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Физика Стандартной Модели элементарных частиц

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Распады векторных мезонов

According to Quantum Mechanics the Green function of a composite state has a pole in the energy E at the resonance mass:

$$G \sim \frac{1}{E - M + i\frac{1}{2}\Gamma}$$

Because the vector bosons are unstable particles, their Green function in the physical gauge has the pole singularity

$$D_{\mu\nu}(q) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{M^2})\, D(q)\,, \;\; D(q) = \frac{1}{q^2 - M^2 + \Pi(q^2)} \sim \frac{1}{E - M + i\frac{1}{2}\Gamma}\,,$$

where $\Pi(q^2)$ is the renormalized polarization operator and therefore we obtain the following relation between its imaginary part $\Im\,\Pi(q^2)$ and the resonance width Γ:

$$\Gamma = \frac{\Im\,\Pi(M^2)}{M}\,.$$

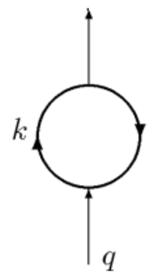

Fig 10 Polarization operator for the photon

Let us begin with the decay of the W^- - boson to the system of the leptons e and $\bar{\nu}$. In this case we can neglect the electron mass in comparison with the W-boson mass. If we compare the polarization operator Π_W of the W-boson with the corresponding operator Π_γ for the virtual photon in the massless QED (see Fig 5 , apart from the substitution $e \to g$ we have the additional factors

$$\frac{1+\gamma_5}{2\sqrt{2}}$$

for each of two vertices. One should also take into account, that the linear term in γ_5 does not give any contribution after integration over the fermion momentum k and the product of two γ_5 matrices is unity. It gives effectively

$$\Pi_W = \frac{1}{4}\frac{g^2}{e^2}\Pi_\gamma = -\frac{1}{4}\frac{g^2}{12\pi^2}\, q^2\, \ln(-\frac{q^2}{m_e^2} - i\epsilon)\,,$$

because the running coupling constant in QED at large q^2 is

$$\alpha(q^2) = \frac{\alpha}{1 - \frac{\alpha}{3\pi}\,\ln(-\frac{q^2}{m_e^2})}\ , \ \alpha = \frac{e^2}{4\pi}\ .$$

Thus, using the relation $\Im\,\ln\left(-q^2 - i\epsilon\right) = -\pi$ for $q^2 \approx M_W^2$, we can obtain the partial width for the decay $W^- \to e\,\overline{\nu^e}$

$$\Gamma_{e\,\overline{\nu^e}} = \frac{g^2}{48\,\pi}\,M_W = \frac{G}{6\,\sqrt{2}\,\pi}\,M_W^3 \simeq 226\,MeV\ .$$

The same result is valid for the W-decay to the $\mu\,\overline{\nu^\mu}$ and $\tau\,\overline{\nu^\tau}$. As for the decay to the quark system $u_i\,\overline{d_j}$, we obtain in analogous way

$$\Gamma_{u_i\,\overline{d_j}} = C\,\frac{G}{6\,\sqrt{2}\,\pi}\,M_W^3\ \ |V_{ij}|^2 \simeq |V_{ij}|^2\ 705\,MeV\ , \qquad (1)$$

where V_{ij} is the corresponding element of the CKM matrix. Due to its unitarity it is cancelled after summation over j. The factor

$$C = 3\,\left(1 + \frac{\alpha_s(M_W^2)}{\pi} + 1.409\,\left(\frac{\alpha_s(M_W^2)}{\pi}\right)^2 - 12.77\,\left(\frac{\alpha_s(M_W^2)}{\pi}\right)^3\right)$$

for quarks takes into account their 3 colour degrees of freedom and the QCD interaction with the strong running coupling constant $\alpha_s = g_s^2/(4\pi) \sim 0.2$ calculated at the scale equal to M_W^2.

Taking into account, that the t-quark has the mass $m_t \simeq 175\,MeV$ and therefore the W-boson does not have the decay channels with this quark, we obtain, that the total width of the W-boson is

$$\Gamma_{tot}^W = 3\,\Gamma_{e\,\overline{\nu^e}} + 2\,\left(\Gamma_{u\bar{d}} + \Gamma_{u\bar{s}} + \Gamma_{u\bar{b}}\right) \simeq 2.09\,GeV$$

in a good agreement with the experimental data. Note, that because the quarks do not exist in the free state, the theoretically obtained Γ_{tot}^W is compared with the decay width in leptons and hadrons. The agreement of the experimental data with the above formulas means, that the hadronization effects for quarks are not very essential, although for light quarks they can be significant.

W WIDTH

The *W* width listed here corresponds to the width parameter in a Breit-Wigner distribution with mass-dependent width. To obtain the world average, common systematic uncertainties between experiments are properly taken into account. The LEP-2 average *W* width based on published results is 2.195 ± 0.083 GeV [SCHAEL 13A]. The combined Tevatron data yields an average W width of 2.046 ± 0.049 GeV [FERMILAB-TM-2460-E].

