Possible textures of the fermion mass matrices

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Abstract

Texture specific fermion mass matrices have played an important role in understanding several features of fermion masses and mixings. In the present work, we have given an overview of all possible cases of Fritzsch-like as well as non Fritzsch-like texture 6 and 5 zero fermion mass matrices. Further, for the case of texture 4 zero Fritzsch-like quark mass matrices, the issue of the hierarchy of the elements of the mass matrices and the role of their phases have been discussed. Furthermore, the case of texture 4 zero Fritzsch-like lepton mass matrices has also been discussed with an emphasis on the hierarchy of neutrino masses for both Majorana and Dirac neutrinos.

1 Introduction

Understanding fermion masses and mixings is one of the outstanding problem of present day particle physics. The idea of quark mixing phenomena was initiated by Cabibbo in 1963 [1], subsequently generalized to two generations by Glashow, Illiopoulos, Maiani [2] and finally to three generations by Kobayashi and Maskawa [3]. This has been tested to a great accuracy and is well accommodated by the Standard Model (SM). Recently, flavor mixing has also been observed in the case of neutrinos implying the existence of non zero, non degenerate neutrino masses necessitating the need to look beyond SM. Also, one has to go beyond the SM in order to understand the pattern of quark masses and mixing parameters as in the SM the quark mass matrices are completely arbitrary. In view of the relationship of fermion mixing phenomena with that of fermion mass matrices, the understanding of the above mentioned issues of flavor physics essentially implies formulating fermion mass matrices.

While on the one hand, Grand unified Theories (GUTs) have provided vital clues for understanding the relationship of fermion mass matrices between quarks and leptons, on the other hand, horizontal symmetries [4] have given clues for the relationship between different generation of fermions. Ideas such as extra dimensions [5] have also been invoked to understand the flavor puzzle. Unfortunately, at present it seems that we do not have any theoretical framework which provides a viable and satisfactory description of fermion masses and mixings.

The lack of a convincing fermion flavor theory from the 'top down' perspective necessitates the need for formulating fermion mass matrices from a 'bottom up' approach. The essential idea behind this approach is that one tries to find the phenomenological fermion mass matrices which are in tune with the low energy data and can serve as guiding stone for developing more ambitious theories. In this context, initially several *ansätze* [6, 7] were suggested for quark mass matrices. One of the successful *ansätze* incorporating the "texture zero" approach was initiated by Fritzsch [6]. *A particular texture structure is said to be texture n zero, if it has n number of non-trivial zeros, for example, if the sum of the number of diagonal zeros and half the number of the symmetrically placed off diagonal zeros is n.*

The detailed plan of the article is as follows. In Section (2), we discuss some of the broad features pertaining to quark and lepton texture specific mass matrices. The relationships of the fermion mass matrices and mixing matrices have been presented in Section (3). Present status of the quark and neutrino mass and mixing parameters have been given in Section (4). The details pertaining to texture 6, 5, 4 zero quark and lepton mass matrices have respectively been presented in Sections (5) and (6). Finally, in Section (7) we summarize and conclude.

2 Texture specific mass matrices

2.1 Quark mass matrices

The mass matrices, having their origin in the Higg's fermion couplings, are arbitrary in the SM, therefore the number of free parameters available with a general mass matrix is larger than the physical observables. For example, if no restrictions are imposed, there are 36 real free parameters in the two 3×3 general complex mass matrices, M_U and M_D, which in the quark sector need to describe ten physical observables, i.e., six quark masses, three mixing angles and one CP violating phase. Similarly, in the leptonic sector, physical observables described by lepton mass matrices are six lepton masses, three mixing angles and one CP violating phase for Dirac neutrinos (two additional phases in case neutrinos are Majorana particles). Therefore, to develop viable phenomenological fermion mass matrices one has to limit the number of free parameters in the mass matrices.

In this context, it is well known that in the SM and its extensions wherein the right handed fields in the Lagrangian are SU(2) singlets, without loss of generality, the mass matrices can be considered as hermitian. This immediately brings down the number of real free parameters from 36 to 18, which however, is still a large number compared to the number of observables. To this end, Fritzsch [6] initiated the idea of texture specific mass matrices which on the one hand imparted predictability to mass matrices while on the other hand, it paved the way for the phenomenology of texture specific mass matrices.

To define the various texture specific cases, we present the typical Fritzsch like texture specific hermitian quark mass matrices, for example,

$$M_U = \begin{pmatrix} 0 & A_U & 0 \\ A_U^* & D_U & B_U \\ 0 & B_U^* & C_U \end{pmatrix}, \qquad M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D^* & D_D & B_D \\ 0 & B_D^* & C_D \end{pmatrix}, \tag{1}$$

where M_U and M_D correspond to up and down mass matrices respectively. It may be noted that each of the above matrix is texture 2 zero type with $A_i = |A_i|e^{i\alpha_i}$ and $B_i = |B_i|e^{i\beta_i}$, where $i = U, D$.

The texture 6 zero Fritzsch mass matrices can be obtained from the above mentioned matrices by taking both D_U and D_D to be zero, which reduces the matrices M_U and M_D each to texture 3 zero type. This Fritzsch *ansätze* [6] as well as some other *ansätze* [7] were ruled out because of the large value predicted for $|V_{cb}|$ due to the high 't' quark mass.

Further, a few other texture 6 zero mass matrices were analyzed by Ramond, Roberts and Ross [8] revealing that these matrices were again ruled out because the predicted value of $|V_{cb}|$ came out to be much larger than the available data at that time. They also explored the question of connection between phenomenological quark mass matrices considered at low energies and the possible mass patterns at the GUT scale and showed that the texture structure of mass matrices is maintained as we come down from GUT scale to m_Z scale. This important conclusion also leads to the fact that the texture zeros of fermion mass matrices can be considered as phenomenological zeros, thereby implying that at all energy scales the corresponding matrix elements are sufficiently suppressed in comparison with their neighboring counterparts. This, therefore, opens the possibility of considering lesser number of texture zeros.

Besides Ramond, Roberts and Ross [8], several authors [9]-[11] then tried to explore the texture 5 zero quark mass matrices. Fritzsch-like texture 5 zero matrices can be obtained by taking either $D_U = 0$ and $D_D \neq 0$ or $D_U \neq 0$ and $D_D = 0$ in Eq. (1), thereby giving rise to two possible cases of texture 5 zero mass matrices pertaining to either M_U or M_D being texture 3 zero type while the other being texture 2 zero type. These analyses reveal that texture 5 zero mass matrices although not ruled out unambiguously yet are not able to reproduce the entire range of data. Further, the issue of the phases of the mass matrices, responsible for CP violation, was not given adequate attention in these analyses.

