Fermion mass matrices, textures and beyond

Manmohan Gupta[*], Priyanka Fakay, Samandeep Sharma[@], Gulsheen Ahuja

Department of Physics, Panjab University, Chandigarh, India.
[@]*Department of Physics, GGDSD College, Chandigarh, India.*
[*]*mmgupta@pu.ac.in*

September 16, 2018

Abstract

The issue of texture specific fermion mass matrices have been examined briefly from the 'bottom-up' perspective. In case no conditions are imposed, the texture *ansätze* leads to a large number of viable possibilities. However, besides textures, if in case one incorporates the ideas of 'natural mass matrices' and uses the facility of Weak Basis Transformations, then one is able to arrive at a minimal finite set of viable mass matrices in the case of quarks.

Understanding fermion masses and mixings is one of the biggest challenges in the present day High Energy Physics. One of the key difficulties in this area is the fact that the fermion masses and mixings span several orders of magnitude. In the case of charged fermions, the range of masses is from 10^5 eV to 10^{12} eV, corresponding respectively to the electron mass and the mass of the top quark. Further, the absolute masses of the neutrinos are not known, however, two of the lightest neutrino masses can be of the order of a fraction of an eV, with no lower limit for the third neutrino mass. In case the theory requires the existence of right handed neutrinos, responsible for see-saw mechanism [1]-[6] with the mass range of $10^{12}-10^{15}$ GeV, the fermion masses would then cover almost 25 orders of magnitude.

The problem gets further complicated when one notices that the pattern of mixings are also quite different in case of quarks and leptons. In fact, in the case of quarks we have clearly hierarchical structure of the mixing angles, for example, $s_{12} \sim 0.22, s_{23} \sim 0.04, s_{13} \sim 0.004$. In contrast, the two of the mixing angles in case of neutrinos are quite large, whereas the third angle although small as compared to the other two angles yet it is of the order of the Cabibbo angle. Similarly, the pattern of masses in the case of charged leptons has a very well defined hierarchy, whereas in the case of neutrino we may have normal/inverted hierarchy or degenerate scenario of neutrino masses. Since the mixing matrices are related to the corresponding mass matrices therefore formulating viable fermion mass matrices becomes all the more complicated.

In the absence of fundamental theory of flavor physics wherein fermion masses and mixings can be understood, the present day phenomenological approaches can

be broadly categorized as 'top-down' and 'bottom-up'. The top-down approach essentially starts with the formulation of mass matrices at the GUT scale, whereas, the bottom-up approach starts with the phenomenological mass matrices at the weak scale. Despite large number of attempts from the top-down perspective [7] yet we are not in a position to incorporate the vast amount of data related to fermion mixing within a consistent framework. In this context, therefore, it is desirable to look at bottom-up approach [8]-[11] consisting of finding the phenomenological fermion mass matrices which are in tune with the low energy data, i.e., observables like quark and lepton masses, mixing angles in both the sectors, angles of the unitarity triangle in the quark sector, etc.. Also, successful phenomenological formulation of mass matrices may provide clues for appropriate dynamical models, in particular, important clues for their formulation at the GUT scale.

The purpose of the present work is to explore the essentials, from a 'bottom-up' approach perspective, needed to arrive at a minimal set of fermion matrices which are compatible with the latest mixing data. To this end, we have not gone into a detailed and comprehensive analysis rather would like to present a brief overview related to the issue mentioned above. Further, we would like to discuss the possibility of arriving at a minimal set of viable mass matrices using textures and other ideas.

To begin with, we discuss the earliest *ansätz* made in the context of quark mass matrices. The first step in this direction was taken by Fritzsch [12, 13], essentially laying down the path for future investigations in this direction. According to his hypothesis, the 3×3 mass matrices for the up and down sectors, M_U and M_D, are hermitian and are given by

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & 0 & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \qquad M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & 0 & B_D \\ 0 & B_D^* & C_D \end{pmatrix}. \tag{1}$$

Another *ansätz* proposed by Stech [14] has the following form for the mass matrices in the up and down sectors

$$M_U = S, \qquad M_D = \beta S + A, \tag{2}$$

where S and A are symmetric and antisymmetric 3×3 matrices respectively. Yet another *ansätz*, proposed by Gronau [15], had the features of both Fritzsch's and Stech's *ansätze*, e.g,

$$M_U = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix}, \qquad M_D = \beta \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} + \begin{pmatrix} 0 & ia & 0 \\ -ia & 0 & ib \\ 0 & -ib & 0 \end{pmatrix}. \tag{3}$$

