Credit Valuation Adjustment for an Interest Rate Swap

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1. Introduction

The report mainly focuses on the credit valuation adjustment (CVA), taken by Counterparty A for an interest rate swap (IRS) using credit spreads for counterparty B. To calculate CVA, it is necessary to calculate the following inputs in the first place:

- Forward LIBOR rates
- Probability of Default
- Expected Exposure
- Discount Factors

2. Forward LIBOR rates

In the project, Heath–Jarrow–Morton (HJM) model and Monte Carlo method are introduced to simulate the forward LIBOR rates.

2.1 Data Preparation

The data is gathered from the official website of Bank of England. In order not to get affected by the regime changes, it only covers the time period between 01-January-2016 and 30-April-2018. Since Bank Liability Curve is built from short sterling futures, it is more suitable for pricing IR derivatives, so GLC is ignored this time. The data structure is 588 rows and 50 columns (with 6 months as the interval). The first entry is $\tau = 0.08$ and $\tau = 0.5$ the second. Please note that all the empty values have been eliminated for the sake of convenience and accuracy.

2.2 PCA

Principal component analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables clarification needed into a set of values of linearly uncorrelated variables called principal components. In a word, it provides systematic factors that describe movement of a curve as a whole and is widely used to project yield curve in the banking industry.

First of all, daily differences in forward rates have been calculated at each tenor and then comes the covariance matrix which is symmetric, and it can be expressed in such a form

$$\Sigma = V\Lambda V'$$

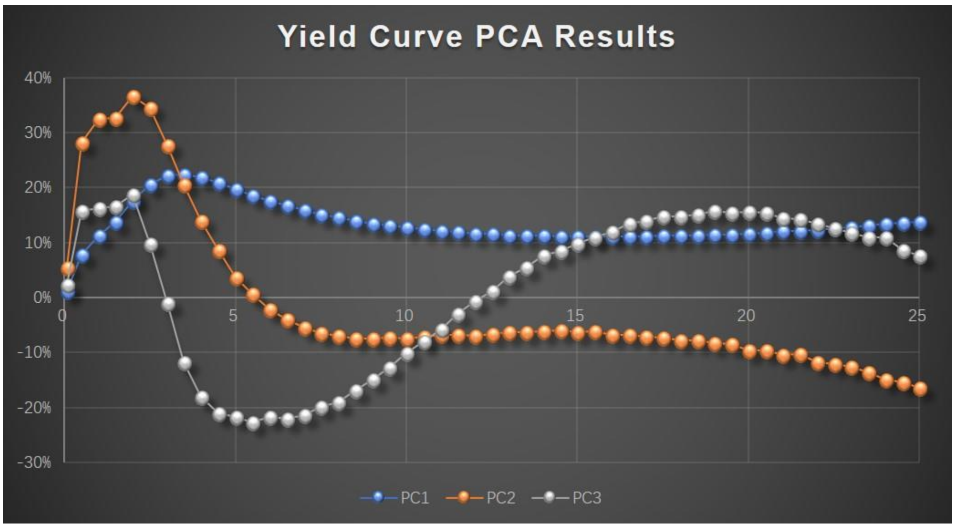
- Λ is a diagonal matrix with eigenvalues $\lambda_1 > \lambda_2 > \dots \lambda_n > 0$ positive and should be expressed as

$$\Lambda = \begin{pmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{pmatrix}$$

- V is a vectorized matrix of eigenvectors.
- The calculation details of the three principal components can be found in *HJM_MY_PCA. xlsx*, the results are

PC	Tenor	Eigenvalues	Cum. R-Square
3rd largest	0.5	4.08239E-05	0.94938
2nd largest	24.5	9.86004E-05	0.91585
1st largest	1.0	0.001016278	0.83485

Then the yield curves can be constructed using the three PCs.



