What is the Expected Return on a Stock?

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What is the expected return on a stock?

- In a factor model, $\mathbb{E}_t R_{i,t+1} - R_{f,t+1} = \sum_{j=1}^{K} \beta_{i,t}^{(j)} \lambda_t^{(j)}$
  - Eg, in the CAPM, $\mathbb{E}_t R_{i,t+1} - R_{f,t+1} = \beta_{i,t}^{(m)} \left(\mathbb{E}_t R_{m,t+1} - R_{f,t+1} \right)$
- But how to measure factor loadings $\beta_{i,t}^{(j)}$ and factor risk premia $\lambda_t^{(j)}$?
- No theoretical or empirical reason to expect either to vary smoothly, given that news sometimes arrives in bursts
  - Scheduled (or unscheduled) release of firm-specific or macro data, monetary or fiscal policy, LTCM, Lehman, Trump, Brexit, Black Monday, 9/11, war, virus, earthquake, nuclear disaster...
  - Level of concern / market focus associated with different types of events can also vary over time

What is the expected return on a stock?

Not easy even in the CAPM

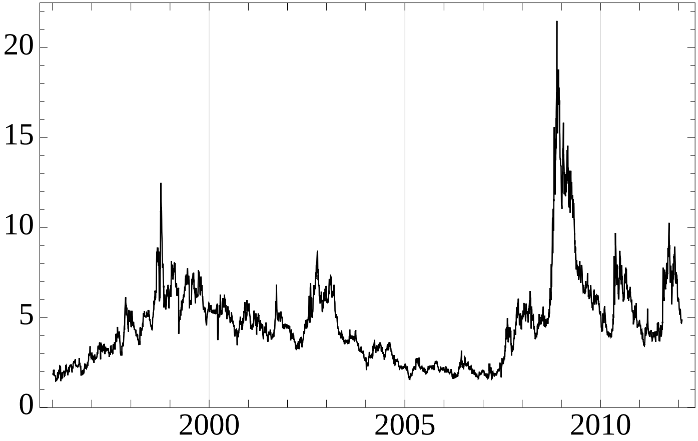

Figure: Martin (2017, *QJE*, "What is the Expected Return on the Market?")

What we do

- We derive a formula for a stock's expected excess return:

$$\mathbb{E}_t \frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \text{SVIX}_t^2 + \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$$

- SVIX indices are similar to VIX and measure risk-neutral volatility
  - market volatility: SVIX_t
  - volatility of stock i: $\text{SVIX}_{i,t}$
  - average stock volatility: $\overline{\text{SVIX}}_t$
- Our approach works in real time at the level of the individual stock
- The formula requires observation of option prices but no estimation
- The formula performs well empirically in and out of sample

What we do

- We derive a formula for a stock's expected return in excess of the market:

$$\mathbb{E}_t \frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$$

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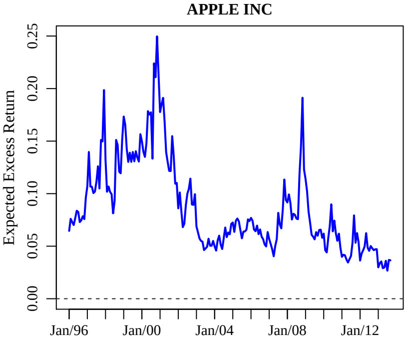
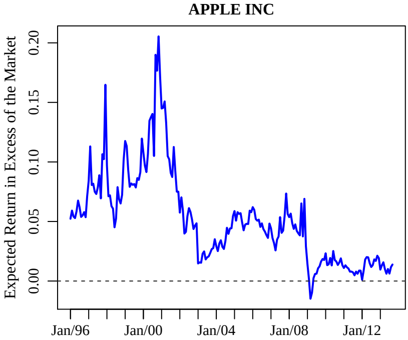
What is the expected return on Apple?

