Risky Copulas

Stephany Flores-Ramos, Carey Hiuhu, Azka Javaid

STAT 360 Probability Group I
December 11, 2015

Risk Management: *It's all about the tails!*

- Tails in finance dictate where money to be made or lost is
- Heavier the tails in a portfolio, the more probable that the investment may move more than three standard deviations from the mean
  - Positive move will increase the returns
  - Negative move will decrease the returns
- Understanding tail behavior leads to a **diversified** portfolio
- Tail behavior enables us to account for **downward movement** (worst-case scenario)

Copulas in Finance

- Copulas should capture both **downside risk** and **diversification** opportunities for assets.
- Traditional dependence has been based on the Gaussian Copula which falters when it comes to capturing dependence on **extreme events**.
- Student's t captures dependence in the tails but is also flexible enough to capture dependence in the center.

Copula Overview

Gaussian copula

$$C_n^{\Phi}(\boldsymbol{u}; \boldsymbol{\Omega}^{\Phi}) = \Phi_n(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n); \boldsymbol{\Omega}^{\Phi})$$

u--vector of marginal probabilities
$\boldsymbol{\Omega}^{\boldsymbol{\Phi}}$--correlation matrix (parameter ρ)
$\boldsymbol{\Phi}^{-1}$--the inverse standard normal cumulative probability
$\boldsymbol{\Phi}_n$--cdf for n-variate standard normals

Student's t copula

$$C_n^{\Psi}(\boldsymbol{u}; \boldsymbol{\Omega}^{\Psi}, \nu) = \Psi_n(\Psi^{-1}(u_1; \nu), \ldots, \Psi^{-1}(u_n; \nu); \boldsymbol{\Omega}^{\Psi}, \nu)$$

u--vector of marginal probabilities
$\boldsymbol{\Omega}^{\Psi}$--correlation matrix (parameter ρ)
v--degrees of freedom
$\boldsymbol{\Psi}^{-1}$--inverse of cdf for Student's t distribution with mean zero, dispersion one
$\boldsymbol{\Psi}_n$--cdf of n-variate Student's t distribution

Our Objectives

- Explore the financial applications of two copula types
- Fit bivariate copula models on pairs of stocks, bonds and real estate returns
- Simulate returns from the copula models
- Assess copula fit
    - Estimate parameter values
    - Visual copula assessment
    - Goodness of fit tests

Data Summary

1499 Returns from Standard & Poor's 500 Composite Index (stocks), JP Morgan's US Government Bond Index (bonds) and the NAREIT All Index (real estate) from January 1, 1999 to December 17, 2004

Summary statistics and tail indices

	Stocks	Bonds	Real estate
Mean	0.012	0.022	0.065
Volatility	1.26	0.34	0.81
Skewness	0.18	−0.34	−0.33
Kurtosis	4.60	3.83	7.34