- The volatility vectors can be calculated using

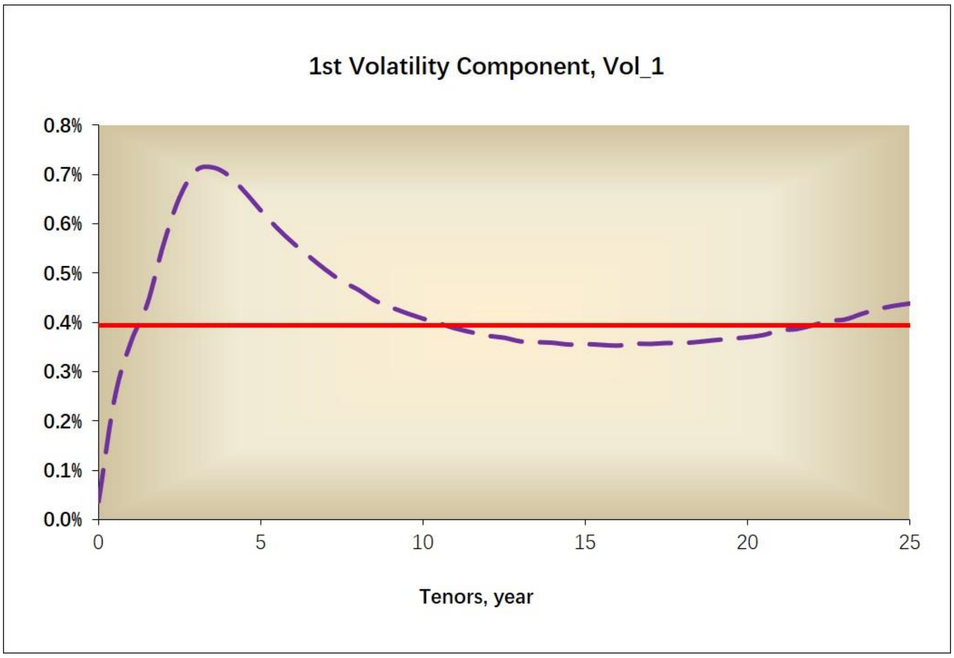
$$\bar{v}_i(t^*,\tau)=\sqrt{\lambda_i}e_\tau^{(i)}$$

The calculation details of the three volatility vectors can be found in *HJM_MY_MC. xlsx*

2.3 Cubic Spline

The fitting is done by a single cubic spline wrt tenor τ

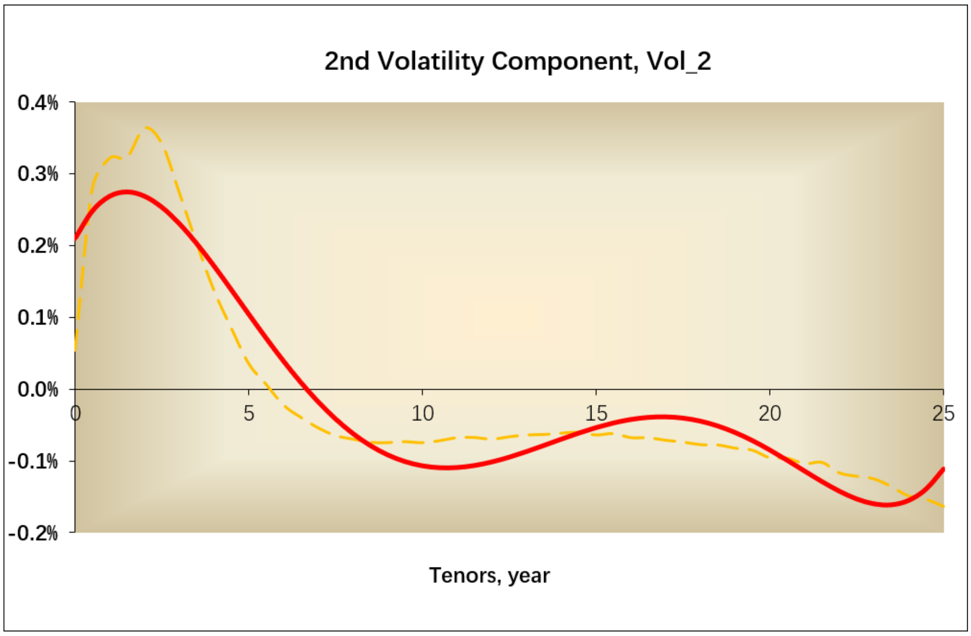
$$v(t,\tau)=\beta_0+\beta_1\tau+\beta_2\tau^2+\beta_3\tau^3+\ldots$$



