

NEUTRINO MASSES AND MIXING: LEPTONS VERSUS QUARKS ^a

ALEXEI YU. SMIRNOV

Physik-Department T30d, Technische Universität München, James-Frank-Strasse,

D-85748 Garching, Germany

International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy

Institute for Nuclear Research, RAS, Moscow, Russia

E-mail: smirnov@ictp.trieste.it, smirnov@ph.tum.de

ABSTRACT

Comparison of properties of quark and leptons as well as understanding their similarities and differences is one of the milestones on the way to underlying physics. Several observations, if not accidental, can strongly affect the implications: (i) nearly tri-bimaximal character of lepton mixing, (ii) special neutrino symmetries, (iii) the QLC-relations. We consider possible connections between quarks and leptons which include the quark-lepton symmetry and unification, approximate universality, and quark-lepton complementarity. Presence of new neutrino states and their mixing with the left or/and right handed neutrinos can be the origin of additional differences of quarks and leptons.

1. Introduction

One of the key issues on the way to underlying physics is a comparison of properties of quarks and leptons and understanding their similarities and differences. This comparison has two aspects of the fundamental importance:

- understanding the fermion masses and mixings;
- uncovering the path of further unification - unification of quarks and leptons, particles and forces.

Are quarks and leptons similar or fundamentally different? Still whole spectrum of possibilities exists from the weakly broken quark-lepton universality to existence of different structures and symmetries in these two sectors.

In this paper we confront properties of quarks and leptons. We then discuss their possible connections:

- symmetry and unification;
- universality;
- complementarity;

^aTalk given at the III International Workshop on: NO-VE "Neutrinos in Venice" Fifty years after the Neutrino discovery, 7-10 Feb 2006, Venice, Italy.

angles	quarks	leptons	sum
θ_{12}	12.8°	33.9°	$46.7^\circ \pm 2.4^\circ$
θ_{23}	2.3°	41.6°	$43.9^\circ \begin{smallmatrix} +5.1^\circ \\ -3.6^\circ \end{smallmatrix}$
θ_{13}	0.5°	$< 8.0^\circ$	$< 8.5^\circ$

Table 1: The best fit values of mixing angles in the quark and lepton sectors at m_Z scale in degrees. Shown are also the sums of the angles with 1σ error bars.

- diversity, that is, existence of new structures which can produce difference in the two sectors.

2. Leptons versus quarks

2.1. Confronting mixing and masses

To compare mixings in the quark and lepton sector we use the standard parametrization of mixing matrices:

$$V_f = V_{23}(\theta_{23})I_\delta V_{13}(\theta_{13})V_{12}(\theta_{12}), \quad f = CKM, \quad PMNS, \quad (1)$$

where V_{ij} is the matrix of rotations in the ij - plane, and I_δ is the diagonal matrix of the CP-violating phases. (Notice that V_{PMNS} corresponds to V_{CKM}^\dagger).

The Table I presents the mixing angles in the quark and lepton sectors from the analysis of ref. ¹⁾. Similar results have been obtained by other groups ^{2,3)}. Shown are also the sums of the corresponding angles. Apparently, the mixing patterns in these two sectors are strongly different. The only common feature is that the 1-3 mixings (between the “remote” generations) are small in both cases.

Several comments are in order.

The b.f. value of the 1-2 mixing angle, $\theta_{12} = 33.9^\circ$, deviates from the maximal mixing by more than 6σ ²⁾.

The 2-3 mixing is consistent with maximal one. A small shift of θ_{23} from 45° is related to the excess of e-like atmospheric neutrino events in the sub-GeV range detected by SuperKamiokande (SK) ⁴⁾. It has been found when effects of 1-2 sector were included in the analysis ⁵⁾. According to ⁶⁾ $\sin^2 \theta_{23} = 0.47$ and slightly larger shift, $\sin^2 \theta_{23} = 0.44$, follows from the analysis ¹⁾. The deviation of the b.f. value

from maximal mixing is characterized by

$$D_{23} \equiv 0.5 - \sin^2 \theta_{23} = 0.03 - 0.06. \quad (2)$$

Still large deviation is allowed: $-0.17 < D_{23} < 0.21$ and relative shift can be as large as

$$D_{23}/\sin^2 \theta_{23} \sim 0.4 \quad (2\sigma). \quad (3)$$

The 1-3 leptonic mixing is consistent with zero. The most conservative 3σ bound is $\sin^2 \theta_{13} < 0.048$ ¹⁾, and at 1σ we have $\sin \theta_{13} < 0.13$. The 1-3 mixing is small in a sense that

$$\sin \theta_{13} \ll \sin \theta_{12} \sin \theta_{23} \approx 0.37. \quad (4)$$

So, apparently the quark feature $\theta_{13} \sim \theta_{12} \times \theta_{23}$ does not work here. Another interesting benchmark is the ratio of the solar and atmospheric neutrino mass scales,

$$\sin \theta_{13} = \sqrt{r} \equiv \sqrt{\frac{\Delta m_{21}^2}{\Delta m_{31}^2}} = 0.17, \quad (5)$$

which is allowed at about 2σ level. An additional (model dependent) factor of the order 0.3 - 2 may appear in this relation. Much smaller values of $\sin \theta_{13}$ would imply most probably certain symmetry of the mass matrix.

Let us consider now the masses.

The latest analysis of the cosmological data (including the WMAP 3 years result) gives the upper bound on the sum of masses of active neutrinos ⁷⁾

$$\sum_i m_i < 0.14 \text{ eV}, \quad 95\% \text{C.L.} \quad (6)$$

which already starts to disfavor the degenerate spectrum of neutrinos.

On the other hand, if the Heidelberg-Moscow result ⁸⁾ is confirmed and if it is due to exchange of the light Majorana neutrinos, the neutrino mass spectrum should be strongly degenerate with a common mass $m_0 \sim (0.2 - 0.6) \text{ eV}$. This would be in conflict with the bound (6).