As an extension of texture 5 zero mass matrices, several authors [9, 12]-[14] carried out the study of the implications of the Fritzsch-like texture 4 zero mass matrices. It may be noted that Fritzsch-like texture 4 zero mass matrices can be obtained by considering both M_U and M_D, with non zero $D_i (i = U, D)$ in Eq. (1), to be texture 2 zero type. Although from the above mentioned analyses one finds that texture 4 zero mass matrices were able to accommodate the quark mixing data quite well, however it may be noted that these analyses assumed 'strong hierarchy' of the elements of the mass matrices as well as explored only their limited domains. Further, in the absence of any precise information about CP violating phase δ and related parameters, again adequate attention was not given to the phases of the mass matrices.

Recent refinements in quark mixing data motivated several authors [15]-[22] to have a re-look at the compatibility of Fritzsch like texture 4 zero mass matrices with the quark mixing data. In particular, using assumption of 'strong hierarchy' of the elements of the mass matrix defined as $D_i < |B_i| < C_i, (i = U, D)$, having its motivation in the hierarchy of the quark mixing angles several attempts [15]-[20] were made to predict the value of precisely known parameter $\sin 2\beta$. Unfortunately, the value of $\sin 2\beta$ predicted by these analyses came out to be in quite disagreement with its precisely known value. A somewhat detailed and comprehensive analyses of texture 4 zero quark mass matrices for the first time was carried out by Xing and Zhang [13], in particular they attempted to find the parameter space available to the elements of mass matrices. Their analysis has also given valuable clues about the phase structure of the mass matrices, in particular for the strong hierarchy case they conclude that only one of the two phase parameters plays a dominant role. Subsequently, attempts [14, 22] have been made to update and broaden the scope of the analysis carried out by Xing and Zhang [13], in particular regarding the structural features of the mass matrices having implications for the value of parameter $\sin 2\beta$.

2.2 Lepton mass matrices

In the leptonic sector, one would like to mention that the observation of neutrino oscillations has added another dimension to the issue of fermion masses and mixing. In fact, the pattern of neutrino masses and mixings seems to be vastly different from that of quarks. At present, the available neutrino oscillation data does not throw any light on the neutrino mass hierarchy, which may be normal/ inverted and may even be degenerate. Further, the situation becomes complicated when one realizes that neutrino masses are much smaller than charged fermion masses as well as it is not clear whether neutrinos are Dirac or Majorana particles. The situation becomes more complicated in case one has to understand the quark and neutrino mixing phenomena in a unified manner.

In this context, to understand the pattern of neutrino masses and mixings, texture zero approach has also been tried with good deal of success [23]-[29]. An early attempt to formulate lepton mass matrices was carried out by Frampton, Glashow and Marfatia [23], wherein assuming a complex symmetric Majorana mass matrix and considering seven possible texture 2 zero cases, they carried out the implications of these for the neutrino oscillation data. Thereafter, several attempts were made using texture specific lepton mass matrics to explain the pattern of neutrino masses and mixings. In particular, for normal hierarchy of neutrino masses, Fukugita, Tanimoto and Yanagida [25] carried out an analysis of Fritzsch-like texture 6 zero mass matrices. Similarly, Zhou and Xing [26] also carried out a systematic analysis of all possible texture 6 zero mass matrices for Majorana neutrinos with an emphasis on normal hierarchy of neutrino masses. Recently, [27, 28] for all possible hierarchies of neutrino masses, for both Majorana as well as Dirac neutrinos, detailed analyses of Fritzsch-like texture 6, 5 and 4 zero mass matrices was carried out.

From the above discussion, one finds that texture specific mass matrices are able to accommodate the quark as well as neutrino mixing data. This brings into fore the issue of quark-lepton unification, advocated by Smirnov [30], by considering similar structures for quark and lepton mass matrices. Keeping this in mind as well as in view of absence of any theoretical justification for Fritzsch-like mass matrices, recent attempts [31, 32] were made to consider non Fritzsch-like mass matrices for quarks as well as neutrinos.

3 Relationship of fermion mass matrices and mixing matrices

3.1 Quark mass matrices and mixing matrix

In the SM, the quark mass terms for three generations of quarks can be expressed as

$$\overline{q}_{U_L} M_U \; q_{U_R} + \overline{q}_{D_L} M_D \; q_{D_R} \,, \tag{2}$$

where $q_{U_{L(R)}}$ and $q_{D_{L(R)}}$ are the left-handed (right-handed) quark fields for the up sector (u, c, t) and down sector (d, s, b) respectively. M_U and M_D are the mass matrices for the up and the down sector of quarks. In order to re-express above equation in terms of the physical quark fields, one can diagonalize the mass matrices by the following bi-unitary transformations

$$V_{U_L}^{\dagger} M_U V_{U_R} = M_U^{diag} \equiv \text{Diag}\,(m_u, m_c, m_t)\,, \tag{3}$$

$$V_{D_L}^{\dagger} M_D V_{D_R} = M_D^{diag} \equiv \text{Diag}\,(m_d, m_s, m_b)\,, \tag{4}$$

where $M_{U,D}^{diag}$ are real and diagonal, while V_{U_L} and V_{U_R} etc. are complex unitary matrices. The quantities m_u, m_d etc. denote the eigenvalues of the mass matrices, i.e. the physical quark masses. Using Eqs. (3) and (4), one can rewrite (2) as

$$\overline{q}_{U_L} V_{U_L} M_U^{diag} V_{U_R}^{\dagger} q_{U_R} + \overline{q}_{D_L} V_{D_L} M_D^{diag} V_{D_R}^{\dagger} q_{D_R} \,, \tag{5}$$

which can be re-expressed in terms of physical quark fields as

$$\overline{q}_{U_L}^{phys} M_U^{diag} q_{U_R}^{phys} + \overline{q}_{D_L}^{phys} M_D^{diag} q_{D_R}^{phys} \,, \tag{6}$$

where $q_{U_L}^{phys} = V_{U_L}^{\dagger} q_{U_L}$ and $q_{D_L}^{phys} = V_{D_L}^{\dagger} q_{D_L}$ and so on.

The mismatch of diagonalizations of up and down quark mass matrices leads to the quark mixing matrix V_{CKM}, referred to as the Cabibbo-Kobayashi-Maskawa (CKM) [1, 3] matrix given as

$$V_{\text{CKM}} = V_{U_L}^{\dagger} V_{D_L}. \tag{7}$$

The CKM matrix expresses the relationship between quark mass eigenstates d, s, b which participate in the strong q−q and q−$\overline{\text{q}}$ interactions and the interaction eigenstates or flavor eigenstates d', s', b' which participate in the weak interactions and are the linear combinations of mass eigenstates, for example,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{8}$$

where V_{ud}, V_{us} etc. describe the transition of u to d, u to s respectively, and so on.

