# What is the Expected Return on a Stock?

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**Abstract**

We derive a formula for the expected return on a stock in terms of the risk-neutral variance of the market and the stock's excess risk-neutral variance relative to the average stock. These quantities can be computed from index and stock option prices; the formula has no free parameters. The theory performs well empirically both in and out of sample. Our results suggest that there is considerably more variation in expected returns, over time and across stocks, than has previously been acknowledged.

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In this paper, we derive a new formula that expresses the expected return on an individual stock in terms of the risk-neutral variance of the market, the risk-neutral variance of the individual stock, and the value-weighted average of individual stocks' risk-neutral variance. Then we show that the formula performs well empirically.

The inputs to the formula—the three measures of risk-neutral variance—are computed directly from option prices. As a result, our approach has some distinctive features that separate it from more conventional approaches to the cross-section.

First, as it is based on current market prices rather than, say, accounting information, it can in principle be implemented in real time. Nor does it require us to use any historical information: it represents a parsimonious alternative to pooling data on many firm characteristics (as, for instance, in Lewellen, 2015).

Second, it provides conditional forecasts at the level of the individual stock. Rather than asking, say, what the unconditional average expected return is on a portfolio of small value stocks, we can ask, what is the expected return on Apple, today?

Third, the formula makes specific, quantitative predictions about the relationship between expected returns and the three measures of risk-neutral variance; it does not require estimation of any parameters. This can be contrasted with factor models, in which both factor loadings and the factors themselves are estimated from the data (with all the associated concerns about data-snooping). There is a closer comparison with the CAPM, which makes a specific prediction about the relationship between expected returns and betas, but even the CAPM requires the forward-looking betas that come out of theory to be estimated based on historical data.

Our approach does not have this deficiency and, as we will show, it performs better empirically than the CAPM. But—like the CAPM—it requires us to take a stance on the conditionally expected return on the market. We do so by applying the results of Martin (2017), who argues that the risk-neutral variance of the market provides a lower bound on the equity premium. In fact, we exploit Martin's more aggressive claim that, empirically, the lower bound is approximately tight, so that risk-neutral variance directly measures the equity premium. We also present results that avoid any dependence on this claim, however, by forecasting expected returns *in excess of the market*. In doing so, we isolate the purely cross-sectional predictions of our framework that are independent from the market-timing issue of forecasting the equity premium. As these predictions exploit the cross-section as well as the time-series, the associated empirical results are stronger in a statistical sense than those of Martin (2017).

We introduce the theoretical framework in Section I; then we show how to construct the three risk-neutral variance measures, and discuss some of their properties, in Section II.

Our main empirical results are presented in Section III. We test the framework for S&P 100 and S&P 500 stocks, at forecast horizons ranging from one month to two years. It may be worth pointing out that papers in the predictability literature typically aim to uncover variables that are statistically significant in forecasting regressions. We share this goal, of course, but as our model makes predictions about the quantitative relationship between expected returns and risk-neutral variances, we hope also to find that the estimated coefficients on the predictor variables are close to *specific numbers* that come out of the theory. For most specifications we find that that we do not reject the model, whereas we can reject the null hypothesis of no predictability at the six-month, one-year and two-year horizons.

In Section IV, we study how our findings relate to stock characteristics. Notably, we run panel regressions of realized returns onto beta, size, book-to-market, and past returns. In our stock universe, and over our sample period, size and book-to-market are statistically significant forecasters of excess returns, though not of returns in excess of the market. When we include our predictive variables based on risk-neutral variance, the characteristics become statistically insignificant. But the risk-neutral variance variables themselves are significant predictors (of both excess returns and excess-of-market returns); moreover, they enter with coefficients that are insignificantly different than those predicted by the theory. In a similar vein, we show that the returns on portfolios sorted on the characteristics are consistent with the model.

Section V assesses the out-of-sample predictive performance of the formula when its coefficients are constrained to equal the values implied by the theory. We compute out-of-sample $R^2$ coefficients that compare the formula's predictions to those of a range of competitors, as in Goyal and Welch (2008). We start by comparing against competitors that are themselves out-of-sample predictors (in the sense of being based on a priori considerations, without in-sample information). The formula outperforms all such competitors at horizons of three, six, 12 and 24 months, both for expected returns and for expected returns in excess of the market.

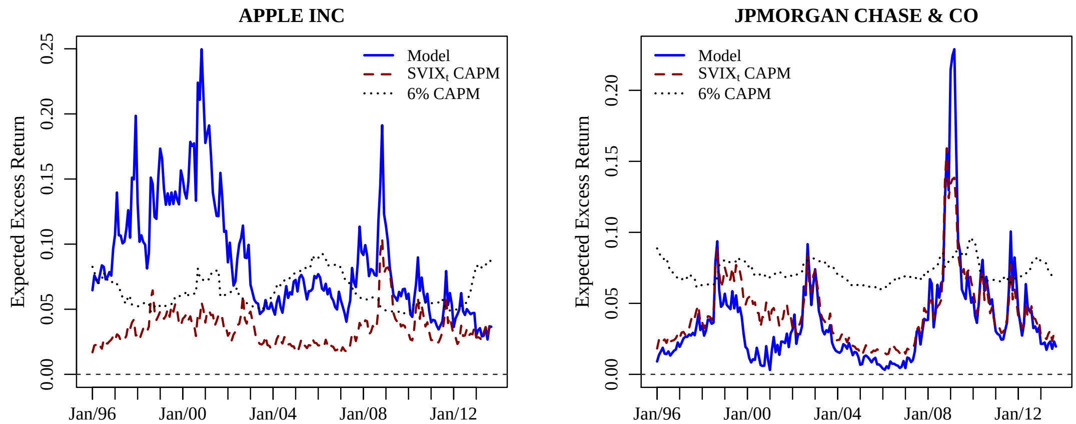
We go on to compare, more ambitiously, against competitors that have in-sample information. At the six- and 12-month horizons, the only case in which our model of expected excess returns 'loses' is when we allow the competitor predictor to know both the in-sample average (across stocks) realized return *and* the multivariate in-sample relationships between realized returns and beta, size, book-to-market, and past returns. (When we allow the competitor to know only the in-sample average and the univariate relationship between realized returns and any one of the characteristics, our formula outperforms.) Even more strikingly, in the purely cross-sectional case in which we forecast returns in excess of the market, the formula outperforms the competitor armed with knowledge of the *multivariate* relationship.

The empirical success of our formula is particularly notable because it makes some dramatic predictions about stock returns. Figure 1 plots the time-series of expected excess returns, relative to the riskless asset and relative to the market, for Apple and for JPMorgan Chase & Co. over the period from January 1996 to October 2014. According to our model, expected returns spiked for both stocks in the depths of the financial crisis of 2008–9. In the case of Apple, this largely reflected a high market-

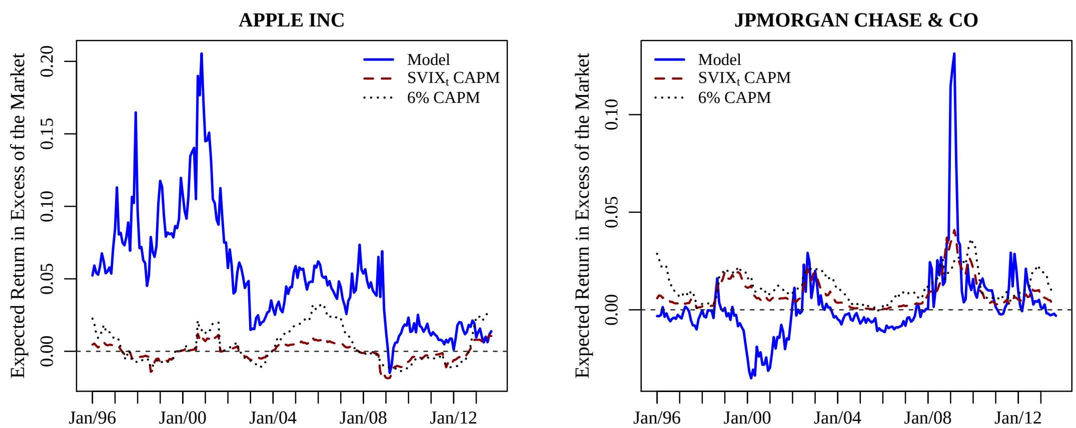
Figure 1: Expected excess returns and expected returns in excess of the market. Annual horizon.

This Figure illustrates our results. It shows the time series of expected excess returns and expected returns in excess of the market for Apple Inc. and JP Morgan Chase & Co. at an annual horizon (solid); and, for comparison, the corresponding time series using the CAPM with a constant equity premium of 6% (dotted) or an equity premium calculated using the SVIX index (dashed).

*Panel A. Expected excess returns*




*Panel B. Expected returns in excess of the market*



wide equity premium rather than an Apple-specific phenomenon, whereas JP Morgan Chase's expected excess return was high even relative to the market risk premium. The figure also plots expected excess returns computed using the CAPM with one-year rolling historical betas (and the equity premium computed from the SVIX index of Martin (2017), or fixed at 6%), to illustrate the point—which, as we will show, holds more generally—that our model generates more volatility in expected returns, both over time and in the cross-section, than does the CAPM.

*Related literature.* A large literature has documented the importance of idiosyncratic volatility for future stock returns, though with varying conclusions. For instance, Ang et al. (2006) find a negative relation both for total volatility and for idiosyncratic volatility (defined as the residual variance of Fama–French three factor regressions on daily returns over the past month). By contrast, Fu (2009) finds a positive relation when idiosyncratic volatility is measured by the conditional variance obtained from fitting an EGARCH model to residuals of Fama–French regressions on monthly returns.

Our model attributes an important role to average stock variance (measured as the value-weighted sum of individual stock risk-neutral variances), a prediction that we confirm empirically. This result echoes the finding of Herskovic et al. (2016) that idiosyncratic volatility (measured from past returns) exhibits a strong factor structure and that firms' loadings on the common component predict equity returns. Furthermore, our measure of average stock variance may capture a potential factor structure in the cross-section of equity options, as documented by Christoffersen et al. (2017) across 29 Dow Jones firms.

Various authors have explored the forecasting power of options-based measures. An et al. (2014) find that increases in implied volatilities of at-the-money call and put options have opposing implications, predicting high and low subsequent stock returns, respectively. Conrad et al. (2013) study the relationship between risk-neutral moments and realized returns, and find a negative, though not statistically significant, relationship between risk-neutral variance and subsequent stock returns; they work with the risk-neutral variance of log returns (following Bakshi et al. (2003)), so their volatility indices load particularly strongly on the prices of deep out-of-the-money put options. In contrast to both these papers, our theoretical results lead us to focus on the risk-neutral variance of index- and stock-level *simple* returns; the resulting volatility indices load equally on the prices of options of all strikes.

Other papers work within the CAPM and attempt to estimate betas more accurately by incorporating forward-looking information from options. French et al. (1983) estimate beta using a stock's historical return correlation with the market and option-implied volatilities for the stock and the market. Buss and Vilkov (2012) take a similar approach, but estimate correlation from a parametric model that links correlation under the risk-neutral and the objective measure. Chang et al. (2012) make assumptions under which expected correlation can be computed from the ratio of option-implied stock to market skewness; this implies, however, that a firm's implied beta will only be positive if its skewness has the same sign as market skewness, and it will typically not provide a meaningful CAPM beta for firms with positive skewness.

In a more closely related, and contemporaneous, paper, Kadan and Tang (2016) adapt an idea of Martin (2017) to derive a lower bound on expected stock returns. To understand the main differences between their approach and ours, recall that Martin starts from an identity that relates the equity premium to a risk-neutral variance term and a (real-world) covariance term; he exploits the identity by arguing that a *negative correlation condition* (NCC) holds for the market return, so that the covariance term is nonpositive in quantitatively reasonable models of financial markets. If so, the risk-neutral variance of the market provides a lower bound on the equity premium.[1] Kadan and Tang (2016) modify this approach to derive a lower bound for expected

[1]Schneider and Trojani (2018) propose a related approach to forecasting the equity premium based on (among other things) variants of the NCC.

stock returns based on a negative correlation condition for individual stocks. But it is trickier to make the argument that the NCC should hold at the individual stock level, so Kadan and Tang's approach only applies for a subset of S&P 500 stocks.

# I Theory

Our starting point is the gross return with maximal expected log return: call it $R_{g,t+1}$, so $\mathbb{E}_t \log R_{g,t+1} \geq \mathbb{E}_t \log R_{i,t+1}$ for any gross return $R_{i,t+1}$. This *growth-optimal return* has the special property, unique among returns, that $1/R_{g,t+1}$ is a stochastic discount factor. To see this, note that it is attained by choosing portfolio weights $\{g_n\}_{n=1}^N$ on the tradable returns (on stocks, stock options, index options, and the riskless asset) to solve

$$\max_{\{g_n\}_{n=1}^N} \mathbb{E} \log \sum_{n=1}^N g_n R_{n,t+1} \quad \text{such that} \sum_{n=1}^N g_n = 1.$$

The first-order conditions for this problem are that

$$\mathbb{E}\left(\frac{R_{i,t+1}}{\sum_{n=1}^N g_n R_{n,t+1}}\right) = \psi \quad \text{for all } i,$$

where $\psi$ is a Lagrange multiplier; we follow Roll (1973) and Long (1990) in assuming that these first-order conditions have an interior solution. Multiplying by $g_i$ and summing over $i$, we see that $\psi = 1$, and hence that the reciprocal of $R_{g,t+1} \equiv \sum_{n=1}^N g_n R_{n,t+1}$ is an SDF.

We write $\mathbb{E}_t^*$ for the associated risk-neutral expectation (more precisely, the time-

$t+1$-forward-neutral expectation) that is defined via[2]

$$\frac{1}{R_{f,t+1}}\,\mathbb{E}_t^*\,X_{t+1} = \mathbb{E}_t\left(\frac{X_{t+1}}{R_{g,t+1}}\right). \tag{1}$$

In these terms, the key property of the growth-optimal portfolio, which follows directly from (1), is that

$$\mathbb{E}_t\,\frac{R_{i,t+1}}{R_{f,t+1}} - 1 = \text{cov}_t^*\left(\frac{R_{i,t+1}}{R_{f,t+1}}, \frac{R_{g,t+1}}{R_{f,t+1}}\right) \quad \text{for all stocks } i. \tag{2}$$

Thus risk-neutral covariances with the growth-optimal return determine risk premia.

We start by projecting stock returns onto the growth-optimal portfolio under the risk-neutral measure. That is, for every stock $i$ we decompose

$$\frac{R_{i,t+1}}{R_{f,t+1}} = \alpha_{i,t}^* + \beta_{i,t}^*\frac{R_{g,t+1}}{R_{f,t+1}} + \varepsilon_{i,t+1} \tag{3}$$

where

$$\beta_{i,t}^* = \frac{\text{cov}_t^*\left(\frac{R_{i,t+1}}{R_{f,t+1}}, \frac{R_{g,t+1}}{R_{f,t+1}}\right)}{\text{var}_t^*\,\frac{R_{g,t+1}}{R_{f,t+1}}} \tag{4}$$

$$\mathbb{E}_t^*\,\varepsilon_{i,t+1} = 0 \tag{5}$$

$$\text{cov}_t^*(\varepsilon_{i,t+1}, R_{g,t+1}) = 0. \tag{6}$$

Equations (3)–(5) define $\varepsilon_{i,t+1}$, $\beta_{i,t}^*$ and $\alpha_{i,t}^*$; and equation (6) is a consequence of (3)–(5). Thus the only assumption embodied in (3)–(6) is that the appropriate risk-neutral

[2] A helpful perspective to keep in mind is that of an unconstrained log investor who is marginal in all markets, including option markets, but chooses to invest his wealth fully in the market. (See Martin (2017) and Kremens and Martin (2018) for a similar approach in the context of the stock market and of currencies, respectively.) Such an investor must perceive the market itself as growth-optimal, so that if $\mathbb{E}_t$ represents the expectations of the log investor, (1) and subsequent equations hold with $R_{g,t+1} = R_{m,t+1}$.

moments exist and are finite, and that $\text{var}_t^* \, R_{g,t+1}/R_{f,t+1}$ is non-zero. (This last assumption is needed for (4) to be well defined: it rules out the theoretically interesting, but empirically implausible, possibility that the risk-neutral and true probability measures coincide, as in that case the growth-optimal portfolio is riskless.)

It may be helpful to compare this approach to that of Hansen and Richard (1987), who also projected arbitrary returns onto a 'distinguished' return—in their case, the minimum-second-moment return, $R_{*,t+1}$, which is proportional to an SDF so has the key property that $\mathbb{E}_t \left(R_{*,t+1} R_{i,t+1}\right) = \mathbb{E}_t \left(R_{*,t+1}^2\right)$ for all tradable returns $R_{i,t+1}$, and hence that[3]

$$\mathbb{E}_t \, \frac{R_{i,t+1}}{R_{f,t+1}} - 1 = -\frac{R_{f,t+1}}{\mathbb{E}_t \, R_{*,t+1}} \, \text{cov}_t \left( \frac{R_{i,t+1}}{R_{f,t+1}}, \frac{R_{*,t+1}}{R_{f,t+1}} \right) \quad \text{for all stocks } i. \tag{2$'$}$$

This equation says that *true* covariances with a tradable payoff determine risk premia. It motivates the decomposition

$$\frac{R_{i,t+1}}{R_{f,t+1}} = \alpha_{i,t} + \beta_{i,t} \frac{R_{*,t+1}}{R_{f,t+1}} + u_{i,t+1} \tag{3$'$}$$

[3] Using one of the results of Hansen and Jagannathan (1991), this can also be written as

$$\mathbb{E}_t \, \frac{R_{i,t+1}}{R_{f,t+1}} - 1 = -\left(1 + S_t^2\right) \text{cov}_t \left( \frac{R_{i,t+1}}{R_{f,t+1}}, \frac{R_{*,t+1}}{R_{f,t+1}} \right),$$

where $S_t$ is the maximal conditional Sharpe ratio at time $t$, using the facts that (i) $R_{f,t+1}/\,\mathbb{E}_t \, R_{*,t+1} = \mathbb{E}_t \left(R_{*,t+1}^2\right) / \left(\mathbb{E}_t \, R_{*,t+1}\right)^2$ by the key property of $R_{*,t+1}$; and (ii) $\mathbb{E}_t \left(R_{*,t+1}^2\right) / \left(\mathbb{E}_t \, R_{*,t+1}\right)^2 = 1 + S_t^2$, which follows because $R_{*,t+1}$ lies (by definition) at the tangency point of an origin-centered circle to the lower edge of the minimum-variance frontier in a mean–standard-deviation diagram.

where

$$\beta_{i,t} = \frac{\operatorname{cov}_t\left(\frac{R_{i,t+1}}{R_{f,t+1}}, \frac{R_{*,t+1}}{R_{f,t+1}}\right)}{\operatorname{var}_t \frac{R_{*,t+1}}{R_{f,t+1}}} \tag{4$'$}$$

$$\mathbb{E}_t\, u_{i,t+1} = 0 \tag{5$'$}$$

$$\operatorname{cov}_t(u_{i,t+1}, R_{*,t+1}) = 0. \tag{6$'$}$$

We spell this out explicitly to emphasize the analogy between the two approaches. As before, equations (3′)–(5′) define $u_{i,t+1}$, $\beta_{i,t}$, and $\alpha_{i,t}$; and equation (6′) follows from them.[4] Equations (2′)–(6′) can be viewed as the theoretical foundation of the factor pricing literature. But as forward-looking real-world covariances are not directly observable, they must be estimated from time-series data. Such estimates will only approximate the true forward-looking covariances if the econometric environment is sufficiently stable (ergodic, stationary) in a statistical sense. Thus to make these equations empirically useful, one needs to make further assumptions about the stochastic properties of $u_{i,t+1}$ across assets and over time, about the stability of conditional betas over appropriate time horizons, and about the factors that must be included to provide a tolerable approximation to the true minimum-second-moment return.

Very broadly speaking, our approach may have a particular advantage at times when information arrives suddenly and in lumps, whether as the result of an earnings announcement, macroeconomic news, a terrorist attack, natural or unnatural disaster, or something else. Backward-looking historical covariances will adjust sluggishly at such times—which may be of particular interest to investors, decision-makers inside firms, and policymakers who must respond rapidly to changing conditions—whereas option prices, and hence our formulas, will react almost instantly.

[4]By taking risk-neutral expectations of (3) we see that $\alpha^*_{i,t} = 1 - \beta^*_{i,t}$. Similarly, by taking real-world expectations of (3′) and using (2′) together with the properties of $R_{*,t+1}$ mentioned in footnote 3, we find that $\alpha_{i,t} = 1 - \beta_{i,t}$.

That said, we will also need to make assumptions to make our approach implementable in practice. Equations (2) and (4) together imply that

$$\mathbb{E}_t \frac{R_{i,t+1}}{R_{f,t+1}} - 1 = \beta^*_{i,t} \operatorname{var}^*_t \frac{R_{g,t+1}}{R_{f,t+1}}. \tag{7}$$

We also have, from (3) and (6),

$$\operatorname{var}^*_t \frac{R_{i,t+1}}{R_{f,t+1}} = \beta^{*2}_{i,t} \operatorname{var}^*_t \frac{R_{g,t+1}}{R_{f,t+1}} + \operatorname{var}^*_t \varepsilon_{i,t+1}. \tag{8}$$

What we would *like* to measure is the right-hand side of (7). What we *can* measure is the left-hand side of (8) (as we will show in the next section). To connect the two, we make two assumptions.

First, we approximate the $\beta^{*2}_{i,t}$ term in (8) by linearizing $\beta^{*2}_{i,t} \approx 2\beta^*_{i,t} - k$, where $k$ is a constant. This approximation is reasonable if $\beta^*_{i,t}$ is not too far from 1 for a typical stock.[5] In Internet Appendix IA.A, we explicitly derive the residual that the approximation neglects, and argue that it is small for most stocks in our sample. We

[5]If $k = 1$ this linearization is the tangent to $\beta^{*2}_{i,t}$ at $\beta^*_{i,t} = 1$. Alternatively, if, say, the cross-section of betas has mean 1 and standard deviation $\sigma$ then one could set $k = 1 - \sigma^2$ in order to minimize the mean squared approximation error. As we will shortly see, the precise value of $k$ turns out not to be important. The choice to linearize around $\beta^*_{i,t} = 1$ is not critical, though we think it is natural: if the equal-weighted portfolio of stocks is approximately growth-optimal, then $\beta^*_{i,t}$ is close to 1 on average, while if the market is approximately growth-optimal, then $\beta^*_{i,t}$ is close to 1 on value-weighted average. More generally, we could linearize $\beta^{*2}_{i,t} \approx c\beta^*_{i,t} + d$ for appropriately chosen $c$ and $d$. For example, the tangent to $\beta^{*2}_{i,t}$ at $\beta^*_{i,t} = \beta_0$, some constant, corresponds to $c = 2\beta_0$ and $d = -\beta_0^2$; or one might want to choose $c$ and $d$ to achieve some other goal (e.g., to minimize the mean squared error for a given distribution of $\beta^*_{i,t}$). If one takes this approach, equations (14) and (15) are unaltered except that $1/2$ is replaced by $1/c$; in particular, the value of $d$ drops out. (See Internet Appendix IA.A.) Our empirical results suggest that it is reasonable to linearize around 1, that is, to set $c = 2$.

therefore replace (8) with

$$\mathrm{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} = (2\beta_{i,t}^* - k)\,\mathrm{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} + \mathrm{var}_t^*\,\varepsilon_{i,t+1}. \tag{9}$$

Using (7) and (9) to eliminate the dependence on $\beta_{i,t}^*$, we have

$$\mathbb{E}_t \frac{R_{i,t+1}}{R_{f,t+1}} - 1 = \frac{1}{2}\,\mathrm{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} + \frac{k}{2}\,\mathrm{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} - \frac{1}{2}\,\mathrm{var}_t^*\,\varepsilon_{i,t+1}. \tag{10}$$

To make further progress, let $w_{i,t}$ be the market capitalization weight of stock $i$ in the index. Value-weighting the above equation, we find that

$$\mathbb{E}_t \frac{R_{m,t+1}}{R_{f,t+1}} - 1 = \frac{1}{2}\sum_j w_{j,t}\,\mathrm{var}_t^* \frac{R_{j,t+1}}{R_{f,t+1}} + \frac{k}{2}\,\mathrm{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} - \frac{1}{2}\sum_j w_{j,t}\,\mathrm{var}_t^*\,\varepsilon_{j,t+1}. \tag{11}$$

Subtracting (11) from (10),

$$\mathbb{E}_t \frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \frac{1}{2}\left(\mathrm{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} - \sum_j w_{j,t}\,\mathrm{var}_t^* \frac{R_{j,t+1}}{R_{f,t+1}}\right) - \frac{1}{2}\left(\mathrm{var}_t^*\,\varepsilon_{i,t+1} - \sum_j w_{j,t}\,\mathrm{var}_t^*\,\varepsilon_{j,t+1}\right). \tag{12}$$

Our second assumption is that the final term on the right-hand side of (12), which is zero on value-weighted average, can be captured by a time-invariant stock fixed effect $\alpha_i$. This fixed-effects formulation, which is econometrically convenient, would follow immediately if, for example, the risk-neutral variances of residuals decompose separably, $\mathrm{var}_t^*\,\varepsilon_{i,t+1} = \phi_i + \psi_t$, and value weights are constant over time.

It will be convenient to define three different measures of risk-neutral variance:

$$\begin{aligned} \mathrm{SVIX}_t^2 &= \mathrm{var}_t^*\left(R_{m,t+1}/R_{f,t+1}\right) \\ \mathrm{SVIX}_{i,t}^2 &= \mathrm{var}_t^*\left(R_{i,t+1}/R_{f,t+1}\right) \\ \overline{\mathrm{SVIX}}_t^2 &= \sum_i w_{i,t}\,\mathrm{SVIX}_{i,t}^2\,. \end{aligned} \tag{13}$$

These measures can be computed directly from option prices, as we show in the next section. The $\text{SVIX}_t$ index was introduced by Martin (2017)—the name echoes the related VIX index—but the definitions of stock-level $\text{SVIX}_{i,t}$ and of $\overline{\text{SVIX}}_t$, which measures average stock volatility, are new to this paper. Introducing these definitions into (12) we arrive at our first, purely relative, prediction about the cross-section of expected returns in excess of the market (excess-of-market returns, for short):

$$\frac{\mathbb{E}_t\, R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right). \tag{14}$$

We test this prediction by running a panel regression of excess-of-market returns of individual stocks $i$ onto stock fixed effects and excess stock variance $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$.