The solar and the atmospheric mass differences squared give the lower bound on ratio of the second and third neutrino masses:

$$\frac{m_2}{m_3} \geq \sqrt{r} = 0.15 - 0.20. \quad (7)$$

This should be compared with ratios for charged leptons and quarks (at m_Z scale): $m_\mu/m_\tau = 0.06$, $m_s/m_b = 0.02 - 0.03$, $m_c/m_t = 0.005$. Apparently, the neutrino hierarchy (7) is the weakest one. This is consistent with possible mass-mixing relation: large mixings are associated to weak mass hierarchy.

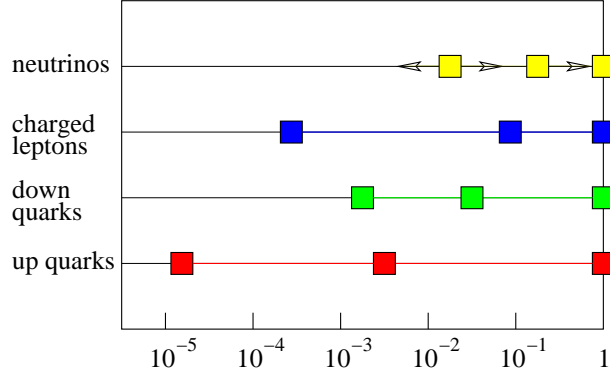


Figure 1: Mass hierarchies of quarks and leptons. The mass of heaviest fermion of a given type is taken to be 1.

In fig. 1 we show the mass ratios for three generations. The strongest hierarchy and geometric relation $m_u \times m_t \sim m_c^2$ exist for the upper quarks. It seems the observed pattern of masses is an interplay of some regularities (flavor alignment) and randomness (“anarchy”). That may indicate the perturbative picture when the lowest order masses and mixing are universal, whereas corrections have more complicated (“random”) flavor structure.

In what follows we will discuss certain observed features which can strongly affect interpretation of the results.

2.2. Tri-bimaximal mixing

Experimental results are in a very good agreement with the so called tri-bimaximal mixing ⁹⁾. The corresponding mixing matrix is

$$U_{tbm} = U_{23}^m U_{12}(\theta_{12}) = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}, \quad (8)$$

where $\sin^2 \theta_{12} = 1/3$ is about 1σ larger than the best experimental fit value. Here ν_2 is tri-maximally mixed: in the middle column three flavors mix maximally, whereas ν_3 (third column) is bi-maximally mixed. Mixing parameters turn out to be some simple numbers 0, $1/\sqrt{3}$, $1/\sqrt{2}$ and can appear as Clebsh-Gordan coefficients.

In the case of normal mass hierarchy ($m_1 \approx 0$) the mass matrix which leads to the tri-bimaximal mixing has the following form

$$m_\nu \approx \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \frac{m_2}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (9)$$

where $m_2 \approx \sqrt{\Delta m_{21}^2}$ and $m_3 \approx \sqrt{\Delta m_{31}^2}$. It is the sum of two singular matrices with certain symmetries. The later gives a hint of its origin.

Matrix (8) was motivated by certain geometric consideration. If description of the data by (8) is not accidental and certain principle/symmetry is behind, we should conclude on substantial differences in the quark and lepton sectors. Though some models have been constructed which reproduce the tri-bimaximal mixing and include also quark ¹⁰⁾.

2.3. Complementarity

According to the Table I, the sums of the mixing angles of quarks and leptons for the 1-2 and 2-3 generations agree with 45° . The quark and lepton mixings sum up to maximal mixing ^{11,12)}. Possible implications of this result called the quark-lepton complementarity relation (QLC) will be considered in sect. 3.3. Notice that the QLC relations written for angles are essentially parametrization independent. Indeed, due to smallness of 1-3 mixings in the quark and lepton sectors the relations can be written as $\arcsin(V_{us}) + \arcsin(V_{e2}) = \pi/4$. The mixing matrix elements V_{us} and V_{e2} are physical parameters.

2.4. Neutrino symmetry

Several observations may testify for special symmetry(ies) associated to neutrinos. In particular,

- maximal or nearly maximal 2-3 mixing,
- zero 1-3 mixing,

both indicate toward the same underlying symmetry. Both features can be consequences of the $\nu_\mu - \nu_\tau$ permutation symmetry of the neutrino mass matrix ¹³⁾ in the flavor basis. The permutation symmetry can be a part of, *e.g.*, discrete S_3 , A_4 or D_4 groups which in turn, are the subgroups of continuous $SO(3)$.

Important fact is that the symmetry is realized for neutrinos only, and only in the flavor basis where the charge lepton mass matrix is diagonal. The symmetry is broken in the charged lepton sector by inequality of masses of muon and tau lepton. Realization of this symmetry in specific gauge models faces some generic problems. Model should be constructed in such a way that the symmetry is weakly broken in the neutrino sector but strongly broken for the charged leptons. This implies different transformation properties of the right handed components of neutrinos and charged leptons, since the left components form the $SU(2)$ doublets. This, in turn, contradicts the L-R symmetry, and consequently, the $SO(10)$ type of unification. Still such symmetry transformations can be consistent with the $SU(5)$ unification. Alternatively, one can consider more sophisticated fermionic or/and Higgs sectors.

It is also non-trivial to extend the symmetry to the quark sector which prevents from any simple Grand Unification. A modification of the $\nu_\mu \leftrightarrow \nu_\tau$ symmetry has been proposed recently that can be the universal symmetry of quarks and leptons¹⁴⁾. The symmetry is formulated in the basis which differs from the flavor basis and therefore should be considered as the 2 – 3 family symmetry. It is argued that beside maximal (large) 2-3 leptonic mixing, smallness of the V_{cb} element of the CKM-mixing matrix testifies for this symmetry as well.