Interestingly, these *ansätze* were ruled out by the "high" value of the t quark mass and these continue to be ruled out even with subsequent refinements in the data. To this end, we discuss, in somewhat detail, the case of Fritzsch *ansätz*. The essentials of the methodology usually used to carry out the analysis include diagonalizing the mass matrices M_U and M_D by unitary transformations and obtaining a Cabibbo-Kobayashi-Maskawa (CKM) matrix from these transformations. To ensure

the viability of the considered mass matrices, this CKM matrix should be compatible with the quark mixing data, for details regarding this we refer the readers to [11]. Following this methodology for the above mentioned *ansätz* considered by Fritzsch, the CKM matrix so obtained by considering latest inputs from PDG 2014 [17] is given by

$$V_{\text{CKM}} = \begin{pmatrix} 0.9837 - 0.9872 & 0.2248 - 0.2268 & 0.0053 - 0.0075 \\ 0.2203 - 0.2264 & 0.9160 - 0.9721 & 0.0601 - 0.2037 \\ 0.0302 - 0.0308 & 0.0043 - 0.0194 & 0.9991 - 0.9999 \end{pmatrix}. \quad (4)$$

A look at this matrix immediately reveals that the ranges of most of the CKM elements show no overlap with those obtained by recent global analyses [17]. This, therefore, leads to the conclusion that the Fritzsch *ansätz* is not compatible with the recent quark mixing data.

The above conclusion can be explicitly understood by studying the analytical expressions of the elements $|V_{ub}|$ and $|V_{cb}|$, e.g.,

$$V_{ub} = -\sqrt{\frac{m_d}{m_s}}\left(\frac{m_s}{m_d}\right)^{\frac{3}{2}}e^{i\phi_1} - \sqrt{\frac{m_u}{m_c}}\sqrt{\frac{m_s}{m_b}} + \sqrt{\frac{m_u}{m_c}}\sqrt{\frac{m_c}{m_t}}e^{i\phi_2}, \quad (5)$$

$$V_{cb} = \sqrt{\frac{m_u}{m_c}}\sqrt{\frac{m_d}{m_s}}\left(\frac{m_s}{m_b}\right)^{\frac{3}{2}}e^{i\phi_1} - \sqrt{\frac{m_s}{m_b}} + \sqrt{\frac{m_c}{m_t}}e^{i\phi_2}, \quad (6)$$

where phases ϕ_1 and ϕ_2 are related to the phases associated with the elements of the mass matrices [10]. In Fig.1, we have plotted the dependence of these elements with respect to the strange quark mass m_s. While plotting the allowed ranges of the matrix elements $|V_{ub}|$ and $|V_{cb}|$, all other parameters have been given full variation within the allowed ranges. A general look at the figure immediately shows that the plotted values both $|V_{ub}|$ and $|V_{cb}|$ have no overlap with the allowed experimental ranges of these. Thus, one can again conclude that Fritzsch *ansätz* is not viable.
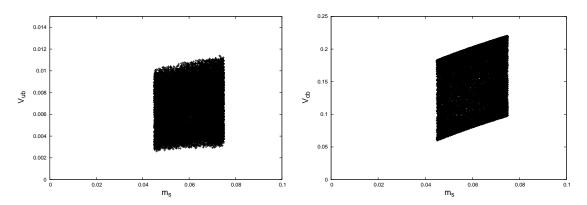

Figure 1: Plots showing the allowed range of $|V_{ub}|$ and $|V_{cb}|$ w.r.t the light quark mass m_s for the Fritzsch mass matrix.

The generalization of the Fritzsch *ansätze* led to the idea of textures. *A particular texture structure is said to be texture n zero, if it has n number of non-trivial zeros,*

for example, if the sum of the number of diagonal zeros and half the number of the symmetrically placed off diagonal zeros is n. Therefore, if both M_U and M_D have n texture zeros each, together these are called texture $2n$ zero mass matrices. For example, the Fritzsch *ansätz*, mentioned in equation (1), corresponds to texture 6 zero quark mass matrices.

Apart from texture 6 zero mass matrices considered by Fritzsch, some other versions of these were also analyzed and consequently ruled out by Ramond *et.al.* [16], these continue to be ruled out even by the present quark mixing data. In this context, Ramond *et.al.* [16] also arrived at an important conclusion that the texture structure of a matrix as well as its hermiticity property are not 'affected' when one scales down from GUT scale to weak scale, justifying the formulation of texture specific mass matrices. This important conclusion also leads to the fact that the texture zeros of fermion mass matrices can be considered as phenomenological zeros, thereby implying that at all energy scales the corresponding matrix elements are sufficiently suppressed in comparison with their neighboring counterparts. This, therefore, opens the possibility of considering less than six texture zeros [10] for the quark mass matrices.