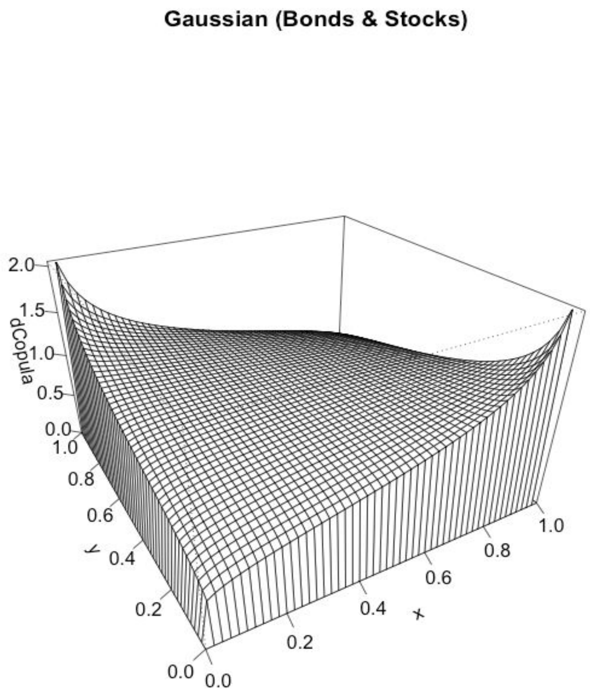
Copula Parameters

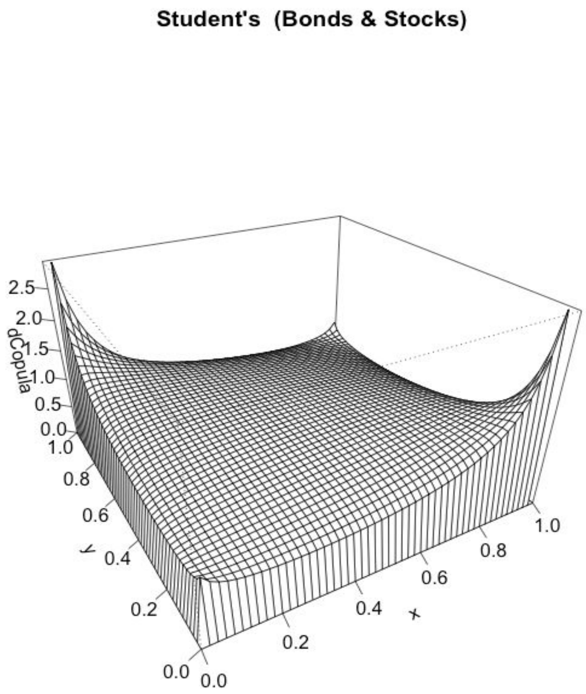
Table 1: Estimated parameter values for bivariate copula relationships

Gaussian		Student's t			
$\rho_{s,b}$	-0.200	$\rho_{s,b}$	-0.189	$df_{s,b}$	7.24
$\rho_{s,r}$	0.472	$\rho_{s,r}$	0.477	$df_{s,r}$	10.9
$\rho_{b,r}$	-0.0722	$\rho_{b,r}$	-0.0759	$df_{b,r}$	14.1
logL	-316.50	logL	-286.09		

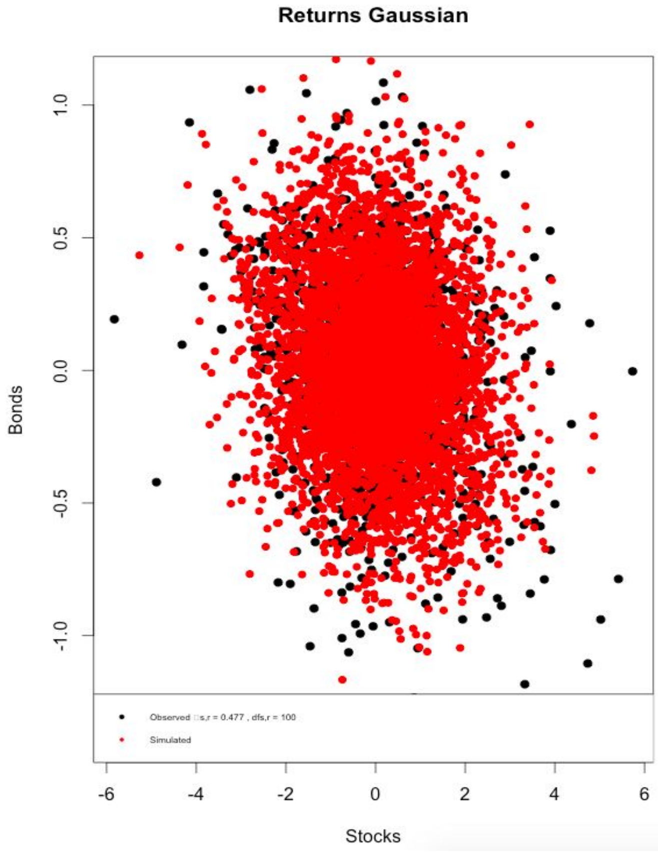
*Where ρ is the correlation parameter between two returns. The log likelihood (logL) is the value used to assess the reliability of the derived parameters given the returns data (high values show better fit). The returns are stocks (s), bonds (b), and real estate (r).