The 2-3 symmetry implies the following universal form of the mass matrices:

$$M = \begin{pmatrix} X & A & A \\ A & B & C \\ A & C & B \end{pmatrix} + \delta m, \quad (10)$$

where small corrections, $\delta m \ll B$, are of the same order for leptons and quarks.

The 2-3 symmetry does not contradict mass hierarchy which depends on particular values of parameters in the matrix (10). To get the hierarchical mass spectrum of the charged fermions (quarks and leptons) one should take

$$B_{q,l} \approx C_{q,l}, \quad X_{q,l} \ll A_{q,l} \ll B_{q,l}. \quad (11)$$

The corresponding matrices are diagonalized by nearly maximal 2-3 rotation. The physical CKM mixing is small (zero in the limit $\delta m \rightarrow 0$). Large lepton mixing requires small 2-3 rotation from the neutrino mass matrix. This can be achieved if

$$C_\nu \ll B_\nu, \quad C_\nu < |\delta m_{22} - \delta m_{33}|. \quad (12)$$

Furthermore, correct neutrino mass split can be obtained if $X_\nu \approx B$, and neutrinos have quasi-degenerate spectrum. So, essentially the mass matrices of neutrinos and charged fermions are strongly different; moreover, large lepton mixing is not the consequence of the 2-3 symmetry but result of tuning of parameters of the zero order matrix and corrections. Apparently additional symmetries/principles should be introduced to explain properties (11, 12).

Generic feature is that introduction of symmetry is motivated by maximal or nearly maximal lepton mixing. However realizations of the symmetry in a majority of gauge models show that large mixing appears eventually as a result of tuning of parameters and *not as consequence of symmetry*. This clearly makes whole context to be inconsistent.

Two remarks are in order.

(i) Symmetry is realized in terms of the mass (Yukawa coupling) matrices. It turns out that structure of the mass matrix is very sensitive to even small deviations of the 2-3 mixing from maximal and 1-3 mixing from zero. Taking the best fit values

of parameters from ¹⁾ $\sin^2 \theta_{13} = 0.01$, $\sin^2 \theta_{23} = 0.44$, we obtain the matrix of the absolute values of masses in meV ¹⁵⁾:

$$M = \begin{pmatrix} 3.2 & 6.0 & 0.6 \\ \dots & 24.8 & 21.4 \\ \dots & \dots & 30.7 \end{pmatrix} \quad (13)$$

which should be compared with the symmetry matrix (10). Notice that in contrast to (10) the 12 and 13 elements are strongly different and 33- element is greater than 22 element by 20 – 25%.

(ii) The present measurements admit substantial deviations of θ_{23} from maximal and θ_{13} from zero. That, in turn, allows even stronger deviation of the matrix from the symmetric form.

So, it is not excluded that neutrino symmetry approach is simply misleading.

2.5. Additional structure?

The features discussed above: tri-bimaximal mixing, neutrino symmetry, quark-lepton complementarity may indicate that quarks and leptons are fundamentally different and some additional structures exist that lead to this difference.

The main question here is whether these features/relations are real or accidental? “Real” in a sense that simple and direct symmetry or principle exist which lead to the relations. “Accidental” in a sense that relations are an interplay (sum) of several independent effects or contributions.

Quarks and leptons have similar gauge structure, which establishes clear correspondence of the leptons and quarks. On the other hand, the quarks and leptons have strongly different mass and mixing patterns.

The hope is that all particular features of neutrino mass spectrum and lepton mixing can be reduced eventually to the neutrality of neutrinos: zero electric and color charges. This neutrality opens unique possibility for neutrinos to

- have the Majorana mass terms, and
- mix with singlets of the SM symmetry group.

Both features are realized in the seesaw mechanism ¹⁶⁾. As we will see, the second one may have two different effects: (i) modify the mass matrix of active neutrinos, (ii) produce certain dynamical effects on the neutrino conversion (if new states are light).

Is this enough to explain all salient properties of neutrinos? Do the data really indicate existence of new physics structure (new particles, interactions, symmetries)? Is this additional structure the seesaw, or something beyond seesaw is involved?

In this connection a general context could be that beyond the SM apart from the RH neutrinos some other fermions (singlets of the SM symmetry group) exist. These fermions can have various origins in physics beyond the SM, being related to

Grand Unification, supersymmetry, existence of extra dimension, *etc.*. Existence of large number of singlets is a generic consequence of string theory. Masses of these singlets can be essentially at any scale, from zero to the Planck mass. They can mix in general with both LH and RH neutrino components.

The singlets and their mixing with SM neutrinos may be a missed structure which explains the difference of quark and lepton properties on the top of strong interactions.

3. Quark-lepton connections

3.1. Quark-lepton symmetry

There is an apparent correspondence between quarks and leptons. Each quark has its own counterpart in the leptonic sector. Leptons can be treated as the 4th color following the Pati-Salam $SU(4)_C$ unification symmetry¹⁷⁾.

Further unification is possible, when quarks and leptons form multiplets of larger gauge group. The most appealing possibility is $SO(10)$ ¹⁸⁾, where all known components of quarks and leptons as well as the RH neutrinos form unique 16-plet. It is difficult to believe that these features are accidental. Though, it is not excluded that the quark-lepton connection has rather complicated form.

The quark-lepton symmetry is not equivalent to the quark-lepton unification. Indeed, in the $SU(5)$ GU models the quark-lepton correspondence ($\nu \leftrightarrow u$, $d \leftrightarrow l$) is explicitly broken by different $SU(5)$ -gauge transformation properties: $u, u^c \sim \mathbf{10}$, whereas $\nu \sim \bar{\mathbf{5}}$, $\nu^c \sim \mathbf{1}$, then $d \sim \mathbf{10}$, $d^c \sim \bar{\mathbf{5}}$ but $l \sim \bar{\mathbf{5}}$, $l^c \sim \mathbf{10}$. This unification leads to diversity which is not seen in the low energy effective theory.