Expected excess returns

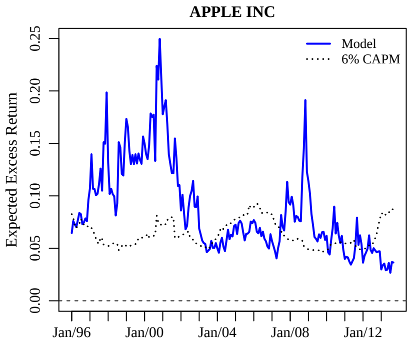


Expected returns in excess of the market

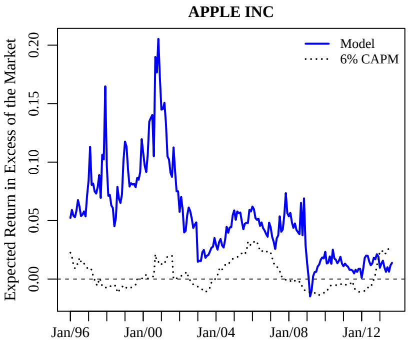


What is the expected return on Apple?

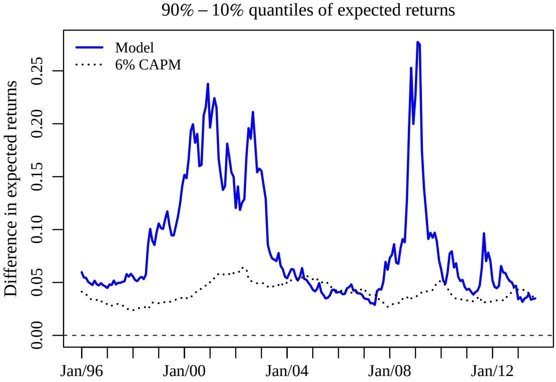
Expected excess returns



Expected returns in excess of the market



Cross-sectional variation in expected returns



- Expected returns based on our model imply much more cross-sectional variation across stocks than benchmark forecasts

Outline

- Where do the formulas come from?
- Construction and properties of volatility indices
- Panel regressions and the relationship with characteristics
- The factor structure of unexpected stock returns
- Out-of-sample analysis

Theory (1)

- $R_{g,t+1}$: the gross return with maximal expected log return
- This growth-optimal return has the special property that $1/R_{g,t+1}$ is a stochastic discount factor (Roll, 1973; Long, 1990)
- Write $\mathbb{E}_t^*$ for the associated risk-neutral expectation,

$$\frac{1}{R_{f,t+1}} \mathbb{E}_t^* X_{t+1} = \mathbb{E}_t \left(\frac{X_{t+1}}{R_{g,t+1}} \right)$$

- Using the fact that $\mathbb{E}_t^* R_{i,t+1} = R_{f,t+1}$ for any gross return $R_{i,t+1}$, this implies the key property of the growth-optimal return that

$$\mathbb{E}_t \frac{R_{i,t+1}}{R_{f,t+1}} - 1 = \text{cov}_t^* \left(\frac{R_{i,t+1}}{R_{f,t+1}}, \frac{R_{g,t+1}}{R_{f,t+1}} \right)$$

Theory (2)

- For each stock i, we decompose

$$\frac{R_{i,t+1}}{R_{f,t+1}} = \alpha_{i,t} + \beta_{i,t}\frac{R_{g,t+1}}{R_{f,t+1}} + u_{i,t+1} \tag{1}$$

where

$$\beta_{i,t} = \frac{\mathrm{cov}_t^*\left(\frac{R_{i,t+1}}{R_{f,t+1}}, \frac{R_{g,t+1}}{R_{f,t+1}}\right)}{\mathrm{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}}} \tag{2}$$

$$\mathbb{E}_t^* u_{i,t+1} = 0 \tag{3}$$

$$\mathrm{cov}_t^*(u_{i,t+1}, R_{g,t+1}) = 0 \tag{4}$$

- Equations (2) and (3) define $\beta_{i,t}$ and $\alpha_{i,t}$; and (4) follows from (1)–(3)
- Only assumption so far: first and second moments exist and are finite

Theory (3)

- The key property, and the definition of $\beta_{i,t}$, imply that

$$\mathbb{E}_t \frac{R_{i,t+1}}{R_{f,t+1}} - 1 = \beta_{i,t} \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} \tag{5}$$

- We also have, from (1) and (4),

$$\operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} = \beta_{i,t}^2 \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} + \operatorname{var}_t^* u_{i,t+1} \tag{6}$$

- We connect the two by linearizing $\beta_{i,t}^2 \approx 2\beta_{i,t} - 1$, which is appropriate if $\beta_{i,t}$ is sufficiently close to one, i.e. replace (6) with

$$\operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} = (2\beta_{i,t} - 1) \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} + \operatorname{var}_t^* u_{i,t+1} \tag{7}$$