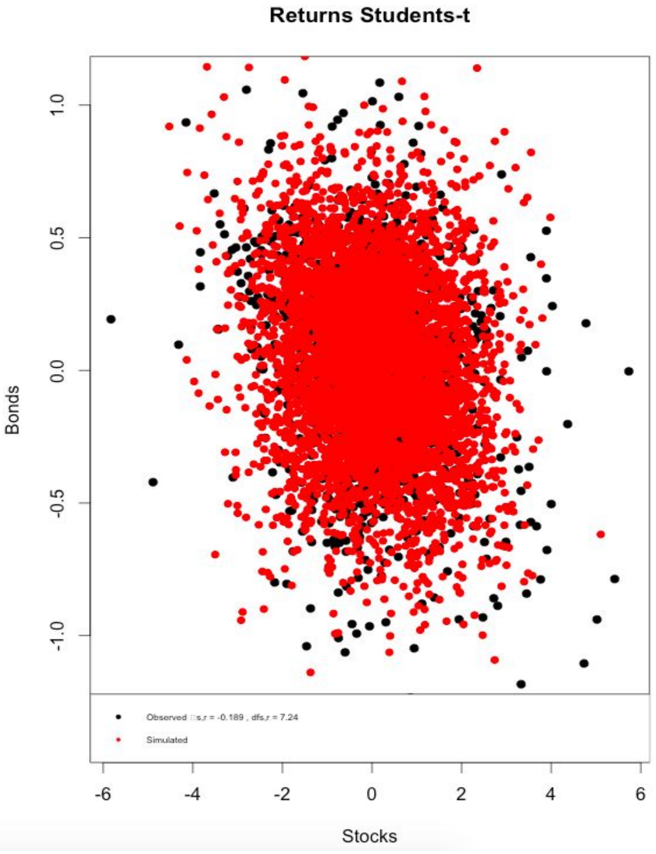
Density Plots



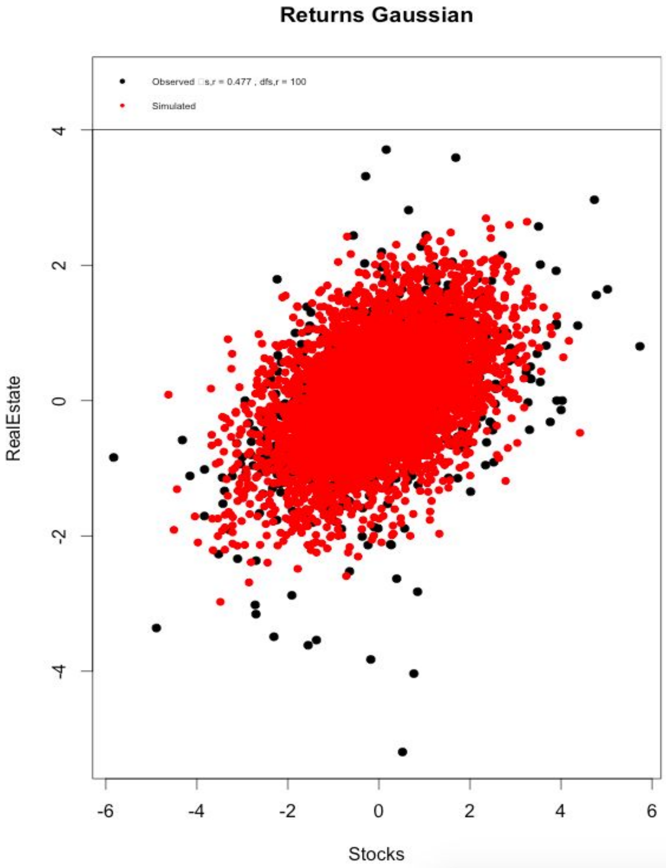


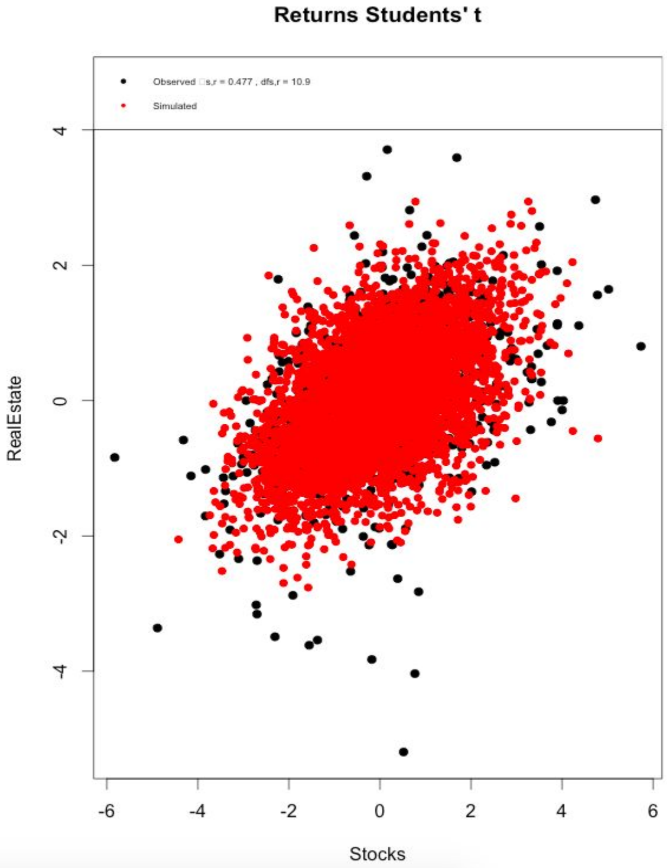
Modeling Stocks and Bonds Returns



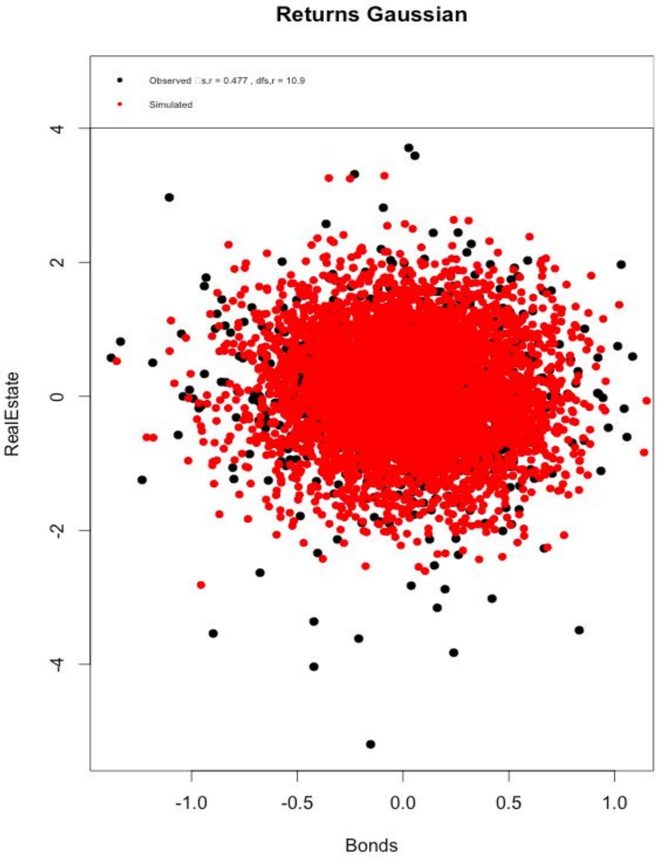


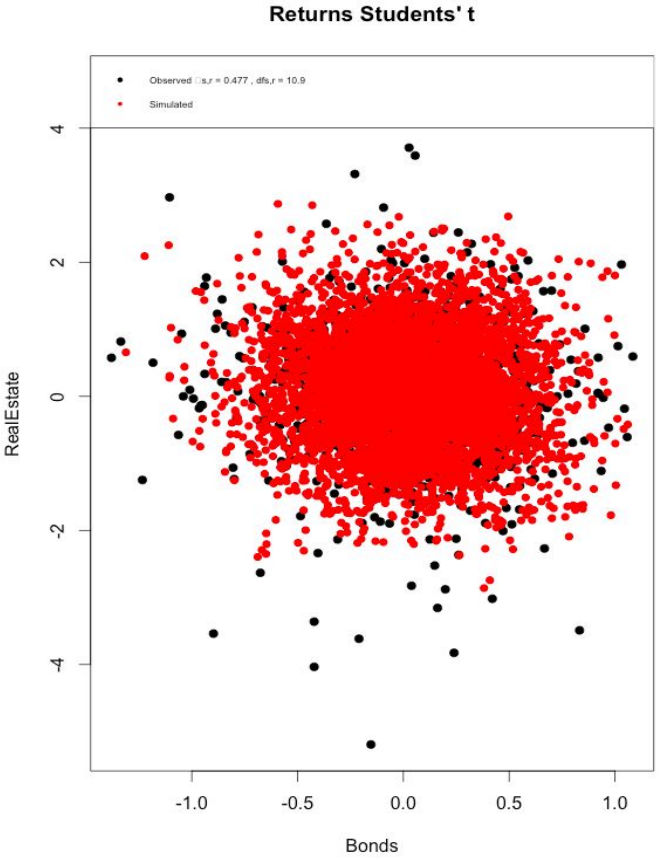
Modeling Stocks and Real Estate Returns





Modeling Bonds and Real Estate Returns





Goodness of Fit Tests

Anderson-Darling Test

- Tests if data sample is drawn from a population with a specified distribution
- Gives more weight to the tails
    - H_o: The data follows the specified distribution
    - H_a: The data does not follow the specified distribution

White Test

- Tests for homoscedasticity (constant variance of errors)