The difference of the gauge properties can lead to

- (i) different mass hierarchies of upper and down quarks, and also charge leptons and neutrinos¹⁹⁾;
- (ii) different mixings of quarks and leptons. In fact, the loopsided mechanism of large mixing realizes this possibility²⁰⁾.

Generically, GUT's provide with all ingredients necessary for the seesaw mechanism:

- RH neutrino components;
- large mass scale;
- lepton number violation.

Besides this, generically GUT's give relations between masses and mixings of leptons and quarks. They lead to equalities of masses if a single Higgs multiplet is involved in the Yukawa couplings, with well known example being the $b-\tau$ unification, $m_b \approx m_\tau$, at the GUT scale. In general, when several different Higgs multiplets are involved, one gets "sum rules" between masses and mixings of quarks and leptons

21).

However, GUT's do not explain the flavor structures. Apart from some exceptional cases (*e.g.*, antisymmetric representations) no flavor structure is produced by GUT's. Existing attempts to combine GUT's and various horizontal or family symmetries (especially neutrino symmetries) have not produced yet substantial results.

3.2. Quark-lepton universality

Can we speak on the quark-lepton universality in a complete theory, in spite of big differences of mass and mixing patterns? Is it possible that not only the gauge but also Yukawa interactions of quark and leptons are very similar?

The idea behind is that the matrix of Yukawa couplings, Y , has the following form

$$Y = Y_0 + \delta Y_f, \quad f = u, d, D, l, \quad (14)$$

where $\delta Y_f \ll Y_0$ and Y_0 is the universal matrix for all fermions. The similarity (universality) of quarks and leptons is realized in terms of the matrices of Yukawa couplings and not of observables - mass ratios and mixing angles. The key point is that similar mass matrices can lead to substantially different mixing angles and masses (eigenvalues) if the matrices are nearly singular (rank-1)^{22,23}. The singular matrices are “unstable” in a sense that small perturbations can lead to strong variations of mass ratios and mixing angles (the latter - in the context of seesaw).

Let us consider the universal structure for the mass matrices of all quarks and leptons²³:

$$Y_u \sim Y_d \sim Y_D \sim Y_M \sim Y_l \sim Y_0, \quad (15)$$

where Y_D is the Dirac type neutrino Yukawa matrix, Y_M is the Majorana type matrix for the RH neutrinos and Y_0 is the singular matrix. As an important example we take

$$Y_0 = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda \\ \lambda^2 & \lambda & 1 \end{pmatrix}, \quad \lambda \sim 0.2 - 0.3. \quad (16)$$

This matrix has only one non-zero eigenvalue and no physical mixing appears at this stage.

Let us introduce perturbations, ϵ , in the following form

$$Y_{ij}^f = Y_{ij}^0(1 + \epsilon_{ij}^f), \quad f = u, d, e, \nu, N, \quad (17)$$

where Y_{ij}^0 is the element of the original singular matrix. This form can be justified, *e.g.*, in context of the Froggatt-Nielsen mechanism²⁴. (The key element is the form of perturbations (17) which distinguishes the ansatz (16) from other possible schemes with singular matrices.) It has been shown that small perturbations $\epsilon \leq 0.25$ are

enough to explain large difference in mass hierarchies and mixings of quarks and leptons ²³⁾.

The seesaw plays crucial role here: It generates not only small neutrino masses but also large lepton mixing. Indeed, according to the seesaw $m \propto M_R^{-1}$, and nearly singular matrix of the RH neutrinos leads to enhancement of the lepton mixing ²⁵⁾.

In this approach maximal lepton mixing is accidental.

The quark-lepton universality can be introduced differently as universality of the *mixing matrices* ²⁶⁾. One can postulate that in certain “universality” basis in the first approximation the mass matrices of all fermions are diagonalized by the same matrix V or its charge conjugate V^* .

Such a possibility is inspired by the $SU(5)$ unification where leptons and down antiquarks enter the same 5-plet. All the matrices but the matrix for the charged leptons, M_l , are diagonalized by V :

$$V^\dagger M_f V = D_f, \quad f = u, d, \nu, \quad (18)$$

where D_f are the diagonal mass matrices. For the charged leptons we have

$$V^T M_l V^* = D_l. \quad (19)$$

From (18) and (19) one obtains the $SU(5)$ relation: $M_l = M_d^T$. (Another version is when neutrino mass matrix is diagonalized by V^* .)

According to (18, 19) in the first approximation one obtains for the physical mixing matrices

$$V_{CKM} = V^\dagger V = I, \quad V_{PMNS} = V^T V. \quad (20)$$

The quark mixing is absent, whereas the lepton mixing is non-trivial and can be large.

In general, the upper and down fermions are diagonalized by different matrices V' and V . In this case we obtain

$$V_{CKM} = V'^\dagger V, \quad V_{PMNS} = V^T V'. \quad (21)$$

Now the quark mixing is non-zero in the lowest order. Furthermore, (21) leads to the following relation between mixing matrices:

$$V_{PMNS} = V^T V V_{CKM}^\dagger. \quad (22)$$

So, the quark and lepton mixings are complementary to $V_{PMNS}^0 = V^T V$. The matrix V_{PMNS}^0 is symmetric and characterized by two angles $\phi_1/2$ and ϕ_2 . It is close to phenomenological matrix for relatively small values of the angles: $\phi_1/2 \sim \phi_2 \sim 20-25^\circ$. With the CKM type corrections, as in eq. (22), V_{PMNS} gives good description of data and predicts $\sin \theta_{13} > 0.08$ ²⁶⁾.

The universal mixing can originate from the mass matrices of particular form which are related to the universal real matrix A :

$$M_{u,\nu} \approx mD^*AD^*, \quad M_d \approx mD^*AD, \quad M_l \approx mDAD^*. \quad (23)$$

Here $D \equiv \text{diag}(1, i, 1)$. It happens that the phenomenologically required structure of the matrix A is very similar to that in (16). Such structures can be embedded into $SU(5)$ and $SO(10)$ models ²⁶⁾.