Parameter of the first volatility is

$$\beta_0=0.003938$$

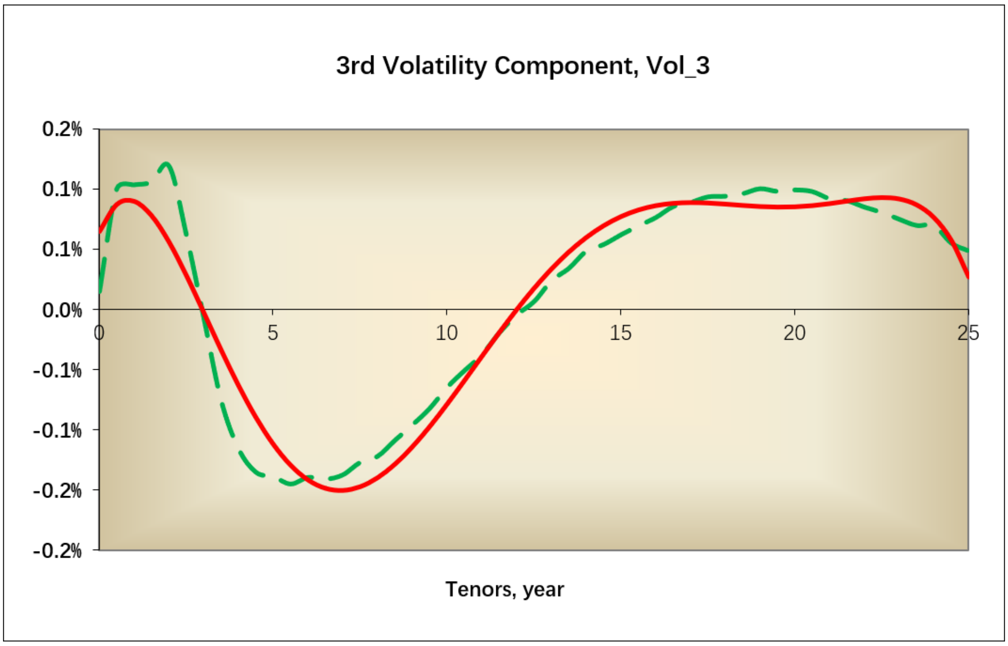
And the cumulative R^2 is 100%.



Parameters of the second volatility are

$$\begin{cases} \beta_0 = 0.002094642 \\ \beta_1 = 0.000967322 \\ \beta_2 = \text{-}0.000420678 \\ \beta_3 = 4.63897\text{E-}05 \\ \beta_4 = \text{-}2.02504\text{E-}06 \\ \beta_5 = 3.08963\text{E-}08 \end{cases}$$

And the cumulative R^2 is 90.41%.



Parameters of the third volatility are

$$\begin{cases} \beta_0 = 0.000646671 \\ \beta_1 = 0.000687907 \\ \beta_2 = \text{-}0.000521378 \\ \beta_3 = 9.02256\text{E-}05 \\ \beta_4 = \text{-}6.52249\text{E-}06 \\ \beta_5 = 2.15058\text{E-}07 \\ \beta_6 = \text{-}2.67803\text{E-}09 \end{cases}$$

And the cumulative R^2 is 94.49%.