In order to answer the question posed in the title of the paper, we must also take a view on the expected return on the market itself. To do so, we exploit a result of Martin (2017), who argues that the SVIX index can be used as a forecast of the equity premium: specifically, that $\mathbb{E}_t\, R_{m,t+1} - R_{f,t+1} = R_{f,t+1}\,\text{SVIX}_t^2$. Substituting this into equation (12), we have

$$\frac{\mathbb{E}_t\, R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \text{SVIX}_t^2 + \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right). \tag{15}$$

We test (15) by running a panel regression of realized excess returns on individual stocks $i$ onto stock fixed effects, risk-neutral variance $\text{SVIX}_t^2$, and excess stock variance $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$.

As noted above, the fixed effects in (14) and (15) should be zero on value-weighted average. We test this prediction in two ways: first, in a weaker form, that $\sum_i w_i \alpha_i = 0$ (where $w_i = \frac{1}{T_i}\sum_t w_{i,t}$ is the average value weight of stock $i$ and $T_i$ the number of time-series observations for stock $i$). We also consider, and test, the stronger assumption that $\alpha_i = 0$ for all $i$, which would hold if risk-neutral residual variance is constant across stocks, though not necessarily across time. In this form, we are imposing a tight relationship between a stock's risk-neutral variance and its risk-neutral beta: by (8), stocks with high variances must also have high risk-neutral betas. Making this assumption in (14), for example, we have[6]

$$\frac{\mathbb{E}_t\, R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \frac{1}{2}\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right). \tag{16}$$

Correspondingly, if we assume that the fixed effects are constant across $i$ in (15), we end up with a formula for the expected return on a stock that has no free parameters:

$$\frac{\mathbb{E}_t\, R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \mathrm{SVIX}_t^2 + \frac{1}{2}\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right). \tag{17}$$

In Section V, we exploit the fact that (16) and (17) require no parameter estimation—only observation of contemporaneous prices—to conduct an out-of-sample analysis, and show that the formulas outperform a range of plausible competitors.

Before we turn to the data, it is worth pausing to restate that we have made two key assumptions. First, we assumed that for stocks in our universe, risk-neutral betas $\beta_{i,t}^*$ are sufficiently close to 1 to justify our linearization (9). Second, we assumed that the risk-neutral variances of residuals—the second term on the right-hand side of equation (12)—can be captured by a fixed-effect formulation.

We emphasize that these assumptions are not appropriate for all assets. Suppose, for example, that asset $j$ is genuinely idiosyncratic—and hence has zero risk premium—but has extremely high, and perhaps wildly time-varying, variance $\mathrm{SVIX}_{j,t}^2$. Then equation (15) cannot possibly hold for asset $j$. Our assumptions reflect a judgment that

[6]At first sight, (16) appears to lead to an inconsistency: if we "set $i = m$," it seems to imply that $\mathrm{SVIX}_t^2 = \overline{\mathrm{SVIX}}_t^2$, which is not true (as we discuss in Section II below). The right way to "set $i = m$" here is to replace $\mathrm{SVIX}_{i,t}^2$ not with $\mathrm{SVIX}_t^2$ but with its value-weighted sum, $\overline{\mathrm{SVIX}}_t^2$. By contrast, it *is* legitimate to "set $i = m$" in linear factor models in which risk premia are expressed in terms of covariances of returns with factors.

such cases are not typical within the universe of stocks that we study (namely, members of the S&P 100 or S&P 500 indices).[7] This is an empirically testable judgment, and we put it to the test below.

# II Three measures of risk-neutral variance

The risk-neutral variance terms that appear in our formulas can be calculated from option prices using the approach of Breeden and Litzenberger (1978). Our measure of market risk-neutral variance, $\text{SVIX}_t^2$, is determined by the prices of index options:

$$\text{SVIX}_t^2 = \frac{2}{R_{f,t+1} S_{m,t}^2} \left[ \int_0^{F_{m,t}} \text{put}_{m,t}(K)\, dK + \int_{F_{m,t}}^{\infty} \text{call}_{m,t}(K)\, dK \right],$$

where we write $S_{m,t}$ and $F_{m,t}$ for the spot and forward (to time $t+1$) prices of the market, and $\text{put}_{m,t}(K)$ and $\text{call}_{m,t}(K)$ for the time-$t$ prices of European puts and calls on the market, expiring at time $t+1$ with strike $K$. (The length of the period from time $t$ to time $t+1$ varies according to the horizon of interest. Thus we will be forecasting 1-month returns using the prices of 1-month options, 3-month returns using the prices of 3-month options, and so on. Throughout the paper, we annualize returns and volatility indices by scaling by horizon length measured in years.) The SVIX index (squared) therefore represents the price of a portfolio of out-of-the-money puts and calls equally weighted by strike. This definition is closely related to that of the VIX index, the key difference being that VIX weights option prices in inverse-square proportion to their

[7] There is an analogy with an earlier debate on the testability of the arbitrage pricing theory (APT). Shanken (1982) showed, under the premise of the APT that asset returns are generated by a linear factor model, that it is possible to construct portfolios that violate the APT prediction that assets' expected returns are linear in the factor loadings. Dybvig and Ross (1985) endorsed the mathematical content of Shanken's results but disputed their interpretation, arguing that the APT can be applied to certain types of asset (for example, stocks), but not to arbitrary portfolios of assets.

strike.

The corresponding index at the individual stock level is defined in terms of individual stock option prices:

$$\mathrm{SVIX}_{i,t}^2 = \frac{2}{R_{f,t+1} S_{i,t}^2} \left[ \int_0^{F_{i,t}} \mathrm{put}_{i,t}(K)\, dK + \int_{F_{i,t}}^{\infty} \mathrm{call}_{i,t}(K)\, dK \right],$$

where the subscripts $i$ indicate that the reference asset is stock $i$ rather than the market.

Finally, using $\mathrm{SVIX}_{i,t}^2$ for all firms available at time $t$, we calculate the risk-neutral average stock variance index as $\overline{\mathrm{SVIX}}_t^2 = \sum_i w_{i,t}\, \mathrm{SVIX}_{i,t}^2$.

We pause to note two facts about these volatility indices. First, average stock volatility must exceed market volatility, that is, $\overline{\mathrm{SVIX}}_t > \mathrm{SVIX}_t$. Given the definitions above, this is an illustration of the slogan that a portfolio of options is more valuable than an option on a portfolio. More formally, it is a consequence of the fact that $\sum_i w_{i,t} \operatorname{var}_t^* R_{i,t+1} > \operatorname{var}_t^* \sum_i w_{i,t} R_{i,t+1}$ or, equivalently, that $\mathbb{E}_t^* \sum_i w_{i,t} R_{i,t+1}^2 > \mathbb{E}_t^* \left[ \left( \sum_i w_{i,t} R_{i,t+1} \right)^2 \right]$, which follows from Jensen's inequality.

Second, risk-neutral variance is, as a rule of thumb, increasing in the time-to-maturity of the underlying options (equivalently, in the length of the period from $t$ to $t+1$). Formally, assume that the underlying asset does not pay dividends and use put-call parity to rewrite

$$\mathrm{SVIX}_{i,t}^2 = \operatorname{var}_t^* \left( R_{i,t+1} / R_{f,t+1} \right) = \frac{2}{R_{f,t+1} S_t^2} \int_0^{\infty} \underbrace{\mathrm{call}_{i,t}(K)}_{\uparrow \text{ in maturity}} \, dK - 1.$$

As is well-known, if the underlying asset does not pay dividends—a tolerable approximation to reality for the stocks and horizons we consider—a European call and an American call have the same value, and hence call prices are increasing in time-to-maturity. Assuming this is not offset by the countervailing effect of increased interest rates $R_{f,t+1}$ over longer horizons, $\mathrm{SVIX}_{i,t}$ should be expected to be monotonic in horizon length. We have found nonmonotonicity to be a useful flag for detecting a small number of extreme outliers in our data, as we discuss further below.

In our empirical work, we start with daily data from OptionMetrics for equity index options on the S&P 100 and on the S&P 500, providing us with time series of implied volatility surfaces from January 1996 to October 2014. We obtain daily equity index price and return data from CRSP and information on the index constituents from Compustat. We also obtain data on the firms' number of shares outstanding and their book equity to compute their market capitalizations and book-to-market ratios. Using the lists of index constituents, we search the OptionMetrics database for all firms that were included in the S&P 100 or S&P 500 during our sample period, and obtain volatility surface data for these individual firms, where available.

We face the issue that S&P 100 index options and individual stock options are American-style rather than European-style. The distinction is likely to be relatively minor at the horizons we consider, as the options whose prices we require are out-of-the-money; in any case, the volatility surfaces reported by OptionMetrics deal with this issue via binomial tree calculations that aim to account for early exercise premia. We take the resulting volatility surfaces as our measures of European implied volatility, following Carr and Wu (2009) among others.

We compute the three measures of risk-neutral variance given in equation (13) for horizons (i.e., option maturities) of one, three, six, 12, and 24 months. We then filter out a small number of extreme outliers in our data that violate the monotonicity property in $\mathrm{SVIX}_{i,t}$ across horizons described above.[8] As summarized in Panel A of Table I, we end up with more than two million firm-day observations for each of the five horizons, covering a total of 869 firms over our sample period from January 1996 to

[8] In the daily data, we end up with 2,106,711 firm-day observations after removing 9,648 observations based on nonmonotonicity. In our monthly data for S&P 500 firms, we end up with 102,198 firm-month observations after removing 401 observations based on nonmonotonicity.

October 2014. Across horizons, we have data on 451 firms on average per day, meaning that we cover slightly more than 90% of the firms included in the S&P 500 index. From the daily data, we also compile data subsets at a monthly frequency for firms included in the S&P 100 (Panel B) and the S&P 500 (Panel C).

[Table I about here]

Figure 2 plots the time series of risk-neutral market variance ($\text{SVIX}_t^2$) and average risk-neutral stock variance ($\overline{\text{SVIX}}_t^2$) for the S&P 500; for the S&P 100 we present these results in Figure IA.1 in the Internet Appendix. The dynamics of $\text{SVIX}_t^2$ and $\overline{\text{SVIX}}_t^2$ are similar for both indices and across horizons. All the time series spike dramatically during the financial crisis of 2008. While the average levels of the (annualized) SVIX measures are similar across horizons, their volatility is higher at short than at long horizons. Similarly, the peaks in $\text{SVIX}_t^2$ and $\overline{\text{SVIX}}_t^2$ during the crisis and other periods of heightened volatility are most pronounced in short-maturity options.[9]

[Figure 2 about here]

Figures 3 and 4 show the relationships between risk-neutral stock variances and various firm characteristics, on average and in the time series. To construct the figures, we sort S&P 500 stocks into portfolios based on their CAPM beta, size, book-to-market ratio, or momentum, and compute the (equally-weighted) average $\text{SVIX}_{i,t}^2$ for

[9] In Appendix A, we show that the ratio of market variance to average stock variance, $\text{SVIX}_t^2/\overline{\text{SVIX}}_t^2$, can be interpreted as a measure of average risk-neutral correlation between stocks. Figure IA.2 in the Internet Appendix plots the time-series of $\text{SVIX}_t^2/\overline{\text{SVIX}}_t^2$ at one-month and one-year horizons for the S&P 100 and S&P 500. Average stock variance was unusually high relative to market variance over the period from 2000 to 2002, indicating that the correlation between stocks was unusually low at that time.

each portfolio, at the 12-month horizon.[10] $\text{SVIX}_{i,t}^2$ is positively related to CAPM beta and inversely related to firm size, on average and throughout our sample period. In contrast, there is a U-shaped relationship between $\text{SVIX}_{i,t}^2$ and book-to-market that reflects an interesting time-series relationship between the two. Growth and value stocks had similar levels of volatility during periods of low index volatility, but value stocks were more volatile than growth stocks during the recent financial crisis and less volatile from 2000 to 2002. We also find a non-monotonic relationship between momentum and $\text{SVIX}_{i,t}^2$. Interestingly, loser stocks exhibited particularly high $\text{SVIX}_{i,t}^2$ from late 2008 until the momentum crash in early 2009.[11]

[Figures 3 and 4 about here]

# III Testing the model

In this section, we use $\text{SVIX}_t^2$, $\text{SVIX}_{i,t}^2$ and $\overline{\text{SVIX}}_t^2$ to test the predictions of our model using full sample information. But before turning to formal tests, we conduct a preliminary exploratory exercise. Specifically, we ask whether, on time-series average, stocks' excess-of-market returns line up with their excess stock variances in the manner predicted by equation (16). To do so, we temporarily restrict to firms that were

[10] We measure momentum by the return over the past twelve months, skipping the most recent month's return (see, e.g., Jegadeesh and Titman, 1993). Our estimation of conditional CAPM betas based on past returns follows Frazzini and Pedersen (2014): we estimate volatilities by one-year rolling standard deviations of daily returns and correlations from five-year rolling windows of overlapping three-day returns.

[11] We find similar results at the 1-month horizon: see Figures IA.3 and IA.4 in the Internet Appendix. Figure IA.5 plots (equally-weighted) average $\text{SVIX}_i$ at the 12-month horizon for portfolios double-sorted on size and value.

included in the S&P 500 throughout our sample period. For each such firm, we compute time-averaged excess-of-market returns and risk-neutral excess stock variance, $\text{SVIX}_i^2 - \overline{\text{SVIX}}^2$. Equation (16) implies that for each percentage point difference in $\text{SVIX}_i^2 - \overline{\text{SVIX}}^2$, we should see half that percentage point difference in excess returns.

The results of this exercise are shown in Figure 5, which is analogous to the security market line of the CAPM. The return horizon matches the maturity of the options used to compute the SVIX-indices. We regress average excess-of-market returns on $0.5 \times (\text{SVIX}_i^2 - \overline{\text{SVIX}}^2)$. Our theory predicts zero intercept and a slope coefficient of one; we find intercepts close to zero and slope coefficients of 0.60, 0.79, 1.00, 1.10, and 1.01 at forecasting horizons of one, three, six, 12, and 24 months, with $R^2$ ranging from 0.09 to 0.18. Using the same subset of firms, the figures also show decile portfolios sorted by $\text{SVIX}_{i,t}$ (indicated by diamonds) and $3 \times 3$ portfolios sorted by size and book-to-market (indicated by triangles).

[Figure 5 about here]

We repeat this exercise for portfolios sorted on firms' risk-neutral variance $\text{SVIX}_{i,t}$, using all available firms (lifting the requirement of full sample period coverage). Figure 6 shows that average portfolio returns in excess of the market are broadly increasing in portfolios' average volatility relative to aggregate stock volatility, and that $\text{SVIX}_i^2 - \overline{\text{SVIX}}^2$ captures a sizeable fraction of the cross-sectional variation in returns.

[Figure 6 about here]

To test the model formally, we start by estimating the pooled panel regression

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left( \text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}. \tag{18}$$

Based on the formula (16), we would ideally hope to find that $\alpha = 0$ and $\gamma = 1/2$. At a given point in time $t$, our sample includes all firms that are time-$t$ constituents of the index. We compute $R_{m,t+1}$ as the return on the value-weighted portfolio of all index constituent firms included in our sample at time $t$.

We run the regression using monthly data for the S&P 100 and S&P 500 indices, at return horizons (and hence also option maturities) of one, three, six, 12, and 24 months. Throughout the paper, we calculate standard errors (reported in parentheses) and $p$-values using a block bootstrap procedure that accounts for time-series and cross-sectional dependencies in the data. Appendix B provides further detail about the bootstrap procedure and presents Monte Carlo simulation evidence on the reliability of the procedure in finite samples.

The regression results are shown in Table II. The headline result is that when we conduct a Wald test of the joint hypothesis that $\alpha = 0$ and $\gamma = 0.5$, we do not reject our model at any horizon, with $p$-values ranging from 0.44 to 0.84 for S&P 100 firms (Panel A) and from 0.49 to 0.63 for S&P 500 firms (Panel B). By contrast, we can reject the hypothesis that $\gamma = 0$ with some confidence in most cases (with $p$-values of 0.079, 0.020, 0.015, and 0.007 for S&P 100 firms at three-, six-, 12-, and 24-month horizons; and $p$-values of 0.072, 0.068, and 0.077 for S&P 500 firms at six-, 12-, and 24-month horizons).

[Table II about here]

We test the prediction (14) by running a panel regression with firm fixed effects

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left( \mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}. \tag{19}$$

and testing the hypothesis that $\gamma = 1/2$ and $\sum_i w_i \alpha_i = 0$.

The results are in Table III. Now $\gamma$ is significantly different from zero even at the shorter horizons—and, in most cases, not significantly different from 0.5. We also find, however, that the value-weighted sum of firm fixed effects is statistically different from zero, though we note that the estimates are fairly small in economic terms (and, consistent with the pooled panel results, we will see below that the model performs well when we drop firm fixed effects entirely, as we do in our out-of-sample analysis).[12]

[Table III about here]

Turning to excess returns (as opposed to excess-of-market returns), we test the prediction of equation (17) by running the regression

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta \, \mathrm{SVIX}_t^2 + \gamma \left( \mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}, \tag{20}$$

and the prediction of equation (15) by running a regression with stock fixed effects,

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta \, \mathrm{SVIX}_t^2 + \gamma \left( \mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}. \tag{21}$$

Our model predicts that $\alpha = 0$, $\beta = 1$ and $\gamma = 1/2$ in equation (20), and that $\beta = 1$, $\gamma = 1/2$ and $\sum_i w_i \alpha_i = 0$ in equation (21).

The pooled panel regression results are shown in Table IV. For S&P 100 firms (Panel A), the headline result is again that we do not reject our model at any horizon: $p$-values of the joint hypothesis test that $\alpha = 0$, $\beta = 1$, and $\gamma = 0.5$ range from 0.55 to 0.69. By contrast, we can reject the joint hypothesis that $\beta = 0$ and $\gamma = 0$ with moderate confidence for six-, 12-, and 24-month returns (with $p$-values of 0.064, 0.045, and 0.012, respectively). Notice that as the estimated coefficient $\gamma$ exploits

[12] Moreover, the fixed effects are not statistically significant if we use portfolios sorted on $\mathrm{SVIX}_{i,t}^2$ as test assets: see Tables IA.1, IA.2, IA.3 and IA.4 in the Internet Appendix.

cross-sectional information, it is estimated more precisely than is $\beta$: our results are therefore consistently stronger, in a statistical sense, than those of Martin (2017).

[Table IV about here]

The corresponding results for S&P 500 firms are reported in Panel B. We do not reject the joint hypothesis that $\alpha = 0$, $\beta = 1$, and $\gamma = 0.5$ at horizons of one, three, six, and 12 months (with $p$-values between 0.169 and 0.267). We do however reject the model at the 24-month horizon: the estimated $\beta$ is even higher than the theory predicts. We can cautiously reject the joint null that $\beta = 0$ and $\gamma = 0$ at horizons of six, 12, and 24 months (with $p$-values of 0.071, 0.092, and 0.036).

The coefficient estimates remain fairly stable, and we draw similar conclusions, when we allow for firm fixed effects in Table V. For S&P 100 firms (Panel A), a Wald test of the joint null hypothesis that $\sum_i w_i \alpha_i = 0$, $\beta = 1$, and $\gamma = 0.5$ does not reject the model (with $p$-values between 0.11 and 0.36), and we can strongly reject the joint null that $\beta = \gamma = 0$ for horizons of six, 12, and 24 months (with $p$-values below 0.01). The $\beta$ estimates are little changed compared to the pooled panel regressions, while the $\gamma$ estimates are somewhat higher. The statistical results are more clear cut for S&P 500 firms when we include firm fixed effects (Panel B). We do not reject the joint null hypothesis implied by our model at horizons up to and including 12 months, and can strongly reject the null that $\beta = \gamma = 0$ at horizons of six, twelve, and 24 months (with $p$-values of 0.019, 0.008, and 0.002).

[Table V about here]

We have also run these regressions on subsamples of the data. Figure 7 plots the estimated coefficients $\beta$ and $\gamma$ using successive yearly and three-yearly subsamples over our sample period, and shows that our results are not driven by any one subperiod. It also helps to emphasize the point that the cross-sectional coefficient $\gamma$, which exploits the information in the entire cross-section of stocks, is estimated more precisely than the 'market' coefficient $\beta$.

[Figure 7 about here]

# IV Risk premia and stock characteristics

The results of the previous section show that the model performs well in forecasting stock returns. Nonetheless, we would like to know whether there is return-relevant information in other firm characteristics—notably CAPM beta, (log) size, book-to-market, and past returns—that is not captured by our predictor variables (see, e.g., Fama and French, 1993; Carhart, 1997; Lewellen, 2015).

As a preliminary check, Figure 8 shows that average realized excess returns line up fairly well with our cross-sectional excess return predictor, $0.5(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2)$, for characteristic-sorted portfolios. The return predictor for a portfolio is calculated by averaging over its constituent stocks. Unless otherwise noted, we work with S&P 500 stocks and at an annual horizon throughout this section.

[Figure 8 about here]

We test formally whether our framework is able to explain differences in risk premia associated with the various characteristics in two ways: we run regressions of individual stock excess returns onto our predictor variables and the characteristics; and we re-run the regressions of the previous section using portfolios double-sorted on characteristics and on $\text{SVIX}_{i,t}^2$ as test assets.

Consider, first, the regressions on characteristics and our predictors. Table VI reports the results for returns in excess of the market. The first column shows the estimated coefficients in a regression of realized excess-of-market returns onto characteristics. We do not find a statistically significant relationship between the characteristics and realized returns in excess of the market (consistent with the findings of Nagel (2005), who documents limited cross-sectional variation in returns on S&P 500 stocks sorted on book-to-market, for example), and we cannot reject the joint hypothesis that the coefficients on all characteristics are zero. In the second column, we add our predictor $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$. We find that its estimate *is* statistically significant individually, and we do not reject the joint hypothesis that it enters with a coefficient of 0.5 while the coefficients on all characteristics are zero; adjusted $R^2$ increases from 1.0% to 4.0% when we add our predictor variable.

[Table VI about here]

Table VII reports the corresponding results for excess returns. In the absence of our predictor variables, we find that size and book-to-market characteristics are individually statistically significant, and we can reject the joint hypothesis that the coefficients on all characteristics are zero. But once we add $\text{SVIX}_t^2$ and $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$, we do not reject the joint hypothesis that the coefficients on the characteristics are all zero while those on the volatility measures are equal to their theoretical values of 1 and 0.5. Moreover, adjusted $R^2$ increases from 1.9% to 5.3% when our predictor variables are added.

[Table VII about here]

The next columns of Tables VI and VII address the relationships between expected excess returns and characteristics, with expected excess returns calculated in two ways:

using the coefficients estimated in regressions (18) or (20), and using the theory-implied coefficients given in equations (17) or (16). (We do so for interest: our theory makes no predictions about these regressions.) The characteristics capture a sizeable fraction of the variation in theory-implied expected returns in excess of the market ($R^2 = 37.8\%$) and theory-implied expected excess returns ($R^2 = 30.5\%$). In both cases there is a significantly positive relationship between expected returns and beta and a significantly negative relationship between expected returns and size, but the other characteristics do not exhibit a statistically significant relationship to expected returns. When we calculate expected returns using the estimated coefficients from (18) and (20) rather than the theoretical values, the point estimates of the regression coefficients for the characteristics are similar but are estimated less precisely, so are not significantly different from zero.

The last two columns of the tables show that there is little evidence of a systematic relationship between *unexpected* (that is, realized minus expected) returns and characteristics.

For our second test, we sort stocks into quintile portfolios based on their beta, size, book/market, or momentum, and then within each characteristic portfolio we sort firms into quintile portfolios based on $\mathrm{SVIX}_{i,t}^2$. We run regressions (18) and (19) using the $5 \times 5$ portfolios as test assets, and calculate portfolio-level expected returns in excess of the market as the equal-weighted average of the constituent stocks' expected returns in excess of the market. The results are shown in Table VIII. Our model is never rejected. In the specification that is *least* favorable to our theory—the fixed-effects regression with size-sorted portfolios—we find a $p$-value of 0.07 for the joint hypothesis test; all other $p$-values are above 0.2, and the estimates of $\gamma$ are close to 0.5. The corresponding results for excess returns are in the Internet Appendix, Table IA.5. We find similar results when we conduct the double sort in the opposite direction, first sorting on $\mathrm{SVIX}_{i,t}^2$ and then on the other characteristic: see Tables IA.6 and IA.7.

[Table VIII about here]

# V Out-of-sample analysis

The formulas (16) and (17) have no free parameters, so it is reasonable to hope that they may be well suited to out-of-sample forecasting. In this section, we show that they are. This fact is particularly striking given the substantial variability of the forecasts both in the time series and in the cross-section. The former point is consistent with Martin (2017); the latter is new to this paper. It is illustrated in Figure 9, which plots the evolution of the cross-sectional differences in one-year expected excess returns generated by our model.

We compare the performance of the formulas (16) and (17) to various competitor forecasting benchmarks using an out-of-sample R-squared measure along the lines of Goyal and Welch (2008). We define

$$R^2_{OS} = 1 - \frac{\sum_i \sum_t FE^2_{M,it}}{\sum_i \sum_t FE^2_{B,it}},$$

where $FE_{M,it}$ and $FE_{B,it}$ denote the forecast errors for stock $i$ at time $t$ based on our model and on a benchmark prediction, respectively. Our model outperforms a given benchmark if the corresponding $R^2_{OS}$ is positive.

What are the natural competitor benchmarks? One possibility is to give up on trying to make differential predictions across stocks, and simply to use a forecast of the expected return on the market as a forecast for each individual stock. We consider various ways of doing so. We use the market's historical average excess return as an equity premium forecast, following Goyal and Welch (2008) and Campbell and Thompson (2008), and we use the S&P 500 ($\overline{\text{S\&P500}}_t$) and the CRSP value-weighted index ($\overline{\text{CRSP}}_t$) as proxies for the market. We also use the risk-neutral variance of the market, $\text{SVIX}_t^2$, to proxy for the equity premium, as suggested by Martin (2017). Lastly, we consider a constant excess return forecast of 6% *p.a.*, corresponding to long-run estimates of the equity premium used in previous research.