Theory (4)

- Using (5) and (7) to eliminate the dependence on $\beta_{i,t}$,

$$\mathbb{E}_t \frac{R_{i,t+1}}{R_{f,t+1}} - 1 = \frac{1}{2} \operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} + \frac{1}{2} \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} - \frac{1}{2} \operatorname{var}_t^* u_{i,t+1}$$

- Value-weighting,

$$\mathbb{E}_t \frac{R_{m,t+1}}{R_{f,t+1}} - 1 = \frac{1}{2} \sum_j w_{j,t} \operatorname{var}_t^* \frac{R_{j,t+1}}{R_{f,t+1}} + \frac{1}{2} \operatorname{var}_t^* \frac{R_{g,t+1}}{R_{f,t+1}} - \frac{1}{2} \sum_j w_{j,t} \operatorname{var}_t^* u_{j,t+1}$$

- Now take differences...

Theory (5)

- Now take differences:

$$\mathbb{E}_t \frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \frac{1}{2}\left(\operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} - \sum_j w_{j,t} \operatorname{var}_t^* \frac{R_{j,t+1}}{R_{f,t+1}}\right) \underbrace{-\frac{1}{2}\left(\operatorname{var}_t^* u_{i,t+1} - \sum_j w_{j,t} \operatorname{var}_t^* u_{j,t+1}\right)}_{\alpha_i}$$

- Second term is zero on value-weighted average: we assume it can be captured by a time-invariant stock fixed effect α_i
- Follows immediately if the risk-neutral variances of residuals decompose separably, $\operatorname{var}_t^* u_{i,t+1} = \phi_i + \psi_t$, and value weights are constant over time

Theory (6)

So,

$$\mathbb{E}_t \frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \frac{1}{2}\left(\underbrace{\operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}}}_{\mathrm{SVIX}_{i,t}^2} - \underbrace{\sum_j w_{j,t} \operatorname{var}_t^* \frac{R_{j,t+1}}{R_{f,t+1}}}_{\overline{\mathrm{SVIX}}_t^2} \right) + \alpha_i$$

where fixed effects α_i are zero on value-weighted average

Theory (6)

So,

$$\mathbb{E}_t \frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \alpha_i$$

where fixed effects α_i are zero on value-weighted average

Theory (7)

- For the expected return on a stock, we must take a view on the expected return on the market
- Exploit an empirical claim of Martin (2017) that

$$\mathbb{E}_t \frac{R_{m,t+1} - R_{f,t+1}}{R_{f,t+1}} = \operatorname{var}_t^* \frac{R_{m,t+1}}{R_{f,t+1}}$$

- Substituting back,

$$\mathbb{E}_t \frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \underbrace{\operatorname{var}_t^* \frac{R_{m,t+1}}{R_{f,t+1}}}_{\text{SVIX}_t^2} + \frac{1}{2}\left(\underbrace{\operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}}}_{\text{SVIX}_{i,t}^2} - \underbrace{\sum_j w_{j,t} \operatorname{var}_t^* \frac{R_{j,t+1}}{R_{f,t+1}}}_{\overline{\text{SVIX}}_t^2} \right) + \alpha_i$$

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- Substituting back,

$$\mathbb{E}_t \frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \text{SVIX}_t^2 + \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right) + \alpha_i$$

Theory (8)

- Three different variance measures:

$$
\begin{aligned}
\mathrm{SVIX}_t^2 &= \mathrm{var}_t^* \left(R_{m,t+1}/R_{f,t+1} \right) \\
\mathrm{SVIX}_{i,t}^2 &= \mathrm{var}_t^* \left(R_{i,t+1}/R_{f,t+1} \right) \\
\overline{\mathrm{SVIX}}_t^2 &= \sum_i w_{i,t} \, \mathrm{SVIX}_{i,t}^2
\end{aligned}
$$

- SVIX can be calculated from option prices using the approach of Breeden and Litzenberger (1978)

Theory (9)