In view of the relationship of the mixing matrix with the mass matrix, a knowledge of the $V_{\rm CKM}$elements would have important implications for the mass matrices. The $V_{\rm CKM}$, by definition, is a unitary matrix, hence can be expressed in terms of three real angles and six phases. Out of the six phases, five can be re-absorbed into the quark fields, therefore, one is left with only one non-trivial phase which is responsible for CP violation in the SM. There are several parameterizations of the SM, however the most commonly used parameterization is the standard parameterization given by Particle Data Group (PDG) [33]. The PDG representation of the $V_{\rm CKM}$is given as

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{9}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. The angles θ_{12}, θ_{23} and θ_{13} can be chosen to lie in the first quadrant, whereas the quadrant of δ has physical significance, therefore cannot be fixed. In the PDG representation, sin$\delta \neq 0$ implies the existence of CP violation. A precise measurement of $V_{\rm CKM}$elements and the parameters characterizing the PDG representation e.g., mixing angles and CP violating phase δ will undoubtedly have implications for the mass matrices.

3.2 Lepton mass matrices and mixing matrix

The observation of neutrino oscillation phenomenon which essentially implies the flavor conversion of neutrinos is similar to the quark mixing phenomenon. This possibility of flavor conversion was originally examined by B. Pontecorvo and further generalized by Maki, Nakagawa and Sakata [34]. The emerging picture that neutrinos are massive and therefore mix has been proved beyond any doubt and provides an unambiguous signal of NP.

In the case of neutrinos, the generation of masses is not straight-forward as they may have either the Dirac masses or the more general Dirac-Majorana masses. A Dirac mass term can be generated by the Higgs mechanism with the standard Higgs doublet. In this case, the neutrino mass term can be written as

$$\overline{\nu}_{a_L} M_{\nu D} \nu_{a_R} + h.c., \tag{10}$$

where $a = e$, μ, τ and ν_e, ν_μ, ν_τ are the flavor eigenstates. $M_{\nu D}$ is a complex 3×3 Dirac mass matrix. The mass term mentioned above would also be characterized by the same symmetry breaking scale such as that of charged leptons or quarks, therefore, in this case very small masses of neutrinos would be very unnatural from the theory point of view. On the other hand, the neutrino might be a Majorana particle which is defined as is its own antiparticle and is characterized by only two independent particle states of the same mass ($\nu_{\rm L}$ and $\bar{\nu}_{\rm R}$ or $\nu_{\rm R}$ and $\bar{\nu}_{\rm L}$). A Majorana mass term, which violates both the law of total lepton number conservation and that of individual lepton flavor conservation, can be written either as

$$\frac{1}{2}\overline{\nu}_{a_L} M_L \nu^c_{a_R} + h.c. \qquad \text{or as} \qquad \frac{1}{2}\overline{\nu}^c_{a_L} M_R \nu_{a_R} + h.c., \tag{11}$$

where M_l and M_R are complex symmetric matrices.

A simple extension of the SM is to include one right handed neutrino in each of the three lepton families, while the Lagrangian of the electroweak interactions is kept invariant under $SU(2)_L \times U(1)_Y$ gauge transformations. This can be shown to lead to Dirac-Majorana mass terms which further lead to the famous seesaw mechanism [35] for the generation of small neutrino masses, e.g.,

$$M_\nu = -M^T_{\nu D}\,(M_R)^{-1}\,M_{\nu D}, \tag{12}$$

where $M_{\nu D}$ and M_R are respectively the Dirac neutrino mass matrix and the right-handed Majorana neutrino mass matrix.

The seesaw mechanism is based on the assumption that, in addition to the standard Higgs mechanism of generation of the Dirac mass term, there exists a beyond the SM mechanism of generation of the right-handed Majorana mass term, which changes the lepton number by two and is characterized by a mass $M \gg m$. The Dirac mass term mixes the left-handed field ν_L, the component of a doublet, with a single field $(\nu^c)_R$. As a result of this mixing the neutrino acquires Majorana mass, which is much smaller than the masses of leptons or quarks.

Similar to the quark sector, the lepton mass matrices can be diagonalized by bi-unitary transformations and the corresponding mixing matrix obtained, known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) or lepton mixing matrix [34], is given as

$$V_{\rm PMNS} = V^\dagger_{l_L} V_{\nu_L}. \tag{13}$$

The PMNS matrix expresses the relationship between the neutrino mass eigenstates and the flavor eigenstates, e.g.,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{14}$$

where ν_e, ν_μ, ν_τ are the flavor eigenstates, ν_1, ν_2, ν_3 are the mass eigenstates and the 3×3 mixing matrix is the leptonic mixing matrix [34]. For the case of three Dirac neutrinos, in the standard PDG parametrization [33], involving three angles θ_{12}, θ_{23}, θ_{13} and the

Dirac-like CP violating phase δ_l the mixing matrix has the form

$$V_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_l} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_l} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_l} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_l} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_l} & c_{23}c_{13} \end{pmatrix}, \qquad (15)$$

with $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$. In the case of the Majorana neutrinos, there are extra phases which cannot be removed. Therefore, the above matrix takes the following form

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_l} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_l} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_l} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_l} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_l} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (16)$$

where α_1 and α_2 are the Majorana phases which do not play any role in neutrino oscillations.

4 Experimental status of fermion masses and mixing parameters

To carry out any analysis regarding exploring the compatibility of fermion mass matrices with the recent data, one needs to keep in mind the experimental constraints imposed by the relationship between mass matrices and their corresponding mixing matrices. To facilitate our discussion in this regard, we first present the status of relevant data in the quark as well as in the lepton sector.

