A Formal Proof of the Pythagorean Theorem

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Abstract

The Pythagorean theorem is a foundational result typically encountered early in mathematical education, where it is accepted through diagrams, folklore, and social pressure. Despite being one of the best-known theorems in human history, it is rarely presented to students in a mechanized, machine-checkable form. We remedy this gap by providing a complete formal proof in the Rocq/Coq proof assistant. Our proof is intentionally minimal: we avoid geometry, avoid square roots, and reduce the statement to algebraic normalization in $\mathbb{R}$. The main technical contribution is the observation that `ring` is more consistent than most classrooms.

Keywords: formal proof, Pythagorean theorem, Rocq, Coq, ring tactic

1 Introduction

The Pythagorean theorem states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the legs. Most readers first encounter this theorem via a diagram containing at least one line that is not quite straight. They then proceed to use it for years without ever asking whether the proof was formally correct, or merely convincing.

Meanwhile, proof assistants have matured to a point where many deep results can be mechanized. This naturally raises an urgent question: can we also mechanize the theorem that everyone already knows?

This paper answers "yes", in the most literal and least surprising way possible.

2 Statement and Setup

Rather than formalizing Euclidean geometry, we place the right triangle on the coordinate axes. Let

$$O = (0, 0), \quad A = (a, 0), \quad B = (0, b),$$

with $a, b \in \mathbb{R}$. Then OA and OB are perpendicular, and the squared length of the segment AB is

$$AB^2 = (a - 0)^2 + (0 - b)^2 = a^2 + b^2.$$

To avoid square roots entirely, we work with *squared distance*.

Definition 1 (Squared distance) For points $p = (x_1, y_1)$ and $q = (x_2, y_2)$ in $\mathbb{R}^2$, define

$$\mathrm{dist2}(p, q) = (x_1 - x_2)^2 + (y_1 - y_2)^2.$$

Theorem 1 (Pythagoras on the axes) *For all $a, b \in \mathbb{R}$,*

$$\mathrm{dist2}((a, 0), (0, b)) = a^2 + b^2.$$

3 Formalization in Rocq/Coq

We now present a complete mechanized proof of Theorem 1. The proof uses only the standard real numbers library and the `ring` tactic for normalization.

3.1 Code Listing

Listing 1 A minimal formalization of Pythagoras (squared form) in Rocq/Coq.

```
From Coq Require Import Reals.
From Coq Require Import Ring.

Open Scope R_scope.

Definition Point : Type := (R * R)%type.

Definition dist2 (p q : Point) : R :=
  let '(x1, y1) := p in
  let '(x2, y2) := q in
  (x1 - x2) * (x1 - x2) + (y1 - y2) * (y1 - y2).

Definition sq (x : R) : R := x * x.

Theorem pythagore_axes (a b : R) :
  dist2 (a, 0) (0, b) = sq a + sq b.
```

```
Proof.
  unfold dist2, sq.
  simpl.
  ring.
Qed.
```

4 Results

Theorem 1 is verified by the proof assistant with no additional axioms beyond those already present in the standard real number development. No counterexamples were found.

5 Conclusion

We provided a complete formal proof of a classroom theorem that almost no one asked to see formalized. In doing so, we confirm that the Pythagorean theorem is compatible with mechanized reasoning, and that parentheses can be trusted more than diagrams.

6 Contribution

Declarations

- Funding: Not applicable.
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- Code availability: Provided in Listing 1.