More ambitious competitor models would seek to provide differential forecasts of individual firm stock returns, as we do. Again, we consider several alternatives. One natural thought is to use historical average of firms' stock excess returns ($\overline{\text{RX}}_{i,t}$). Another is to estimate firms' conditional CAPM betas from historical return data and combine the beta estimates with the aforementioned market premium predictions. We also consider firm-level risk-neutral variance ($\text{SVIX}_{i,t}^2$) as a competitor forecasting variable, motivated by Kadan and Tang (2016), who show that under certain conditions $\text{SVIX}_{i,t}^2$ provides a lower bound on stock $i$'s risk premium.

The results for expected excess returns are shown in Panel A of Table IX. Our formula (17) outperforms all the above competitors at the 3-, 6-, 12-, and 24-month horizons, and its relative performance (as measured by $R_{OS}^2$) almost invariably increases with forecast horizon, at least up to the one-year horizon.[13] At the one-year horizon, $R_{OS}^2$ ranges from 1.68% to 3.82% depending on the competitor benchmark, with the exception of the historical average stock return $\overline{\text{RX}}_{i,t}$, which it outperforms by a wide margin, with an $R_{OS}^2$ above 27%. This dramatic outperformance reflects an advantage of our approach: it does not rely on historical data. This is particularly important for stocks with short return histories that may not be representative of future returns. For example, at the peak of the dotcom bubble, young tech firms had extremely high historical average returns over their short histories. In such cases, employing the historical average as a predictor may lead to large forecast errors for subsequent returns.

[Table IX about here]

[13]The $R_{OS}^2$ results are based on expected excess returns defined as $\mathbb{E}_t\, R_{i,t+1} - R_{f,t+1}$, i.e. we multiply the left and the right side of equations (16) and (17) by $R_{f,t+1}$.

The results for expected returns in excess of the market are shown in Panel B and are, if anything, even stronger. We adjust the conditional CAPM predictions appropriately (by multiplying the equity premium by beta minus one); and we add a 'random walk' forecast of zero. In doing so, we focus on the cross-sectional dimension of firms' equity returns, net of (noisy) market return forecasts. The formula (16) outperforms all the competitors at every horizon, and the outperformance increases with forecast horizon up to one year. At the one-year horizon, $R^2_{OS}$ is around 3% relative to each of the benchmarks.

More surprisingly, our model is competitive with—and at horizons of six months or more, typically outperforms—a range of predictions based on *in-sample* information. The first three lines of Panel A of Table X compare the performance of the excess-return formula (17) to the in-sample average equity premium and the in-sample average excess return on a stock (each of which makes the same forecast for every stock's return); and to estimated beta multiplied by the in-sample equity premium (which does differentiate across stocks). In each case, $R^2_{OS}$ is increasing with forecast horizon up to one year and is positive at horizons of three, six, 12, and 24 months.

[Table X about here]

The next five lines compare the model forecasts to in-sample predictions based on firm characteristics: more precisely, to the fitted values from pooled univariate regressions of excess returns onto conditional betas, onto (log) size, onto book-to-market ratios, or onto the stock's past return. The formula outperforms each of the characteristics at horizons of six and twelve months, and is competitive with the model that knows the in-sample multivariate relationship between expected returns and all four characteristics.

Remarkably, the model performs even better for returns in excess of the market. The results are shown in Panel B of Table X. The formula (16) outperforms the univariate characteristics-based competitors at all horizons; it even beats the in-sample *multivariate* model at horizons from 1 month to 1 year.

# VI Conclusions

This paper has presented new theoretical and empirical results on the cross-section of expected stock returns. We would like to think that our approach to this classic topic is idiosyncratic in more than one sense.

In sharp contrast with the factor model approach to the cross-section—which has both the advantage and the disadvantage of imposing almost no structure, and therefore says *ex ante* little about the anticipated signs, and nothing about the sizes, of coefficient estimates—we make specific predictions both for the signs and sizes of coefficients, and test these numerical predictions in the data. In this dimension, a better comparison is with the CAPM, which makes the quantitative prediction that the slope of the security market line should equal the market risk premium. But (setting aside the fact that it makes no prediction for the market risk premium) even the CAPM requires betas to be estimated if this prediction is to be tested. At times when markets are turbulent, it is far from clear that historical betas provide robust measures of the idealized forward-looking betas called for by the theory; and if the goal is to forecast returns over, say, a one-year horizon, one cannot respond to this critique by taking refuge in the last five minutes of high-frequency data. In contrast, our predictive variables, which are based on option prices, are observable in real time and inherently forward-looking.

Our approach performs well in and out of sample, particularly over six-, 12-, and 24-month horizons. The model does a good job of accounting for realized returns on portfolios sorted on characteristics (beta, size, book-to-market, and past returns) known to be problematic for previous generations of asset-pricing models. When we run stock-level panel regressions of realized returns onto characteristics and our volatility predictor variables, our volatility variables drive out the characteristics and are themselves statistically significant; and we do not reject the hypothesis that the associated coefficients take the values predicted by the theory.

As the coefficients in the formula for the expected return on a stock are theoretically motivated, we need only observe the market prices of certain options to implement the formula: no estimation is required. Our approach therefore avoids the critique of Goyal and Welch (2008), and we show that it outperforms a range of competitor predictors out of sample—even including competitors with knowledge of the in-sample relationship between expected returns and characteristics.

Our real-time measure of the expected return on a stock has many potential applications in asset pricing and corporate finance: for example, we are currently exploring the reaction of expected stock returns to macroeconomic and firm-specific news announcements. As expected (or "required") rates of return are a key determinant of investment decisions, our results also have important implications for macroeconomics more generally—notably because our approach generates considerably more variation in expected returns, both over time and across stocks, than does, say, the CAPM. This points toward a quantitatively and qualitatively new view of risk premia.

# Appendix

## A A measure of correlation

This section shows that the ratio $\mathrm{SVIX}_t^2/\overline{\mathrm{SVIX}}_t^2$ can be interpreted as an approximate measure of average risk-neutral correlation between stocks. Note first that

$$\mathrm{var}_t^* R_{m,t+1} - \sum_i w_{i,t}^2 \,\mathrm{var}_t^* R_{i,t+1} = \sum_{i\neq j} w_{i,t} w_{j,t} \,\mathrm{corr}_t^*(R_{i,t+1}, R_{j,t+1}) \sqrt{\mathrm{var}_t^* R_{i,t+1} \,\mathrm{var}_t^* R_{j,t+1}},$$

so we can define a measure of average correlation, $\rho_t$, as

$$\rho_t = \frac{\mathrm{var}_t^* R_{m,t+1} - \sum_i w_{i,t}^2 \,\mathrm{var}_t^* R_{i,t+1}}{\sum_{i\neq j} w_{i,t} w_{j,t} \sqrt{\mathrm{var}_t^* R_{i,t+1} \,\mathrm{var}_t^* R_{j,t+1}}}.$$

Now, we have

$$\rho_t \approx \frac{\mathrm{var}_t^* R_{m,t+1}}{\sum_i w_{i,t}^2 \,\mathrm{var}_t^* R_{i,t+1} + \sum_{i\neq j} w_{i,t} w_{j,t} \sqrt{\mathrm{var}_t^* R_{i,t+1} \,\mathrm{var}_t^* R_{j,t+1}}} = \frac{\mathrm{var}_t^* R_{m,t+1}}{\left(\sum_i w_{i,t} \sqrt{\mathrm{var}_t^* R_{i,t+1}}\right)^2}.$$

This last expression features the square of average stock *volatility*, rather than average stock variance, in the denominator, but we can approximate $\left(\sum_i w_{i,t} \sqrt{\mathrm{var}_t^* R_{i,t+1}}\right)^2 \approx \sum_i w_{i,t} \,\mathrm{var}_t^* R_{i,t+1}$. (The approximation neglects a Jensen's inequality term: the left-hand side is strictly smaller than the right-hand side.) This leads us to the correlation measure

$$\rho_t \approx \frac{\mathrm{var}_t^* R_{m,t+1}}{\sum_i w_{i,t} \,\mathrm{var}_t^* R_{i,t+1}} = \frac{\mathrm{SVIX}_t^2}{\overline{\mathrm{SVIX}}_t^2}. \tag{A.1}$$

## B Bootstrap procedure

Our empirical analysis uses a large set of panel data, in which residuals may be correlated across firms and across time. Petersen (2009) provides an extensive discussion of how such cross-sectional and time-series dependencies in panel data may bias standard errors in OLS regressions and suggests using two-way clustered standard errors. In a further analysis, he finds that standard errors obtained from a bootstrap procedure based on firm clusters are identical to the two-way clustered standard errors in his panel data. We choose to work with bootstrap standard errors because this is the more conservative approach in our setup for two reasons. First, our monthly data generates overlapping observations at return horizons exceeding one month. Second, our data is characterized by high but less than perfect coverage of the cross-section of index constituent firms, due to limited availability of option data.

To alleviate biases in standard errors that arise from applying asymptotic theory to finite samples, we use a non-parametric bootstrap procedure based on resampling. More specifically, because our data is characterized by time-series dependence, we use an overlapping block resampling scheme (originally proposed by Kuensch, 1989) to handle serial correlation and heteroskedasticity; in that block bootstrap procedure, we also take cross-sectional dependencies into account. Using a large number of bootstrap samples, we estimate the bootstrap covariance matrix and estimate Wald statistics, as we describe in more detail in Section B.1 below. In Section B.2, we provide simulation evidence on the finite-sample properties of the block bootstrap procedure; the detailed results are in the Internet Appendix.

## B.1 Implementation of the block bootstrap procedure

We first describe the details of the block bootstrap procedure that we apply for pooled panel regressions of returns in excess of the market. Second, we discuss the adjustments to the procedure in regressions of excess returns (instead of excess-of-market returns) and, third, the adjustments to the procedure when using firm fixed effects regressions (instead of pooled panel regressions). Fourth, we discuss the adjustments for portfolio regressions (compared to regressions on the individual firm level).

**Pooled panel regressions of returns in excess of the market.** Use a block bootstrap approach to generate $b = 1, ..., B$ bootstrap samples by resampling from the actual panel data, as suggested by Kuensch (1989). From the actual data, we need dates, firm identifiers, firms' stock returns in excess of the risk-free rate, firms' risk-neutral variances ($\text{SVIX}^2_{i,t}$), and firms' market capitalizations.

1. We generate $B = 1,000$ bootstrap samples of panel data, where the number of time periods in each sample matches the number of time periods in the actual data. More specifically, we generate a bootstrap sample $b$ as follows:

   (a) Start the resampling procedure by randomly drawing a block of time-length $T$, i.e. corresponding to the return prediction horizon and the maturity of the options used to compute the $SVIX$-quantities.[14] From the block drawn, randomly select a subset of firms.[15]

[14]In time-series bootstraps, it is possible to implement automated procedures that determine the block-length based on the properties of the time- series (e.g., Politis and White, 2004; Patton et al., 2009). These procedures are not implementable in our panel data setup as different firm time-series may suggest different block lengths but we need to choose a single block length across all firms to account for the cross-sectional dependencies in the data across time. For instance, for $T = 12$ months we find that applying such a procedure for different firm time-series of $\text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t$ would suggest block lengths between approximately 8 and 24 months. We repeat our bootstrap procedure with these block lengths 8 and 24 months, instead of 12 months, and find that our conclusions remain unchanged. We therefore set the block-length equal to return horizon $T$ to account for overlapping observations and follow the suggestion of Lahiri (1999) to keep the block length fixed and to allow for overlaps in the blocks.

[15]The idea is to account for the empirical reality that options data may not be available for all firms. For the large number of bootstrap samples $B = 1,000$ that we use, the results of randomly selecting a subset of firms or including all firms that are available in a drawn block leads to identical results. Conceptually, our approach is similar to the bootstrap using firm clusters described by Petersen (2009) in his footnote 12.

(b) Draw further (overlapping) blocks, with replacement, until the bootstrap sample has the same number of time periods as the actual data.

(c) For every point in time in the bootstrap sample $b$, determine the firms' market weights and compute the value-weighted average of individual stocks' risk-neutral variance, that is $\overline{\text{SVIX}}_t^2 = \sum_i w_{i,t}\,\text{SVIX}_{i,t}^2$, the market return as the return on the value-weighted portfolio, and the stocks' returns in excess of the market.

2. For each bootstrap sample, run the pooled panel regression of returns in excess of the market onto risk-neutral excess stock variance,

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left( \text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2 \right) + \varepsilon_{i,t+1},$$

and collect the $B = 1,000$ bootstrap estimates of $\alpha$ and $\gamma$.

3. Using the $B = 1,000$ bootstrap estimates of $\alpha$ and $\gamma$, compute the bootstrap covariance matrix of $\alpha$ and $\gamma$. Using this bootstrap covariance matrix, we compute Wald statistics for hypothesis tests.[16] Building on the asymptotic refinement achieved from bootstrapping the covariance matrix, we use the Wald tests' asymptotic distribution to compute the $p$-values. We explore the finite-sample properties of this bootstrap procedure in Section B.2; our simulation evidence

[16] We prefer to compute the Wald statistic based on the bootstrap covariance matrix rather than to bootstrap the Wald statistic because our approach explicitly takes cross-sectional dependencies as well as overlapping observations and other time-dependencies into account. Qualitatively, our results are very similar when we bootstrap Wald statistics that are computed using a double-clustered covariance matrix as suggested by Petersen (2009). The quantitative bootstrap results of the Wald tests can be quite different when using a non-clustered covariance matrix, but we would still not reject the model. As a further sanity check, we also verified that the $p$-values of bootstrapped likelihood ratio test statistics are identical to those of the bootstrapped Wald statistics computed from non-clustered covariance matrices.

suggests that the approach works well.

**Pooled panel regressions of excess returns.** The bootstrap procedure for pooled panel regressions of excess returns is essentially the same as the one described for returns in excess of the market above. The only modifications are:

- In step 1, also include the risk-neutral market variance ($\text{SVIX}_t^2$) in the resampling procedure.
- In step 2, run the regression of excess returns on $\text{SVIX}_t^2$ and $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$, and collect the $B = 1,000$ bootstrap estimates of $\alpha$, $\beta$, and $\gamma$.
- In step 3, compute the bootstrap covariance matrix for $\alpha$, $\beta$, and $\gamma$ and use it to compute standard errors and to conduct hypothesis tests.

**Regressions with firm fixed effects.** For the bootstraps of the firm fixed effects regressions we adjust the procedure for the pooled panel regressions described above as follows:

- In step 2, run the regression with firm fixed effects $\alpha_i$ (instead of the intercept $\alpha$) and
  - compute the value-weighted sum of firm fixed effects at every date in every bootstrap sample, that is $\alpha_t = \sum_i w_{i,t}\alpha_i$
  - in each bootstrap sample, compute $\overline{\alpha}$ as the time-series average of $\alpha_t$
  - collect the $B = 1,000$ estimates of $\overline{\alpha}$ (instead of intercept $\alpha$)
- In step 3, compute the bootstrap covariance matrix with $\overline{\alpha}$ (instead of intercept $\alpha$) and use it to compute standard errors and to conduct hypothesis tests.

**Portfolio regressions.** The bootstraps for pooled panel and fixed effects regressions using excess returns and excess-of-market returns of portfolios follow the corresponding firm-level procedures described above. The only difference is that in step 1(a) we use all portfolios rather than resampling in the cross-section, as we have a balanced panel of portfolio data.

## B.2 Finite sample properties of the block bootstrap procedure

To provide evidence for the reliability of our bootstrap procedure in finite samples, we conduct a simulation study. We simulate $S$ samples on which we impose the null hypothesis and within each sample we repeat the bootstrap procedure from Section B.1 above with $B$ iterations. We then, first, compare the empirical quantiles of the Wald statistic in the simulated data to the quantiles of $\chi^2$-distribution, that is the Wald statistic's asymptotic distribution. These results suggest that our procedure, using the bootstrap covariance matrix to compute the Wald statistic and then using the asymptotic distribution to infer its $p$-value, is reasonable. Second, we compare the rejection frequency for the null hypothesis in the simulated data (on which we imposed the null hypothesis) to the nominal size of the test, and these results provide further support for our empirical approach.

Given the enormous computational demand of this exercise with an additional $S \times B$ bootstrap samples to be generated and evaluated, we focus on the pooled panel regressions of S&P 100 firms' returns in excess of the market. We simulate data under the null hypothesis by imposing $\alpha = 0$ and $\gamma = 0.5$ and drawing blocks of innovations from the regression residuals (from the specification in Panel A of Table II). The block resampling scheme follows the approach described above in Section B.1 and again serves to account for cross-sectional and time-series dependencies. We start by setting the number of simulations $S = 200$ and the number of bootstrap iterations to $B = 99$, thereby following the choice of Piatti and Trojani (2014) in a similar double-bootstrap exercise; we also show that the results are similar when we increase the number of simulations to $S = 400$ and the number of bootstrap iterations to $B = 198$. Our subsequent discussion is based on the results for the one-year horizon and we then show that our conclusions are very similar for other horizons.

**Empirical and asymptotic quantiles of the Wald statistic.** Panel A in Figure IA.6 compares the empirical quantiles of the Wald statistic in the simulated data to the quantiles of the Wald statistic's asymptotic $\chi^2$-distribution. With the vertical lines marking the 90%-, 95%-, and 99%-quantiles, the plot shows that the empirical quantiles are virtually identical to the quantiles of the $\chi^2$-distribution beyond the 95%-quantile; only in the very far tails of the distribution the critical values from the empirical distribution exceed those from the chi-squared distribution. These results suggest that our approach to, first, use the bootstrap covariance matrix to compute the Wald statistic, and then, second, to use the asymptotic distribution to infer the $p$-value of the Wald statistic, should work well.

**Nominal size and empirical rejection frequencies.** Panel B in Figure IA.6 compares the empirical rejection frequencies of our bootstrap approach when applied to simulated data (on which we impose the null hypothesis) to the corresponding nominal size of the test. That is, we compute the fraction of samples in which the bootstrap procedure leads to a rejection of the hypothesis when using the nominal size given on the x-axis. Similar as in Panel A, the dotted and dashed lines plot the 90%/10%-, 95%/5%-, and 99%/1%- quantiles to mark the economically interesting regions, where we care about rejections. We find that empirical rejection frequencies are well aligned with nominal size, in particular within these economically interesting regions, and that differences in empirical rejections frequencies and nominal size should be too small to lead to incorrect inference in our empirical analysis. To illustrate this, the big symbol in the plot indicates the $p$-value of the Wald statistic that we obtain from our empirical test of the model in the data; this $p$-value is 0.437 as reported in Panel A of Table II. These results suggest that we are very far away from making an inference error.

Using horizons shorter and longer than one year, Figure IA.7 shows that the empirical quantiles of the Wald statistic in the simulated data also line up well with quantiles of the Wald statistic's asymptotic $\chi^2$-distribution for horizons of three and six months. For the shortest (longest) horizon of one (24) month(s), the empirical quantiles appear somewhat too low (high) compared to the asymptotic quantiles. Nonetheless, the comparison of empirical rejection frequencies in the simulated data to the nominal sizes used in the tests in Figure IA.8 suggests that we are unlikely to make an inference error at any horizon. All results are very similar when increasing the number of simulations and bootstrap iterations to $S = 400$ and $B = 198$ as we show in Figure IA.9; overall the alignment of empirical rejection frequencies in the simulated data with nominal sizes used in the tests slightly improves when increasing $S$ to 400 and $B$ to 198.

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## Table I: Sample data

This Table summarizes the data used in the empirical analysis. We search the OptionMetrics database for all firms that have been included in the S&P 100 or S&P 500 during the sample period from January 1996 to October 2014 and obtain all available volatility surface data. We use this data to compute firms' risk-neutral variances $\left(\mathrm{SVIX}_{i,t}^2\right)$ for horizons of one, three, six, 12, and 24 months. Panel A summarizes the number of total observations, the number of unique days and unique firms in our sample, as well as the average number of firms for which options data is available per day. For some econometric analysis, we also compile data subsets at a monthly frequency for firms included in the S&P 100 (summarized in Panel B) and the S&P 500 (Panel C).

*Panel A. Daily data*

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| Observations | 2,106,711 | 2,106,711 | 2,106,711 | 2,106,711 | 2,106,711 |
| Sample days | 4,674 | 4,674 | 4,674 | 4,674 | 4,674 |
| Sample firms | 869 | 869 | 869 | 869 | 869 |
| Average firms/day | 451 | 451 | 451 | 451 | 451 |

*Panel B. Monthly data for S&P 100 firms*

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| Observations | 21,205 | 20,820 | 20,247 | 19,100 | 16,896 |
| Sample months | 224 | 222 | 219 | 213 | 201 |
| Sample firms | 177 | 176 | 176 | 171 | 167 |
| Average firms/month | 95 | 94 | 92 | 90 | 84 |

*Panel C. Monthly data for S&P 500 firms*

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| Observations | 102,198 | 100,252 | 97,340 | 91,585 | 80,631 |
| Sample months | 224 | 222 | 219 | 213 | 201 |
| Sample firms | 877 | 869 | 863 | 832 | 770 |
| Average firms/month | 456 | 452 | 444 | 430 | 401 |

Table II: Expected returns in excess of the market: Pooled panel regressions

This Table presents results from regressing equity returns in excess of the market on the stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$ for S&P 100 firms (Panel A) and for S&P 500 firms (Panel B). The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\text{SVIX}_{i,t}^2$ and $\overline{\text{SVIX}}_t^2$. We report estimates of the pooled panel regression specified in equation (18),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (joint test of zero intercept and $\gamma = 0.5$), for a test whether $\gamma = 0.5$, and for a test whether $\gamma$ is equal to zero. The rows labelled 'theory adj-$R^2$ (%)' report the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| *Panel A. S&P 100 firms* | | | | | |
| $\alpha$ | 0.008 | 0.008 | 0.005 | 0.007 | 0.010 |
| | (0.015) | (0.014) | (0.015) | (0.016) | (0.016) |
| $\gamma$ | 0.541 | 0.551 | 0.761 | 0.819 | 0.723 |
| | (0.345) | (0.313) | (0.328) | (0.337) | (0.270) |
| Adjusted $R^2$ (%) | 0.473 | 1.185 | 3.527 | 6.070 | 6.665 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.841 | 0.832 | 0.609 | 0.437 | 0.439 |
| $H_0 : \gamma = 0.5$ | 0.906 | 0.871 | 0.427 | 0.344 | 0.409 |
| $H_0 : \gamma = 0$ | 0.118 | 0.079 | 0.020 | 0.015 | 0.007 |
| Theory adj-$R^2$ (%) | 0.463 | 1.151 | 3.054 | 5.005 | 5.712 |
| *Panel B. S&P 500 firms* | | | | | |
| $\alpha$ | 0.016 | 0.016 | 0.013 | 0.014 | 0.019 |
| | (0.015) | (0.015) | (0.016) | (0.019) | (0.019) |
| $\gamma$ | 0.301 | 0.414 | 0.551 | 0.553 | 0.354 |
| | (0.285) | (0.273) | (0.306) | (0.302) | (0.200) |
| Adjusted $R^2$ (%) | 0.135 | 0.617 | 1.755 | 2.892 | 1.901 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.489 | 0.560 | 0.630 | 0.600 | 0.596 |
| $H_0 : \gamma = 0.5$ | 0.486 | 0.752 | 0.869 | 0.862 | 0.467 |
| $H_0 : \gamma = 0$ | 0.291 | 0.129 | 0.072 | 0.068 | 0.077 |
| Theory adj-$R^2$ (%) | 0.068 | 0.547 | 1.648 | 2.667 | 1.235 |

Table III: Expected returns in excess of the market: Panel regressions with fixed effects

This Table presents results from regressing equity returns in excess of the market on the stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$ for S&P 100 firms (Panel A) and for S&P 500 firms (Panel B). The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\text{SVIX}_{i,t}^2$ and $\overline{\text{SVIX}}_t^2$. We report estimates of the panel regression with firm fixed effects specified in equation (19),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i \alpha_i$ reports the time-series average of the value-weighted sum of firm fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (joint test of zero intercept and $\gamma = 0.5$), for a test whether $\gamma = 0.5$, and for a test whether $\gamma$ is equal to zero.