3.3. Quark-lepton complementarity (QLC)

As it was mentioned in sec. 2.3, within 1σ the data are in agreement with the quark-lepton complementary relations

$$\theta_{12} + \theta_C = \frac{\pi}{4}, \quad \theta_{23} + \arcsin V_{cb} = \frac{\pi}{4}, \quad (24)$$

For various reasons it is difficult to expect exact equalities (24). However certain correlation clearly shows up:

- the 2-3 leptonic mixing is close to maximal one because the 2-3 quark mixing is very small;
- the 1-2 leptonic mixing deviates from maximal one substantially because the 1-2 quark mixing (*i.e.*, Cabibbo angle) is relatively large.

Can it be accidental? A general scheme for the QLC relations is that

$$\text{“lepton mixing} = \text{bi-maximal mixing} - \text{CKM”}, \quad (25)$$

where the bi-maximal mixing matrix is ²⁷⁾:

$$U_{bm} = U_{23}^m U_{12}^m = \frac{1}{2} \begin{pmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & \sqrt{2} \\ 1 & -1 & \sqrt{2} \end{pmatrix}. \quad (26)$$

Here U_{ij}^m is maximal mixing rotation in the ij -plane.

Let us consider two possible QLC scenarios which differ by origin of the bi-maximal mixing and lead to different predictions.

1). QLC1: The bi-maximal mixing is generated by the neutrino mass matrix, presumably due to seesaw. The charged lepton mass matrix produces the CKM mixing as a consequence of the q-l symmetry: $m_l \approx m_d$. Therefore

$$U_{PMNS} = U_{CKM}^\dagger \Gamma_\alpha U_{bm}, \quad (27)$$

where $\Gamma_\alpha \equiv \text{diag}(1, 1, e^{i\alpha})$ is the phase matrix which appears in general at diagonalization. In this case exact relation (24) is not realized since the U_{12}^{CKM} rotation matrix should be permuted with U_{23}^m in (27) to reduce (27) to the standard parametrization form (1). As a consequence, the QLC relation is modified:

$$\sin \theta_{12} = \sin(\pi/4 - \theta_C) + 0.5 \sin \theta_C (\sqrt{2} - 1 - V_{cb} \cos \alpha). \quad (28)$$

Numerically (without the RGE effects) we find $\sin^2 \theta_{12} = 0.3345$ for $\alpha \sim 90^\circ$ and $\sin^2 \theta_{12} = 0.330$ for $\alpha = 0$. This is practically indistinguishable from the tri-bimaximal mixing prediction $\sin^2 \theta_{12} = 0.3333$.

Let us stress that practically the same predictions for 1-2 mixing are obtained from two different combinations of matrices:

$$U_{23}^m U_{12}(\arcsin(1/\sqrt{3})) \quad \text{and} \quad U_{12}(\theta_C) U_{23}^m U_{12}^m \quad (29)$$

which are completely independent. Therefore an equality of the predictions is just accidental coincidence. This means that one of the two approaches (QLC1 or tri-bimaximal mixing) is wrong. To some extent that can be tested by measuring the 1-3 mixing. In the QLC1-scenario one obtains

$$\sin^2 \theta_{13} = 0.5 \sin^2 \theta_C \approx 0.0245, \quad (30)$$

whereas the tri-bimaximal mixing implies $\sin^2 \theta_{13} = 0$ unless some corrections are introduced.

2). QLC2: Maximal mixing comes from the charged lepton mass matrix and the CKM mixing originates from the neutrino mass matrix due to the q-l symmetry: $m_D \sim m_u$ (assuming also that in the context of seesaw the RH neutrino mass matrix does not influence mixing). Consequently,

$$U_{PMNS} = U_{bm} \Gamma_\alpha U_{CKM}^\dagger. \quad (31)$$

In this case the QLC relation for 1-2 mixing is satisfied precisely: $\sin \theta_{12} = \sin(\pi/4 - \theta_C)$. Now $\sin^2 \theta_{13} \approx \sin^2 \theta_{12} V_{cb}^2$ is extremely small.

All three predictions for 1-2 mixing (from QLC1, QLC2 and tri-bimaximal mixing) are within 1σ errors from the b.f. point. The tri-bimaximal mixing and QLC1 predictions almost coincide, the b.f. value is in between the QLC2 and two other predictions: $\theta_{12}(\text{QLC2}) < \theta_{12}^{exp} < \theta_{12}(\text{QLC1}) \approx \theta_{12}(\text{tbm})$. To disentangle these two possibilities one needs to measure the 1-2 mixing with accuracy $\Delta \theta_{12} \sim 1^\circ$ or $\Delta \sin^2 \theta_{12} \sim 0.015$ (5%).

There are two main issues related to the QLC relations:

(1) origin of the bi-maximal mixing;

(2) mechanism of propagation of the CKM mixing from the quark sector to the lepton one. The problem here is big difference of mass ratios of the quarks and leptons: $m_e/m_\mu = 0.0047$, $m_d/m_s = 0.04 - 0.06$, as well as difference of masses of muon and s-quark at the GU scale. This means that mixing should weakly depend on or be independent of masses.

So, if not accidental, the QLC relation may have the following implications:

- the quark-lepton symmetry,
- existence of some additional structure which produces the bi-maximal mixing,
- mass matrices with weak correlation of the mixing angles and mass eigenvalues.

Alternatively, it may imply certain flavor physics with $\sin \theta_C$ being the “quantum” of this physics.

In majority of models proposed so far, the approximate QLC relation appears as a result of interplay of different independent factors or as sum of several independent contributions. From this point of view the QLC relation is accidental.

4. Effects of new neutrino states

Effects of new neutrino states (singlets of the SM symmetry group) depend on their masses. Superheavy new states essentially decouple. These states are not produced in laboratory experiments, but they can lead to indirect effects:

- modify substantially the mass matrix of active neutrinos;
- violate universality of the weak interactions, *etc.*.

