$

photon part of QED is invariant under charge conjugation one can show (Furry'theorem) that only even powers of photon fields A^μ_{cl} appear in the functional $\mathcal{Z}^{(2)}_\gamma$. The term with odd powers of A^μ_{cl} change sign under charge conjugation and therefore vanish. Thus the perturbative expansion of the one-loop effective action for photons,

$$\Gamma_{1,\gamma} = i\hbar \sum^\infty \frac{1}{n} \mathrm{Tr} \left(\left[\frac{1}{i\,\partial\!\!\!/ - m + i\epsilon} A\!\!\!/_{cl} \right]^{2n} \right).$$

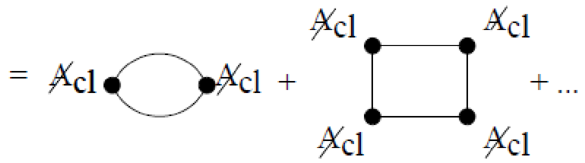

Рис.8. Потенциально расходящиеся вклады

The first term quadratic in photon fields defines the (vacuum) polarization operator $\Pi_{\mu\nu}$,

$$= i\hbar\frac{1}{2}\int d^4x d^4y A^\mu_{cl}(x) \mathrm{Tr}\left(\gamma_\mu S(x-y)\gamma_\nu S(y-x)\right) A^\nu_{cl}(y) = \hbar\frac{1}{2}\int \frac{d^4p}{(2\pi)^4} A^\mu_{cl}(-p)\Pi_{\mu\nu}(p)A^\nu_{cl}(p);$$

$$\Pi_{\mu\nu}(p) \equiv i\int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\gamma_\mu \frac{k\!\!\!/ + m}{k^2 - m^2 + i\epsilon}\gamma_\nu \frac{k\!\!\!/ - p\!\!\!/ + m}{(k-p)^2 - m^2 + i\epsilon}\right).$$

By power counting this integral is quadratically divergent $\sim \int d^4k/k^2$. As the calculation of such integrals uses shifting in momenta it is convenient to employ the Pauli-Villars regularization. Start from one P.-V. field with a mass $M \gg |p_\mu|, m$. As the divergence is mass independent it canceled in the difference $\Pi_{\mu\nu}(m) - \Pi_{\mu\nu}(M)$. Still the related integral remains divergent: the milder divergence is logarithmical and therefore proportional to a mass, $\Pi_{\mu\nu}(m) - \Pi_{\mu\nu}(M) \sim (m^2 - M^2)\int d^4k/k^4$. One has to introduce two P.-V. fields with masses M_1, M_2 and relative weights c_1, c_2 . Then the regularized polarization operator $\Pi^{reg}_{\mu\nu}(p) = \Pi_{\mu\nu}(m) - c_1\Pi_{\mu\nu}(M_1) - c_2\Pi_{\mu\nu}(M_2)$, and the conditions to remove divergences are evidently

$$c_1 + c_2 = 1; \quad c_1M_1^2 + c_2M_2^2 = m^2.$$

These fictitious fields are bosons/fermions with spin 1/2 and fractional internal degrees of freedom that can be implemented directly in the determinant,

$$\Gamma^{reg}_{1,\gamma} = \mathrm{Tr}\log\left(1 - \frac{1}{i\,\partial\!\!\!/ - m + i\epsilon} A\!\!\!/_{cl}\right)$$

$$-c_1\mathrm{Tr}\log\left(1 - \frac{1}{i\,\partial\!\!\!/ - M_1 + i\epsilon} A\!\!\!/_{cl}\right) - c_2\mathrm{Tr}\log\left(1 - \frac{1}{i\,\partial\!\!\!/ - M_2 + i\epsilon} A\!\!\!/_{cl}\right).$$

One does not supply them with any physical meaning but decouple them in the limit $M_{1,2} \to \infty$

$$\Pi^{reg}_{\mu\nu}(p) = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right)\Pi^{reg}(p^2);$$

$$\Pi^{reg}(p^2) = -i\frac{1}{3}\int_0^1 dx \int \frac{d^4k}{(2\pi)^4}\left(\frac{-8k(k-p) + 16m^2}{(k^2 - m^2 - 2kpx + p^2x + i\epsilon)^2} - [\text{P.-V.}]\right)$$

$$\stackrel{M_{1,2}^2 \gg |p^2|, m^2}{=} \frac{1}{6\pi^2} \int_0^1 dx \left(0 + 3p^2 x(1-x) \log \frac{M^2}{m^2 - p^2 x(1-x)} + 0 \right) \stackrel{-p^2 \gg m^2}{\simeq} \frac{1}{12\pi^2} p^2 \log \frac{M^2}{-p^2}.$$

$$\Gamma_\gamma \simeq \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} A_{cl}^\mu(-p)(-g_{\mu\nu}p^2 + p_\mu p_\nu) \frac{1}{e^2(p^2)} A_{cl}^\nu(p$$

$$\frac{1}{e^2(p^2)} \equiv \frac{1}{e_0^2} + \frac{\Pi(p^2)}{p^2} \stackrel{-p^2 \gg m^2}{\simeq} \frac{1}{e_0^2} + \frac{1}{12\pi^2} \log \frac{M^2}{-p^2}.$$

$$\frac{1}{e_{ph}^2(\mu)} = \frac{1}{e_0^2} + \frac{1}{12\pi^2} \log \frac{M^2}{\mu^2}; \quad \frac{d}{d\tau} \left(\frac{1}{e_{ph}^2(\mu)} \right) = -\frac{1}{12\pi^2} \quad \text{or} \quad \frac{d\alpha}{d\tau} = \frac{1}{3\pi}\alpha^2 \equiv \beta(\alpha),$$