- Homoscedastic = better fit
    - H_o: Constant variance (homoscedastic)
    - H_a: Non-constant variance (heteroscedastic)

Goodness of Fit

Table 2: Goodness-of-Fit test results

Gaussian				Student's t			
	p-value		p-value		p-value		p-value
$AD_{s,b}$	0.0035	$White_{s,b}$	0.0004	$AD_{s,b}$	**0.09041**	$White_{s,b}$	**0.9999**
$AD_{s,r}$	**0.123**	$White_{s,r}$	0.0013	$AD_{s,r}$	**0.657**	$White_{s,r}$	**0.9999**
$AD_{b,r}$	**0.1364**	$White_{b,r}$	0.025	$AD_{b,r}$	**0.578**	$White_{b,r}$	**0.9999**

The goodness of fitness test results provide evidence for lack of fit if $p<0.05$. A bootstrap of 1000 was used to find these statistical values.

Conclusion

- Student's t copula vs. Gaussian copula
- Gaussian copula fails
  - Heteroscedasticity
  - Tail dependence
- Student's t copula in financial realm (**downward risks** and **diversification benefits**)

Citations

- Alice, Michy. "Modelling Dependence with Copulas in R." *DataScience RSS*. Web. 10 Dec. 2015.
- Kole, Erik, Kees Koedijk, and Marno Verbeek. "Selecting copulas for risk management." *Journal of Banking & Finance* 31.8 (2007): 2405-2423.
- "1.3.5.11. Measures of Skewness and Kurtosis." *1.3.5.11. Measures of Skewness and Kurtosis*. Web. 10 Dec. 2015.
- "1.3.5.14. Anderson-Darling Test." *1.3.5.14. Anderson-Darling Test*. Web. 10 Dec. 2015.
- 'Heteroscedasticity'. N.p., Okstate.edu, 2015. Web. 10 Dec. 2015.