In sum, with the application of cubic spline method, the cumulative R-square of the three volatilities are all more than 90%, thus providing a relatively ideal fitting of the yield curve. Also, the number of parameters is rather reasonable, so the next step should be using the three volatilities in the Monte Carlo simulation to generate the forward LIBOR rates.

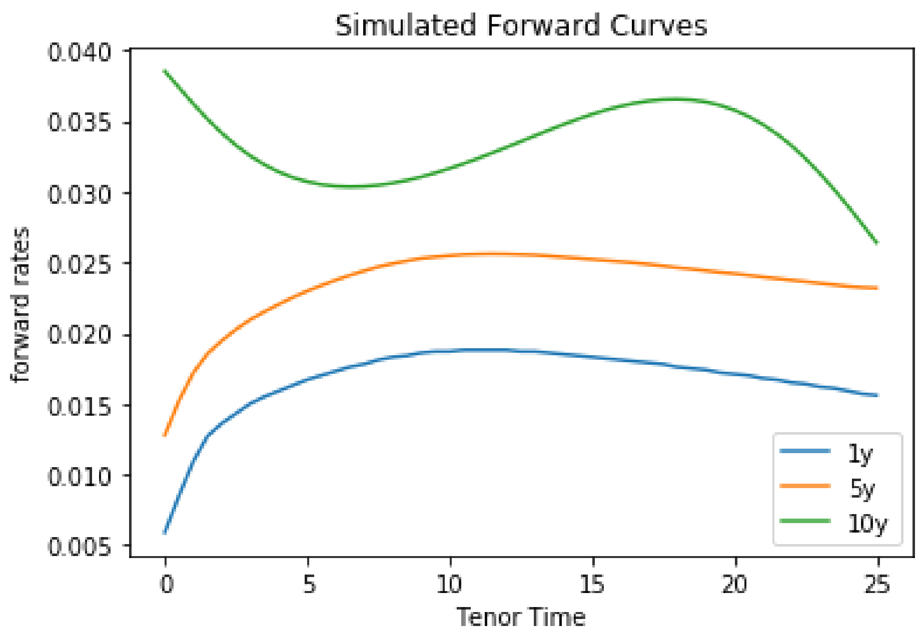
2.4 Monte Carlo Simulation

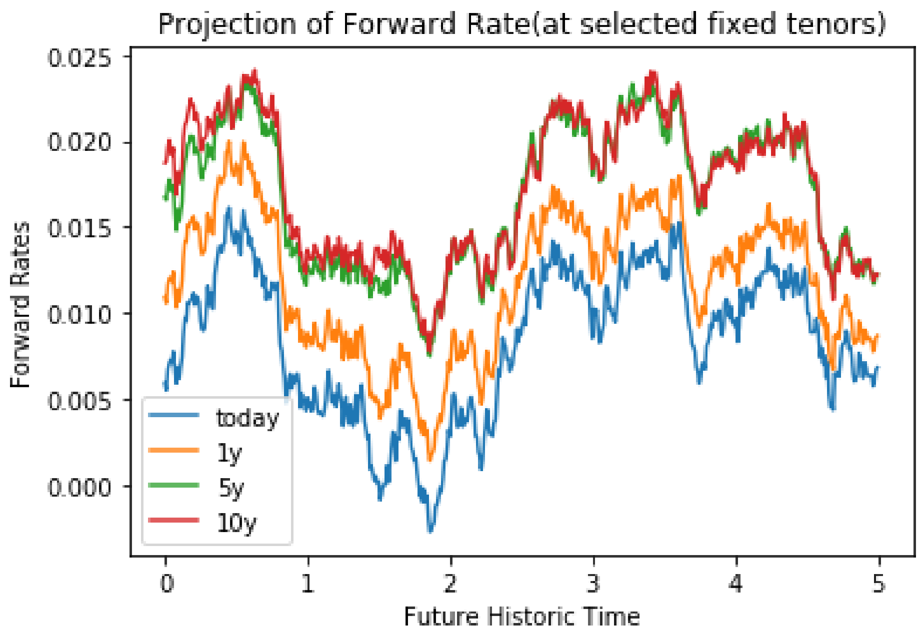
Monte Carlo method is used to estimate an expectation by simulating the evolution of forward rates and implemented under risk-neutral HJM dynamics as below

$$d\overline{f} = \mu(t)dt + \sum_{i=1}^{3} Vol_i \phi_i \sqrt{dt} + \frac{dF}{d\tau} dt$$

The simulation time is set to be 500. Then comes out the simulated forward curves.