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| *Panel A. S&P 100 firms* | | | | | |
| $\sum_i w_i \alpha_i$ | 0.026 | 0.024 | 0.023 | 0.022 | 0.020 |
| | (0.010) | (0.009) | (0.009) | (0.009) | (0.009) |
| $\gamma$ | 0.780 | 0.833 | 1.120 | 1.156 | 1.018 |
| | (0.385) | (0.360) | (0.348) | (0.313) | (0.286) |
| Adjusted $R^2$ (%) | 1.097 | 4.013 | 9.896 | 16.866 | 24.071 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.026 | 0.012 | 0.006 | 0.002 | 0.013 |
| $H_0 : \gamma = 0.5$ | 0.468 | 0.355 | 0.074 | 0.036 | 0.070 |
| $H_0 : \gamma = 0$ | 0.043 | 0.021 | 0.001 | 0.000 | 0.000 |
| *Panel B. S&P 500 firms* | | | | | |
| $\sum_i w_i \alpha_i$ | 0.036 | 0.034 | 0.033 | 0.033 | 0.033 |
| | (0.008) | (0.007) | (0.008) | (0.008) | (0.008) |
| $\gamma$ | 0.560 | 0.730 | 0.949 | 0.917 | 0.637 |
| | (0.313) | (0.313) | (0.319) | (0.291) | (0.199) |
| Adjusted $R^2$ (%) | 0.398 | 3.015 | 7.320 | 12.637 | 17.479 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $H_0 : \gamma = 0.5$ | 0.848 | 0.461 | 0.160 | 0.152 | 0.491 |
| $H_0 : \gamma = 0$ | 0.073 | 0.019 | 0.003 | 0.002 | 0.001 |

## Table IV: Expected excess returns: Pooled panel regressions

This Table presents results from regressing equity excess returns of S&P 100 firms (Panel A) and S&P 500 firms (Panel B) on the risk-neutral variance of the market variance ($\text{SVIX}_t^2$) and the stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\text{SVIX}_t^2$, $\text{SVIX}_{i,t}^2$, and $\overline{\text{SVIX}}_t^2$. We report estimates of the pooled panel regression specified in equation (20),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\,\text{SVIX}_t^2 + \gamma\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept, $\beta = 1$, and $\gamma = 0.5$), for tests whether $\beta$ and $\gamma$ are equal to zero, for a test whether $\gamma = 0.5$, and for a test whether $\gamma$ is equal to zero. The rows labelled 'theory adj-$R^2$ (%)' report the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| *Panel A. S&P 100 firms* | | | | | |
| $\alpha$ | 0.073 | 0.035 | -0.009 | 0.001 | -0.006 |
| | (0.064) | (0.074) | (0.054) | (0.067) | (0.068) |
| $\beta$ | -0.001 | 1.070 | 2.244 | 1.956 | 1.990 |
| | (2.032) | (2.263) | (1.465) | (1.404) | (1.517) |
| $\gamma$ | 0.469 | 0.489 | 0.729 | 0.834 | 0.736 |
| | (0.346) | (0.332) | (0.340) | (0.343) | (0.267) |
| Adjusted $R^2$ (%) | 0.274 | 0.942 | 3.809 | 6.387 | 7.396 |
| $H_0 : \alpha = 0, \beta = 1, \gamma = 0.5$ | 0.550 | 0.687 | 0.660 | 0.566 | 0.608 |
| $H_0 : \beta = \gamma = 0$ | 0.356 | 0.335 | 0.064 | 0.045 | 0.012 |
| $H_0 : \gamma = 0.5$ | 0.929 | 0.974 | 0.500 | 0.330 | 0.376 |
| $H_0 : \gamma = 0$ | 0.175 | 0.140 | 0.032 | 0.015 | 0.006 |
| Theory adj-$R^2$ (%) | 0.099 | 0.625 | 2.509 | 3.896 | 4.830 |
| *Panel B. S&P 500 firms* | | | | | |
| $\alpha$ | 0.057 | 0.019 | -0.038 | -0.021 | -0.054 |
| | (0.074) | (0.079) | (0.059) | (0.071) | (0.076) |
| $\beta$ | 0.743 | 1.882 | 3.483 | 3.032 | 3.933 |
| | (2.311) | (2.410) | (1.569) | (1.608) | (1.792) |
| $\gamma$ | 0.214 | 0.305 | 0.463 | 0.512 | 0.324 |
| | (0.296) | (0.287) | (0.320) | (0.318) | (0.200) |
| Adjusted $R^2$ (%) | 0.096 | 0.767 | 3.218 | 4.423 | 5.989 |
| $H_0 : \alpha = 0, \beta = 1, \gamma = 0.5$ | 0.267 | 0.242 | 0.169 | 0.184 | 0.015 |
| $H_0 : \beta = \gamma = 0$ | 0.770 | 0.553 | 0.071 | 0.092 | 0.036 |
| $H_0 : \gamma = 0.5$ | 0.333 | 0.497 | 0.908 | 0.971 | 0.377 |
| $H_0 : \gamma = 0$ | 0.470 | 0.287 | 0.148 | 0.108 | 0.105 |
| Theory adj-$R^2$ (%) | -0.107 | 0.227 | 1.491 | 1.979 | 1.660 |

Table V: Expected excess returns: Panel regressions with fixed effects

This Table presents results from regressing equity excess returns of S&P 100 firms (Panel A) and S&P 500 firms (Panel B) on the risk-neutral variance of the market variance ($\mathrm{SVIX}_t^2$) and the stock's risk-neutral variance measured relative to stocks' average risk-neutral variance $\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right)$. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\mathrm{SVIX}_t^2$, $\mathrm{SVIX}_{i,t}^2$, and $\overline{\mathrm{SVIX}}_t^2$. We report estimates of the panel regression with firm fixed effects specified in equation (21),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\,\mathrm{SVIX}_t^2 + \gamma\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i \alpha_i$ reports the time-series average of the value-weighted sum of firm fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept, $\beta = 1$, and $\gamma = 0.5$), for tests whether $\beta$ and $\gamma$ are equal to zero, for a test whether $\gamma = 0.5$, and for a test whether $\gamma$ is equal to zero.

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| | *Panel A. S&P 100 firms* | | | | |
| $\sum w_i \alpha_i$ | 0.089 | 0.051 | 0.010 | 0.018 | 0.003 |
| | (0.062) | (0.071) | (0.051) | (0.064) | (0.066) |
| $\beta$ | -0.085 | 0.947 | 2.091 | 1.793 | 1.876 |
| | (2.041) | (2.277) | (1.423) | (1.325) | (1.391) |
| $\gamma$ | 0.734 | 0.801 | 1.126 | 1.225 | 1.083 |
| | (0.392) | (0.387) | (0.370) | (0.314) | (0.273) |
| Adjusted $R^2$ (%) | 1.211 | 4.771 | 11.861 | 20.003 | 27.455 |
| $H_0 : \sum_i w_i \alpha_i = 0, \beta = 1, \gamma = 0.5$ | 0.233 | 0.363 | 0.274 | 0.111 | 0.184 |
| $H_0 : \beta = \gamma = 0$ | 0.128 | 0.103 | 0.008 | 0.000 | 0.000 |
| $H_0 : \gamma = 0.5$ | 0.551 | 0.436 | 0.091 | 0.021 | 0.033 |
| $H_0 : \gamma = 0$ | 0.061 | 0.038 | 0.002 | 0.000 | 0.000 |
| | *Panel B. S&P 500 firms* | | | | |
| $\sum w_i \alpha_i$ | 0.080 | 0.042 | -0.008 | 0.012 | -0.026 |
| | (0.072) | (0.075) | (0.055) | (0.070) | (0.079) |
| $\beta$ | 0.603 | 1.694 | 3.161 | 2.612 | 3.478 |
| | (2.298) | (2.392) | (1.475) | (1.493) | (1.681) |
| $\gamma$ | 0.491 | 0.634 | 0.892 | 0.938 | 0.665 |
| | (0.325) | (0.331) | (0.336) | (0.308) | (0.205) |
| Adjusted $R^2$ (%) | 0.650 | 4.048 | 10.356 | 17.129 | 24.266 |
| $H_0 : \sum_i w_i \alpha_i = 0, \beta = 1, \gamma = 0.5$ | 0.231 | 0.224 | 0.164 | 0.133 | 0.060 |
| $H_0 : \beta = \gamma = 0$ | 0.265 | 0.119 | 0.019 | 0.008 | 0.002 |
| $H_0 : \gamma = 0.5$ | 0.978 | 0.686 | 0.243 | 0.155 | 0.420 |
| $H_0 : \gamma = 0$ | 0.131 | 0.056 | 0.008 | 0.002 | 0.001 |

## Table VI: The relationship between realized, expected, and unexpected excess-of-market returns and characteristics

This Table presents results from regressing realized, expected, or unexpected equity returns in excess of the market ($y_{i,t+1}$) on the firm's CAPM beta, log size, book-to-market, past return, and risk-neutral stock variance measured relative to stocks' average risk-neutral variance, $\text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t$:

$$y_{i,t+1} = a + b_1 \text{Beta}_{i,t} + b_2 \log(\text{Size}_{i,t}) + b_3 \text{B/M}_{i,t} + b_4 \text{Ret}^{(12,1)}_{i,t} + c\left(\text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t\right) + \varepsilon_{i,t+1}.$$

The data is monthly and covers S&P 500 firms from January 1996 to October 2014. The first two columns present results for realized returns, the middle two columns for expected returns, and the last two columns for unexpected returns. In columns labelled 'theory,' we set the parameter values of our model forecast to the values implied by equation (16); in columns labelled 'estimated,' we use parameter estimates of a pooled panel regression (i.e. we use the estimates obtained from the regression specified in equation (18) and reported in Panel B of Table II). The return horizon is one year. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. The last three rows report the regression's adjusted-$R^2$ and the $p$-values of Wald tests on joint parameter significance, testing (i) whether all $b_i$-estimates are zero, (ii) whether all $b_i$-estimates are zero and $c = 0.5$, (iii) whether all non-constant coefficients are jointly zero.

| | Realized returns | | Expected returns | | Unexpected returns | |
|---|---|---|---|---|---|---|
| | | | estimated | theory | estimated | theory |
| const | 0.429 | 0.277 | 0.131 | 0.107 | 0.298 | 0.321 |
| | (0.371) | (0.377) | (0.073) | (0.027) | (0.365) | (0.359) |
| $\text{Beta}_{i,t}$ | 0.016 | -0.131 | 0.113 | 0.105 | -0.097 | -0.088 |
| | (0.075) | (0.062) | (0.066) | (0.016) | (0.046) | (0.078) |
| $\log(\text{Size}_{i,t})$ | -0.018 | -0.006 | -0.009 | -0.009 | -0.009 | -0.010 |
| | (0.014) | (0.015) | (0.006) | (0.002) | (0.015) | (0.013) |
| $\text{B/M}_{i,t}$ | 0.032 | 0.031 | 0.001 | 0.001 | 0.032 | 0.032 |
| | (0.025) | (0.027) | (0.006) | (0.005) | (0.026) | (0.026) |
| $\text{Ret}^{(12,1)}_{i,t}$ | -0.051 | -0.029 | -0.017 | -0.015 | -0.034 | -0.035 |
| | (0.041) | (0.041) | (0.018) | (0.010) | (0.039) | (0.040) |
| $\text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t$ | | 0.705 | | | | |
| | | (0.308) | | | | |
| Adjusted $R^2$ (%) | 1.031 | 3.969 | 37.766 | 37.766 | 1.051 | 0.974 |
| $H_0 : b_i = 0$ | 0.347 | 0.153 | 0.435 | 0.000 | 0.157 | 0.619 |
| $H_0 : b_i = 0, c = 0.5$ | | 0.234 | | | | |
| $H_0 : b_i = 0, c = 0$ | | 0.018 | | | | |

## Table VII: The relationship between realized, expected, and unexpected returns and characteristics

This Table presents results from regressing realized, expected, and unexpected equity excess returns ($y_{i,t+1}$) on the firm's CAPM beta, log size, book-to-market, past return, risk-neutral market variance ($\mathrm{SVIX}_t$), and risk-neutral stock variance measured relative to stocks' average risk-neutral variance ($\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t$):

$$y_{i,t+1} = a + b_1 \mathrm{Beta}_{i,t} + b_2 \log(\mathrm{Size}_{i,t}) + b_3 \mathrm{B/M}_{i,t} + b_4 \mathrm{Ret}^{(12,1)}_{i,t} + c_0\, \mathrm{SVIX}^2_t + c_1 \left( \mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t \right) + \varepsilon_{i,t+1}.$$

The data is monthly and covers S&P 500 firms from January 1996 to October 2014. The first two columns present results for realized returns, the middle two columns for expected returns, and the last two columns for unexpected returns. In columns labelled 'theory', we set the parameter values of our model forecast to the values implied by theory (i.e. we use equation (17)); while in columns labelled 'estimated', we use parameter estimates of a pooled panel regression (i.e. we use the estimates obtained from the regression specified in equation (20) and reported in Panel B of Table IV). The return horizon is one year. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. The last four rows report adjusted-$R^2$ and the $p$-values of Wald tests of joint parameter significance, testing (i) whether all $b_i$-estimates are zero, (ii) whether all $b_i$-estimates are zero, $c_0 = 1$, and $c_1 = 0.5$, (iii) whether all non-constant coefficients are jointly zero.

| | Realized returns | | Expected returns | | Unexpected returns | |
|---|---|---|---|---|---|---|
| | | | estimated | theory | estimated | theory |
| const | 0.721 | 0.452 | 0.259 | 0.164 | 0.462 | 0.557 |
| | (0.341) | (0.320) | (0.133) | (0.035) | (0.332) | (0.331) |
| $\mathrm{Beta}_{i,t}$ | 0.038 | -0.048 | 0.082 | 0.097 | -0.044 | -0.059 |
| | (0.068) | (0.068) | (0.064) | (0.018) | (0.046) | (0.072) |
| $\log(\mathrm{Size}_{i,t})$ | -0.030 | -0.019 | -0.010 | -0.009 | -0.019 | -0.021 |
| | (0.014) | (0.013) | (0.007) | (0.002) | (0.013) | (0.013) |
| $\mathrm{B/M}_{i,t}$ | 0.071 | 0.068 | 0.003 | 0.001 | 0.068 | 0.069 |
| | (0.034) | (0.038) | (0.010) | (0.006) | (0.038) | (0.037) |
| $\mathrm{Ret}^{(12,1)}_{i,t}$ | -0.049 | -0.005 | -0.046 | -0.026 | -0.003 | -0.023 |
| | (0.063) | (0.054) | (0.042) | (0.015) | (0.050) | (0.058) |
| $\mathrm{SVIX}^2_t$ | | 2.792 | | | | |
| | | (1.472) | | | | |
| $\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t$ | | 0.511 | | | | |
| | | (0.357) | | | | |
| Adjusted $R^2$ (%) | 1.924 | 5.265 | 17.277 | 30.482 | 0.973 | 1.197 |
| $H_0 : b_i = 0$ | 0.003 | 0.201 | 0.702 | 0.000 | 0.187 | 0.092 |
| $H_0 : b_i = 0, c_0 = 1, c_1 = 0.5$ | | 0.143 | | | | |
| $H_0 : b_i = 0, c_0 = 0, c_1 = 0$ | | 0.001 | | | | |

Table VIII: Excess-of-market returns of characteristics/$\mathrm{SVIX}_{i,t}$ double-sorted portfolios

This Table presents results from regressing portfolio equity returns in excess of the market on the portfolio stock's risk-neutral variance measured relative to stocks' average risk-neutral variance, $\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2$. The data is monthly from January 1996 to October 2014. At the end of each month, we sort S&P 500 firms into 5x5-double sorted portfolios based on firm characteristics and $\mathrm{SVIX}_{i,t}$. We first assign firms to quintile portfolios based on their CAPM beta, size, book-to-market, or momentum. In the second step, we sort stocks within each of the characteristics portfolios into $\mathrm{SVIX}_{i,t}$-quintiles, providing us with a total of 25 conditionally double-sorted portfolios. The one-year horizon of the portfolio returns matches the 365 day-maturity of the options used to compute $\mathrm{SVIX}_{i,t}^2$ and $\overline{\mathrm{SVIX}}_t^2$. Panel A reports estimates of the pooled panel regression specified in equation (18),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left( \mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with portfolio fixed effects specified in equation (19),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left( \mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i \alpha_i$ reports the time-series average of the value-weighted sum of firm fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept and $\gamma = 0.5$) and for a test whether $\gamma$ is equal to zero. For the pooled panel regressions, the row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled panel regressions* | | | | |
| $\alpha$ | 0.015 | 0.013 | 0.015 | 0.014 |
| | (0.019) | (0.020) | (0.020) | (0.019) |
| $\gamma$ | 0.495 | 0.572 | 0.502 | 0.559 |
| | (0.311) | (0.323) | (0.327) | (0.319) |
| Adjusted $R^2$ (%) | 8.391 | 9.908 | 8.098 | 10.245 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.635 | 0.593 | 0.635 | 0.613 |
| $H_0 : \gamma = 0.5$ | 0.987 | 0.823 | 0.996 | 0.890 |
| $H_0 : \gamma = 0$ | 0.112 | 0.076 | 0.125 | 0.088 |
| Theory adj-$R^2$ (%) | 7.598 | 8.995 | 7.232 | 8.555 |
| *Panel B. Panel regressions with portfolio fixed effects* | | | | |
| $\sum_i w_i \alpha_i$ | 0.015 | 0.008 | 0.014 | 0.019 |
| | (0.017) | (0.005) | (0.016) | (0.017) |
| $\gamma$ | 0.794 | 0.941 | 0.711 | 0.864 |
| | (0.490) | (0.529) | (0.507) | (0.491) |
| Adjusted $R^2$ (%) | 13.010 | 16.419 | 12.679 | 15.020 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.439 | 0.070 | 0.479 | 0.212 |
| $H_0 : \gamma = 0.5$ | 0.549 | 0.405 | 0.677 | 0.459 |
| $H_0 : \gamma = 0$ | 0.106 | 0.075 | 0.161 | 0.079 |

Table IX: Out-of-sample forecast accuracy

This Table presents results on the out-of-sample accuracy of our model relative to benchmark predictions. To compare the forecast accuracy of the model to that of the benchmarks, we compute an out-of-sample R-squared, defined as

$$R^2_{OS} = 1 - \frac{\sum_i \sum_t FE_M^2}{\sum_i \sum_t FE_B^2},$$

where $FE_M$ and $FE_B$ denoted the forecast errors from our model and a benchmark prediction, respectively. Panel A evaluates forecasts of expected equity excess returns, as given in equation (17), and Panel B evaluates forecasts of expected equity returns in excess of the market return, as given in equation (16). The data is monthly and covers S&P 500 stocks from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\text{SVIX}_t^2$, $\text{SVIX}_{i,t}^2$, and $\overline{\text{SVIX}}_t^2$. For Panel A, the benchmark forecasts are the risk-neutral market variance ($\text{SVIX}_t^2$), the time-$t$ historical average excess returns of the S&P 500 ($\overline{\text{S\&P500}}_t$) and the CRSP value-weighted index ($\overline{\text{CRSP}}_t$), a constant prediction of 6% $p.a.$, the stock's risk-neutral variance ($\text{SVIX}_{i,t}^2$), the time-$t$ historical average of the firms' stock excess returns ($\overline{\text{RX}}_{i,t}$), and conditional CAPM implied predictions, where we estimate the CAPM betas from historical return data. For Panel B, we use $\text{SVIX}_{i,t}^2$, a random walk (i.e., zero return forecast), and the conditional CAPM as benchmarks.

*Panel A. Expected excess returns*

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| $\text{SVIX}_t^2$ | 0.09 | 0.57 | 1.77 | 3.08 | 2.77 |
| $\overline{\text{S\&P500}}_t$ | 0.09 | 0.79 | 2.56 | 3.82 | 4.46 |
| $\overline{\text{CRSP}}_t$ | -0.09 | 0.24 | 1.43 | 1.70 | 0.88 |
| 6% $p.a.$ | -0.01 | 0.46 | 1.84 | 2.54 | 2.06 |
| $\text{SVIX}_{i,t}^2$ | 0.95 | 1.87 | 1.55 | 2.17 | 7.64 |
| $\overline{\text{RX}}_{i,t}$ | 1.40 | 4.97 | 11.79 | 27.10 | 56.67 |
| $\widehat{\beta}_{i,t} \times \overline{\text{S\&P500}}_t$ | 0.09 | 0.79 | 2.54 | 3.76 | 4.72 |
| $\widehat{\beta}_{i,t} \times \overline{\text{CRSP}}_t$ | -0.06 | 0.28 | 1.46 | 1.68 | 1.61 |
| $\widehat{\beta}_{i,t} \times \text{SVIX}_t^2$ | 0.04 | 0.46 | 1.58 | 2.87 | 2.91 |
| $\widehat{\beta}_{i,t} \times$ 6% $p.a.$ | 0.00 | 0.47 | 1.84 | 2.48 | 2.58 |

*Panel B. Expected returns in excess of the market*

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| Random walk | 0.16 | 0.76 | 1.92 | 3.07 | 1.99 |
| $(\widehat{\beta}_{i,t} - 1) \times \overline{\text{S\&P500}}_t$ | 0.18 | 0.80 | 1.98 | 3.10 | 2.17 |
| $(\widehat{\beta}_{i,t} - 1) \times \overline{\text{CRSP}}_t$ | 0.21 | 0.89 | 2.14 | 3.35 | 2.83 |
| $(\widehat{\beta}_{i,t} - 1) \times \text{SVIX}_t^2$ | 0.11 | 0.62 | 1.68 | 2.80 | 2.01 |
| $(\widehat{\beta}_{i,t} - 1) \times$ 6% $p.a.$ | 0.19 | 0.83 | 2.04 | 3.19 | 2.49 |

Table X: Model out-of-sample forecasts vs in-sample benchmark predictions

This Table presents results on the out-of-sample accuracy of our model relative to benchmark predictions that also include in-sample information on returns and/or firm characteristics. To compare the forecast accuracy of the model to that of the benchmarks, we compute an out-of-sample R-squared, defined as

$$R^2_{OS} = 1 - \frac{\sum_i \sum_t FE_M^2}{\sum_i \sum_t FE_B^2},$$

where $FE_M$ and $FE_B$ denoted the forecast errors from our model and a benchmark prediction, respectively. Panel A evaluates forecasts of expected equity excess returns, as given in equation (17), and Panel B evaluates forecasts of expected equity returns in excess of the market return, as given in equation (16). The data is monthly and covers S&P 500 stocks from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\text{SVIX}_t^2$, $\text{SVIX}_{i,t}^2$, and $\overline{\text{SVIX}}_t^2$. For Panel A, the benchmark forecasts are the in-sample average market excess return, a conditional CAPM forecast that uses the in-sample average market excess return as an estimate of the equity premium, the in-sample average return across all stocks; and the fitted values of predictive in-sample regressions of stock returns in excess of the market on CAPM betas, log market capitalization, book-to-market ratios, stock momentum, and all four firm characteristics. For Panel B, we use analogous predictions based on returns in excess of the market.

*Panel A. Expected excess returns*

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| in-sample avg mkt | -0.05 | 0.31 | 1.52 | 1.90 | 1.42 |
| in-sample avg all stocks | -0.09 | 0.17 | 1.26 | 1.42 | 0.56 |
| $\widehat{\beta}_{i,t}\times$ in-sample avg mkt | -0.03 | 0.34 | 1.54 | 1.87 | 2.04 |
| $\text{Beta}_{i,t}$ | -0.09 | 0.16 | 1.22 | 1.30 | 0.56 |
| $\log(\text{Size}_{i,t})$ | -0.19 | -0.17 | 0.62 | 0.21 | -1.34 |
| $\text{B/M}_{i,t}$ | -0.18 | -0.03 | 0.89 | 0.77 | 0.00 |
| $\text{Ret}_{i,t}^{(12,1)}$ | -0.10 | 0.15 | 1.09 | 1.05 | -0.76 |
| All | -0.25 | -0.30 | 0.26 | -0.53 | -2.71 |

*Panel B. Expected returns in excess of the market*

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| in-sample avg all stocks | 0.11 | 0.58 | 1.60 | 2.48 | 0.95 |
| $(\widehat{\beta}_{i,t}-1)\times$ in-sample avg mkt | 0.20 | 0.86 | 2.11 | 3.29 | 2.63 |
| $\text{Beta}_{i,t}$ | 0.11 | 0.58 | 1.60 | 2.45 | 0.95 |
| $\log(\text{Size}_{i,t})$ | 0.05 | 0.39 | 1.27 | 1.90 | 0.12 |
| $\text{B/M}_{i,t}$ | 0.07 | 0.50 | 1.47 | 2.31 | 0.88 |
| $\text{Ret}_{i,t}^{(12,1)}$ | 0.10 | 0.56 | 1.47 | 2.05 | 0.03 |
| All | 0.03 | 0.34 | 1.11 | 1.46 | -0.64 |

Figure 2: Option-implied equity variance of S&P 500 firms

This Figure plots the time-series of the risk-neutral variance of the market ($SVIX_t^2$) and of stocks' average risk-neutral variance ($\overline{SVIX}_t^2$). We compute $SVIX_t^2$ from equity index options on the S&P 500. $\overline{SVIX}_t^2$ is the value-weighted sum of S&P 500 stocks' risk-neutral variance computed from individual firm equity options. Panels A through D present the variance series implied by equity options with maturities of one, three, six, 12, and 24 months. The data is daily from January 1996 to October 2014.

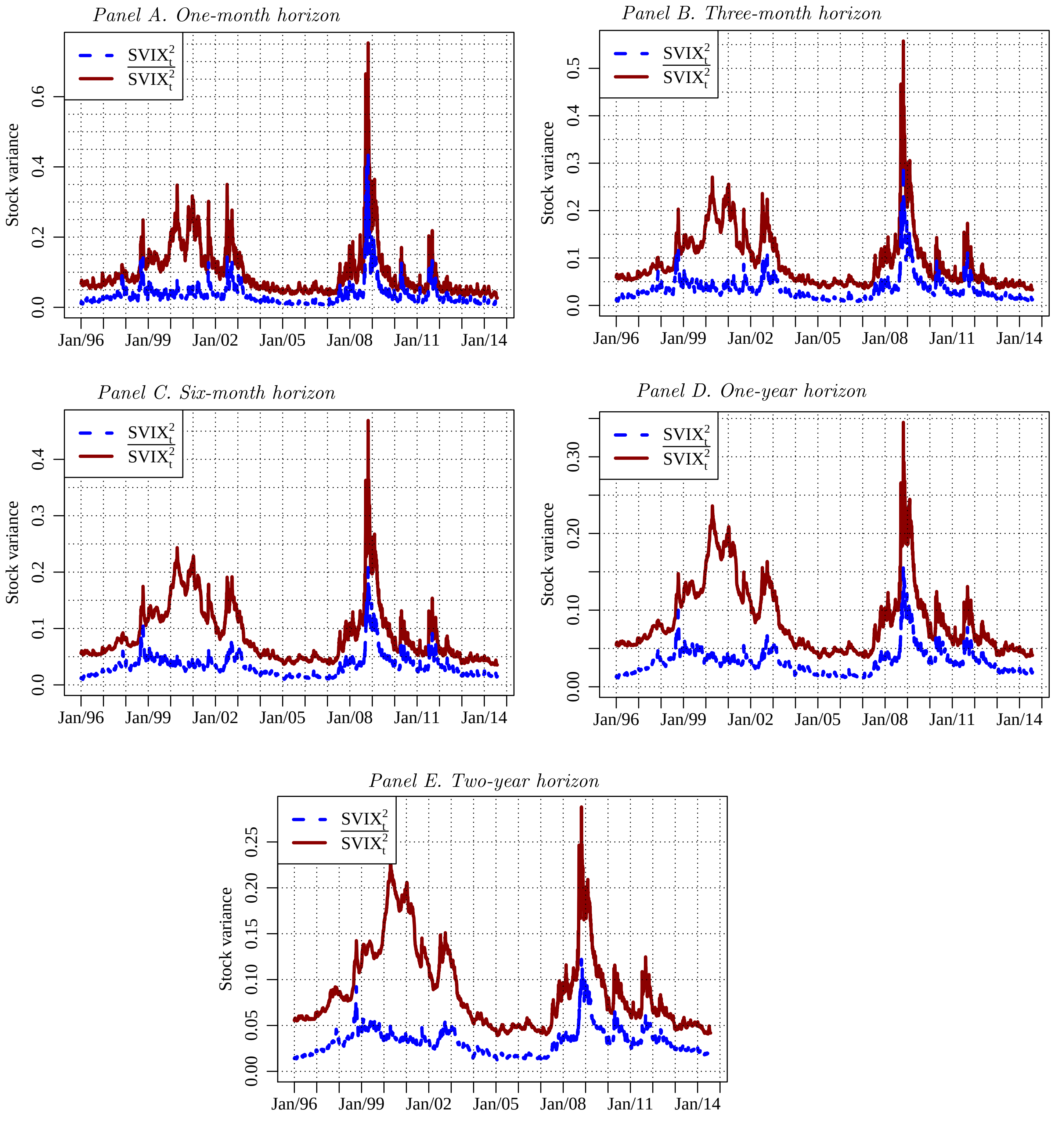


Figure 3: Beta, size, value, momentum, and option-implied equity variance

This Figure reports (equally-weighted) averages of risk-neutral stock variance ($\mathrm{SVIX}^2_{i,t}$, computed from individual firm equity options) of S&P 500 stocks, conditional on firm beta, size, book-to-market, and momentum. At every date $t$, we assign stocks to decile portfolios based on on their characteristics and report the time-series averages of $\mathrm{SVIX}^2_{i,t}$ across deciles using $\mathrm{SVIX}^2_{i,t}$-horizons of one year (Panels A to D).