4.1 Quark sector

In the quark sector, the most important constraints are provided by the directly observed quantities, such as masses of the quarks, $V_{\rm CKM}$elements, CP violating phase δ and $\sin2\beta$, etc.. The quark masses relevant for the present work are the "current" quark masses at m_Z energy scale [36], e.g.,

$$\begin{gathered} m_u = 1.27^{+0.5}_{-0.42}\,\text{MeV}, \quad m_d = 2.90^{+1.24}_{-1.19}\,\text{MeV}, \quad m_s = 55^{+16}_{-15}\,\text{MeV}, \\ m_c = 0.619 \pm 0.084\,\text{GeV}, \quad m_b = 2.89 \pm 0.09\,\text{GeV}, \quad m_t = 171.7 \pm 3.0\,\text{GeV}. \end{gathered} \qquad (17)$$

The light quark masses m_u, m_d and m_s are usually further constrained by using the following mass ratios [37]

$$m_u/m_d = 0.553 \pm 0.043\;, \qquad m_s/m_d = 18.9 \pm 0.8\;. \qquad (18)$$

The analysis of mass matrices yields the CKM mixing matrix elements which can then be compared with mixing matrix obtained from a rigorous data based analysis. For ready reference as well as for the sake of readability, we reproduce here the CKM matrix as per PDG 2010 [33], at 95% C.L. as

$$V_{\rm CKM} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}. \tag{19}$$

Similarly, the precisely measured values [33] of the CP violating parameter $\sin 2\beta$, the Jarlskog rephasing invariant parameter J and the CP violating phase δ respectively are

$$\sin 2\beta = 0.673 \pm 0.023, \quad J = (2.91^{+0.19}_{-0.11}) \times 10^{-5}, \quad \delta = (73^{+22}_{-25})^\circ. \tag{20}$$

4.2 Leptonic sector

Adopting the three neutrino framework, several authors [38]-[40] have presented updated information regarding neutrino mass and mixing parameters obtained by carrying out detailed global analyses. The latest situation regarding these parameters at 3σ C.L. is summarized as follows [39],

$$\Delta m^2_{21} = (7.05 - 8.34) \times 10^{-5}\ {\rm eV}^2, \qquad |\Delta {\rm m}^2_{31}| = (2.07 - 2.75) \times 10^{-3}\ {\rm eV}^2, \tag{21}$$

$$\sin^2\theta_{12} = 0.25 - 0.37, \quad \sin^2\theta_{23} = 0.36 - 0.67, \quad \sin^2\theta_{13} \leq 0.056. \tag{22}$$

For the sake of completion as well as for ready reference, we present the following PMNS matrix determined by taking into account the neutrino oscillation data by Garcia *et al.* [41] at 3σ C.L. as

$$\rm V_{PMNS} = \begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0.00 - 0.20 \\ 0.25 - 0.53 & 0.47 - 0.73 & 0.56 - 0.79 \\ 0.21 - 0.51 & 0.42 - 0.69 & 0.61 - 0.83 \end{pmatrix}. \tag{23}$$

5 Texture specific quark mass matrices

5.1 Texture 6 zero matrices

As mentioned earlier, texture 6 zero Fritzsch mass matrices have already been ruled out [6, 8]. Therefore, for the sake of completion, we would like to discuss all possible non Fritzsch-like combinations of texture 6 zero Hermitian mass matrices as well. Before counting all possibilities, in view of non zero masses of quarks, these matrices have to satisfy the conditions Trace $M_{U,D} \neq 0$ and Det $M_{U,D} \neq 0$. One can easily check that in case of texture 3 zero mass matrices we arrive at 20 different possible texture patterns, out of which 8 are easily ruled out by imposing these conditions. The remaining 12 possible textures break into two classes, as shown in Table (1), depending upon the equations these matrices satisfy. For example, six matrices of class I, mentioned in Table (1), satisfy

	Class I	Class II
a	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & Be^{i\beta} \\ \mathbf{0} & Be^{-i\beta} & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & D & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C \end{pmatrix}$
b	$\begin{pmatrix} \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & C & Be^{i\beta} \\ Ae^{-i\alpha} & B^{-i\beta} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & C & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & D \end{pmatrix}$
c	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & Be^{i\beta} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \\ Be^{-i\beta} & \mathbf{0} & C \end{pmatrix}$	$\begin{pmatrix} D & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C \end{pmatrix}$
d	$\begin{pmatrix} C & Be^{i\beta} & \mathbf{0} \\ Be^{-i\beta} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} C & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & D & Ae^{i\alpha} \\ \mathbf{0} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$
e	$\begin{pmatrix} \mathbf{0} & Be^{i\beta} & Ae^{i\alpha} \\ Be^{-i\beta} & C & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} D & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & C & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \end{pmatrix}$
f	$\begin{pmatrix} C & \mathbf{0} & Be^{i\beta} \\ \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ Be^{-i\beta} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} C & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & Ae^{-i\alpha} & D \end{pmatrix}$

Table 1: Twelve possibilities of texture 3 zero mass matrices categorized into two classes I and II, with each class having six matrices.

the following equations

$$C = m_1 - m_2 + m_3, \quad A^2 + B^2 = m_1 m_2 + m_2 m_3 - m_1 m_3, \quad A^2 C = m_1 m_2 m_3. \tag{24}$$

Similarly, in case of class II, all six matrices satisfy the following equations

$$C + D = m_1 - m_2 + m_3, \quad A^2 - CD = m_1 m_2 + m_2 m_3 - m_1 m_3, \quad A^2 C = m_1 m_2 m_3. \tag{25}$$

The subscripts U and D have not been used as these are valid for both kind of mass matrices. It may be added that these classes are also related through permutation symmetry [42].

Matrices M_U and M_D each can correspond to any of the 12 possibilities, therefore yielding 144 possible combinations which in principle can yield 144 quark mixing matrices. These 144 combinations can be put into 4 different categories, e.g., if M_U is any of the 6 matrices from class I, then M_D can be either from class I or class II yielding 2 categories of 36 matrices each. Similarly, we obtain 2 more categories of 36 matrices each when M_U is from class II and M_D is either from class I or class II. The 36 combinations in each category further can be shown to be reduced to groups of six combinations of mass matrices, each yielding same CKM matrix. Thus, the problem of exploring the compatibility of 144 phenomenologically allowed texture 6 zero combinations with the recent low energy data is reduced only to an examination of 4 groups each having 6 combinations of mass matrices corresponding to the same CKM matrix. Detailed analysis [31] of these four groups of texture 6 zero mass matrices reveals that all possible combinations of texture 6 zero are ruled out as these are not able to reproduce the CKM element $|V_{cb}|$.

5.2 Texture 5 zero matrices

Considering now the case of texture 5 zero mass matrices which consist either of M_U being 2 zero and M_D being 3 zero or vice versa. Texture 3 zero possibilities have already been enumerated, therefore we consider only the possible patterns of texture 2 zero mass matrices. After taking into consideration the Trace and Determinant conditions mentioned earlier, one can check that there are 18 possible texture 2 zero patterns. These textures further break into three classes, detailed in Table (2), depending upon the diagonalization equations satisfied by these matrices, however, it can be shown that the classes IV and V essentially reduce to texture 3 zero patterns. We are therefore left with only class III of texture 2 zero matrices that needs to be explored for texture 5 zero combinations. All matrices of this class satisfy the following equation

$$C+D = m_1-m_2+m_3, \quad A^2+B^2-CD = m_1m_2+m_2m_3-m_1m_3, \quad A^2C = m_1m_2m_3. \quad (26)$$

Considering class III of texture 2 zero mass matrices along with different patterns of class I and class II of texture 3 zero mass matrices we find a total of 144 possibilities of texture 5 zero mass matrices, in sharp contrast to the case if we had considered the classes IV and V also yielding 432 possibilities. Keeping in mind the hierarchy of the elements of the CKM matrix, we observe that out of 144 cases, we are again left with only 4 such groups of texture 5 zero mass matrices leading to mixing matrix having hierarchical structure as that of CKM matrix.