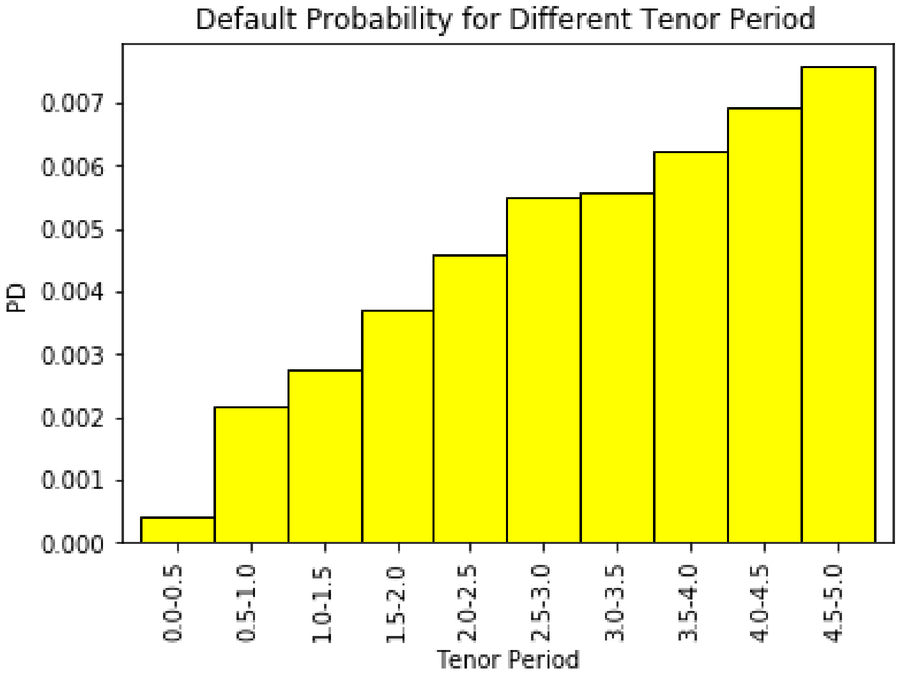
It is very interesting to find that sometimes the forward curves will stretch to the negative side of the y-axis. This is because HJM follows Gaussian distribution, which allows the negative rates to come into view.





3. Probability of Default

In this case, I choose Porsche AG as the Counterparty B to bootstrap the probability of default from its CDS spread gathered from Bloomberg Terminal. The time period is the same as the BLC historical forward rates. The calculation details can be checked in CDS_Boostrapping.xlsx. The recovery rate is 40% and only linear interpolation is involved.



As is widely known, the default probability increases as time goes by. It is also validified by the graph above. As a highly profitable company, Porsche AG never demonstrate a great potential to default. The highly default probability is only slightly above 0.7%.