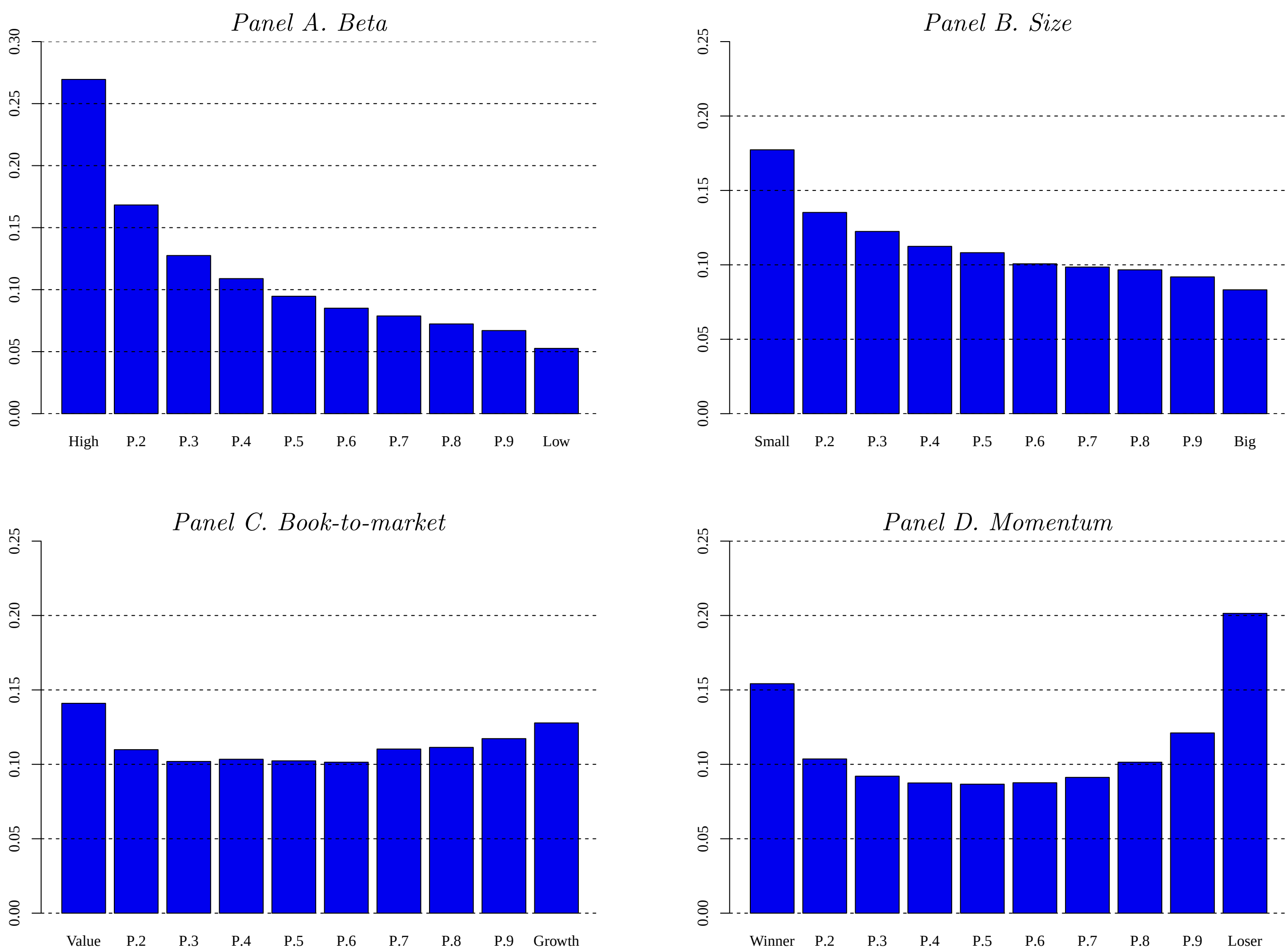
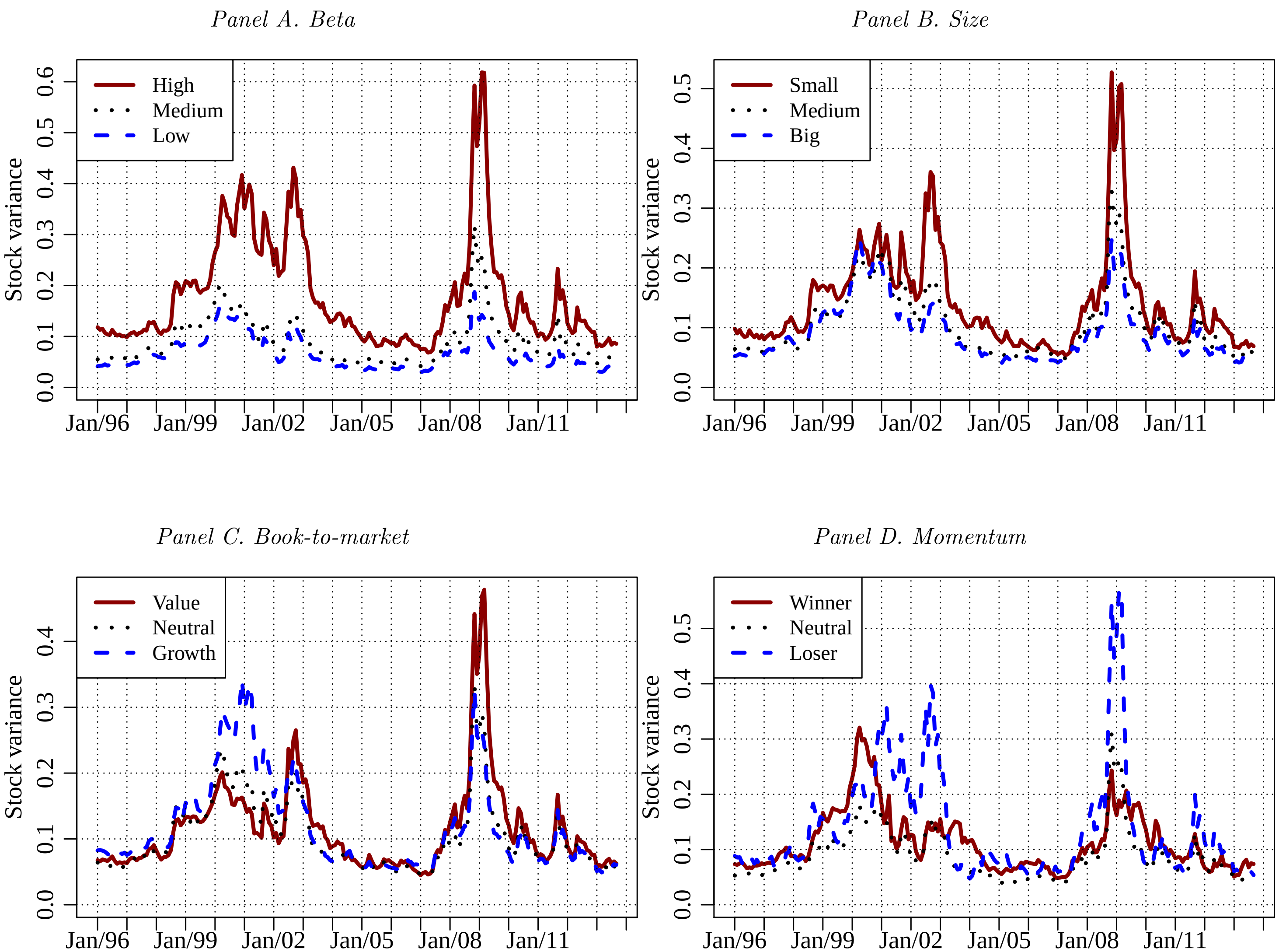


Figure 4: Beta, size, value, momentum, and option-implied equity variance

This Figure plots the time-series of risk-neutral stock variance ($\text{SVIX}_{i,t}^2$) of S&P 500 stocks, conditional on firm beta, size, book-to-market, and momentum. The horizon is one year. At every date $t$, we classify firms as small, medium, or big when their market capitalization is in the bottom, middle, or top tertile of the time-$t$ distribution across all firms in our sample, and compute the (equally-weighted) average $\text{SVIX}_{i,t}^2$. Similarly, we classify firms by their other characteristics at time $t$.



# Figure 5: Average equity returns in excess of the market

This Figure presents results on the relation between a firm's equity returns in excess of the market and its risk-neutral variance measured relative to average risk-neutral stock variance. For firms that were constituents of the S&P 500 index throughout our sample period, we compute time-series averages of their returns in excess of the market and their stock volatility relative to stocks' average volatility ($\text{SVIX}_i^2 - \overline{\text{SVIX}}^2$). We multiply the stock variance estimate by 0.5 and plot the pairwise combinations (blue crosses) for horizons of one, three, six, 12, and 24 months (Panels A to E). The black line represents the regression fit to the individual firm observations with slope coefficient and R-squared reported in the plot legend. Our theory implies that the slope coefficient of this regression should be one and that the intercept should be zero. The red diamonds represent decile portfolios of firms sorted by $\text{SVIX}_{i,t}^2$. Similarly, the triangles in orange represent portfolios of stocks formed according to firms' size and book-to-market.

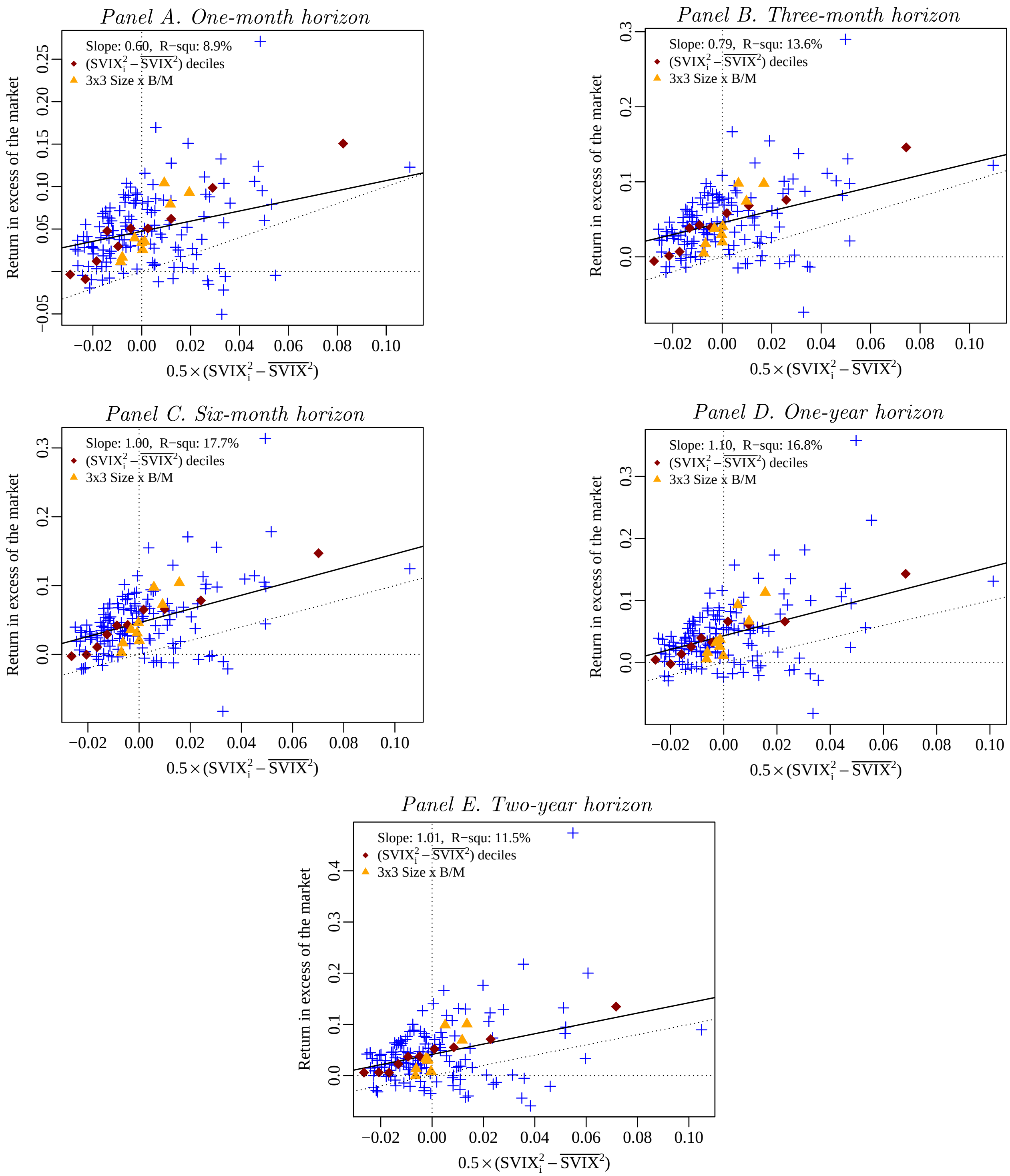


Figure 6: Portfolios sorted by excess stock volatility

This Figure reports results on the relationship between equity portfolio returns in excess of the market and risk-neutral stock variance measured relative to stocks' average risk-neutral variance. At the end of each month, we group all available firms into 10, 25, 50, or 100 portfolios (Panels A to D) based on their individual variance relative to average variance, $\text{SVIX}_i^2 - \overline{\text{SVIX}}^2$; the horizon is one year. For each portfolio, we compute the time-series average return in excess of the market and plot the pairwise combinations with the corresponding stock variance estimate multiplied by 0.5. Our theory implies that the slope coefficient of this regression should be one. The black line represents the regression fit to the portfolio observations with slope coefficient and R-squared reported in the plot legend. The sample period is January 1996 to October 2014.

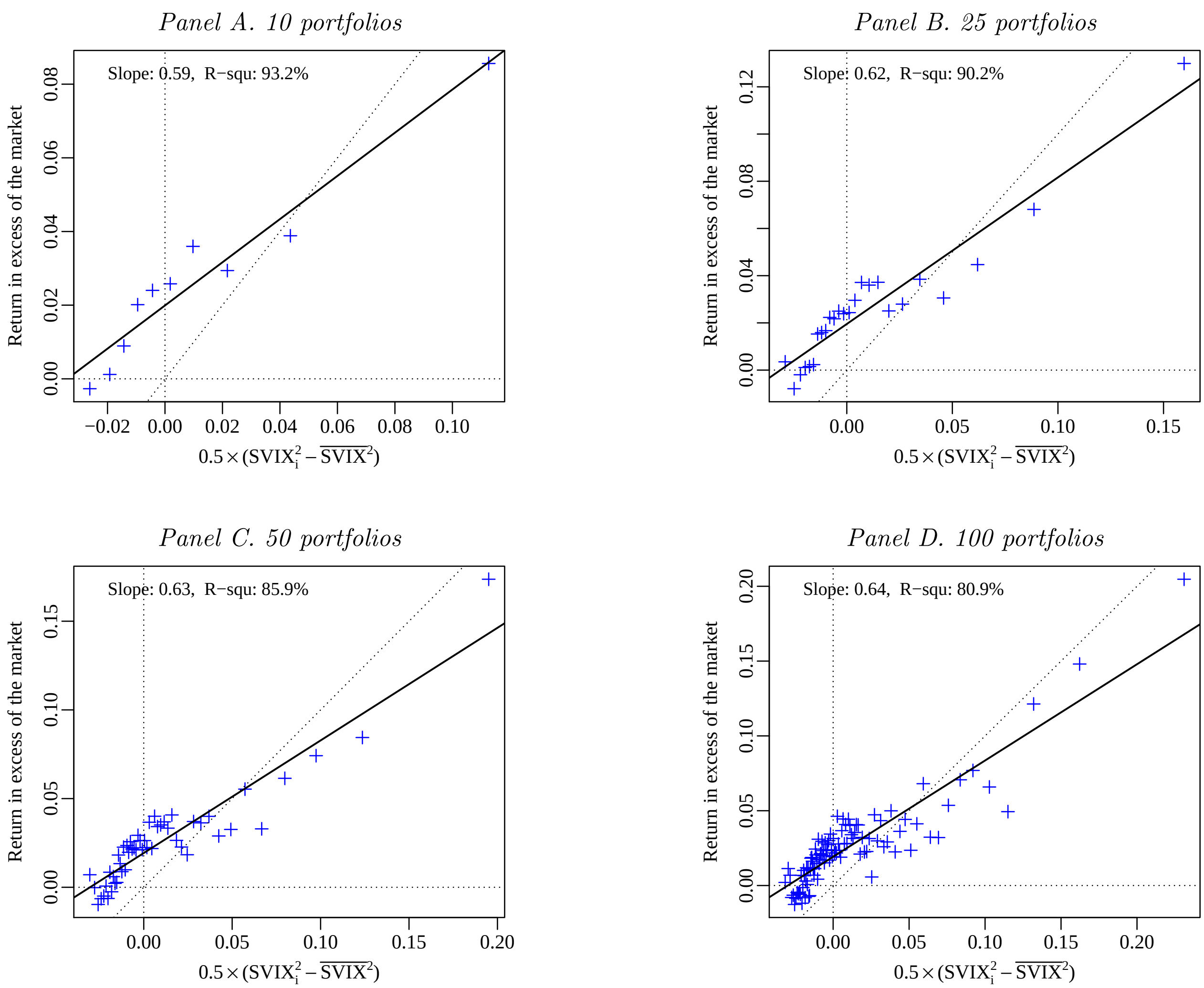


Figure 7: Regression estimates in subsamples

This Figure summarizes results from regressing equity excess returns of S&P 500 firms on the risk-neutral variance of the market variance, $\text{SVIX}_t^2$, and the stock's risk-neutral variance measured relative to stocks' average risk-neutral variance, $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$. The data is monthly from January 1996 to October 2014, and we present results for yearly subsamples in Panel A and for three-year subsamples in Panel B. The return horizon is one year and matches the maturity of the options used to compute $\text{SVIX}_t^2$, $\text{SVIX}_{i,t}^2$, and $\overline{\text{SVIX}}_t^2$. We report estimates for the pooled panel regression (20),

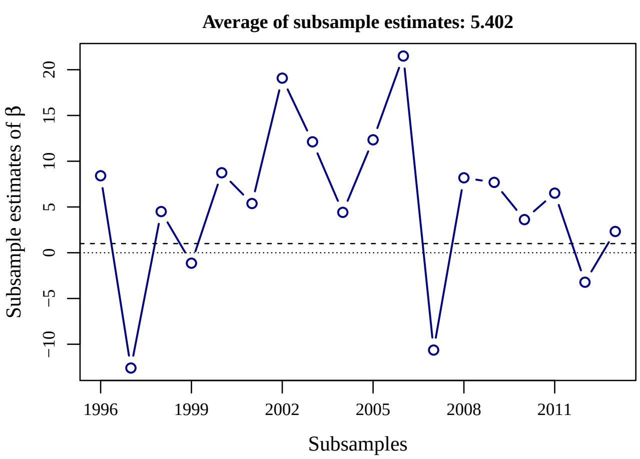
$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\,\text{SVIX}_t^2 + \gamma\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

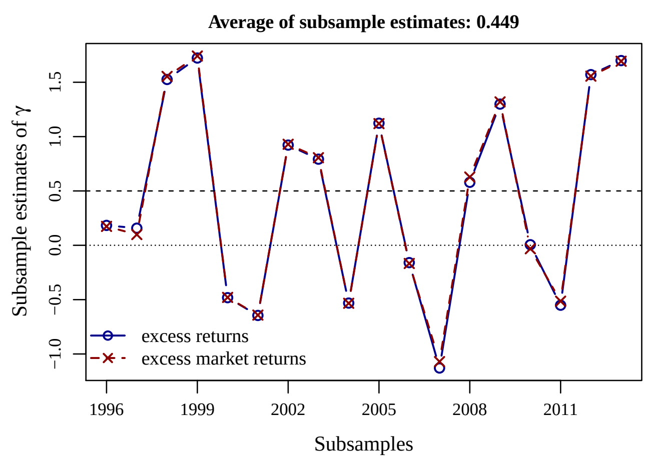
We also report estimates from regressing equity returns in excess of the market on the stock's risk-neutral variance relative to average risk-neutral variance, $\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$, using the pooled panel regression (18),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

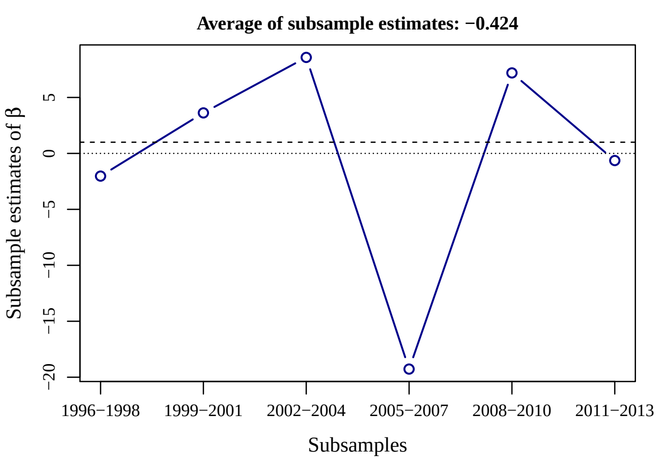
The dashed line in each panel indicates the coefficient value predicted by our theory, that is $\beta = 1$ and $\gamma = 0.5$.

*Panel A. One-year subsamples*




*Panel B. Three-year subsamples*



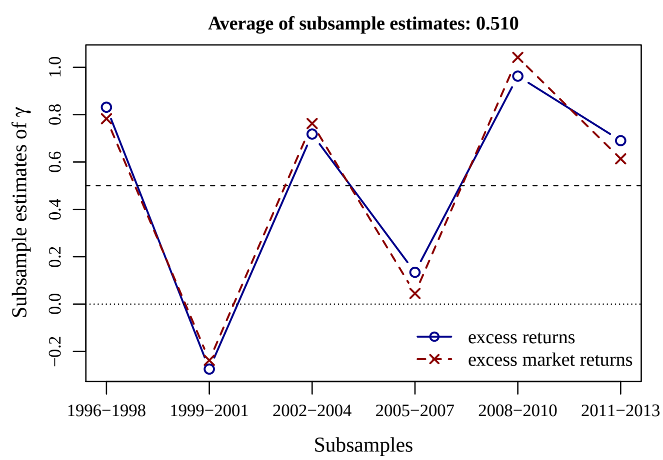
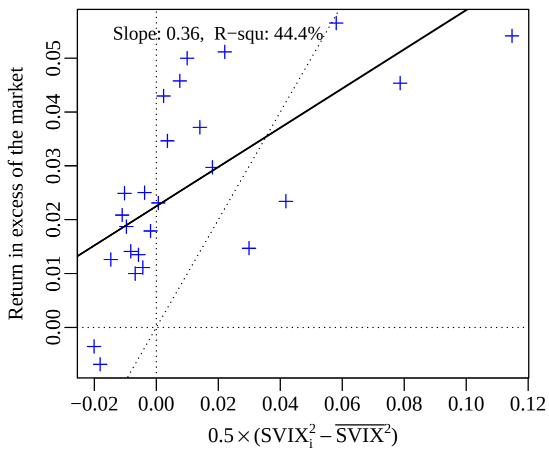



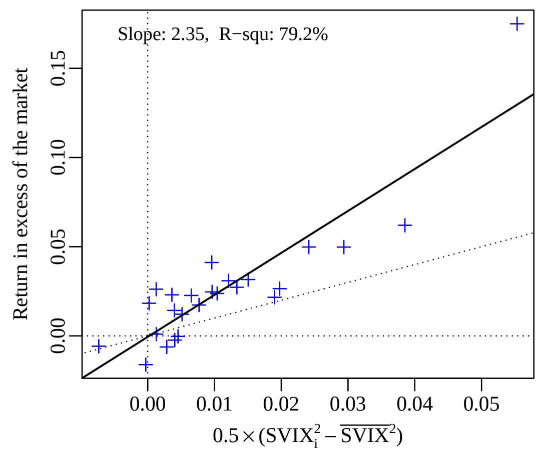
Figure 8: Portfolios sorted by beta, size, book-to-market, and momentum

This Figure reports results on the relationship between equity portfolio returns in excess of the market and risk-neutral stock variance measured relative to average firm-level risk-neutral variance. At the end of each month, we form 25 portfolios based on firms' beta, size, book-to-market, or momentum (Panels A to D) and from a $5 \times 5$ conditional double sort on size and book-to-market (Panel E). For each portfolio, we compute the time-series average return in excess of the market and plot the pairwise combinations with the corresponding stock variance estimate multiplied by 0.5. The black line represents the regression fit to the portfolio observations with slope coefficient and R-squared reported in the plot legend. Our theory implies that the slope coefficient of this regression should be one. The sample period is January 1996 to October 2014.

