MATH584 - Math for Algo Trading Homework 3

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File Descriptions

File Name	Description
PnL_Abs.csv	1a) Absolute PnL process T-cost=0.005
PnL_Abs_noCost.csv	1a) Absolute PnL process T-cost=0
PnL_Rel.csv	1a) Relative PnL process T-cost=0.005
PnL_Rel_noCost.csv	1a) Relative PnL process T-cost=0
PnL_Abs_CV.csv	1b) Absolute PnL process T-cost=0.005, investing $1M*V
PnL_Abs_CV_noCost.csv	1b) Absolute PnL process T-cost=0, investing $1M*V
PnL_Rel_CV.csv	1b) Relative PnL process T-cost=0.005, investing $1M*V
PnL_Rel_CV_noCost.csv	1b) Relative PnL process T-cost=0, investing $1M*V
Log_PnL_Abs.csv	2) Absolute PnL process T-cost=0.005
Log_PnL_Abs_noCost.csv	2) Absolute PnL process T-cost=0
Log_PnL_Rel.csv	2) Relative PnL process T-cost=0.005
Log_PnL_Rel_noCost.csv	2) Relative PnL process T-cost=0

Remark: Data length is 200 days for all files.

1. Pairs Trading

The algorithm can be described as follows…

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For each day, loop through every possible pair
   If the position of the pair is closed, check if pair is cointegrated
   (using the calibration window)
         If cointegrated: open position, record the investment size on
         both long and short, max holding date (T limit), and lambda2,
         bounds

   If position is already opened, check if pair reached lambda2, T
   limit, or bounds (stopping condition)

         If reached, check if the pair is STILL cointegrated
               If still cointegrated,

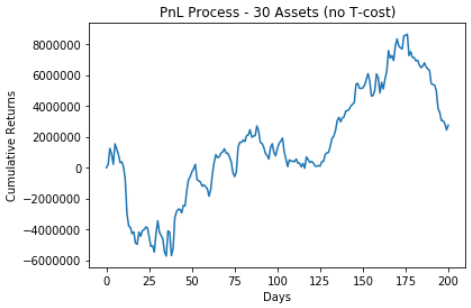
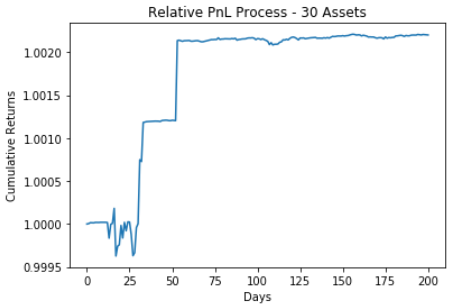
                     If lambda1 on same side (of the mean), continue
                     to invest the pair