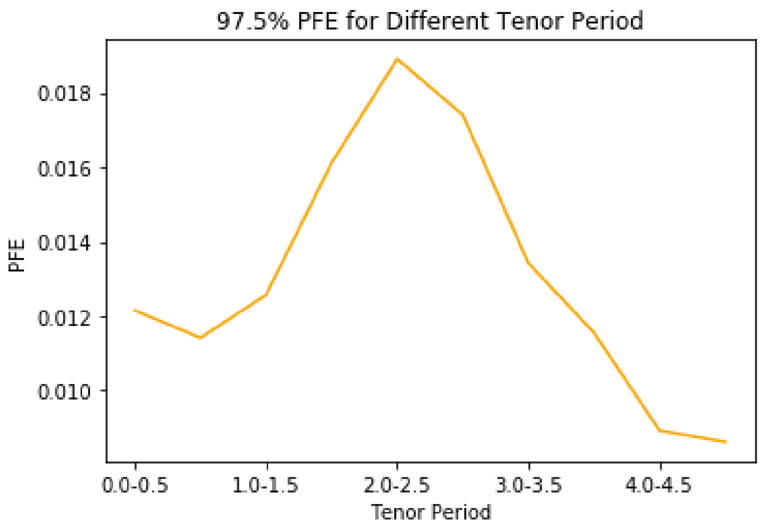
4. Expected Exposure

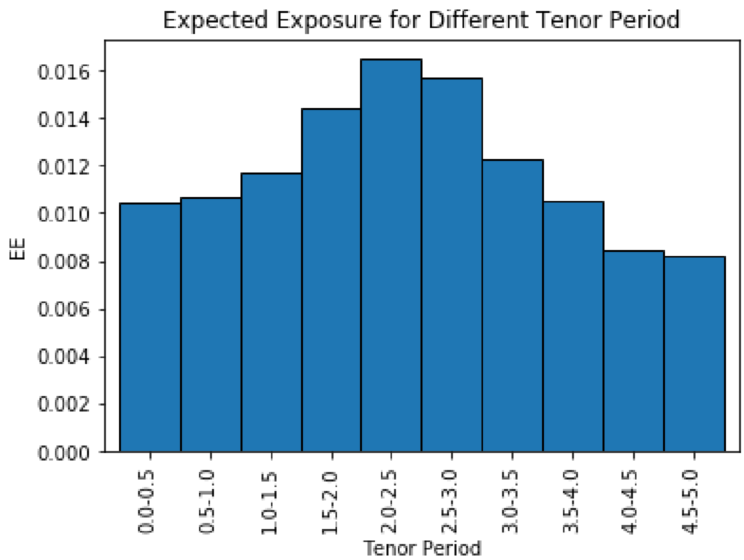
Potential Future Exposure (PFE) is the maximum expected credit exposure over a specified period of time calculated at some level of confidence (i.e. at a given quantile). PFE is a measure of counterparty risk/credit risk. ... It can be called sensitivity of risk with respect to market prices. The Expected Exposure (EE) is defined similarly to the PFE, except that the average is used instead of a specific quantile.

The expected exposure is calculated as

$$\max(MtM, 0)$$

The 97.5% potential future exposure (PFE) generally demonstrates a rather similar trend to EE throughout these 5 years. An apparent humped structure can be detected from the two graphs. In summary, both charts show that the riskiest period lies between year 2 and year 2.5.





5. Discount Factors

Before calculating the discount factors, it is necessary to turn the simulated forward rates into LIBOR rates using

$$LIBOR = \frac{e^{f\tau} - 1}{\tau}$$

Using the discount factor formula, we can get the following discount factor table

$$DF(0, T_{i+1}) = \prod_i \frac{1}{1 + \tau_i L(t; T_i, T_{i+1})}, DF(T_i, T_{i+1}) = \frac{DF(0, T_{i+1})}{DF(0, T_i)}$$

Tenor	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.0	1	0.991289	0.985675	0.979343	0.975078	0.967911	0.96048	0.953961	0.947108	0.943987	0.939872
0.5	0	1	0.994337	0.987949	0.983647	0.976417	0.968921	0.962344	0.955431	0.952282	0.948131
1.0	0	0	1	0.993576	0.989249	0.981978	0.974439	0.967825	0.960872	0.957706	0.953531
1.5	0	0	0	1	0.995645	0.988327	0.980739	0.974083	0.967085	0.963898	0.959696
2.0	0	0	0	0	1	0.99265	0.985029	0.978343	0.971315	0.968114	0.963894
2.5	0	0	0	0	0	1	0.992322	0.985587	0.978507	0.975282	0.971031
3.0	0	0	0	0	0	0	1	0.993213	0.986078	0.982828	0.978544
3.5	0	0	0	0	0	0	0	1	0.992816	0.989544	0.985231
4.0	0	0	0	0	0	0	0	0	1	0.996705	0.99236
4.5	0	0	0	0	0	0	0	0	0	1	0.995641
5.0	0	0	0	0	0	0	0	0	0	0	1