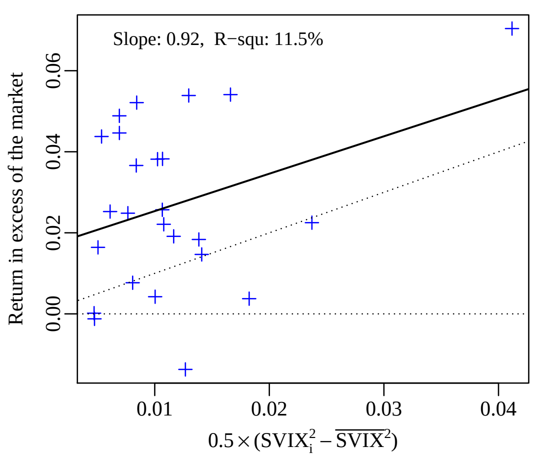
*Panel A. 25 Beta portfolios*



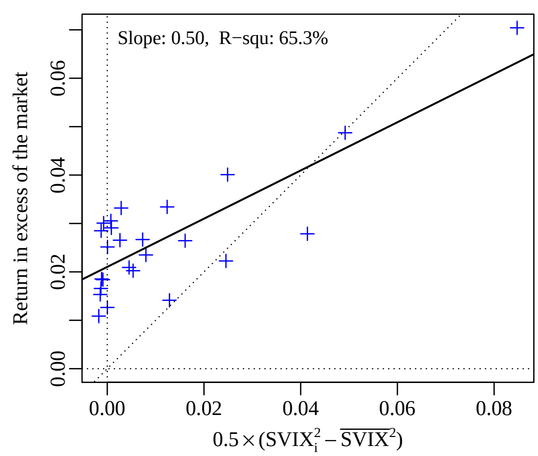

*Panel B. 25 Size portfolios*



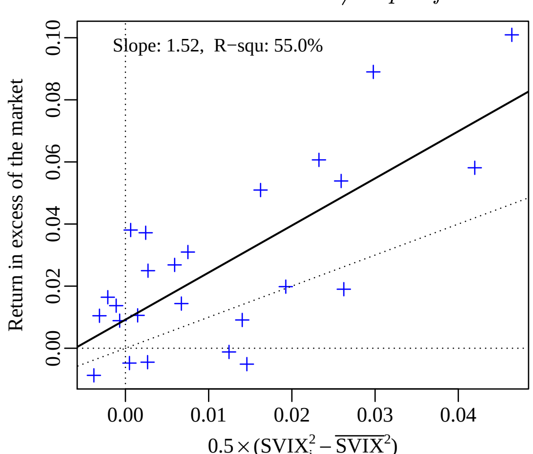

*Panel C. 25 B/M portfolios*




*Panel D. 25 Momentum portfolios*



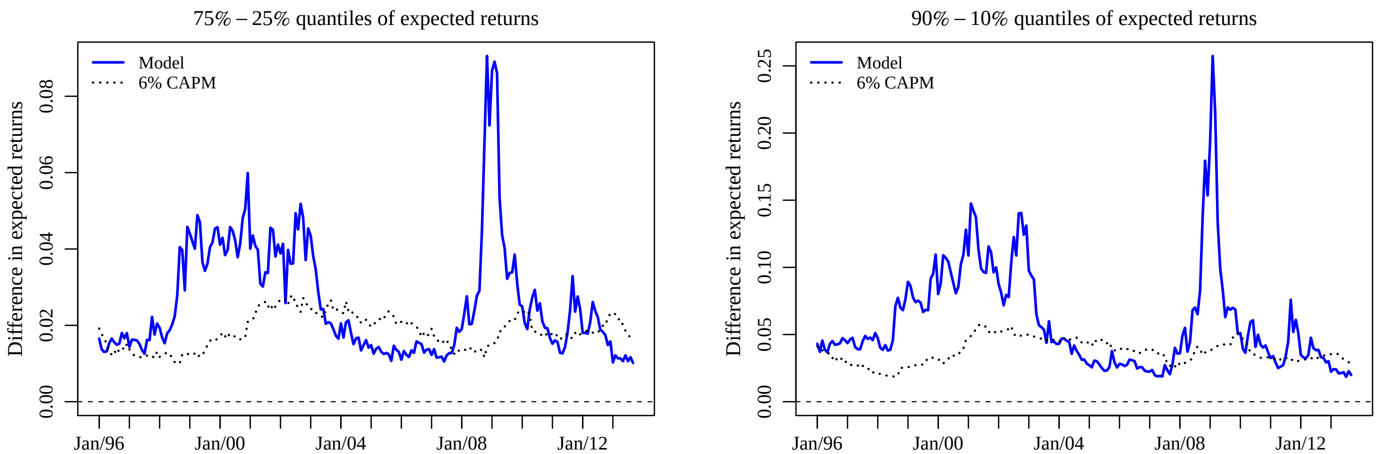

*Panel E. 25 Size × B/M portfolios*



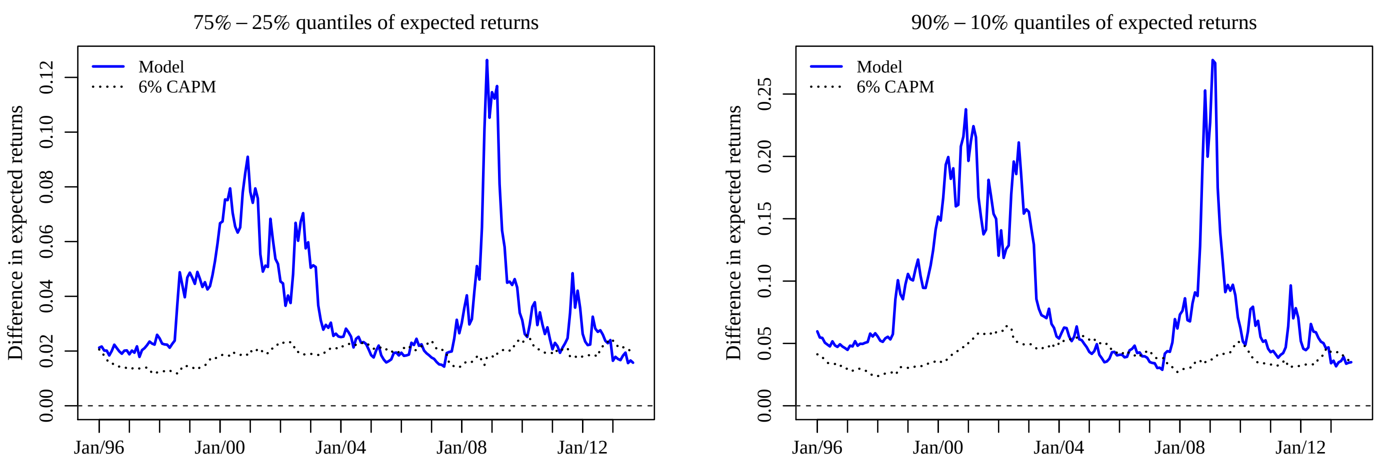
## Figure 9: Cross-sectional variation in expected returns

This Figure plots time-series of cross-sectional differences in one-year expected excess returns generated by our model and by CAPM forecasts. The CAPM forecasts use conditional betas (estimated from historical returns) and a constant 6% *p.a.* equity premium. The plots show the difference in the 75%- and 25%-quantiles of expected returns (on the left) and the difference in the 90%- and 10%-quantiles of expected returns (on the right) for S&P 100 stocks (Panel A) and S&P 500 stocks (Panel B). The data is monthly and covers S&P 500 stocks from January 1996 to October 2014.

*Panel A. Expected returns of S&P 100 firms*




*Panel B. Expected returns of S&P 500 firms*



Internet Appendix for

# What is the Expected Return on a Stock? ‡

Ian Martin Christian Wagner

‡Martin, Ian and Christian Wagner, Internet Appendix to "What is the Expected Return on a Stock?", Journal of Finance [DOI string].

## IA.A The linearization

In the body of the paper, we linearized $\beta_{i,t}^{*2}$ around $\beta_{i,t}^{*} = 1$, that is, approximated $\beta_{i,t}^{*2} \approx 2\beta_{i,t}^{*} - 1$. More generally, we could replace (8) with

$$\operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} = (c\beta_{i,t}^* + d) \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} + \operatorname{var}_t^* \varepsilon_{i,t+1}, \tag{IA.A.1}$$

where $c$ and $d$ are constants that can be chosen to satisfy some other requirement, as discussed in footnote 5. If we do so then, using (7) and (IA.A.1) to eliminate the dependence on $\beta_{i,t}^*$, and imposing $\operatorname{var}_t^* \varepsilon_{i,t+1} = \phi_i + \psi_t$ as in the main text,

$$\mathbb{E}_t \frac{R_{i,t+1}}{R_{f,t+1}} - 1 = \frac{1}{c} \operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} - \frac{d}{c} \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} - \frac{1}{c} \operatorname{var}_t^* \varepsilon_{i,t+1}. \tag{IA.A.2}$$

Value-weighting,

$$\mathbb{E}_t \frac{R_{m,t+1}}{R_{f,t+1}} - 1 = \frac{1}{c} \sum_j w_{j,t} \operatorname{var}_t^* \frac{R_{j,t+1}}{R_{f,t+1}} - \frac{d}{c} \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} - \frac{1}{c} \sum_j w_{j,t} \operatorname{var}_t^* \varepsilon_{j,t+1}. \tag{IA.A.3}$$

Subtracting (IA.A.3) from (IA.A.2),

$$\frac{\mathbb{E}_t R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \frac{1}{c} \left( \mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) \quad \text{where} \quad \sum_i w_{i,t} \alpha_i = 0. \tag{IA.A.4}$$

Similarly, substituting $\mathbb{E}_t R_{m,t+1} - R_{f,t+1} = R_{f,t+1} \mathrm{SVIX}_t^2$ into equation (IA.A.4), we have

$$\frac{\mathbb{E}_t R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \mathrm{SVIX}_t^2 + \frac{1}{c} \left( \mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) \quad \text{where} \quad \sum_i w_{i,t} \alpha_i = 0.$$

We can calculate the residual that the linearization neglects using (7) and (8):

$$\mathbb{E}_t \frac{R_{i,t+1}}{R_{f,t+1}} - 1 = \frac{1}{2} \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} + \frac{1}{2} \operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} - \frac{1}{2} \operatorname{var}_t^* \varepsilon_{i,t+1} - \frac{1}{2} \left( \beta_{i,t}^* - 1 \right)^2 \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}}.$$

Multiplying the above equation by the value weight $w_{i,t}$ and summing over $i$,

$$\mathbb{E}_t \frac{R_{m,t+1}}{R_{f,t+1}} - 1 = \frac{1}{2} \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} + \frac{1}{2} \sum_j w_{j,t} \operatorname{var}_t^* \frac{R_{j,t+1}}{R_{f,t+1}} - \frac{1}{2} \sum_j w_{j,t} \operatorname{var}_t^* \varepsilon_{j,t+1}$$
$$- \frac{1}{2} \sum_j w_{j,t} \left(\beta_{j,t}^* - 1\right)^2 \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}}.$$

Subtracting this from the previous equation and defining $\alpha_i$ as in the main text,

$$\mathbb{E}_t \frac{\mathbb{R}_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \frac{1}{2} \left( \operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} - \sum_j w_{j,t} \operatorname{var}_t^* \frac{R_{j,t+1}}{R_{f,t+1}} \right)$$
$$- \frac{1}{2} \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} \left[ \left(\beta_{i,t}^* - 1\right)^2 - \sum_j w_{j,t} \left(\beta_{j,t}^* - 1\right)^2 \right]. \qquad \text{(IA.A.5)}$$

In our baseline linearized equation (12), we are neglecting the final term on the right-hand side of equation (IA.A.5). Thus our measure will overstate expected returns for stocks $i$ for which $\beta_{i,t}^*$ is unusually far from one, and will understate expected returns for stocks for which $\beta_{i,t}^*$ is unusually close to one.

Unfortunately, $\beta_{i,t}^*$ is not directly observable. But if one is prepared to assume that $\operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}}$ can be proxied by $\text{SVIX}_t^2$, then it becomes possible to get a rough sense of the internal consistency of our approach by using equations (7) and (17) to compute a firm's risk-neutral beta as

$$\beta_{i,t}^* = 1 + \frac{1}{2} \frac{\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)}{\text{SVIX}_t^2}.$$

The third term in equation (IA.A.5) then becomes

$$\text{approximation error} = -\frac{1}{2} \text{SVIX}_t^2 \left[ \left(\beta_{i,t}^* - 1\right)^2 - \sum_j w_{j,t} \left(\beta_{j,t}^* - 1\right)^2 \right].$$

Figure IA.10 in the Internet Appendix plots the empirical distributions of $(\beta_{i,t}^* - 1)^2$

and of approximation errors, defined in this way, at horizons of one, three, six, 12, and 24 months. At the one year horizon, we find that the implied approximation errors are within $\pm 2.5\%$ *p.a.* for over 90% of our observations of S&P 100 firms and around 80% of observations of S&P 500 firms.

Table IA.1: Expected excess returns of S&P 100 stock portfolios sorted by SVIX

This Table presents results from regressing portfolio equity excess returns on the risk-neutral variance of the market variance ($\mathrm{SVIX}_t^2$) and on the portfolio's risk-neutral variance measured relative to portfolios' average risk-neutral variance $\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right)$. At the end of every month, we sort S&P 100 firms into 25 portfolios based on their $\mathrm{SVIX}_{i,t}^2$. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\mathrm{SVIX}_t^2$, $\mathrm{SVIX}_{i,t}^2$, and $\overline{\mathrm{SVIX}}_t^2$. Panel A reports estimates of the pooled panel regression specified in equation (20),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\,\mathrm{SVIX}_t^2 + \gamma\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports results for the panel regression with portfolio fixed effects specified in equation (21),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\,\mathrm{SVIX}_t^2 + \gamma\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i\alpha_i$ reports the time-series average of the value-weighted sum of portfolio fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept, $\beta = 1$, and $\gamma = 0.5$) and for tests whether $\beta$ and $\gamma$ are equal to zero. For the pooled panel regressions, the row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| *Panel A. Pooled panel regressions* | | | | | |
| $\alpha$ | 0.074 | 0.040 | -0.008 | -0.002 | -0.011 |
| | (0.063) | (0.074) | (0.053) | (0.068) | (0.070) |
| $\beta$ | -0.098 | 0.857 | 2.140 | 1.954 | 2.033 |
| | (1.997) | (2.272) | (1.426) | (1.372) | (1.522) |
| $\gamma$ | 0.438 | 0.497 | 0.728 | 0.769 | 0.690 |
| | (0.279) | (0.334) | (0.327) | (0.292) | (0.214) |
| Adjusted $R^2$ (%) | 0.390 | 1.460 | 6.124 | 9.738 | 11.524 |
| $\alpha$, $\beta$, $\gamma$ | 0.523 | 0.693 | 0.713 | 0.619 | 0.669 |
| $\beta = \gamma = 0$ | 0.174 | 0.302 | 0.062 | 0.030 | 0.003 |
| $\gamma = 0.5$ | 0.823 | 0.993 | 0.486 | 0.358 | 0.374 |
| $\gamma = 0$ | 0.117 | 0.137 | 0.026 | 0.009 | 0.001 |
| Theory adj-$R^2$ (%) | 0.016 | 0.922 | 4.087 | 6.174 | 7.951 |
| *Panel B. Panel regressions with portfolio fixed effects* | | | | | |
| $\sum w_i\alpha_i$ | 0.082 | 0.048 | 0.006 | 0.010 | -0.004 |
| | (0.060) | (0.070) | (0.049) | (0.063) | (0.067) |
| $\beta$ | -0.200 | 0.731 | 1.903 | 1.745 | 1.918 |
| | (1.963) | (2.210) | (1.377) | (1.299) | (1.451) |
| $\gamma$ | 0.600 | 0.726 | 1.162 | 1.141 | 0.922 |
| | (0.377) | (0.501) | (0.460) | (0.402) | (0.310) |
| Adjusted $R^2$ (%) | 1.222 | 4.286 | 11.942 | 19.436 | 23.417 |
| $\sum w_i\alpha_i$, $\beta$, $\gamma$ | 0.305 | 0.536 | 0.416 | 0.325 | 0.509 |
| $\beta = \gamma = 0$ | 0.147 | 0.308 | 0.039 | 0.017 | 0.012 |
| $\gamma = 0.5$ | 0.792 | 0.652 | 0.150 | 0.111 | 0.173 |
| $\gamma = 0$ | 0.112 | 0.147 | 0.012 | 0.005 | 0.003 |

## Table IA.2: Expected returns in excess of the market of S&P 100 stock portfolios sorted by SVIX

This Table presents results from regressing portfolio returns in excess of the market on the portfolio's risk-neutral variance measured relative to portfolios' average risk-neutral variance $\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$. At the end of every month, we sort S&P 100 firms into 25 portfolios based on their $\text{SVIX}_{i,t}^2$. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\text{SVIX}_{i,t}^2$ and $\overline{\text{SVIX}}_t^2$. Panel A reports estimates of the pooled panel regression specified in equation (18),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with portfolio fixed effects specified in equation (19),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i \alpha_i$ reports the time-series average of the value-weighted sum of portfolio fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept and $\gamma = 0.5$) and for a test whether $\gamma$ is equal to zero. For the pooled panel regressions, the row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| *Panel A. Pooled panel regressions* | | | | | |
| $\alpha$ | 0.008 | 0.008 | 0.006 | 0.008 | 0.011 |
| | (0.014) | (0.013) | (0.014) | (0.015) | (0.015) |
| $\gamma$ | 0.514 | 0.569 | 0.768 | 0.754 | 0.672 |
| | (0.268) | (0.309) | (0.308) | (0.287) | (0.220) |
| Adjusted $R^2$ (%) | 1.043 | 2.826 | 7.690 | 11.814 | 12.436 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.831 | 0.764 | 0.496 | 0.347 | 0.265 |
| $H_0 : \gamma = 0.5$ | 0.957 | 0.822 | 0.385 | 0.376 | 0.433 |
| $H_0 : \gamma = 0$ | 0.055 | 0.065 | 0.013 | 0.009 | 0.002 |
| Theory adj-$R^2$ (%) | 1.023 | 2.714 | 6.593 | 10.093 | 10.840 |
| *Panel B. Panel regressions with portfolio fixed effects* | | | | | |
| $\sum_i w_i \alpha_i$ | 0.013 | 0.013 | 0.012 | 0.012 | 0.014 |
| | (0.014) | (0.013) | (0.014) | (0.015) | (0.015) |
| $\gamma$ | 0.731 | 0.860 | 1.232 | 1.096 | 0.869 |
| | (0.343) | (0.436) | (0.395) | (0.363) | (0.308) |
| Adjusted $R^2$ (%) | 1.333 | 4.123 | 11.017 | 15.493 | 16.729 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.498 | 0.413 | 0.089 | 0.105 | 0.283 |
| $H_0 : \gamma = 0.5$ | 0.501 | 0.408 | 0.064 | 0.100 | 0.232 |
| $H_0 : \gamma = 0$ | 0.033 | 0.048 | 0.002 | 0.003 | 0.005 |

## Table IA.3: Expected excess returns of S&P 500 stock portfolios sorted by SVIX

This Table presents results from regressing portfolio equity excess returns on the risk-neutral variance of the market variance ($\text{SVIX}_t^2$) and on the portfolio's risk-neutral variance measured relative to portfolios' average risk-neutral variance $\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$. At the end of every month, we sort S&P 500 firms into 100 portfolios based on their $\text{SVIX}_{i,t}^2$. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\text{SVIX}_t^2$, $\text{SVIX}_{i,t}^2$, and $\overline{\text{SVIX}}_t^2$. Panel A reports estimates of the pooled panel regression specified in equation (20),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\,\text{SVIX}_t^2 + \gamma\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports results for the panel regression with portfolio fixed effects specified in equation (21),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\,\text{SVIX}_t^2 + \gamma\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i \alpha_i$ reports the time-series average of the value-weighted sum of portfolio fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept, $\beta = 1$, and $\gamma = 0.5$) and for tests whether $\beta$ and $\gamma$ are equal to zero. For the pooled panel regressions, the row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| | | *Panel A. Pooled panel regressions* | | | |
| $\alpha$ | 0.058 | 0.022 | -0.037 | -0.021 | -0.055 |
| | (0.074) | (0.079) | (0.059) | (0.071) | (0.077) |
| $\beta$ | 0.693 | 1.813 | 3.420 | 2.978 | 3.856 |
| | (2.318) | (2.409) | (1.562) | (1.614) | (1.813) |
| $\gamma$ | 0.254 | 0.285 | 0.446 | 0.490 | 0.327 |
| | (0.296) | (0.285) | (0.318) | (0.313) | (0.200) |
| Adjusted $R^2$ (%) | 0.236 | 1.421 | 6.065 | 8.247 | 12.003 |
| $\alpha$, $\beta$, $\gamma$ | 0.308 | 0.206 | 0.168 | 0.190 | 0.021 |
| $\beta = \gamma = 0$ | 0.686 | 0.593 | 0.077 | 0.104 | 0.040 |
| $\gamma = 0.5$ | 0.405 | 0.451 | 0.865 | 0.975 | 0.386 |
| $\gamma = 0$ | 0.391 | 0.317 | 0.160 | 0.117 | 0.102 |
| Theory adj-$R^2$ (%) | -0.143 | 0.314 | 2.804 | 3.735 | 3.833 |
| | | *Panel B. Panel regressions with portfolio fixed effects* | | | |
| $\sum w_i \alpha_i$ | 0.063 | 0.027 | -0.023 | -0.009 | -0.051 |
| | (0.069) | (0.072) | (0.054) | (0.067) | (0.073) |
| $\beta$ | 0.571 | 1.705 | 3.168 | 2.740 | 3.779 |
| | (2.245) | (2.293) | (1.459) | (1.506) | (1.714) |
| $\gamma$ | 0.426 | 0.429 | 0.772 | 0.796 | 0.423 |
| | (0.477) | (0.500) | (0.551) | (0.552) | (0.384) |
| Adjusted $R^2$ (%) | 1.084 | 4.389 | 11.833 | 18.108 | 24.333 |
| $\sum w_i \alpha_i$, $\beta$, $\gamma$ | 0.438 | 0.402 | 0.269 | 0.255 | 0.017 |
| $\beta = \gamma = 0$ | 0.655 | 0.668 | 0.082 | 0.134 | 0.085 |
| $\gamma = 0.5$ | 0.877 | 0.886 | 0.622 | 0.591 | 0.841 |
| $\gamma = 0$ | 0.371 | 0.391 | 0.161 | 0.149 | 0.271 |

## Table IA.4: Expected returns in excess of the market of S&P 500 stock portfolios sorted by SVIX

This Table presents results from regressing portfolio returns in excess of the market on the portfolio's risk-neutral variance measured relative to portfolios' average risk-neutral variance $\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right)$. At the end of every month, we sort S&P 500 firms into 100 portfolios based on their $\mathrm{SVIX}_{i,t}^2$. The data is monthly from January 1996 to October 2014. The column labels indicate the return horizons ranging from one month to two years. The return horizons match the maturities of the options used to compute $\mathrm{SVIX}_{i,t}^2$ and $\overline{\mathrm{SVIX}}_t^2$. Panel A reports estimates of the pooled panel regression specified in equation (18),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with portfolio fixed effects specified in equation (19),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i \alpha_i$ reports the time-series average of the value-weighted sum of portfolio fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept and $\gamma = 0.5$) and for a test whether $\gamma$ is equal to zero. For the pooled panel regressions, the row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| Horizon | 30 days | 91 days | 182 days | 365 days | 730 days |
|---|---|---|---|---|---|
| *Panel A. Pooled panel regressions* | | | | | |
| $\alpha$ | 0.015 | 0.016 | 0.013 | 0.014 | 0.018 |
| | (0.014) | (0.015) | (0.016) | (0.019) | (0.019) |
| $\gamma$ | 0.345 | 0.399 | 0.537 | 0.533 | 0.357 |
| | (0.286) | (0.271) | (0.304) | (0.297) | (0.202) |
| Adjusted $R^2$ (%) | 0.506 | 1.635 | 4.517 | 7.087 | 5.125 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.552 | 0.537 | 0.626 | 0.619 | 0.654 |
| $H_0 : \gamma = 0.5$ | 0.586 | 0.710 | 0.904 | 0.912 | 0.479 |
| $H_0 : \gamma = 0$ | 0.228 | 0.140 | 0.077 | 0.073 | 0.078 |
| Theory adj-$R^2$ (%) | 0.378 | 1.403 | 4.243 | 6.553 | 3.509 |
| *Panel B. Panel regressions with portfolio fixed effects* | | | | | |
| $\sum_i w_i \alpha_i$ | 0.019 | 0.021 | 0.020 | 0.019 | 0.019 |
| | (0.014) | (0.014) | (0.016) | (0.019) | (0.020) |
| $\gamma$ | 0.630 | 0.705 | 0.993 | 0.898 | 0.516 |
| | (0.437) | (0.445) | (0.479) | (0.478) | (0.351) |
| Adjusted $R^2$ (%) | 0.785 | 2.686 | 7.289 | 10.396 | 8.080 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.362 | 0.271 | 0.180 | 0.257 | 0.578 |
| $H_0 : \gamma = 0.5$ | 0.766 | 0.644 | 0.303 | 0.406 | 0.964 |
| $H_0 : \gamma = 0$ | 0.149 | 0.113 | 0.038 | 0.060 | 0.141 |

Table IA.5: Excess returns of characteristics/$\text{SVIX}_{i,t}$ double-sorted portfolios