For relatively small masses, say $M_S \ll m_W$, these new states can be produced in reactions thus leading to direct effects but also they modify the mass matrix of active neutrinos. Light new states with $m_S \sim m_\nu$ can lead to non-trivial oscillation effects.

Here we consider two applications of possible existence of new neutrinos states. They realize an idea that these states play the role of additional structures which lead to substantial difference of quark and lepton properties.

4.1. Screening of Dirac structure

Let us introduce one heavy neutral state S for each generation and consider mass matrix in the basis (ν, N^c, S) of the following form

$$m = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_D^T \\ 0 & M_D & M_S \end{pmatrix}. \quad (32)$$

Here M_S is the Majorana mass matrix of new fermions. Such a structure can be formed by a lepton number violated in the M_S and some additional symmetry which forbids also 13-element.

For $m_D \ll M_D \ll M_S$ the matrix leads to the double (cascade) seesaw mechanism²⁸⁾:

$$m_\nu = m_D^T M_D^{-1T} M_S M_D^{-1} m_D, \quad (33)$$

and the mass matrix of RH neutrinos becomes $M_R = -M_D M_S^{-1} M_D^T$. If two Dirac mass matrices are proportional each other,

$$M_D = A^{-1} m_D, \quad A \equiv v_{EW}/V_{GU}, \quad (34)$$

they cancel in (33) and we obtain

$$m_\nu = A^2 M_S. \quad (35)$$

That is, the structure of light neutrino mass matrix is determined by M_S immediately and does not depend on the Dirac mass matrix (the later is screened). The seesaw mechanism provides scale of neutrino masses but not the flavor structure of the mass matrix.

Notice that screening does not depend on the scale of M_S and in fact $M_S \ll M_D$ is also possible. However it is natural to assume that M_D is at the GUT scale, and M_S is at the Planck scale M_{Pl} which leads to correct values of the light neutrino masses. It can be shown that at least in SUSY version the radiative corrections do not destroy screening²⁹⁾. The relation (34) can be a consequence of Grand Unification with extended gauge group or/and certain flavor symmetry^{29,30)}.

Structure of the light neutrino mass matrix depends now on M_S which can be related to some physics at the Planck scale, and consequently, lead to “unusual” properties of neutrinos. In particular,

- (i) certain symmetry of M_S can be the origin of “neutrino” symmetry;
- (ii) the matrix $M_S \propto I$ leads to the quasi-degenerate mass spectrum;
- (iii) M_S can be the origin of bi-maximal mixing thus leading to the QLC relations, if the charged lepton mass matrix generates the CKM rotation.

4.2. New states and induced mass matrix

Suppose the active neutrinos acquire (*e.g.*, via seesaw) the Majorana mass matrix m_a . Consider one sterile neutrino, S , with Majorana mass m_S and mixing with active neutrinos characterized by “vector” of masses $\bar{m}_S \equiv (m_{eS}, m_{\mu S}, m_{\tau S})$. Essentially in the basis (ν, N^c, S) this corresponds to the mass matrix of the form

$$m = \begin{pmatrix} 0 & m_D & \bar{m}_S \\ m_D^T & M_R & 0 \\ \bar{m}_S & 0 & m_S \end{pmatrix}. \quad (36)$$

If $m_S \gg m_{iS}$, then after decoupling of S the mass matrix of active neutrinos becomes

$$m_\nu = m_a + m_I, \quad (37)$$

where the last term is the matrix induced by S :

$$m_I = \frac{1}{m_S} \bar{m}_S^T \bar{m}_S. \quad (38)$$

The induced matrix has zero determinant and therefore can be an origin of singular structures.

Introducing the active-sterile mixing angle θ_S as

$$\sin \theta_S = \bar{m}_S / m_S, \quad (39)$$

we can rewrite the elements of induced matrix as

$$m_I \sim \sin^2 \theta_S m_S. \quad (40)$$

The induced matrix may turn out to be the “missed” element which leads to the difference of mixings of quarks and leptons. Let us consider several possibilities.

1). Suppose $\bar{m}_S \propto (0, 1, 1)$, then the induced matrix reproduces the dominant block of the active neutrino mass matrix for the normal mass hierarchy:

$$m_\nu = \frac{\sqrt{\Delta m_{32}^2}}{2} \begin{pmatrix} \dots & \dots & \dots \\ \dots & 1 & 1 \\ \dots & 1 & 1 \end{pmatrix}, \quad (41)$$

where “dots” denote small parameters. In this case one can realize a possibility that the original active neutrino mass matrix, m_a , has hierarchical structure with small mixings being similar to the quark mass matrices. From eqs. (41) and (40) we find

$$\sin^2 \theta_S m_S = \frac{1}{2} \sqrt{\Delta m_{32}^2} \sim 0.025 \text{ eV}. \quad (42)$$

2). Let us assume that couplings of S with active neutrinos are universal - flavor “blind”:

$$\bar{m}_S \propto (1, 1, 1). \quad (43)$$

Then the induced matrix has form: $m_I \propto D$, where D is the democratic matrix - the second matrix in (9). Suppose that the original active neutrino mass matrix has structure of the first matrix in (9). Then the sum, $m_\nu = m_a + m_I$, reproduces the mass matrix for the tri-bimaximal mixing (9). In this case, according to (9), the parameters of S should satisfy relation

$$\sin^2 \theta_S m_S = \frac{1}{3} \sqrt{\Delta m_{21}^2} \sim 0.003 \text{ eV}. \quad (44)$$

With two sterile neutrinos whole structure (9) can be obtained.

3). New neutrino states are irrelevant if $m_{iS} m_{jS} / m_S \ll (m_a)_{ij}$ or

$$\sin^2 \theta_S m_S < 0.001 \text{ eV}. \quad (45)$$

Clearly, the presence of induced contribution changes implications of the neutrino results^{31,15)}. Since S is beyond the SM structure extended by RH neutrinos, it may be easier to realize “neutrino” symmetries as a consequence of certain symmetry of S couplings with active neutrinos.