A detailed analysis of these texture 5 zero mass matrices have been carried out [31] which shows that interestingly only one possibility, corresponding to the usual Fritzsch-like texture 5 zero mass matrix where M_U is of texture 2 zero and M_D is of texture 3 zero type, appears to be viable. Also it may be added that the viability of this combination depends on the light quark masses used as inputs.

5.3 Texture 4 zero matrices

As is well known, Fritzsch-like texture 4 zero mass matrices are compatible with the quark mixing data, however, there are two issues which need to be addressed in this context. Firstly, one needs to carry out a detailed analysis of the various possible non Fritzsch-like texture 4 zero mass matrices. In fact, the texture two zero possibilities, presented in Table (2), result into 324 texture 4 zero possibilities, the analysis of such a large number of possibilities is yet to be carried out. The other issue is whether we can consider 'weakly' hierarchical mass matrices to reproduce 'strongly' hierarchical mixing angles. This issue has been explored in a detailed manner [14, 22] and in the present work we reproduce some of the essential details regarding this.

	Class III	Class IV	Class V
a	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & D & Be^{i\beta} \\ \mathbf{0} & Be^{-i\beta} & C \end{pmatrix}$	$\begin{pmatrix} D & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & Be^{i\beta} \\ \mathbf{0} & Be^{-i\beta} & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & Fe^{i\gamma} \\ Ae^{-i\alpha} & \mathbf{0} & Be^{i\beta} \\ Fe^{-i\gamma} & Be^{-i\beta} & C \end{pmatrix}$
b	$\begin{pmatrix} \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & C & Be^{i\beta} \\ Ae^{-i\alpha} & Be^{-i\beta} & D \end{pmatrix}$	$\begin{pmatrix} D & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & C & Be^{i\beta} \\ Ae^{-i\alpha} & Be^{-i\beta} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Fe^{i\gamma} & Ae^{i\alpha} \\ Fe^{-i\gamma} & C & Be^{i\beta} \\ Ae^{-i\alpha} & Be^{-i\beta} & \mathbf{0} \end{pmatrix}$
c	$\begin{pmatrix} D & Ae^{i\alpha} & Be^{i\beta} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \\ Be^{-i\beta} & \mathbf{0} & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & Be^{i\beta} \\ Ae^{-i\alpha} & D & \mathbf{0} \\ Be^{-i\beta} & \mathbf{0} & C \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & Be^{i\beta} \\ Ae^{-i\alpha} & \mathbf{0} & Fe^{i\gamma} \\ Be^{-i\beta} & Fe^{-i\gamma} & C \end{pmatrix}$
d	$\begin{pmatrix} C & Be^{i\beta} & \mathbf{0} \\ Be^{-i\beta} & D & Ae^{i\alpha} \\ \mathbf{0} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} C & Be^{i\beta} & \mathbf{0} \\ Be^{-i\beta} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & Ae^{-i\alpha} & D \end{pmatrix}$	$\begin{pmatrix} C & Be^{i\beta} & Fe^{i\gamma} \\ Be^{-i\beta} & \mathbf{0} & Ae^{i\alpha} \\ Fe^{-i\gamma} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$
e	$\begin{pmatrix} D & Be^{i\beta} & Ae^{i\alpha} \\ Be^{-i\beta} & C & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Be^{i\beta} & Ae^{i\alpha} \\ Be^{-i\beta} & C & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & D \end{pmatrix}$	$\begin{pmatrix} \mathbf{0} & Be^{i\beta} & Ae^{i\alpha} \\ Be^{-i\beta} & C & Fe^{i\gamma} \\ Ae^{-i\alpha} & Fe^{-i\gamma} & \mathbf{0} \end{pmatrix}$
f	$\begin{pmatrix} C & \mathbf{0} & Be^{i\beta} \\ \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ Be^{-i\beta} & Ae^{-i\alpha} & D \end{pmatrix}$	$\begin{pmatrix} C & \mathbf{0} & Be^{i\beta} \\ \mathbf{0} & D & Ae^{i\alpha} \\ Be^{-i\beta} & Ae^{-i\alpha} & 0 \end{pmatrix}$	$\begin{pmatrix} C & Fe^{i\gamma} & Be^{i\beta} \\ Fe^{-i\gamma} & \mathbf{0} & Ae^{i\alpha} \\ Be^{i\beta} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$

Table 2: Texture 2 zero possibilities categorized into three classes III, IV and V, with each class having six matrices.

In this context, on the one hand, many authors [15]-[20] have shown that on using strong hierarchy of the elements of the mass matrices, texture 4 zero mass matrices appear to be incompatible with the recent value of $\sin 2\beta$. While on the other hand, recently [14, 22] extension of the parameter space of the elements of these matrices has been carried out to include the case of 'weak hierarchy' amongst them along with the usually considered 'strong hierarchy' case and thereby they have been shown to be compatible with the parameter $\sin 2\beta$.

It may be noted that although hierarchy of the elements of the mass matrices has been considered often while carrying out the analysis, however it has been explicitly defined only in [22]. The various relations between the elements of the mass matrices, given in Eq. (1), A_i, B_i, C_i, D_i $(i = U, D)$ essentially correspond to the structural features of the mass matrices including their hierarchies. As is usual the element $|A_i|$ takes a value much smaller than the other three elements of the mass matrix which can assume different relations amongst each other, defining different hierarchies. For example, in case $D_i < |B_i| < C_i$ it would lead to a strongly hierarchical mass matrix whereas a weaker hierarchy of the mass matrix implies $D_i \lesssim |B_i| \lesssim C_i$. It may also be added that for the purpose of numerical work, one can conveniently take the ratio $D_i/C_i \sim 0.01$ characterizing strong hierarchy whereas $D_i/C_i \gtrsim 0.2$ implying weak hierarchy.

The analysis carried out by [14] incorporates the quark masses and their ratios mentioned in Eqs. (17)and (18) as well as by imposing the constraints given in Eqs. (19) and

(20). Further, full variation has been given to the phases associated with the mass matrices ϕ_1 and ϕ_2, the parameters D_U and D_D have been given wide variation in conformity with the hierarchy of the elements of the mass matrices e.g., $D_i < C_i$ for $i = U, D$. The extended range of these parameters allows the calculations for the case of weak hierarchy of the elements of the mass matrices as well.