- To see it more directly, note that we want to measure

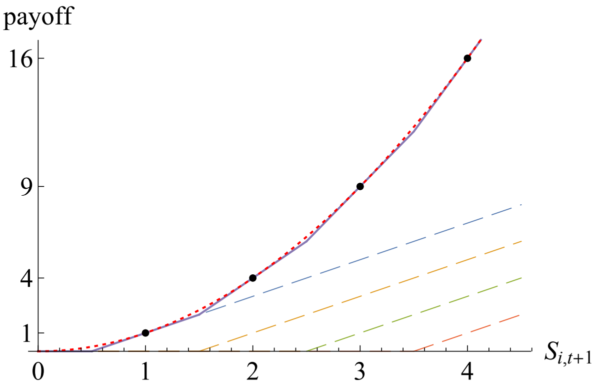
$$\frac{1}{R_{f,t+1}} \operatorname{var}_t^* R_{i,t+1} = \frac{1}{R_{f,t+1}} \mathbb{E}_t^* R_{i,t+1}^2 - \frac{1}{R_{f,t+1}} \left(\mathbb{E}_t^* R_{i,t+1}\right)^2$$

- Since $\mathbb{E}_t^* R_{i,t+1} = R_{f,t+1}$, this boils down to calculating $\frac{1}{R_{f,t+1}} \mathbb{E}_t^* S_{i,t+1}^2$
- That is: how can we price the 'squared contract' with payoff $S_{i,t+1}^2$?

Theory (10)

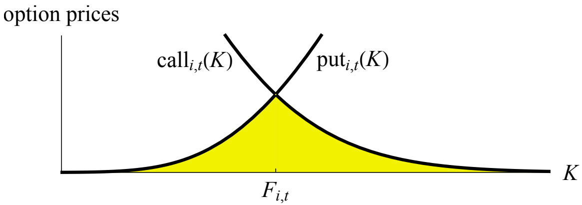
- How can we price the 'squared contract' with payoff $S_{i,t+1}^2$?
- Suppose you buy:
  - 2 calls on stock i with strike $K = 0.5$
  - 2 calls on stock i with strike $K = 1.5$
  - 2 calls on stock i with strike $K = 2.5$
  - 2 calls on stock i with strike $K = 3.5$
  - etc ...

Theory (11)



- So, $\frac{1}{R_{f,t+1}} \mathbb{E}_t^* S_{i,t+1}^2 \approx 2 \sum_K \mathrm{call}_{i,t}(K)$
- In fact, $\frac{1}{R_{f,t+1}} \mathbb{E}_t^* S_{i,t+1}^2 = 2 \int_0^\infty \mathrm{call}_{i,t}(K)\, dK$

Theory (12)



$$\operatorname{var}_t^* \frac{R_{i,t+1}}{R_{f,t+1}} = \frac{2R_{f,t+1}}{F_{i,t}^2}\left[\int_0^{F_{i,t}} \operatorname{put}_{i,t}(K)\, dK + \int_{F_{i,t}}^{\infty} \operatorname{call}_{i,t}(K)\, dK\right]$$

- Closely related to VIX definition, so call this $\text{SVIX}_{i,t}^2$
- $F_{i,t}$ is forward price of stock i, known at time t, $\approx$ spot price
- For SVIX_t^2, use index options rather than individual stock options

Theory: summary

- Expected return on a stock:

$$\frac{\mathbb{E}_t R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \text{SVIX}_t^2 + \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$$

- Pure cross-sectional prediction:

$$\frac{\mathbb{E}_t R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \frac{1}{2}\left(\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2\right)$$

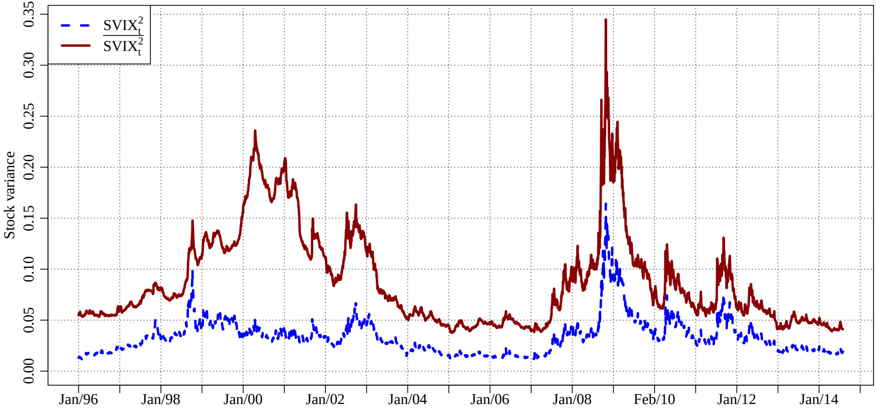
- Also consider the possibility that $\alpha_i = \text{constant} = 0$