This Table presents results from regressing portfolio equity excess returns on the risk-neutral variance of the market variance ($\text{SVIX}_t^2$) and on the portfolio's risk-neutral variance measured relative to portfolios' average risk-neutral variance $\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$. At the end of each month, we sort S&P 500 firms into 5x5-double sorted portfolios based on firm characteristics and $\text{SVIX}_{i,t}$. We first assign firms to quintile portfolios based on their CAPM beta, size, book-to-market, or momentum. In the second step, we sort stocks within each of the characteristics portfolios into $\text{SVIX}_{i,t}$-quintiles, providing us with a total of 25 conditionally double-sorted portfolios. The one-year horizon of the portfolio returns matches the 365 day-maturity of the options used to compute $\text{SVIX}_{i,t}^2$ and $\overline{\text{SVIX}}_t^2$. Panel A reports estimates of the pooled panel regression specified in equation (20),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\, \text{SVIX}_t^2 + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports results for the panel regression with portfolio fixed effects specified in equation (21),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\, \text{SVIX}_t^2 + \gamma \left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i \alpha_i$ reports the time-series average of the value-weighted sum of portfolio fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept, $\beta = 1$, and $\gamma = 0.5$) and for tests whether $\beta$ and $\gamma$ are equal to zero. For the pooled panel regressions, the row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled panel regressions* | | | | |
| $\alpha$ | -0.020 | -0.021 | -0.021 | -0.021 |
| | (0.071) | (0.071) | (0.071) | (0.071) |
| $\beta$ | 2.974 | 2.963 | 3.024 | 2.960 |
| | (1.603) | (1.600) | (1.619) | (1.606) |
| $\gamma$ | 0.450 | 0.520 | 0.446 | 0.511 |
| | (0.326) | (0.340) | (0.342) | (0.335) |
| Adjusted $R^2$ (%) | 9.184 | 9.879 | 9.178 | 10.036 |
| $\alpha$, $\beta$, $\gamma$ | 0.170 | 0.203 | 0.159 | 0.185 |
| $\beta = \gamma = 0$ | 0.119 | 0.107 | 0.127 | 0.116 |
| $\gamma = 0.5$ | 0.877 | 0.954 | 0.874 | 0.983 |
| $\gamma = 0$ | 0.168 | 0.126 | 0.193 | 0.141 |
| Theory adj-$R^2$ (%) | 3.468 | 4.237 | 3.152 | 3.943 |
| *Panel B. Panel regressions with portfolio fixed effects* | | | | |
| $\sum w_i \alpha_i$ | -0.014 | -0.019 | -0.019 | -0.009 |
| | (0.068) | (0.072) | (0.069) | (0.067) |
| $\beta$ | 2.790 | 2.723 | 2.908 | 2.756 |
| | (1.502) | (1.503) | (1.563) | (1.525) |
| $\gamma$ | 0.688 | 0.826 | 0.593 | 0.772 |
| | (0.554) | (0.599) | (0.563) | (0.542) |
| Adjusted $R^2$ (%) | 21.174 | 22.404 | 21.481 | 21.908 |
| $\sum w_i \alpha_i$, $\beta$, $\gamma$ | 0.250 | 0.314 | 0.232 | 0.249 |
| $\beta = \gamma = 0$ | 0.153 | 0.132 | 0.152 | 0.133 |
| $\gamma = 0.5$ | 0.734 | 0.586 | 0.868 | 0.616 |
| $\gamma = 0$ | 0.214 | 0.168 | 0.292 | 0.154 |

## Table IA.6: Excess-of-market returns of $\text{SVIX}_{i,t}$/characteristics double-sorted portfolios

This Table presents results from regressing portfolio equity returns in excess of the market on the portfolio stock's risk-neutral variance measured relative to stocks' average risk-neutral variance, $\text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t$. The data is monthly from January 1996 to October 2014. At the end of each month, we sort S&P 500 firms into 5x5-double sorted portfolios based on $\text{SVIX}_{i,t}$ and firm characteristics. We first assign firms to quintile portfolios based on $\text{SVIX}_{i,t}$. In the second step, we sort stocks within each $\text{SVIX}_{i,t}$-portfolio in quintiles based on their CAPM beta, size, book-to-market, or momentum, providing us with a total of 25 conditionally double-sorted portfolios. The one-year horizon of the portfolio returns matches the 365 day-maturity of the options used to compute $\text{SVIX}^2_{i,t}$ and $\overline{\text{SVIX}}^2_t$. Panel A reports estimates of the pooled panel regression specified in equation (18),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha + \gamma \left( \text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t \right) + \varepsilon_{i,t+1}.$$

Panel B reports estimates of the panel regression with portfolio fixed effects specified in equation (19),

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left( \text{SVIX}^2_{i,t} - \overline{\text{SVIX}}^2_t \right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i \alpha_i$ reports the time-series average of the value-weighted sum of firm fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept and $\gamma = 0.5$) and for a test whether $\gamma$ is equal to zero. For the pooled panel regressions, the row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled panel regressions* | | | | |
| $\alpha$ | 0.016 | 0.014 | 0.015 | 0.014 |
| | (0.020) | (0.020) | (0.020) | (0.019) |
| $\gamma$ | 0.455 | 0.565 | 0.507 | 0.555 |
| | (0.332) | (0.334) | (0.328) | (0.341) |
| Adjusted $R^2$ (%) | 6.343 | 8.954 | 7.967 | 9.223 |
| $H_0 : \alpha = 0, \gamma = 0.5$ | 0.652 | 0.596 | 0.633 | 0.629 |
| $H_0 : \gamma = 0.5$ | 0.891 | 0.846 | 0.982 | 0.937 |
| $H_0 : \gamma = 0$ | 0.171 | 0.091 | 0.122 | 0.122 |
| Theory adj-$R^2$ (%) | 5.483 | 8.065 | 7.092 | 7.249 |
| *Panel B. Panel regressions with portfolio fixed effects* | | | | |
| $\sum_i w_i \alpha_i$ | 0.016 | 0.007 | 0.015 | 0.019 |
| | (0.017) | (0.007) | (0.017) | (0.018) |
| $\gamma$ | 0.803 | 0.964 | 0.846 | 0.960 |
| | (0.539) | (0.551) | (0.530) | (0.555) |
| Adjusted $R^2$ (%) | 11.181 | 15.302 | 13.339 | 14.485 |
| $H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$ | 0.377 | 0.264 | 0.384 | 0.216 |
| $H_0 : \gamma = 0.5$ | 0.574 | 0.400 | 0.514 | 0.408 |
| $H_0 : \gamma = 0$ | 0.137 | 0.080 | 0.111 | 0.084 |

Table IA.7: Excess returns of $\mathrm{SVIX}_{i,t}$/characteristics double-sorted portfolios

This Table presents results from regressing portfolio equity excess returns on the risk-neutral variance of the market variance ($\mathrm{SVIX}_t^2$) and on the portfolio's risk-neutral variance measured relative to portfolios' average risk-neutral variance $\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right)$. At the end of each month, we sort S&P 500 firms into 5x5-double sorted portfolios based on $\mathrm{SVIX}_{i,t}$ and firm characteristics. We first assign firms to quintile portfolios based on $\mathrm{SVIX}_{i,t}$. In the second step, we sort stocks within each $\mathrm{SVIX}_{i,t}$-portfolio in quintiles based on their CAPM beta, size, book-to-market, or momentum, providing us with a total of 25 conditionally double-sorted portfolios. The one-year horizon of the portfolio returns matches the 365 day-maturity of the options used to compute $\mathrm{SVIX}_{i,t}^2$ and $\overline{\mathrm{SVIX}}_t^2$. Panel A reports estimates of the pooled panel regression specified in equation (20),

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha + \beta\,\mathrm{SVIX}_t^2 + \gamma\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1}.$$

Panel B reports results for the panel regression with portfolio fixed effects specified in equation (21),

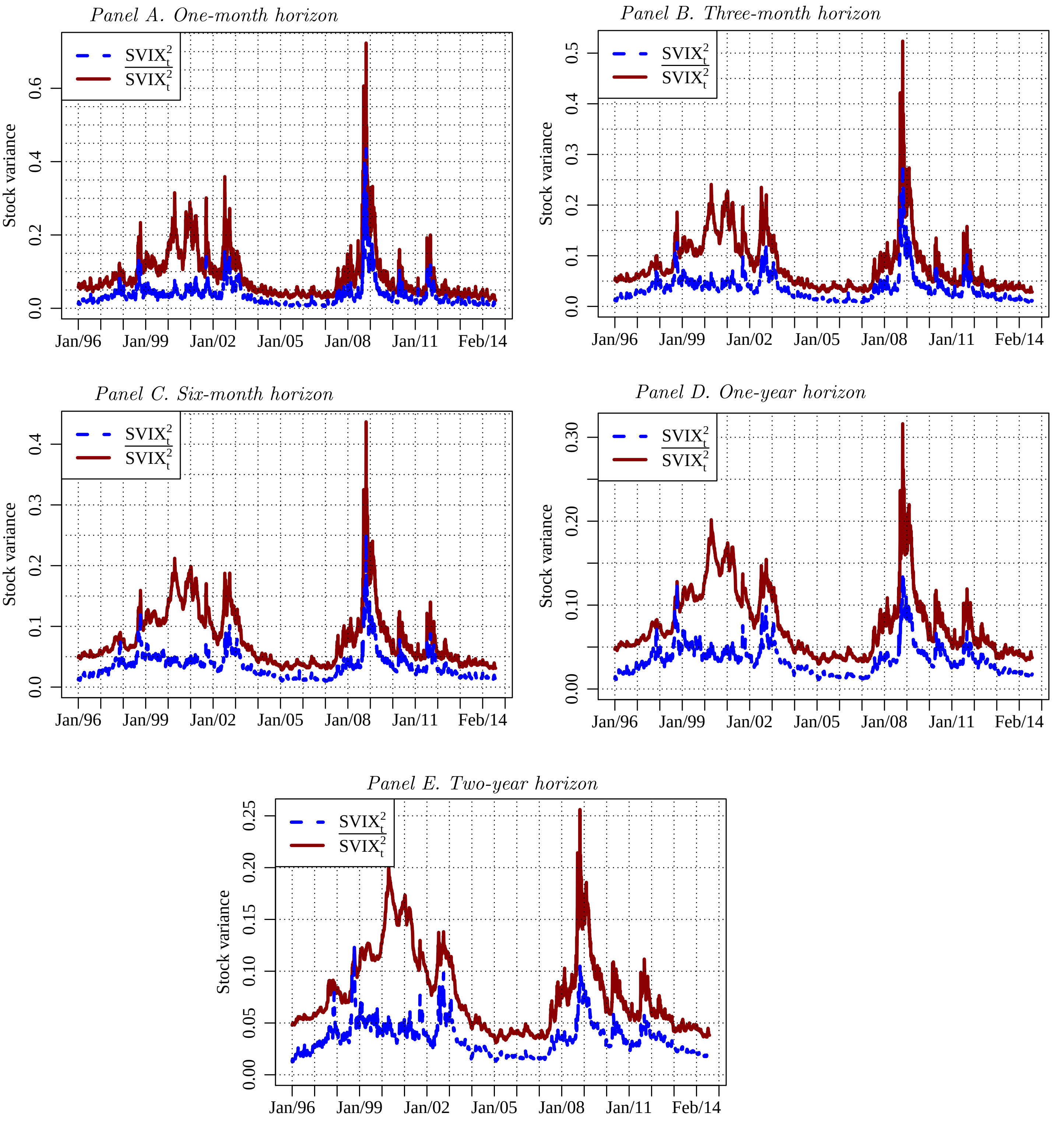
$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\,\mathrm{SVIX}_t^2 + \gamma\left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2\right) + \varepsilon_{i,t+1},$$

where $\sum_i w_i\alpha_i$ reports the time-series average of the value-weighted sum of portfolio fixed effects. Values in parentheses are standard errors obtained from the block bootstrap procedure described in Appendix B. In each panel, we report the regressions' adjusted-$R^2$ and $p$-values of Wald tests testing whether the regression coefficients take the values predicted by our theory (zero intercept, $\beta = 1$, and $\gamma = 0.5$) and for tests whether $\beta$ and $\gamma$ are equal to zero. For the pooled panel regressions, the row labelled 'theory adj-$R^2$ (%)' reports the adjusted-$R^2$ obtained when the coefficients are fixed at the values predicted by our theory.

| | Beta | Size | B/M | Mom |
|---|---|---|---|---|
| *Panel A. Pooled panel regressions* | | | | |
| $\alpha$ | -0.020 | -0.021 | -0.021 | -0.021 |
| | (0.071) | (0.071) | (0.071) | (0.071) |
| $\beta$ | 3.007 | 2.971 | 3.011 | 2.968 |
| | (1.621) | (1.607) | (1.627) | (1.605) |
| $\gamma$ | 0.402 | 0.508 | 0.449 | 0.502 |
| | (0.338) | (0.353) | (0.340) | (0.355) |
| Adjusted $R^2$ (%) | 8.243 | 9.433 | 9.091 | 9.556 |
| $\alpha$, $\beta$, $\gamma$ | 0.149 | 0.201 | 0.169 | 0.167 |
| $\beta = \gamma = 0$ | 0.136 | 0.114 | 0.124 | 0.128 |
| $\gamma = 0.5$ | 0.773 | 0.982 | 0.882 | 0.935 |
| $\gamma = 0$ | 0.234 | 0.150 | 0.186 | 0.184 |
| Theory adj-$R^2$ (%) | 2.403 | 3.745 | 3.056 | 3.292 |
| *Panel B. Panel regressions with portfolio fixed effects* | | | | |
| $\sum w_i\alpha_i$ | -0.013 | -0.020 | -0.015 | -0.007 |
| | (0.068) | (0.071) | (0.067) | (0.067) |
| $\beta$ | 2.796 | 2.719 | 2.809 | 2.700 |
| | (1.528) | (1.499) | (1.538) | (1.496) |
| $\gamma$ | 0.678 | 0.831 | 0.709 | 0.845 |
| | (0.573) | (0.638) | (0.588) | (0.616) |
| Adjusted $R^2$ (%) | 20.408 | 21.957 | 21.623 | 21.596 |
| $\sum w_i\alpha_i$, $\beta$, $\gamma$ | 0.250 | 0.347 | 0.269 | 0.254 |
| $\beta = \gamma = 0$ | 0.158 | 0.140 | 0.151 | 0.140 |
| $\gamma = 0.5$ | 0.756 | 0.603 | 0.722 | 0.576 |
| $\gamma = 0$ | 0.237 | 0.192 | 0.228 | 0.170 |

Figure IA.1: Option-implied equity variance of S&P 100 firms

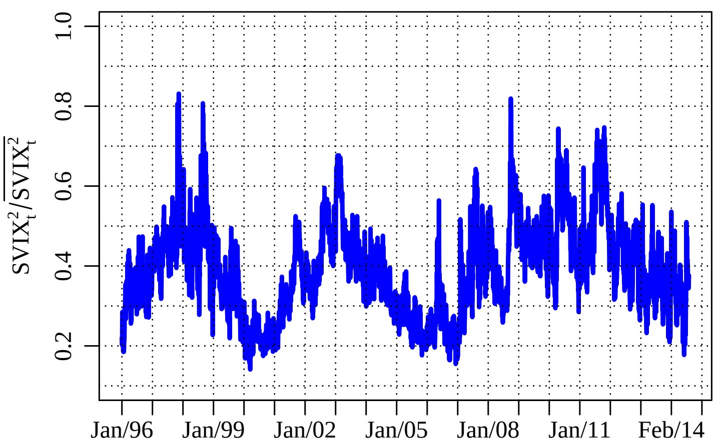
This Figure plots the time-series of the risk-neutral variance of the market ($\mathrm{SVIX}_t^2$) and of stocks' average risk-neutral variance ($\overline{\mathrm{SVIX}}_t^2$). We compute $\mathrm{SVIX}_t^2$ from equity index options on the S&P 100. $\overline{\mathrm{SVIX}}_t^2$ is the value-weighted sum of S&P 100 stocks' risk-neutral variance computed from individual firm equity options. Panels A through E present the variance series implied by equity options with maturities of one, three, six, 12, and 24 months. The data is daily from January 1996 to October 2014.



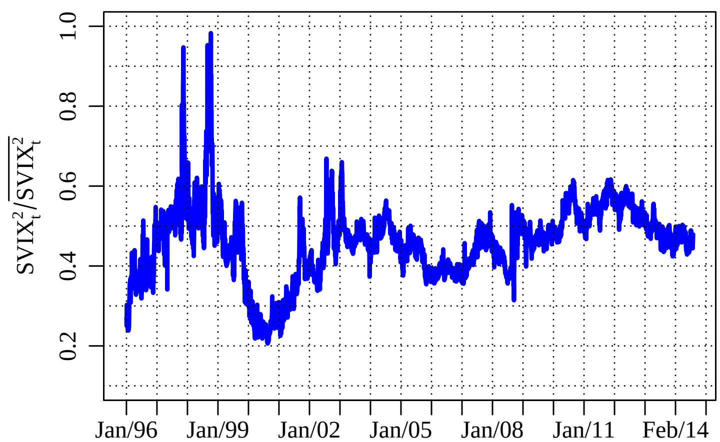
## Figure IA.2: A measure of average risk-neutral correlation between stocks

This Figure plots the time-series of the ratio of the risk-neutral variance of the market to stocks' average risk-neutral variance ($\text{SVIX}_t^2/\overline{\text{SVIX}}_t^2$); in the appendix, we show that this quantity is an approximate measure of average risk-neutral correlation. We compute $\text{SVIX}_t^2$ from equity index options on the S&P 100 (Panels A and B) and S&P 500 (Panels C and D). $\overline{\text{SVIX}}_t^2$ is the corresponding value-weighted sum of S&P 100 or S&P 500 stocks' risk-neutral variance computed from individual firm equity options. The data is daily from January 1996 to October 2014.

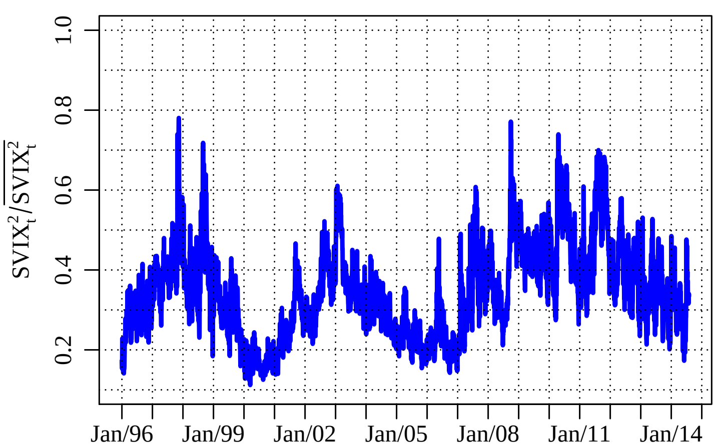
*Panel A. S&P 100, one-month horizon*




*Panel B. S&P 100, one-year horizon*




*Panel C. S&P 500, one-month horizon*




*Panel D. S&P 500, one-year horizon*

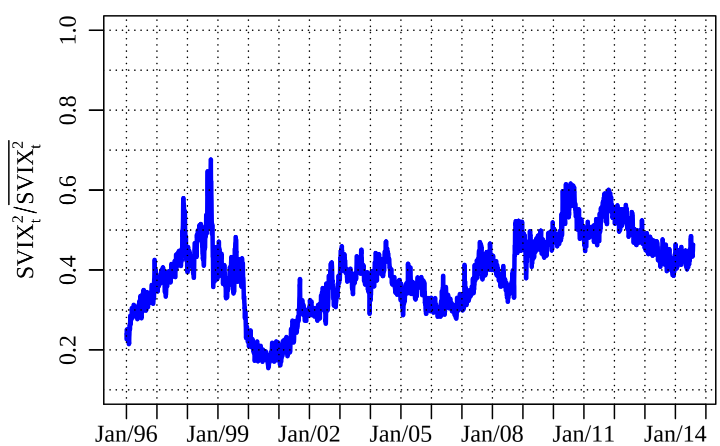


Figure IA.3: Beta, size, value, momentum, and option-implied equity variance

This Figure reports (equally-weighted) averages of risk-neutral stock variance ($\text{SVIX}_{i,t}^2$, computed from individual firm equity options) of S&P 500 stocks, conditional on firm beta, size, book-to-market, and momentum. At every date $t$, we assign stocks to decile portfolios based on on their characteristics and report the time-series averages of $\text{SVIX}_{i,t}^2$ across deciles. The horizon is one month. The sample period is January 1996 to October 2014.

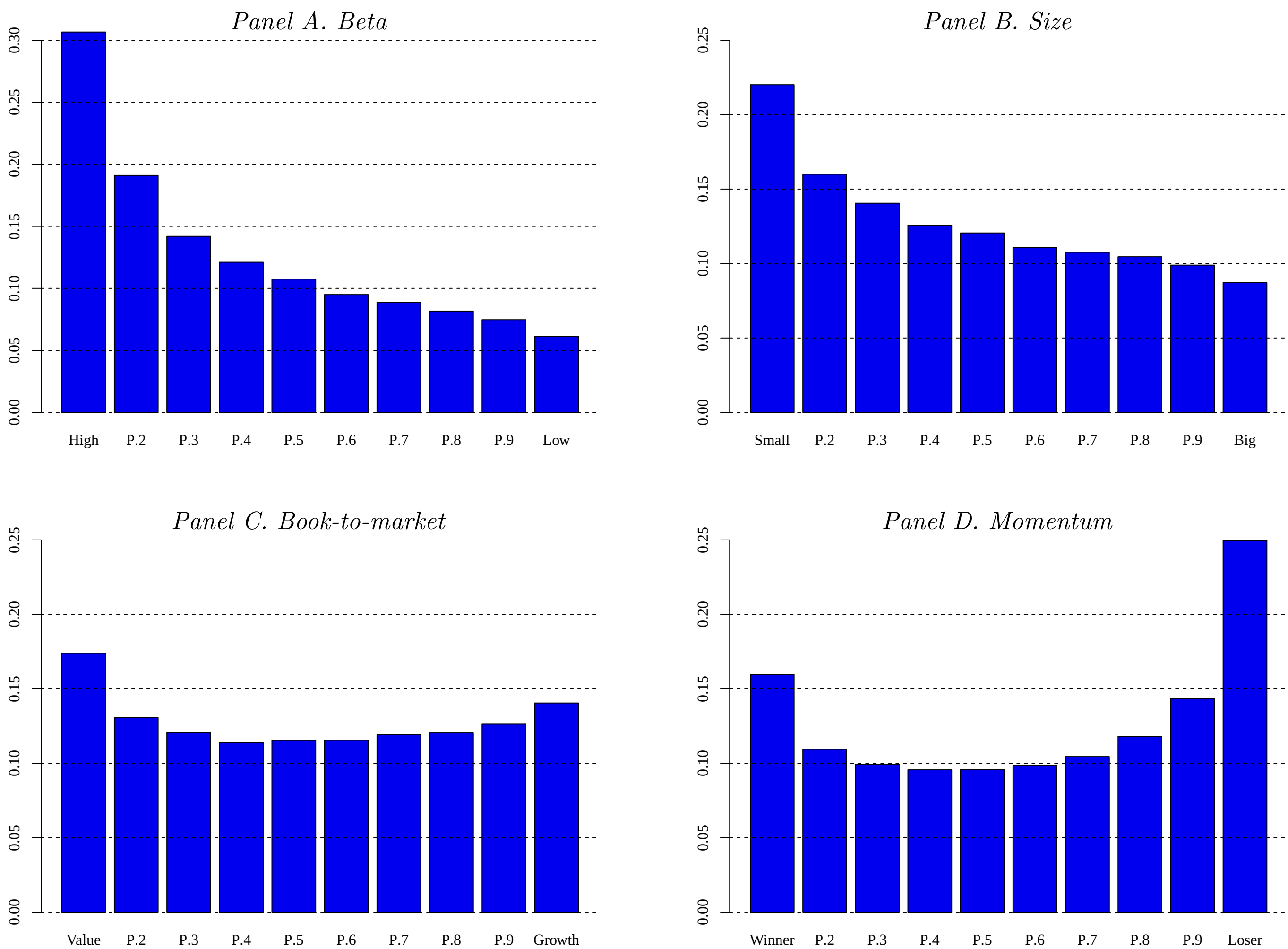
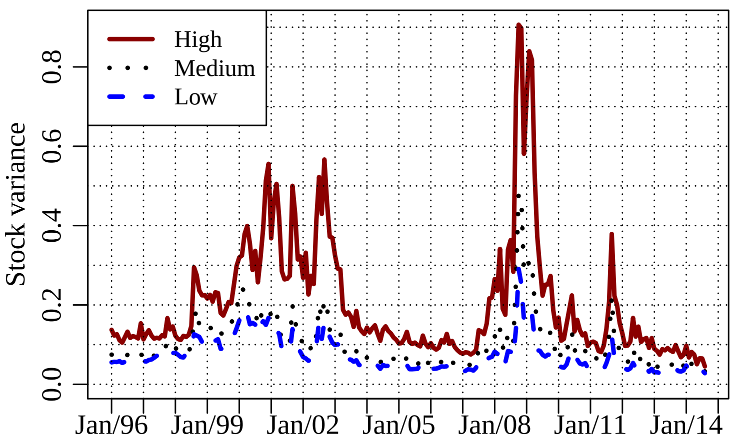


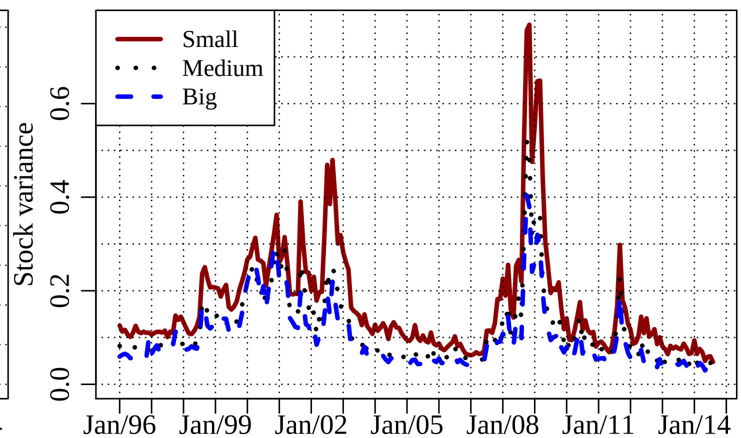
Figure IA.4: Beta, size, value, momentum, and option-implied equity variance

This Figure plots the time-series of risk-neutral stock variance ($SVIX^2_{i,t}$, computed from individual firm equity options) of S&P 500 stocks, conditional on firm beta, size, book-to-market, and momentum. At each date $t$, we classify firms as small, medium, or big when their market capitalization is in the bottom, middle, or top tertile of the time-$t$ distribution across all firms in our sample, and compute the (equally-weighted) average of $SVIX^2_{i,t}$. We classify firms by other characteristics at time $t$ in a similar way. The horizon is monthly. The sample period is from January 1996 to October 2014.