To begin with, in Fig. 1(a) C_U/m_t versus C_D/m_b has been plotted. A look at the figure reveals that both C_U/m_t as well as C_D/m_b take values from $\sim 0.55 - 0.95$, which interestingly indicates the ratios being almost proportional. Also, the figure gives interesting clues regarding the role of strong and weak hierarchy. In particular, one finds that in case one restricts to the assumption of strong hierarchy then these ratios take large values around 0.95. However, for the case of weak hierarchy, the ratios C_U/m_t and C_D/m_b take much larger number of values, in fact almost the entire range mentioned above, which are compatible with the data.

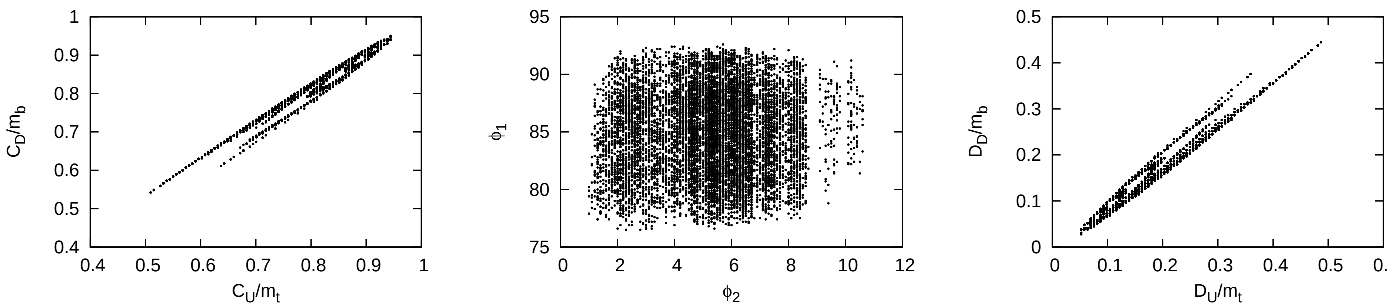


Figure 1: Plots showing the allowed ranges of (a) $C_{\rm U}/m_t$ versus $C_{\rm D}/m_b$, (b) ϕ_1 versus ϕ_2 and (c) $D_{\rm U}/m_t$ versus $D_{\rm D}/m_b$

In Fig. 1(b), the plot of ϕ_1 versus ϕ_2 has been presented. Interestingly, the present refined inputs limit the ranges of the two phases to $\phi_1 \sim 76^{\rm o} - 92^{\rm o}$ and $\phi_2 \sim 1^{\rm o} - 11^{\rm o}$. Keeping in mind that full variation has been given to the free parameters D_U and D_D, corresponding to both strong as well as weak hierarchy cases, it may be noted that the allowed ranges of the two phases come out to be rather narrow. In particular, for the strong hierarchy case one gets $\phi_2 \sim 10^{\rm o}$, whereas for the case of weak hierarchy ϕ_2 takes almost its entire range mentioned above. Also, the analysis indicates that although $\phi_1 \gg \phi_2$, still both the phases are required for fitting the mixing data.

As a next step, the role of the hierarchy defining parameters D_U and D_D has been emphasized. To this end, in Fig. 1(c) D_U/m_t versus D_D/m_b has been given, representing an extended range of the parameters D_U and D_D. A closer look at the figure reveals both D_U/m_t as well as D_D/m_b take values $\sim 0.05 - 0.5$. The lower limit of the range i.e. when the ratios D_U/m_t and D_D/m_b are around 0.05 corresponds to strong hierarchy amongst the elements of the mass matrices, whereas when the elements have weak hierarchy then these ratios take a much larger range of values. From this one may conclude that in the case of strongly hierarchical elements of the texture 4 zero mass matrices, one has limited compatibility of these matrices with the quark mixing data, whereas the weakly hierarchical ones indicate the compatibility for much broader range of the elements.

6 Texture specific lepton mass matrices

6.1 Texture 6 zero matrices

Having discussed the texture specific quark mass matrices, in the light of quark lepton unification hypothesis advocated by Smirnov [30], one would also like to know the status of texture 6 zero Fritzsch as well as non Fritzsch-like lepton mass matrices. A detailed analysis of texture 6 zero mass matrices have been carried out by several authors [27, 32]. In particular, for normal hierarchy of neutrino masses Zhou and Xing [26] have carried out an analysis of all possible Fritzsch as well as non Fritzsch-like texture 6 zero lepton mass matrices. Their analysis has been extended further to include inverted hierarchy and non degenerate scenario of neutrino masses [32].

As already shown for the case of quarks in Section(5), there are a total number of 144 possible cases of texture 6 zero mass matrices. For the case of lepton mass matrices, there are 6 cases for each of the 144 combinations corresponding to normal/ inverted hierarchy and degenerate scenario of neutrino masses for Majorana neutrinos as well as Dirac neutrinos, leading to a total of 864 cases. The analysis carried out in [32] reveals several interesting points. In particular, their investigations for Dirac neutrinos show that there are no viable texture 6 zero lepton mass matrices for normal/ inverted hierarchy as well as degenerate scenario of neutrino masses. For the case of Majorana neutrinos for texture 6 zero lepton mass matrices, again all the cases pertaining to inverted hierarchy and degenerate scenario of neutrino masses are also ruled out. Assuming normal hierarchy of Majorana neutrinos, the analysis reveals that out of 144, only 16 combinations are compatible with current neutrino oscillation data at 3σ C.L..

6.2 Texture 5 zero matrices

Similar to the case of texture 6 zero lepton mass matrices, the implications for different hierarchies in the case of texture 5 zero lepton mass matrices have also been investigated for both Majorana and Dirac neutrinos [43]. For the two types of neutrinos, corresponding to normal/ inverted hierarchy and degenerate scenario of neutrino masses 360 cases each have been considered for carrying out the analysis, making it a total of 2160 cases.

For Majorana neutrinos with normal hierarchy of neutrino masses, out of the 360 combinations, 67 are compatible with the neutrino mixing data. Most of the phenomenological implications of combinations of different categories are similar, however, still these can be experimentally distinguished with more precise measurements of θ_{13} and θ_{23}. Interestingly, degenerate scenario of Majorana neutrinos is completely ruled out by the existing data. In the case of inverted hierarchy, 24 combinations out of 360 are compatible with the neutrino mixing data.