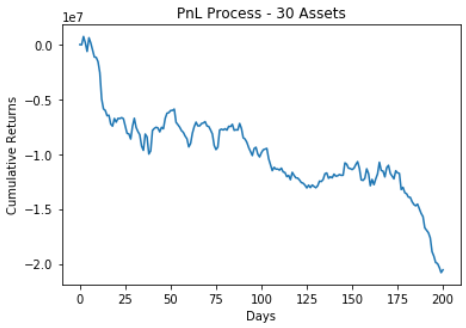
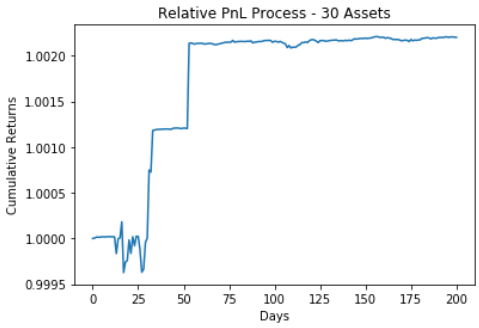
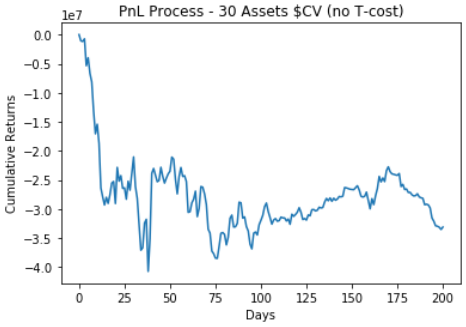
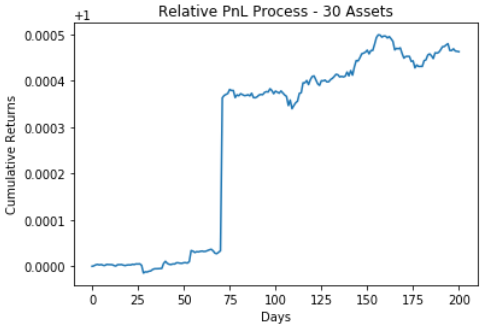
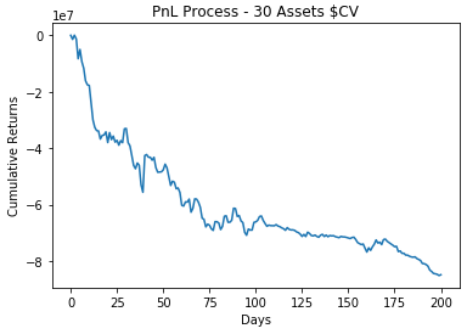
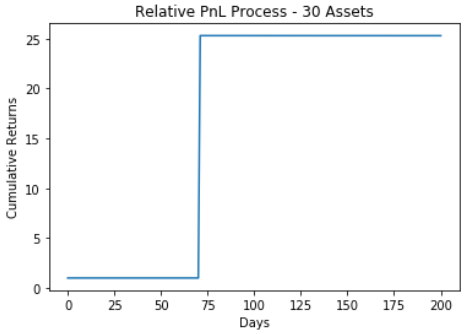

