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Лекция 9 (дополнение к лекции 8), 26.04.2019

Рождение векторных бозонов в электрон-позитронных столкновениях

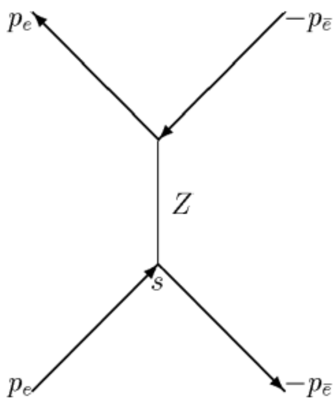


Fig 11 Feynman graph for the $e\bar{e}$ forward scattering through the Z-production

The Z and W bosons can be produced in the $e\bar{e}$ colliding beams and at high energy hadro collisions. To begin with, let us consider the Z-boson production at the e^-e^+ collisions. Th total cross-section according to the optical theorem is proportional to the discontinuity c the elastic scattering amplitude in the forward direction. At large energies $s = W^2 \ll m$ we have

$$\sigma_{tot} = \frac{1}{2s}\frac{1}{i}\Delta\ A(s,0)\,,\ \ \Delta A(s,0) = A(s+i\epsilon,0) - A(s-i\epsilon,0)\ .$$

Using the Feynman rules for the Standard Model we can write the elastic amplitude with the virtual Z-boson in the s channel as follows (see Fig. 5)

$$A(s,0) = -\left(\frac{g}{2\cos\theta_w}\right)^2 \frac{\frac{1}{4}Tr\left(\gamma_\sigma(g_v + g_a\gamma_5)\widehat{p_e}\,\gamma_\sigma\,(g_v + g_a\gamma_5)\,\widehat{p_{\bar{e}}}\right)}{s - M_Z^2 + i\,\Gamma_{tot}\;M_Z}\,,$$

where g_v and g_a are the effective vector and axial couplings and at large s we neglected the electron mass.

Therefore the total cross-section is

$$\sigma_Z = (g_v^2 + g_a^2)\left(\frac{g}{2\cos\theta_w}\right)^2 \frac{\Gamma_{tot}}{4\,M_Z\,(E_{nr}^2 + \frac{1}{4}\Gamma_{tot}^2)}\,,$$

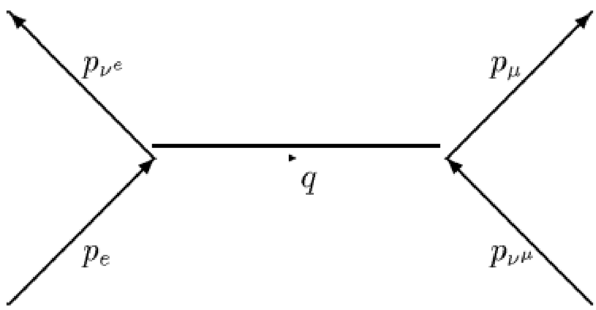
where $E_{nr} = W - M_Z$ ($W = \sqrt{s}$) is the non-relativistic energy of the initial particles and we consider the region near the resonance: $E_{nr} \ll M_Z$. We can rewrite it in the simple form

$$\sigma_Z(s) = \frac{3\pi}{M_Z^2}\;\frac{\Gamma_{e\bar{e}}\,\Gamma_{tot}}{E_{nr}^2 + \frac{1}{4}\Gamma_{tot}^2}$$

with the use of the above expression for the $e\bar{e}$ width of the Z -boson decay

$$\Gamma_{e\bar{e}} = g^2\,\frac{(g_v^2 + g_a^2)}{48\,\pi}\,\frac{M_z}{\cos^2\theta_w}\;.$$

Еще одна реакция с обменом W бозоном



Figu. 12. Feynman graph for the transition $e\nu^\mu \to \nu^e\mu$ with the W-exchange

more complicated process of the transition $e_L \nu^\mu \to \nu^e \mu_L$ is described by the amplitude (see Fig 12

$$M = \left(\frac{g}{\sqrt{2}}\right)^2 \; \overline{u}(p_{\nu^e})\,\gamma_\sigma\,\frac{1+\gamma_5}{2}\,u(p_e)\;\frac{1}{q^2 - m_w^2}\,\overline{u}(p_\mu)\,\gamma_\sigma\,\frac{1+\gamma_5}{2}\,u(p_{\nu^\mu})\;.$$

Here $q = p_e - p_{\nu^e}$ is the momentum transfer and we neglected the small term proportional to $q_\mu q_\nu / m^2$ in the propagator. For low energies, when $q^2 \ll m_w^2$, this amplitude can be constructed from the corresponding term of the Fermi hamiltonian

$$L_{Fermi} = -\frac{G}{\sqrt{2}}\,\overline{\psi}_{\nu^e}\,\gamma_\sigma\,(1+\gamma_5)\psi_e\;\overline{\psi}_\mu\,\gamma_\sigma\,(1+\gamma_5)\psi_{\nu^\mu}\,,$$

if

$$G = \sqrt{2}\,\frac{g^2}{8}\,\frac{1}{m_w^2} = \frac{\pi}{\sqrt{2}}\,\frac{\alpha_e}{\sin^2\theta_w}\,\frac{1}{m_w^2} = \frac{7.235 \times 10^{-2}}{m_w^2}\,,$$

where $\alpha_e = e^2/(4\pi)$ is the fine structure constant. The known value of G equal to $10^{-5}/m_p^2$ is in an agreement with this formula. This agreement is an important achievement of the unified electro-weak theory.

For the calculation of radiative corrections to the physical observables in the Standard Model it is important to know the QED fine structure constant at the scale $q^2 \sim M_Z^2$. Taking into account its renormalization due to the light leptons and hadrons, we obtain

$$\alpha(M_Z)^{-1} = \alpha^{-1}(1-\Delta\alpha) = 127.938 \pm 0.027\,,\;\; \Delta\alpha = \frac{\alpha}{3\pi}\ln\frac{M_Z^2}{m_e^2}\,.$$

Note, that with the use of the Gell-Mann-Low equation for the QCD running coupling constant

$$\frac{d\,\alpha_c}{d\ln\mu} = -\alpha_c\left(\beta_0\frac{\alpha_c}{2\pi} + \beta_1\frac{\alpha_c^2}{4\pi^2} + \beta_2\frac{\alpha_c^3}{64\pi^3} + \ldots\right),$$

where

$$\beta_0 = 11 - \frac{2}{3}n_f\,,\;\; \beta_1 = 51 - \frac{19}{3}n_f\,,\;\; \beta_2 = 2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2\,,$$

we can calculate α_c at arbitrary μ using the experimental information, that

$$\alpha_c(M_z) = 0.118 \pm 0.002\,.$$