In figs. 2 and 3 we show lines of constant induced masses in the plane $\sin^2 \theta_S - m_S$ which are given by the conditions (42), (44), (45) as well as the line $\sin^2 \theta_S m_S < 0.5$ eV which corresponds to maximal allowed value of the matrix elements. We confront these lines with various cosmological, astrophysical and laboratory bounds on the parameters of new neutrino states (see ref.¹⁵⁾ for details).

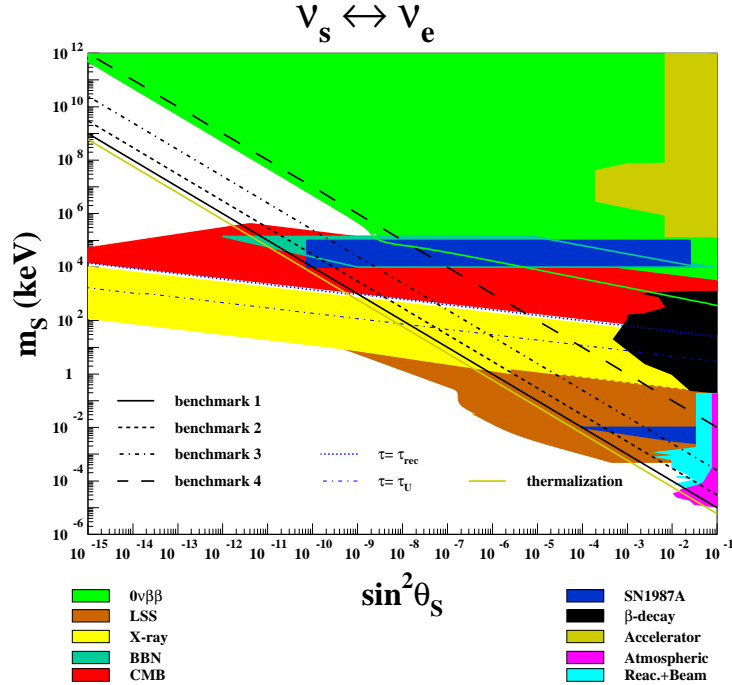


Figure 2: The benchmark lines of induced masses given in eqs. (42), (44), (45) versus the current astrophysical, cosmological and laboratory bounds on $\nu_S - \nu_e$ mixing. The colored regions are excluded. The “thermalization” line and the two decay lines $\tau_S = \tau_{\text{rec}}$ and $\tau_S = \tau_U$ are also shown.

According to figs. 2 and 3 two regions are allowed:

1). Small masses window: $m_S \sim (0.5 - 1)$ eV and $\sin^2 \theta_S = 0.001 - 0.1$, where direct and indirect effects are comparable. This window is disfavored by results of recent analysis of cosmological data⁷⁾, and it is closed if the Big Bang nucleosynthesis bound on the effective number of neutrino species $N_\nu < 4$ is taken.

Notice that there are various ways to avoid the cosmological bounds which however imply an existence of additional physics beyond the Standard model¹⁵⁾.

2). Large masses range: $m_S > 300$ MeV and $\sin^2 \theta_S < 10^{-9}$. Here direct mixing

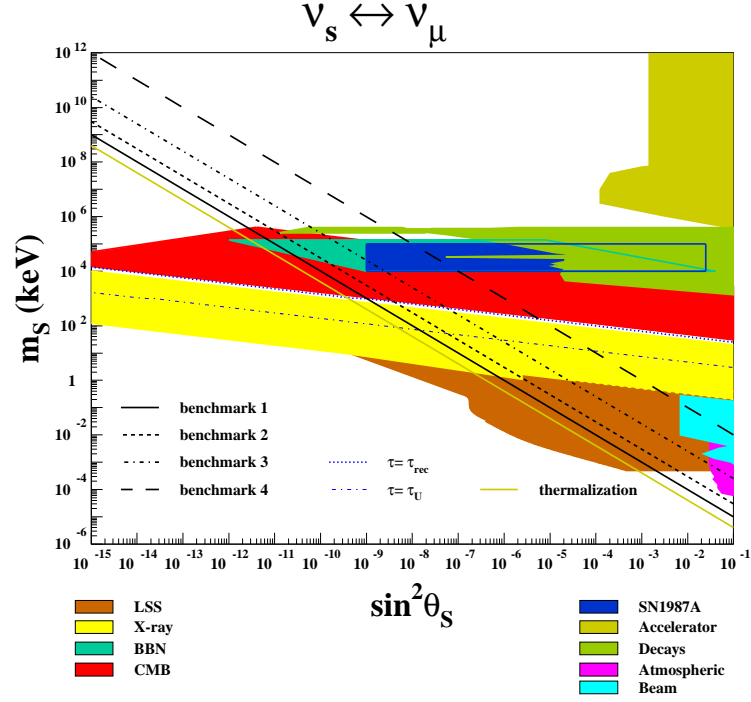


Figure 3: The same as in Fig.2 but for $\nu_s - \nu_\mu$ mixing.

effects are negligible and the presence of new states can not be verified.

5. Summary

Comparison of the properties of the quarks and leptons shows similar gauge characteristics and strong difference of mass and mixing patterns.

There are several observations which (if not accidental) can strongly influence implications of the results. Those include possible presence of special leptonic (neutrino) symmetries; particular (tri-bimaximal) form of neutrino mixing matrix; quark-lepton complementarity relations. These features may indicate that quarks and leptons are fundamentally different and some new structures of theory exist beyond the seesaw.

Mixing with new neutrino states can play the role of this additional structure. In particular, it can

- produce screening of the Dirac structure;
- generate the induced matrix of active neutrinos with certain symmetry properties. The induced matrix can lead to enhancement of lepton mixings, to generation of the dominant block of the mass matrix in the case of normal mass hierarchy, or to various subdominant structures, *e.g.*, for the tri-bimaximal mixing.