                     If lambda1 on opposite side, close and re-open
                     the pair

               If not cointegrated,

                     Close the pair
```

The process is most time consuming during the checking for cointegration where we have to fit the prices to an AR model, retrieve the coefficient $C_{ij,}$ use it to find Z, and do the Dickey-Fuller test to see if it is significantly cointegrated or not. For the purpose of this study, I decreased the time span to only 200 days after the calibration window (day 100 to 299) to save time, while still using all 30 assets.

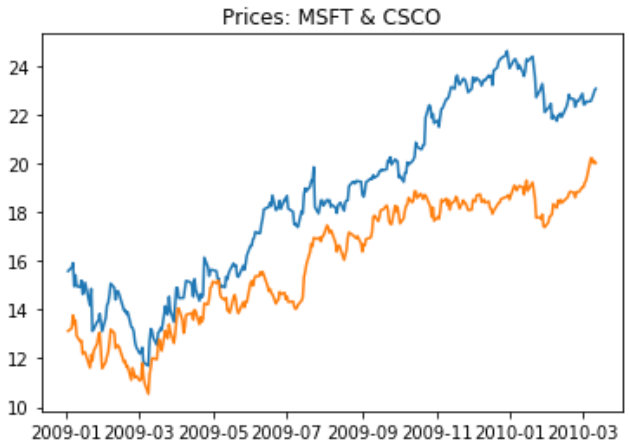
T-cost (λ)	Investment on Long ($)	Absolute PnL process	Relative PnL process
0	1M	 Sharpe Ratio: 0.02549	 Sharpe Ratio: 9259.28

T-cost (λ)	Investment on Long ($)	Absolute PnL process	Relative PnL process
0.005	1M	 Sharpe Ratio: -0.1883	 Sharpe Ratio: 9259.28
0	V*1M	 Sharpe Ratio: -0.0794	 Sharpe Ratio: 41522.5
0.005	V*1M	 Sharpe Ratio: -0.2110	 Sharpe Ratio: 0.6531

The PnL processes displays our cumulative profits over the 200 days. Though with some variance, the λ = 0, $1M strategy was the only one that barely yielded us a positive profit in the end. By adding the T-costs, we expect the PnLs to be lower than their no-cost counterparts. The V*$1M investment strategy which was supposed to scale our investments upon how confident we are on opening a position, had actually cost us even more money. This suggests that the cointegration of most pairs had turned around/ended faster than our expectation.

The relative returns values only vary by a small amount causing them to have small standard deviations and huge relative PnLs as a result. Though high in Sharpe ratio, this is not practical because they only gain us less than 0.5% profit.

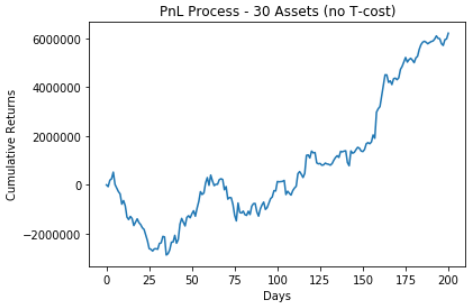
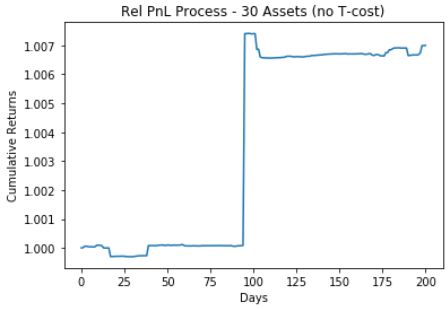
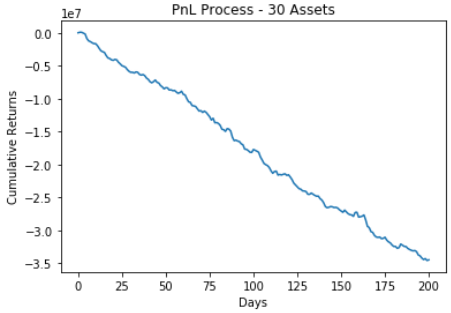
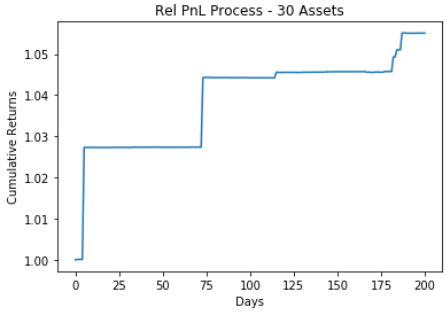
The pairs trading strategies we implemented were profitable at some times and costing us money during others. I decided to examine this phenomenon deeper by looking at a specific pair as an example. The prices of MSFT and CSCO have been found to be cointegrated during several intervals in the first 300 days. Here we can clearly see how the pair of prices started off cointegrated around the first 100 days, suddenly stopped being cointegrated right afterwards, and then repeated this cycle again. This is an example of how our pairs trading strategy failed to work; we opened the position from seeing the initial cointegration, but then lost money because they diverged afterwards. In fact, most of the pairs lost us money in the same fashion and thus our cumulative PnL processes ended up in a deficit.



The results of this assignment were to be expected, there should be much more to implement in a real life pairs trading strategy in order to generate profits. The time span of 200 days may also be too short to make this conclusion, as we might actually see a positive profit if we continued to run this for more years of data.

2. Pairs Trading through daily optimal mean-standard deviation portfolio of log prices

The strategy in this part is different from the first part where we actually check for cointegration between all the pairs every day. We then construct an optimal portfolio using only those pairs that were cointegrated. Note that in the first assignment, we often lost money due to opening a position of a pair that we first found to be cointegrated but diverged afterwards. Not only the log prices are naturally more cointegrated than the prices themselves, recalibrating the portfolio each day would guarantee us that we are always only investing in the cointegrated pairs.

T-cost (λ)	Investment on Long ($)	Absolute PnL process	Relative PnL process
0	1M	 Sharpe Ratio: 2.0595	 Sharpe Ratio: 1917.97
0.005	1M	 Sharpe Ratio: -0.6997	 Sharpe Ratio: 440.08

We can see that this strategy of recalibrating (checking for cointegration) each day is a lot more robust and makes us less susceptible to pairs diverging from being cointegrated. The case where there is no T-cost (λ = 0) yielded us up to $6,000,000 within 200 days, which suggests that the investment decisions are viable. After adding trading costs, however, we end up losing $3,500,000 instead. Reforming the optimal strategy and trading everyday will cost us a lot.

Again the Sharpe ratios for relative PnL tells us nothing but the prices are changing in the direction of our investments and the net gain is very low.