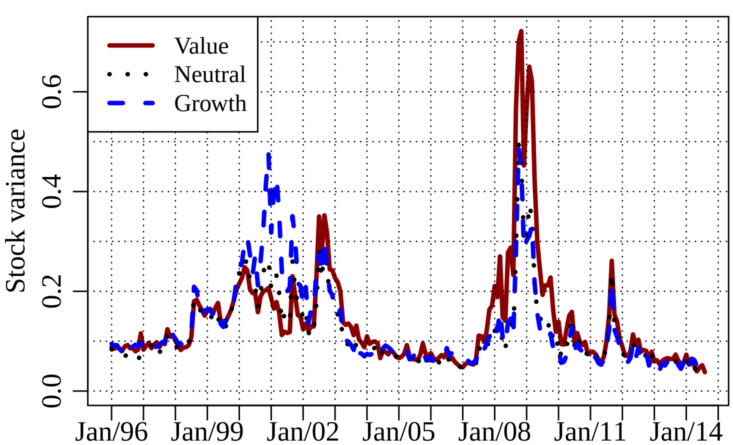
*Panel A. Beta*




*Panel B. Size*




*Panel C. Book-to-market*




*Panel D. Momentum*

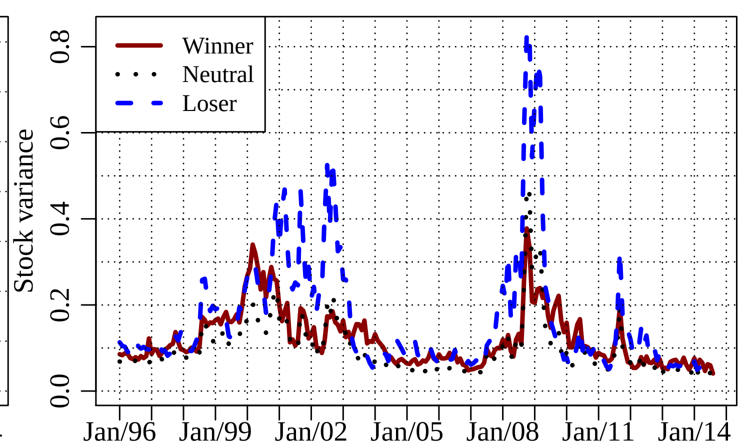


Figure IA.5: Size, value, and option-implied equity variance

This Figure plots the time-series of risk-neutral stock variance ($\mathrm{SVIX}^2_{i,t}$, computed from individual firm equity options) of S&P 500 stocks, conditional on firm size and book-to-market. At each date $t$, we classify firms as small, medium, or big when their market capitalization is in the bottom, middle, or top tertile of the time-$t$ distribution across all firms in our sample, and compute the (equally-weighted) average of $\mathrm{SVIX}^2_{i,t}$. Similarly, we classify firms as value, neutral, or growth stocks when their book-to-market ratio is within the top, middle, or bottom tertile of the book-to-market distribution at time $t$. Panels A and B plot the time-series of $\mathrm{SVIX}^2_{i,t}$-averages for intersections of size and value tertiles. The horizon is annual. The sample period is from January 1996 to October 2014.

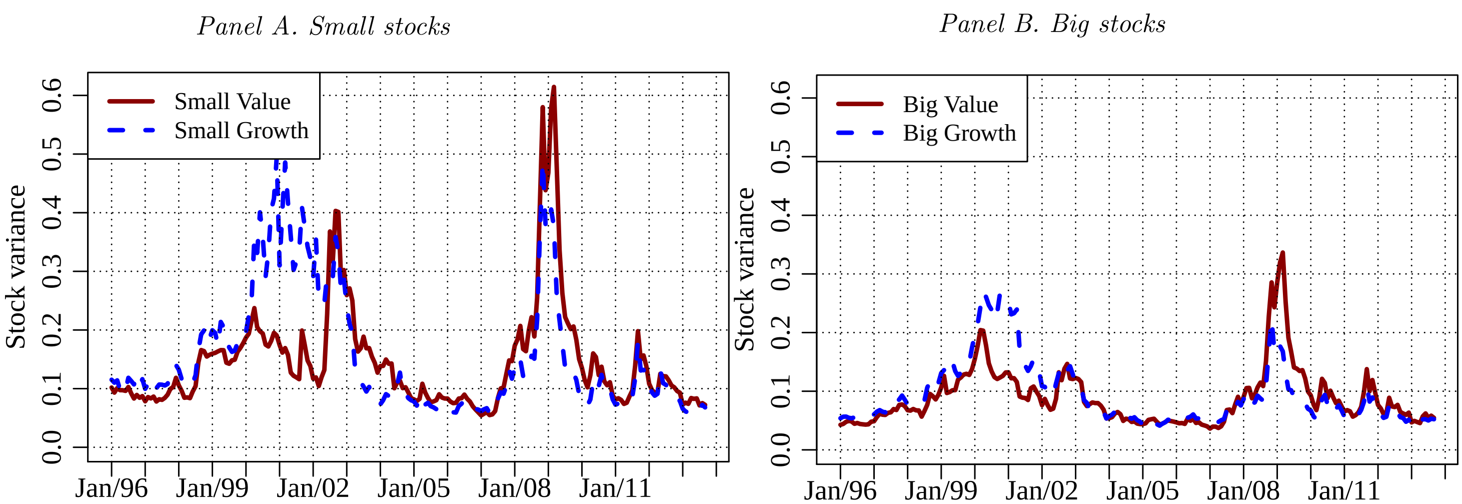
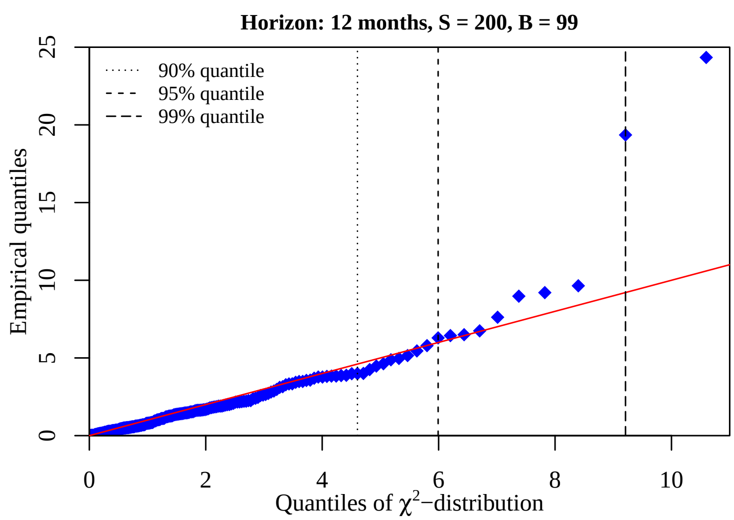


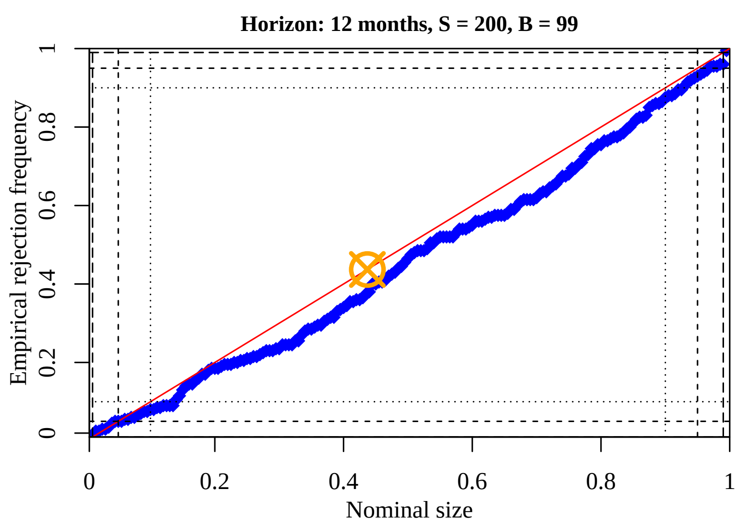
Figure IA.6: Finite-sample properties of the block bootstrap procedure

To provide evidence for the reliability of our bootstrap procedure in finite samples, we simulate $S = 200$ samples on which we impose the null hypothesis ($\alpha = 0$ and $\gamma = 0.5$) and within each sample we repeat the bootstrap procedure described in Appendix B.1 with $B = 99$ iterations. We present results for one-year excess-of-market returns of S&P 100 firms, sampled at a monthly frequency. In Panel A, we compare the empirical quantiles of the Wald statistic in the simulated data to the quantiles of the Wald statistic's asymptotic $\chi^2$-distribution. In Panel B, we compare the rejection frequency for the null hypothesis in the simulated data (on which we imposed the null hypothesis) to the nominal size of the test. The big circle with cross indicates the $p$-value of 0.437 we obtain from applying the bootstrap procedure to the empirical data, as reported in Panel A of Table II.

*Panel A: Empirical and asymptotic quantiles of the Wald statistic*

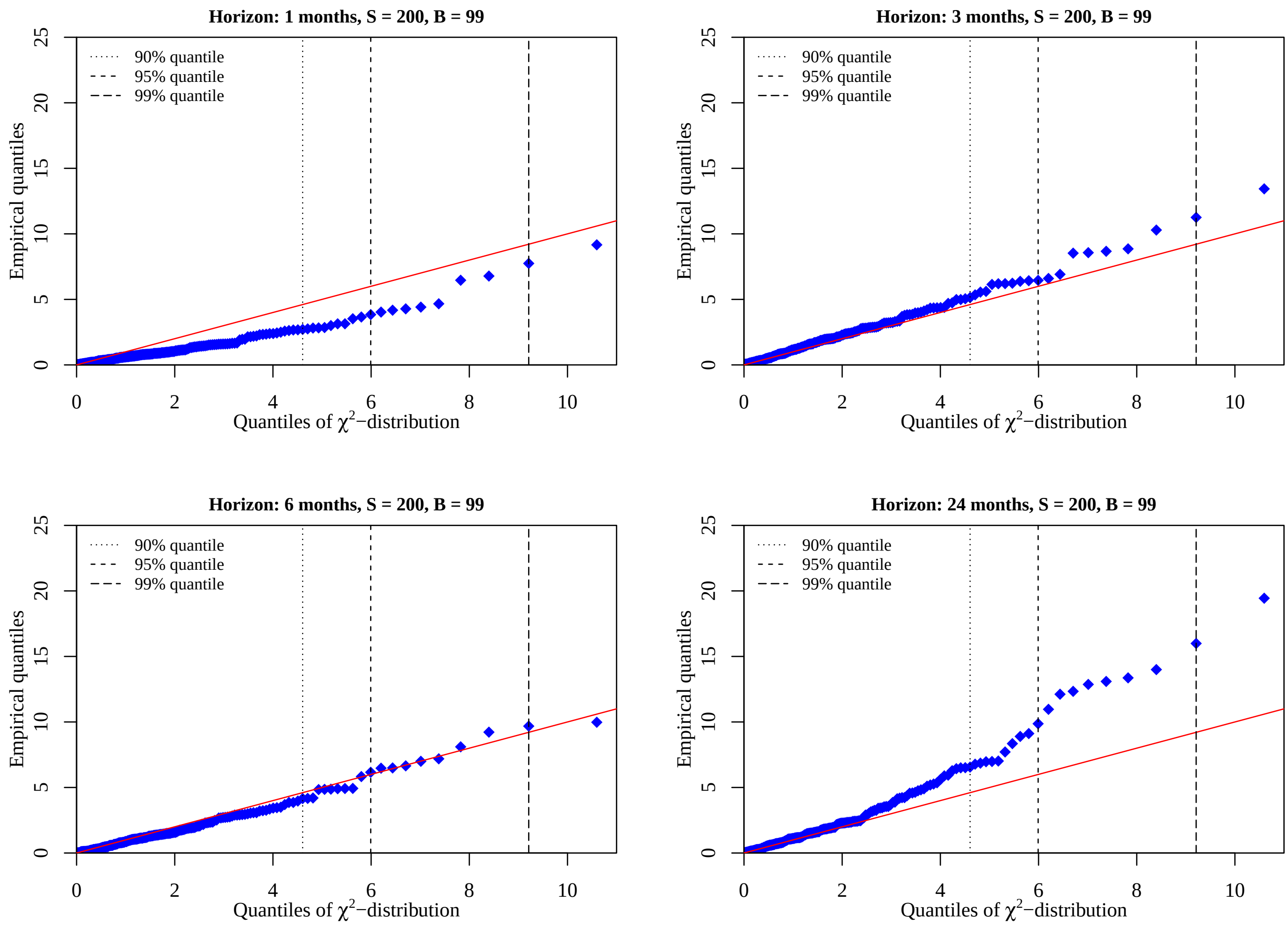



*Panel B: Nominal size and empirical rejection frequencies*



## Figure IA.7: Empirical and asymptotic quantiles of the Wald statistic

To provide evidence for the reliability of our bootstrap procedure in finite samples, we simulate $S = 200$ samples on which we impose the null hypothesis ($\alpha = 0$ and $\gamma = 0.5$) and within each sample we repeat the bootstrap procedure described in Appendix B.1 with $B = 99$ iterations. We present results for one-, three-, six-, and 24-month excess-of-market returns of S&P 100 firms, sampled at a monthly frequency. For each horizon, wee compare the empirical quantiles of the Wald statistic in the simulated data to the quantiles of the Wald statistic's asymptotic $\chi^2$-distribution.



## Figure IA.8: Nominal size and empirical rejection frequencies

To provide evidence for the reliability of our bootstrap procedure in finite samples, we simulate $S = 200$ samples on which we impose the null hypothesis ($\alpha = 0$ and $\gamma = 0.5$) and within each sample we repeat the bootstrap procedure described in Appendix B.1 with $B = 99$ iterations. We present results for one-, three-, six-, and 24-month excess-of-market returns of S&P 100 firms, sampled at a monthly frequency. For each horizon, we compare the rejection frequency for the null hypothesis in the simulated data (on which we imposed the null hypothesis) to the nominal size of the test. The big circles with crosses indicates the $p$-value we obtain from applying the bootstrap procedure to the empirical data, as reported in Panel A of Table II: 0.841 at the one-month horizon, 0.832 at the three-month horizon, 0.609 at the six-month horizon, and 0.439 at the 24-month horizon.

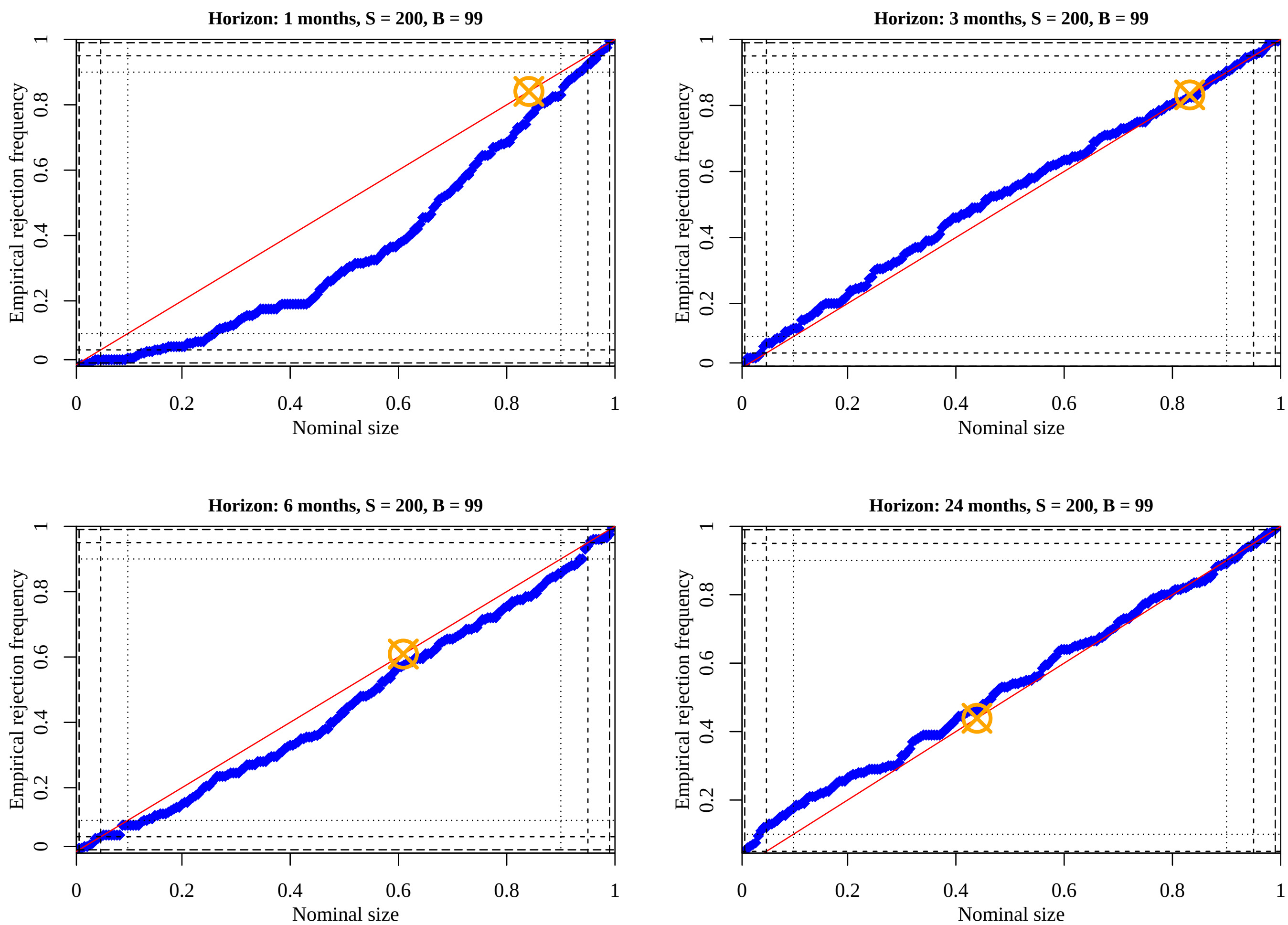


Figure IA.9: Nominal size and empirical rejection frequencies ($S = 400$, $B = 198$)

To provide evidence for the reliability of our bootstrap procedure in finite samples, we simulate $S = 400$ samples on which we impose the null hypothesis ($\alpha = 0$ and $\gamma = 0.5$) and within each sample we repeat the bootstrap procedure described in Appendix B.1 with $B = 198$ iterations. We present results for one-, three-, six-, 12- and 24-month excess-of-market returns of S&P 100 firms, sampled at a monthly frequency. For each horizon, we compare the rejection frequency for the null hypothesis in the simulated data (on which we imposed the null hypothesis) to the nominal size of the test. The big circles with crosses indicates the $p$-value we obtain from applying the bootstrap procedure to the empirical data, as reported in Panel A of Table II: 0.841 at the one-month horizon, 0.832 at the three-month horizon, 0.609 at the six-month horizon, 0.437 at the 12-month horizon, and 0.439 at the 24-month horizon.

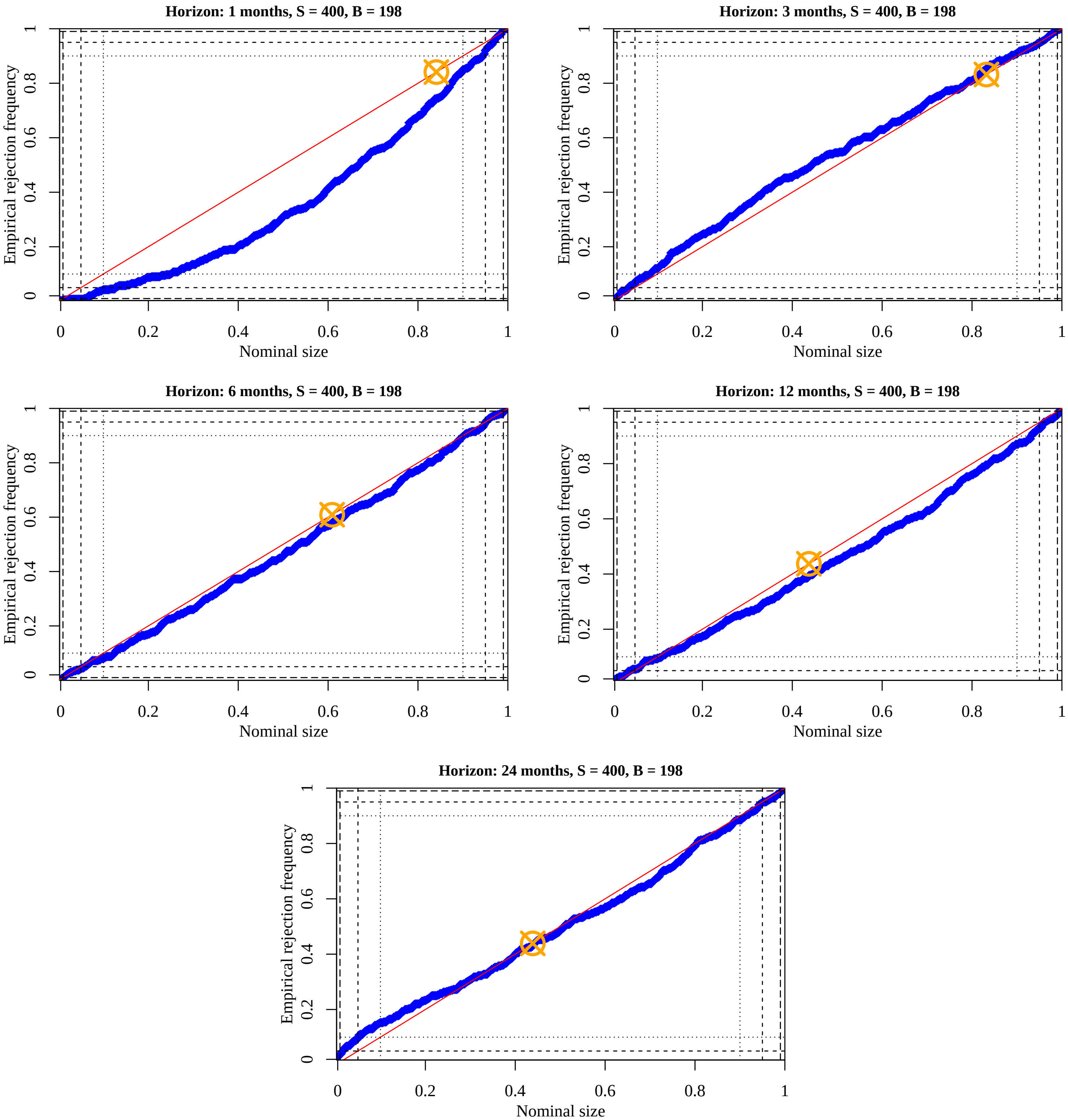
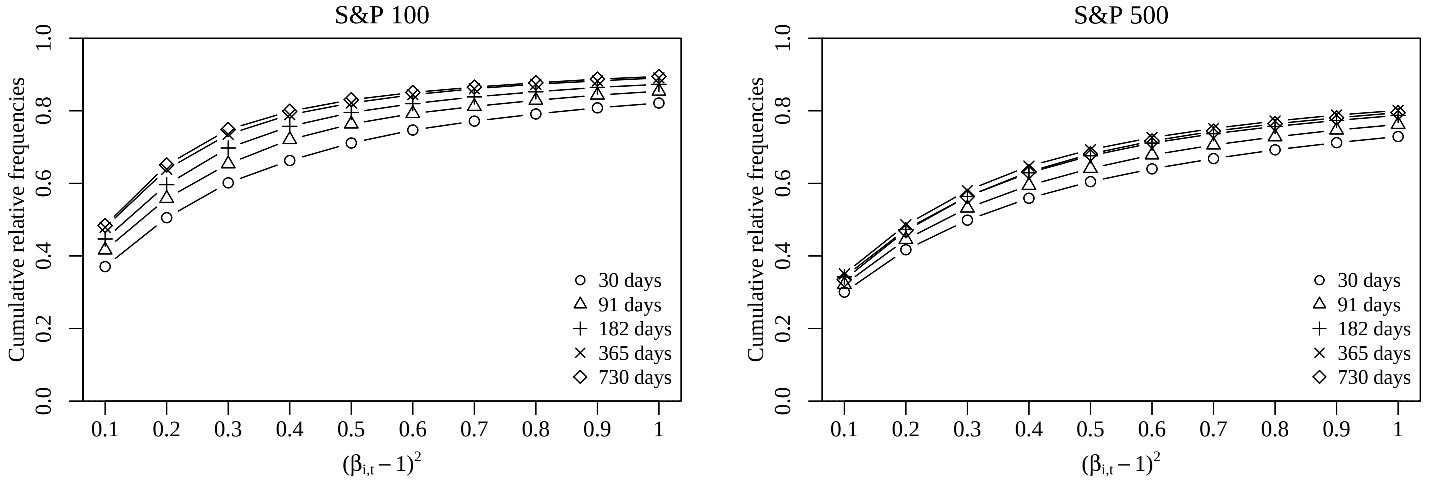


Figure IA.10: Beta Linearization and Approximation Errors

The figures below proxy $\mathrm{var}^*_t(R_{g,t+1}/R_{f,t+1})$ by $\mathrm{var}^*_t(R_{m,t+1}/R_{f,t+1})$ and use equation (7) to compute a firm's implied beta as $\beta^*_{i,t} = 1 + \frac{1}{2}(\mathrm{SVIX}^2_{i,t} - \overline{\mathrm{SVIX}}^2_t)/\mathrm{SVIX}^2_t$ so that, from equation (IA.A.5), approximation error $= -\frac{1}{2}\,\mathrm{SVIX}^2_t[(\beta^*_{i,t}-1)^2 - \sum_j w_{j,t}(\beta^*_{j,t}-1)^2]$. We compute implied betas and approximation errors for all S&P 100 and S&P 500 firms each month from January 1996 to October 2014 for horizons of one, three, six, 12, and 24 months. Panel A reports results for squared deviations of betas from one. Panel B illustrates approximation errors in a range of plus/minus 2.5% *p.a.*

*Panel A. Beta deviations*




*Panel B. Approximation errors*