For Dirac neutrinos with normal hierarchy of neutrino masses, as compared to Majorana cases, out of 360 only 44 combinations are compatible with neutrino mixing data. Interestingly, 6 combinations out of 44 can accommodate degenerate Dirac neutrinos. For inverted hierarchy, 24 combinations are compatible with the existing data.

6.3 Texture 4 zero matrices

Like the case of quarks, the number of viable possibilities for the case of texture 4 zero lepton mass matrices is also quite large, so in the present work we have discussed the essentials of recent detailed analyses [28] regarding only the Fritzsch-like texture 4 zero mass matrices. In particular, they have investigated the implications of different hierarchies of neutrino masses on these matrices for both Majorana and Dirac neutrinos. Interestingly, at 3σ C.L., their analysis rules out both inverted hierarchy and degenerate scenario of neutrino masses for the two types of neutrinos. We reproduce here the essentials of their arguments.

Basically, they have plotted the parameter space corresponding to any of the two mixing angles by constraining the third angle by its experimental values, mentioned in Eq. (22), while giving full allowed variation to other parameters. For ready reference we present these graphs in Fig. (2). These plots immediately reveal that the inverted hierarchy is ruled out at 3σ C.L.. For example, from Fig. (2a), (2b) and (2c), for Majorana neutrinos, one can note that the plotted parameter space of the angles has no overlap with their experimentally allowed 3σ region. Similar plots pertaining to Dirac neutrinos also rule out inverted hierarchy of neutrino masses.

Their analysis also shows that for Majorana or Dirac neutrinos the cases of neutrino masses being degenerate, characterized by either $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3} \lesssim 0.1$ eV or $m_{\nu_3} \sim m_{\nu_1} \lesssim m_{\nu_2} \lesssim 0.1$ eV corresponding to normal and inverted hierarchy respectively, are again ruled out. Considering degenerate scenario corresponding to inverted hierarchy, Fig. (2) can again be used to rule out degenerate scenario at 3σ C.L. for Majorana neutrinos. It needs to be mentioned that while plotting these figures the range of the lightest neutrino mass is taken to be $10^{-8}\,\mathrm{eV} - 10^{-1}\,\mathrm{eV}$, which includes the neutrino masses corresponding to degenerate scenario, therefore by discussion similar to the one given for ruling out inverted hierarchy, degenerate scenario of neutrino masses is ruled out as well.

Coming to degenerate scenario corresponding to normal hierarchy, one can easily show that this is ruled out again. To this end, in Fig. (3), by giving full variation to other parameters, the plot of the mixing angle θ_{12} against the lightest neutrino mass m_{ν_1} have been presented. Fig. (3a) corresponds to the case of Majorana neutrinos and Fig. (3b) to the case of Dirac neutrinos. From the figures one can immediately find that the values of θ_{12} corresponding to $m_{\nu_1} \lesssim 0.1$ eV lie outside the experimentally allowed range, thereby ruling out degenerate scenario for Majorana as well as Dirac neutrinos at 3σ C.L..

After ruling out the cases pertaining to inverted hierarchy and degenerate scenarios, we come the normal hierarchy cases for Majorana as well as Dirac neutrinos. For both types of neutrinos, these yield viable ranges of neutrino masses, mixing angles θ_{12}, θ_{23} and

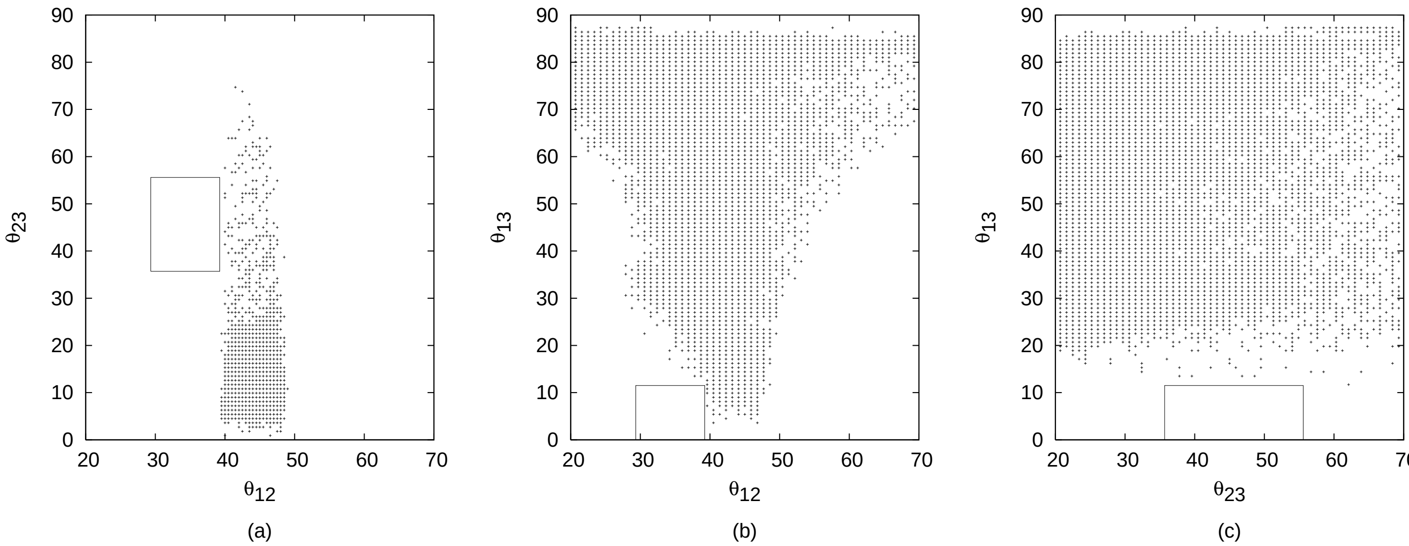
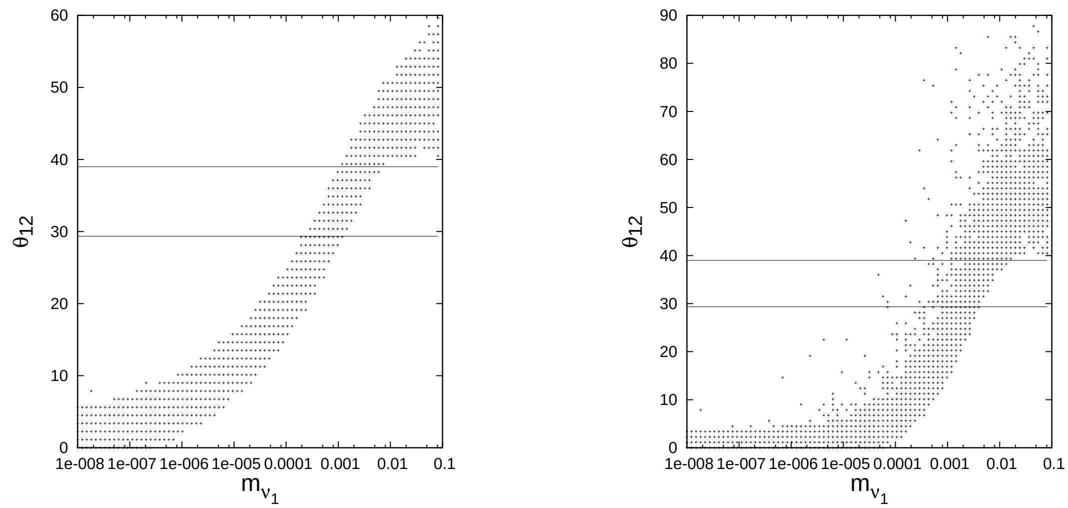


