Risky Copulas: Financial Applications of Copula (Group-I)

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Main Objectives:

- Find an appropriate copula-- Student's t or Gaussian--to forecast downward risk and diversification benefits in financial risk management
- Seek alternatives to the traditional measures of dependence (e.g the Gaussian copula) that fail to account for the tails of the distributions of the returns

Relevance to Risk Management:

- Risk of a portfolio is directly related to the dependence between the elements of a portfolio (stocks, bonds and real estate)
- Copula used to model dependence between the returns of the portfolio elements should capture both the downward risks embedded in the tail distributions and the diversification possibility depicted by the center of the returns distribution

Main Copula differences:

Gaussian :

$$C_n^{\Phi}(\boldsymbol{u}; \boldsymbol{\Omega}^{\Phi}) = \Phi_n(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n); \boldsymbol{\Omega}^{\Phi})$$

- Assigns more probability to central events

Student's t:

$$C_n^{\Psi}(\boldsymbol{u}; \boldsymbol{\Omega}^{\Psi}, \nu) = \Psi_n(\Psi^{-1}(u_1; \nu), \ldots, \Psi^{-1}(u_n; \nu); \boldsymbol{\Omega}^{\Psi}, \nu)$$

- Assigns more probability to tail events

Definition of Descriptive Values:

Skewness: measure of asymmetry in a distribution
Kurtosis: measure of "tailness"; high values signify heavier tails
Volatility: a measure of risk; high values signify riskier returns
Heteroscedasticity: implies non-constant variance of errors

Table 1: Estimated constant values for bivariate copula relationships

Gaussian		Student's t			
$\rho_{s,b}$	-0.200	$\rho_{s,b}$	-0.189	$df_{s,b}$	7.24
$\rho_{s,r}$	0.472	$\rho_{s,r}$	0.477	$df_{s,r}$	10.9
$\rho_{b,r}$	-0.0722	$\rho_{b,r}$	-0.0759	$df_{b,r}$	14.1
logL	-316.50	logL	-286.09		

Where ρ is the correlation parameter between two returns. The log likelihood (logL) is used to assess the reliability of the derived parameters given the returns data (high values show better fit). The returns are stocks (s), bonds (b), and real estate (r).

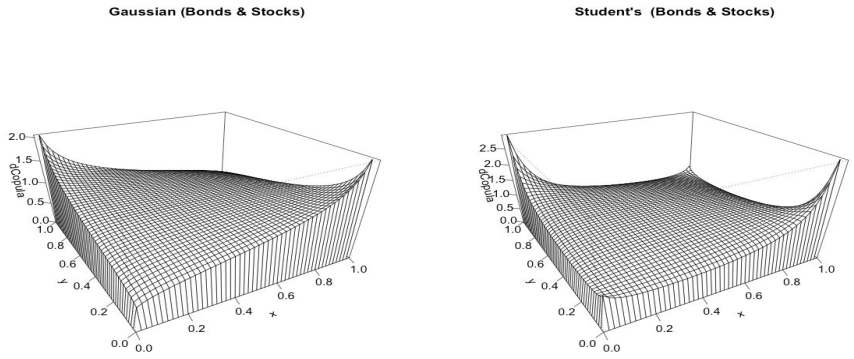

Figure 1: Perspective plots for Gaussian and Student's t copula for Bonds and Stocks returns

Figure 2: Visual fit comparison of Gaussian and Student's t copula to the returns data