OUR FIT uses these average LEP and Tevatron width values and combines them assuming no correlations.

VALUE (GeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
2.085±0.042 OUR FIT					
2.028±0.072	5272	[1] ABAZOV	09AK	D0	$E_{cm}^{p\bar{p}}$ = 1.96 GeV
2.032±0.045±0.057	6055	[2] AALTONEN	08B	CDF	$E_{cm}^{p\bar{p}}$ = 1.96 TeV
2.404±0.140±0.101	10.3k	[3] ABDALLAH	08A	DLPH	E_{cm}^{ee}= 183–209 GeV
1.996±0.096±0.102	10729	[4] ABBIENDI	06	OPAL	E_{cm}^{ee}= 170–209 GeV
2.18 ±0.11 ±0.09	9795	[5] ACHARD	06	L3	E_{cm}^{ee}= 172–209 GeV
[illegible]	[illegible]	[6] [illegible]	[illegible]	[illegible]	[illegible]

W^+ DECAY MODES

W^- modes are charge conjugates of the modes below.

	Mode		Fraction (Γ_i/Γ)	Confidence level
Γ_1	$\ell^+\nu$	[a]	(10.86 ± 0.09) %	
Γ_2	$e^+\nu$		(10.71 ± 0.16) %	
Γ_3	$\mu^+\nu$		(10.63 ± 0.15) %	
Γ_4	$\tau^+\nu$		(11.38 ± 0.21) %	
Γ_5	hadrons		(67.41 ± 0.27) %	
Γ_6	$\pi^+\gamma$		$< 7 \times 10^{-6}$	95%
Γ_7	$D_s^+\gamma$		$< 1.3 \times 10^{-3}$	95%
Γ_8	cX		(33.3 ± 2.6) %	
Γ_9	$c\overline{s}$		$(31 \ ^{+13}_{-11})$ %	

Let us consider now the Z-boson decays. Its interaction Lagrangian can be presented as follows

$$L^Z = \frac{g}{2\cos\theta_w} \sum_k \overline{\psi}_k \hat{Z} \, (g_v + g_a\gamma_5) \ \psi_k \, ,$$

where

$$g_v = T_3 - 2\,Q\sin^2\theta_w \, , \ g_a = T_3 \, .$$

Using the arguments similar to the case of the W-decays, we obtain the relation between the corresponding partial widths of the Z-boson:

$$\Gamma_Z = \Gamma_{W\to e\overline{\nu}} \ (g_v^2 + g_a^2) \, \frac{M_z}{M_w \ \cos^2\theta_w} = \Gamma_{W\to e\overline{\nu}} \ (g_v^2 + g_a^2) \, \frac{M_z^3}{M_w^3} \, .$$

We put approximately $\sin\theta_w = \frac{1}{2}$ and obtain for the $\nu\overline{\nu}$-channel

$$\Gamma_{\nu\overline{\nu}} = \Gamma_{e\overline{\nu^e}} \ (\frac{1}{4} + \frac{1}{4}) \, \frac{M_z^3}{M_w^3} = \frac{1}{2\cos^3\theta_w} \, \Gamma_{e\overline{\nu^e}} \simeq 167 \, MeV,$$

the $e\overline{e}$-channel

$$\Gamma_{e\overline{e}} = \Gamma_{e\overline{\nu^e}} \ ((-\frac{1}{2} + 2\,\frac{1}{4})^2 + \frac{1}{4}) \, \frac{M_z^3}{M_w^3} \simeq 84 \, MeV,$$

the $u\overline{u}$-channel

$$\Gamma_{u\overline{u}} = C\,\Gamma_{e\overline{\nu^e}} \ ((\frac{1}{2} - 2\,\frac{2}{3}\,\frac{1}{4})^2 + \frac{1}{4}) \, \frac{M_z^3}{M_w^3} \simeq 300 \, MeV$$

and the $d\overline{d}$-channel

$$\Gamma_{d\overline{d}} = C\,\Gamma_{e\overline{\nu^e}} \ ((-\frac{1}{2} + 2\,\frac{1}{3}\,\frac{1}{4})^2 + \frac{1}{4}) \, \frac{M_z^3}{M_w^3} \simeq 380 \, MeV \, .$$

Finally for the total width of the Z-boson we have

$$\Gamma_{tot}^Z = 3\,\Gamma_{\nu\,\overline{\nu}} + 3\,\Gamma_{e\overline{e}} + 2\,\Gamma_{u\overline{u}} + 3\,\Gamma_{d\overline{d}} \simeq 2.497 GeV$$

with an excellent agreement with the experimental data.