Extending their analysis of texture 6 zero mass matrices, Ramond *et.al.* [16] have also examined the viability of a few texture 5 zero quark mass matrices. Recently, the compatibility of texture 5 zero mass matrices with the latest mixing data has also been examined in detail [10]. Interestingly, even in this case one finds that there is only marginal compatibility, in particular, out of the large number of possibilities for texture 5 zero mass matrices, only Fritzsch-like mass matrices have limited compatibility with the experimental data. As an extension of texture 5 zero mass matrices, several authors have carried out the study of the implications of the Fritzsch-like texture 4 zero mass matrices [18]-[21]. These analyses reveal that the texture 4 zero mass matrices, undoubtedly, are able to accommodate the quark mixing data quite well.

Very recently, Ludl and Grimus [22] have performed a detailed and comprehensive analysis for general as well as symmetric texture specific quark mass matrices. Without imposing any restrictions on textures and using the facility of 'Weak Basis Transformations', Ludl and Grimus arrive at 243 classes of texture specific mass matrices, related through permutations. To reduce the number of possibilities they use the concept of maximally restrictive classes (one cannot place another texture zero into one of the two mass matrices while keeping the model compatible with the data). Thus, they found 27 viable classes for general mass matrices, however, without any predictive powers. In the case of symmetric mass matrices they have found 15 maximally restrictive textures which are predictive with respect to one or more light quark masses.

The above analysis indicates that in the absence of any additional conditions on textures, even texture 5 zero mass matrices could also be viable and the number of viable possibilities increases rapidly as one goes to lower textures. This therefore, brings us to the conclusion that in case we have to arrive at finite set of mass matrices which may serve as clues for their formulation at fundamental level, one needs to go beyond texture *ansätze*. In this context, two important ideas for the quark matrices

have been considered in the literature, e.g., the concept of 'natural mass matrices', advocated by Peccei and Wang [23] and that of Weak Basis (WB) transformations, considered by Fritzsch and Xing [24] as well as Branco *et.al.* [25].

The essential idea of 'natural mass matrices' consists of formulating quark mass matrices which are able to reproduce hierarchical mixing angles without resorting to fine tuning. This results in considerably constraining the parameter space available to the elements of the mass matrices. Using this concept Peccei and Wang [23] have attempted to reconstruct mass matrices at M_z as well as GUT scale, however without invoking any other condition they are not able to find any finite or viable set of mass matrices. In the context of texture specific mass matrices, the idea of 'natural mass matrices' has been found to be useful in reproducing the data when the following hierarchy is imposed on the elements of the quark mass matrices

$$(1, i) \leq (2, j) \leq (3, 3) \quad i = 1, 2, 3; j = 2, 3. \tag{7}$$

As mentioned earlier, Weak Basis transformations is an another idea to go beyond texture *ansätze*, considered by by Fritzsch and Xing [24] as well as Branco *et.al.* [25]. Within the framework of the SM, the hermitian quark mass matrices, which encode all the information about the quark masses and mixings, have a total of 18 real free parameters, which is a large number compared to only ten physical parameters corresponding to six quark masses and four physical parameters of the CKM matrix. In this context, it is interesting to note that one has the freedom to make a unitary transformation, e.g., $q_L \rightarrow W q_L$, $q_R \rightarrow W q_R$, $q'_L \rightarrow W q'_L$, $q'_R \rightarrow W q'_R$ under which the gauge currents

$$-\mathcal{L}_W^{cc} = \frac{g}{\sqrt{2}} \overline{(u, c, t)}_L \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu + h.c. \tag{8}$$

remain real and diagonal but the mass matrices transform as

$$M_U \longrightarrow M'_U = W^\dagger M_U W, \quad M_D \longrightarrow M'_D = W^\dagger M_D W \tag{9}$$

where W is an arbitrary unitary matrix. Such transformations are referred to as 'Weak Basis (WB) Transformations'.

The WB transformations broadly lead to two possibilities for the texture zero fermion mass matrices. In the first possibility, as observed by Fritzsch and Xing [24], one ends up with texture 2 zero fermion mass matrices, wherein both the fermion mass matrices assume a texture 1 zero hermitian structure of the following form

$$M'_q = \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \qquad q' = U, D. \tag{10}$$

In the second possibility, as observed by Branco *et al.* [25] one ends up with texture 3 zero fermion mass matrices M_U and M_D wherein one of the matrix among these

pairs is a texture 2 zero Fritzsch-like hermitian mass matrix given by

$$M_q = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \qquad q = U, D, \tag{11}$$

while the other mass matrix is a texture 1 zero hermitian mass matrix of the following form

$$M_q' = \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}, \qquad q' = U, D. \tag{12}$$

Further, we would like to emphasize here that although the two approaches for WB transformations are equivalent, but the approach by Branco *et al.* leads to non parallel texture three zero structure while the approach by Fritzsch and Xing leads to parallel texture two zero structure.