Still the approximate quark-lepton universality can be realized. In this case, the dominant mass or mixing matrices are the same for all fermions and small (of the order $\sin \theta_C$) corrections can produce whole difference. The seesaw mechanism plays the key role in getting of large lepton mixing.

6. Acknowledgements

This work has been supported in part by the Alexander von Humboldt Foundation (the Humboldt research award).

7. References

- 1) G. L. Fogli et al, hep-ph/0506083.
- 2) SNO Collaboration (B. Aharmim et al.). *Phys. Rev. C* **72**, 055502 (2005).
- 3) A. Strumia, F. Vissani, *Nucl. Phys. B* **726**, 294 (2005).
- 4) Super-Kamiokande Collaboration (Y. Ashie et al.), *Phys. Rev. D* **71** 112005, (2005)
- 5) O. L. G. Peres, A. Yu. Smirnov, *Phys. Lett. B* **456**, 204 (1999); *Nucl. Phys. B* **680**, 479 (2004).
- 6) M. C. Gonzalez-Garcia, M. Maltoni, A. Yu. Smirnov, *Phys. Rev. D* **70**, 093005 (2004).
- 7) U. Seljak, A. Slosar and P. McDonald, arXiv:astro-ph/0604335.
- 8) H.V. Klapdor-Kleingrothaus, et al, *Phys. Lett. B* **586**, 198 (2004).
- 9) L. Wolfenstein, *Phys. Rev. D* **18**, 958 (1978); P. F. Harrison, D. H. Perkins and W. G. Scott, *Phys. Lett. B* **458**, 79 (1999), *Phys. Lett. B* **530**, 167 (2002).
- 10) E. Ma, *Mod. Phys. Lett. A* **17**, 2361 (2002); E. Ma, G. Rajasekaran, *Phys. Rev. D* **64** 113012, (2001); K.S. Babu, E. Ma, J.W.F. Valle, *Phys. Lett. B* **552**, 207 (2003); W. Grimus and L. Lavoura, *JHEP* **0508**, 013 (2005); K. S. Babu

- and X. G. He, hep-ph/0507217; E. Ma, *Mod. Phys. Lett. A* **20**, 2601 (2005).
 G. Altarelli and F. Feruglio, hep-ph/0507217; H. G. He, Yong-Yeon Keum and R. R. Volkas, hep-ph/0601001.
- 11) A. Yu. Smirnov, hep-ph/0402264; M. Raidal, *Phys. Rev. Lett.* **93**, 161801 (2004).
 - 12) H. Minakata, A. Yu. Smirnov, *Phys. Rev. D* **70**, 073009 (2004).
 - 13) T. Fukuyama and H. Nishiura, hep-ph/9702253; R. N. Mohapatra and S. Nussinov, *Phys. Rev. D* **60**, 013002 (1999); E. Ma and M. Raidal, *Phys. Rev. Lett.* **87**, 011802 (2001); C. S. Lam, *Phys. Lett. B* **507**, 214 (2001).
 - 14) A. S. Joshipura, arXiv:hep-ph/0512252.
 - 15) A. Y. Smirnov and R. Zukanovich Funchal, arXiv:hep-ph/0603009.
 - 16) P. Minkowski, *Phys. Lett. B* **67** 421 (1977); T. Yanagida, in *Proc. of Workshop on Unified Theory and Baryon number in the Universe*, eds. O. Sawada and A. Sugamoto, KEK, Tsukuba, (1979); M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam 1980); P. Ramond, *Sanibel talk*, retroprinted as hep-ph/9809459; S. L. Glashow, in *Quarks and Leptons*, Cargèse lectures, eds M. Lévy, (Plenum, 1980, New York) p. 707; R. N. Mohapatra and G. Senjanović, *Phys. Rev. Lett.* **44**, 912 (1980).
 - 17) J. C. Pati and A. Salam, *Phys. Rev. D* **10**, 275 (1974).
 - 18) H. Georgi, *In Coral Gables 1979 Proceeding, Theory and experiment in high energy physics*, New York 1975, 329 and H. Fritzsch and P. Minkowski, *Annals Phys.* **93** 193 (1975).
 - 19) K. S. Babu and S. M. Barr, *Phys. Lett. B* **381** (1996) 202.
 - 20) C. H. Albright, K. S. Babu and S. M. Barr, *Phys. Rev. Lett.* **81** (1998) 1167.
 - 21) B. Bajc, G. Senjanovic and F. Vissani, *Phys. Rev. Lett.* **90** (2003) 051802.
 - 22) E. K. Akhmedov, et al., *Phys. Lett. B* **498**, 237 (2001); R. Dermisek, *Phys. Rev. D* **70**, 033007 (2004).
 - 23) I. Dorsner, A.Yu. Smirnov, *Nucl. Phys. B* **698**, 386 (2004).
 - 24) C. D. Froggatt and H. B. Nielsen, *Nucl. Phys. B* **147**, 277 (1979).
 - 25) A. Yu. Smirnov, *Phys. Rev. D* **48**, 3264 (1993).
 - 26) A. S. Joshipura and A. Y. Smirnov, arXiv:hep-ph/0512024.
 - 27) F. Vissani, hep-ph/9708483; V. D. Barger, et al, *Phys. Lett. B* **437**, 107 (1998).
 - 28) R. N. Mohapatra, *Phys. Rev. Lett.* **56**, 561 (1986); R. N. Mohapatra and J. W. F. Valle, *Phys. Rev. D* **34**, 1642 (1986).
 - 29) M. Lindner, M. A. Schmidt, A. Yu. Smirnov, *JHEP* **0507**, 048 (2005).
 - 30) O. Vives, hep-ph/0504079; J. E. Kim and J. C. Park, hep-ph/0512130.
 - 31) K.R.S. Balaji, A. Perez-Lorenzana, A.Yu. Smirnov, *Phys. Lett. B* **509**, 111 (2001).