Z WIDTH

OUR FIT is obtained using the fit procedure and correlations as determined by the LEP Electroweak Working Group (see the note "The *Z* boson" and ref. LEP-SLC 06).

VALUE (GeV)	EVTS	DOCUMENT ID		TECN	COMMENT
2.4952±0.0023 OUR FIT					
2.4948±0.0041	4.57M	[1] ABBIENDI	01A	OPAL	E^{ee}_{cm}= 88–94 GeV
2.4876±0.0041	4.08M	[2] ABREU	00F	DLPH	E^{ee}_{cm}= 88–94 GeV
2.5024±0.0042	3.96M	[3] ACCIARRI	00C	L3	E^{ee}_{cm}= 88–94 GeV
2.4951±0.0043	4.57M	[4] BARATE	00C	ALEP	E^{ee}_{cm}= 88–94 GeV

Z DECAY MODES

	Mode		Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	e^+e^-		(3.363 ±0.004) %	
Γ_2	$\mu^+\mu^-$		(3.366 ±0.007) %	
Γ_3	$\tau^+\tau^-$		(3.370 ±0.008) %	
Γ_4	$\ell^+\ell^-$	[a]	(3.3658±0.0023) %	
Γ_5	$\ell^+\ell^-\ell^+\ell^-$	[b]	(3.5 ±0.4) × 10^{-6}	S=1.7
Γ_6	invisible		(20.00 ±0.06) %	
Γ_7	hadrons		(69.91 ±0.06) %	
Γ_8	$(u\overline{u}+c\overline{c})/2$		(11.6 ±0.6) %	
Γ_9	$(d\overline{d}+s\overline{s}+b\overline{b})/3$		(15.6 ±0.4) %	
Γ_{10}	$c\overline{c}$		(12.03 ±0.21) %	
Γ_{11}	$b\overline{b}$		(15.12 ±0.05) %	

Точность древесных расчетов распадов векторных бозонов в СМ

$$R_e = \frac{\Gamma(Z \to \text{адроны})}{\Gamma_e}, \quad R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{адроны})}, \quad R_c = \frac{\Gamma(Z \to c\bar{c})}{\Gamma(Z \to \text{адроны})}.$$

$$\sin^2\theta_W|_{\text{exp}} = 0{,}2255 \pm 0{,}0021.$$

Параметр	Стандартная модель (древесный уровень)	Эксперимент
Γ_Z	2,474	2,4948 ±0,0025
Γ_W	2,09	2,06 ±0,06
R_e	20,29	20,765 ±0,026
R_b	0,219	0,21656 ±0,00074
R_c	0,172	0,1733 ±0,0044

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	$\Gamma_{\tau\to\nu_\tau\mu\overline{\nu}_\mu}/\Gamma_{\mu\to\nu_\tau e\overline{\nu}_e}$	$\Gamma_{\pi\to\mu\overline{\nu}_\mu}/\Gamma_{\pi\to e\overline{\nu}_e}$	$\Gamma_{W\to\mu\overline{\nu}_\mu}/\Gamma_{W\to e\overline{\nu}_e}$
$\lvert g_\mu/g_e\rvert$	$0{,}9999 \pm 0{,}0020$	$1{,}0017 \pm 0{,}0015$	$0{,}997 \pm 0{,}010$

	$\Gamma_{\tau\to\nu_\tau e\overline{\nu}_e}/\Gamma_{\mu\to\nu_\mu e\overline{\nu}_e}$	$\Gamma_{\tau\to\nu_\tau\pi}/\Gamma_{\pi\to\mu\overline{\nu}_\mu}$	$\Gamma_{\tau\to\nu_\tau K}/\Gamma_{K\to\mu\overline{\nu}_\mu}$	$\Gamma_{W\to\tau\overline{\nu}_\tau}/\Gamma_{W\to\mu\overline{\nu}_\mu}$
$\lvert g_\tau/g_\mu\rvert$	$1{,}0004 \pm 0{,}0023$	$0{,}9999 \pm 0{,}0036$	$0{,}979 \pm 0{,}017$	$1{,}037 \pm 0{,}014$

	$\Gamma_{\tau\to\nu_\tau\mu\overline{\nu}_\mu}/\Gamma_{\mu\to\nu_\mu e\overline{\nu}_e}$	$\Gamma_{W\to\tau\overline{\nu}_\tau}/\Gamma_{W\to e\overline{\nu}_e}$
$\lvert g_\tau/g_e\rvert$	$1{,}0002 \pm 0{,}0022$	$1{,}034 \pm 0{,}014$

Число нейтрино по ширине «невидимых» распадов

$$\frac{\Gamma_{\text{inv}}}{\Gamma_l} = \frac{N_\nu \Gamma(Z \to \overline{\nu}\nu)}{\Gamma_l} = \frac{2N_\nu}{(1-4\sin^2\theta_W)^2+1},$$

$$= 5{,}942 \pm 0{,}016$$