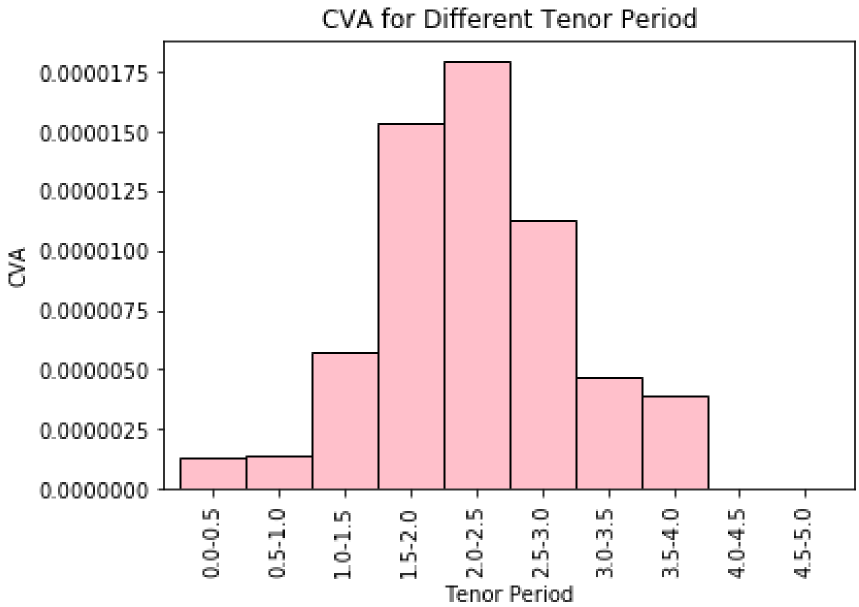
For the sake of convenience, I used the mean value of all discount factors to calculate the CVA.

6. CVA Calculation

Last but least, here it eventually comes to the calculation of credit value adjustment and its equation is expressed as

$$CVA \approx \sum_i (1-R)E\left(\frac{T_{i-1}-T_i}{2}\right)DF\left(\frac{T_{i-1}-T_i}{2}\right)PD\left(\frac{T_{i-1}-T_i}{2}\right)$$

For simplicity, the recovery rate remains 40%.



It can be observed from the chart that there is a humped structure in the CVA plot. Probably it is due to the fact that the default probability is highest between year 2 and year 2.5, as mentioned previously. Given the notional is worth 1 million USD, the ultimate CVA only comes out to 125 USD for Porsche AG. This might be because Porsche is a highly profitable organization and thus not so likely to default.

7. Conclusion

Based on the limited timeframe, I failed to pay attention to many details during my CVA calculation. First, more companies or counterparties B with a wider credit spread or higher probability of default should be tested. Second, my calculation of discount factors is way too simplified, which more or less decrease the calculation accuracy. Third, the shape of PFE is tremendously strange compared, theoretically speaking. Fourth, since HJM is not a market model, it cannot be 'validated' against prices in the strict sense. In a word, a doubt has been cast on its feasibility in the CVA calculation and my future work will be on this part.

References

(1) https://en.wikipedia.org/wiki/Potential_future_exposure

(2) Richard Diamond, CQF Institute, *HJM Model*, 2018

(3) Richard Diamond, CQF Institute, *Model Implementation and Robust Estimation in Rates, Credit and Portfolio Construction*, 2018

(4) Richard Diamond, CQF Institute, *Final Project Brief*, 2018

(5) Alonso Pena, CQF Institute, *Credit Default Swaps*, 2018