Recently an analysis by Costa and Simoes [26] shows that starting from arbitrary matrices M_U and M_D, it is always possible to perform a WB transformation that renders them Hermitian with a particular texture, therefore, resulting in reducing the number of free parameters of general mass matrices. The obtained quark matrices are confronted with the experimental data, reconstructing them at the electroweak scale and at a high scale where the Froggatt-Nielsen mechanism can be implemented. However, in the absence of any constraints on the elements of the mass matrices, it leads to a large number of viable texture zero matrices.

It is therefore evident from the above discussion that neither texture *ansätze* nor Weak Basis transformations or 'naturalness' criteria, on their own, are able to lead to a finite set of viable texture specific mass matrices. In order to obtain the same, perhaps one needs to combine the three as discussed recently by Sharma *et al.*[27]. This analysis shows that one can start with the most general mass matrices and consequently explore the possibility of obtaining a finite set of viable texture specific mass matrices formulated by using weak basis transformations as well as the constraints imposed due to 'naturalness'. Interestingly, the analysis reveals that a particular set of texture 4 zero quark mass matrices can be considered to be a unique viable option for the description of quark mixing data.

A corresponding analysis in the lepton sector, wherein one explores the possibility of arriving at a minimal set of lepton texture specific mass matrices, reveals that this is not possible because of a large number of viable possibilities. The analysis pertaining to texture 4 zero Fritzsch-like mass matrices in the Dirac as well as Majorana neutrino case indicates that these matrices are compatible with the normal hierarchy and degenerate scenario of neutrino masses whereas for inverted hierarchy such matrices are ruled out in case the naturalness conditions are imposed. In conclusion, we can perhaps say that the texture 4 zero Fritzsch-like mass matrices provide an almost unique class of viable fermion mass matrices giving vital clues towards unified textures for model builders.

References

[1] H. Fritzsch, M. Gell-Mann, P. Minkowski, Phys Lett. **B 59**, 256 (1975).

[2] P. Minkowski, Phys. Lett. **B 67**, 421 (1977).

[3] T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, edited by O. Sawada, A. Sugamoto (KEK, Tsukuba, 1979), p. **95**.

[4] M. Gell-Mann, P. Ramond, R. Slansky, in *Supergravity*, edited by F. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. **315**.

[5] S. L. Glashow, in *Quarks and Leptons*, edited by M. Lévy *et al.* (Plenum, New York, 1980), p. **707**.

[6] R. N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[7] Mu-Chen, K. T. Mahanthappa, Int. Jour. of Mod. Phys. **A 18**, 5819 (2003).

[8] H. Fritzsch, Z.Z. Xing, Nucl. Phys. **B 556**, 49 (1999) and references therein.

[9] Z. Z. Xing, H. Zhang, J. Phys. **G 30**, 129 (2004) and refrences therein.

[10] M. Gupta, G. Ahuja, Int. J. Mod. Phys. **A 26**, 2973 (2011) and refrences therein.

[11] M. Gupta, G. Ahuja, Int. J. Mod. Phys. **A 27**, 1230033 (2012) and references therein.

[12] H. Fritzsch, Phys. Lett. **B 70**, 436 (1977).

[13] H. Fritzsch, Phys. Lett. **B 73**, 317 (1978).

[14] B. Stech, Phys. Lett. **B 130**, 189 (1983).

[15] M. Gronau, R. Johnson, J. Schechter, Phys. Rev. Lett. **54**, 2176 (1985).

[16] P. Ramond, R.G. Roberts, G.G. Ross, Nucl. Phys. **B 406**, 19 (1993).

[17] K. A. Olive *et.al.* Particle Data Group, Chin. Phys. **C 38**, 090001 (2014).

[18] D. Du, Z. Z. Xing, Phys. Rev. **D 48**, 2349 (1993).

[19] P. S. Gill, M. Gupta, Pramana **45**, 333 (1995).

[20] P. S. Gill, M. Gupta, Phys. Rev. **D 57**, 3971 (1998).

[21] M. Randhawa, M. Gupta, Phys. Rev. **D 63**, 097301 (2001).

[22] P. Ludl, W. Grimus, arXiv: hep-ph/1501.04942.

[23] R.D. Peccei, K. Wang, Phys. Rev. **D 53**, 5 (1996).

[24] H. Fritzsch, Z. Z. Xing, Phys. Lett. **B 413**, 396 (1997) and references therein.

[25] G. C. Branco *et al.*, Phys. Rev. Lett. **82**, 683 (1999).

[26] D. Emmanuel-Costa, C. Simoes, Phys. Rev. **D 79**, 073006 (2009).

[27] S.Sharma, P. Fakay, G. Ahuja, M. Gupta, Phys. Rev. **D 91**, 053004 (2015).