Figure 2: Plots showing the parameter space corresponding to any of the two mixing angles by constraining the third angle by its experimental limits and giving full allowed variation to other parameters for Majorana neutrinos. The blank rectangular region indicates the experimentally allowed 3σ region of the plotted angles.

θ_{13}, Jarlskog's rephasing invariant parameter in the leptonic sector J_l and the Dirac-like CP violating phase in the leptonic sector δ_l The analysis reveals several interesting points. For both Dirac or Majorana neutrinos, the viable range of the lightest neutrino mass m_{ν_1} is quite different, in particular the range corresponding to Dirac neutrinos is much wider at both the ends as compared to the Majorana neutrinos. Therefore, a measurement of m_{ν_1} could have important implications for the nature of neutrinos. Also, one finds that the lower limit on θ_{13} for the Dirac case is considerably lower than for the Majorana case, therefore a measurement of θ_{13} would have important implications for this case. The different cases of Dirac and Majorana neutrinos do not show any divergence for the ranges of Jarlskog's rephasing invariant parameter.

7 Summary and Conclusion

Fritzsch-like texture specific mass matrices have provided important clues for understanding the pattern of quark mixings and CP violation. Likewise, in the leptonic sector also texture specific mass matrices are useful in explaining the pattern of neutrino masses and mixings. To tackle the larger issue of quark and lepton mixing phenomena together, it is perhaps desirable to take into account the quark-lepton unification hypothesis [30]. This immediately brings forth the issue of finding the simplest texture structure at the leading order, compatible with the quark and lepton mixing phenomena. Further, in the absence of any theoretical justification for Fritzsch-like mass matrices, one also needs to consider



(a) (b)

Figure 3: Plots showing variation of mixing angle θ_{12} with lightest neutrino mass m_{ν_1} by giving full variation to other parameters for (a) Majorana neutrinos and (b) Dirac neutrinos. The parallel lines indicate the 3σ limits of angle θ_{12}.

non Fritzsch-like mass matrices for quarks as well as leptons.

In the present work, we have given an overview of all possible cases of Fritzsch-like as well as non Fritzsch-like texture 6 and 5 zero fermion mass matrices, for details see [27, 31]. Further, for the case of texture 4 zero Fritzsch-like quark mass matrices, the issue of the hierarchy of the elements of the mass matrices and the role of their phases have been discussed, details can be found in [14, 22]. Furthermore, the case of texture 4 zero Fritzsch-like lepton mass matrices has also been discussed with an emphasis on the hierarchy of neutrino masses for both Majorana and Dirac neutrinos, elaborate analyses presented in [28].

These analyses reveal several interesting results. In principle, for the case of quarks [31], there are 144 combinations of texture 6 zero mass matrices whereas in the case of texture 5 zero matrices one can arrive at 360 combinations. Interestingly, all the texture 6 zero combinations are completely ruled out whereas in the case of texture 5 zero mass matrices the only viable possibility looks to be that of Fritzsch-like matrices which shows only limited viability, depending upon the light quark masses used as input.

Further, for the case of texture 4 zero quark mass matrices [14, 22], including the case of 'weak hierarchy' along with the usually considered 'strong hierarchy' case, one finds that the weakly hierarchical mass matrices are able to reproduce the strongly hierarchical mixing angles. Also, both the phases having their origin in the mass matrices have to be non zero to achieve compatibility of these matrices with the quark mixing data.

The same number of combinations have been investigated for the neutrino mixing data considering normal/ inverted hierarchy and degenerate scenario of neutrino masses for Majorana as well as Dirac neutrinos. Texture 6 zero in the case of leptons results into

864 cases to be analyzed [32]. Interestingly, all the possibilities pertaining to normal/ inverted hierarchy and degenerate scenario of neutrino masses for Dirac neutrinos and inverted hierarchy as well as degenerate scenarios in the case of Majorana neutrinos are ruled out. Normal hierarchy of neutrino masses for Majorana neutrinos results into 16 combinations out of 144 which are in accordance with the neutrino oscillation data.

Texture 5 zero entails considering 2160 cases to ascertain their compatibility with the neutrino mixing data [43]. Interestingly, one finds that texture 5 zero lepton mass matrices can accommodate all hierarchies of neutrino masses. In the case of normal hierarchy, 67 combinations for Majorana neutrinos and 44 combinations for Dirac neutrinos are compatible with the neutrino mixing data. There are 6 combinations, out of 44, which can accommodate degenerate Dirac neutrinos. Further, in case of inverted hierarchy, 24 combinations are compatible both for Majorana as well as Dirac neutrinos.

For the Fritzsch-like texture 4 zero neutrino mass matrices [28], analysis pertaining to both Majorana and Dirac neutrinos for different hierarchies of neutrino masses reveals that for both types of neutrinos, all the cases pertaining to inverted hierarchy and degenerate scenarios of neutrino masses are ruled out at 3σ C.L. by the existing data. For the normal hierarchy cases, one gets viable ranges of neutrino masses, mixing angle s_{13}, Jarlskog's rephasing invariant parameter J_l and the CP violating Dirac-like phase δ_l. Interestingly, a measurement of m_{ν_1} and mixing angle θ_{13} could have important implications for the nature of neutrinos.

In conclusion, we would like to remark that on the one hand there is a need to take the analysis of texture specific mass matrices towards completion. For example, besides carrying out the analysis of texture 4 zero non Fritzsch-like fermion mass matrices, one has to consider texture 3 zero cases also, the latter corresponding to general mass matrices after carrying out weak basis rotations. On the other hand, one may also consider breaking the hermiticity condition perturbatively as has been done recently [44] and to go into its detailed implications. Similarly, the issue of phases of mass matrices and their relationship with the CP violating parameters also needs a careful look and detailed investigations.

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