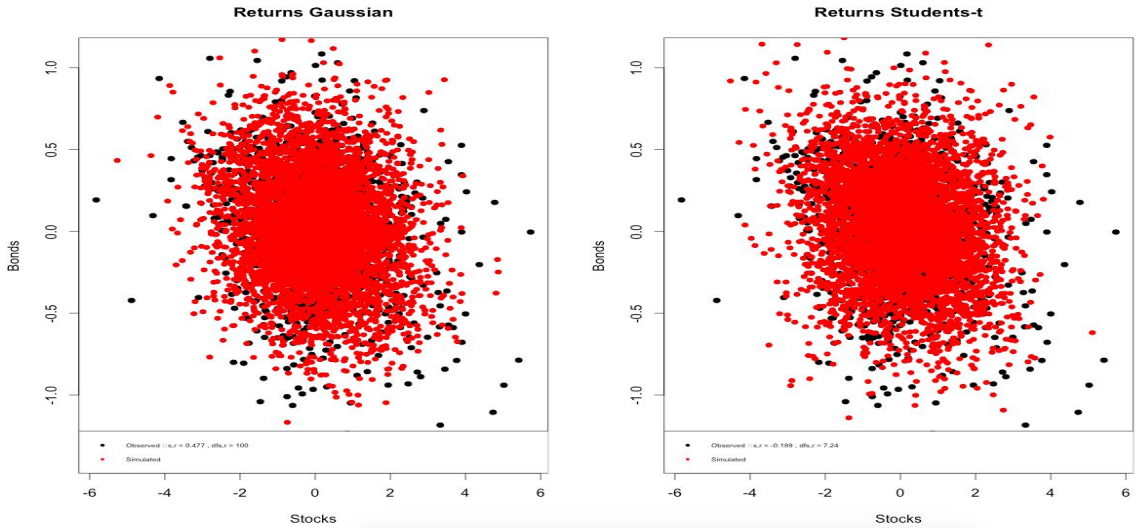

Figure 2 shows the simulated data in red overlaying the observed data in black. The perspective plots (in figure 1) for both copulas are similar. However Student's t copula appears to have a slight better fit on the tails as shown by the greater coverage of the simulated data points on the southeast and northwest ends of the observed returns in Figure 2.

Goodness-of-fit tests: *allow for the selection of an appropriate copula*

White Test:		*Anderson-Darling (AD) Test:*	
H_o:	Constant variance (homoscedastic)	H_o:	The data follows the specified distribution
H_a:	Non-constant variance (heteroscedastic)	H_a:	The data does not follow the specified distribution

Table 2: Goodness-of-Fit test results

Gaussian				**Student's t**			
	p-value		p-value		p-value		p-value
$AD_{s,b}$	0.0035	$White_{s,b}$	0.0004	$AD_{s,b}$	**0.09041**	$White_{s,b}$	**0.9999**
$AD_{s,r}$	**0.123**	$White_{s,r}$	0.0013	$AD_{s,r}$	**0.657**	$White_{s,r}$	**0.9999**
$AD_{b,r}$	**0.1364**	$White_{b,r}$	0.025	$AD_{b,r}$	**0.578**	$White_{b,r}$	**0.9999**

The goodness of fitness test results provide evidence for lack of fit if $p<0.05$. A bootstrap of 1000 was used to find these statistical values.

Conclusions:

- Student's t copula better fits returns than the Gaussian copula for all pairs of returns. Gaussian copula fails because:
  - It appears to be more heteroscedastic than the Student's t
  - It does not account for the tail dependence in stocks and bonds returns
- Since the Student's t copula accounts for both the tails and center of a returns distribution, it describes both safe and risky investments. Consequently, Student's t copula accounts for diversification benefits.

[1]Alice, Michy. "Modelling Dependence with Copulas in R." *DataScience RSS*. Web. 10 Dec. 2015.
[2]'Heteroscedasticity'. N.p., Okstate.edu, 2015. Web. 10 Dec. 2015.
[3]Kole, Erik, Kees Koedijk, and Marno Verbeek. "Selecting copulas for risk management." *Journal of Banking & Finance* 31.8 (2007): 2405-2423.
[4]"1.3.5.14. Anderson-Darling Test." *1.3.5.14. Anderson-Darling Test*. Web. 10 Dec. 2015.
[5]"1.3.5.11. Measures of Skewness and Kurtosis." *1.3.5.11. Measures of Skewness and Kurtosis*. Web. 10 Dec. 2015.