Data

- Prices of index and stock options
  - OptionMetrics data from 01/1996 to 10/2014
  - Maturities from 1 month to 2 years
  - S&P 100 and S&P 500
  - Total of 869 firms, average of 451 firms per day
  - Approx. 2.1m daily observations per maturity
  - Approx. 90,000 to 100,000 monthly observations per maturity
- Other data: CRSP, Compustat, Fama–French library
- A caveat: American-style vs. European-style options
- Today: S&P 500 only unless explicitly noted

SVIX_t^2 and $\overline{\text{SVIX}}_t^2$

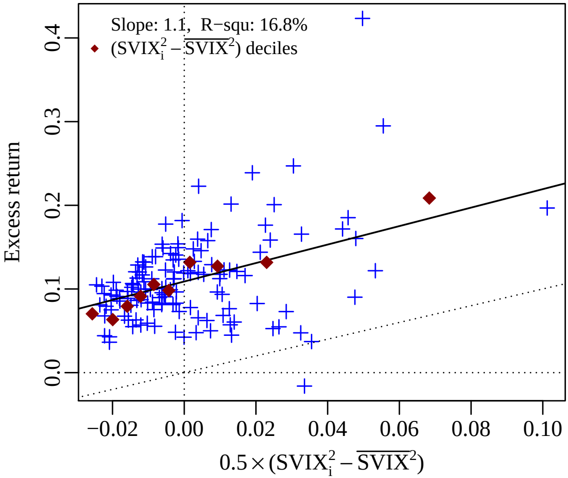
One year horizon



- $\overline{\text{SVIX}}_t > \text{SVIX}_t$ (portfolio of options > option on a portfolio)

Average excess returns on individual stocks

12-month horizon



Empirical analysis

- Excess return panel regression:

$$\frac{R_{i,t+1} - R_{f,t+1}}{R_{f,t+1}} = \alpha_i + \beta\, \mathrm{SVIX}_t^2 + \gamma \left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}$$

  and we hope to find $\sum_i w_i \alpha_i = 0$, $\beta = 1$, and $\gamma = 0.5$

- Excess-of-market return panel regression:

$$\frac{R_{i,t+1} - R_{m,t+1}}{R_{f,t+1}} = \alpha_i + \gamma \left(\mathrm{SVIX}_{i,t}^2 - \overline{\mathrm{SVIX}}_t^2 \right) + \varepsilon_{i,t+1}$$

  and we hope to find $\sum_i w_i \alpha_i = 0$ and $\gamma = 0.5$

- Pooled and firm-fixed-effects regressions
- Block bootstrap to obtain joint distribution of parameters

Expected excess returns

Horizon	30 days	91 days	182 days	365 days	730 days
			Firm fixed-effects regressions		
$\sum w_i \alpha_i$	0.080	0.042	-0.008	0.012	-0.026
	(0.072)	(0.075)	(0.055)	(0.070)	(0.079)
β	0.603	1.694	3.161	2.612	3.478
	(2.298)	(2.392)	(1.475)	(1.493)	(1.681)
γ	0.491	0.634	0.892	0.938	0.665
	(0.325)	(0.331)	(0.336)	(0.308)	(0.205)
Panel adj-R^2 (%)	0.650	4.048	10.356	17.129	24.266
$H_0 : \sum_i w_i \alpha_i = 0, \beta = 1, \gamma = 0.5$	0.231	0.224	0.164	0.133	0.060
$H_0 : \beta = \gamma = 0$	0.265	0.119	0.019	0.008	0.002
$H_0 : \gamma = 0.5$	0.978	0.686	0.243	0.155	0.420
$H_0 : \gamma = 0$	0.131	0.056	0.008	0.002	0.001

Expected excess returns

Horizon	30 days	91 days	182 days	365 days	730 days
			Pooled regressions		
α	0.057	0.019	-0.038	-0.021	-0.054
	(0.074)	(0.079)	(0.059)	(0.071)	(0.076)
β	0.743	1.882	3.483	3.032	3.933
	(2.311)	(2.410)	(1.569)	(1.608)	(1.792)
γ	0.214	0.305	0.463	0.512	0.324
	(0.296)	(0.287)	(0.320)	(0.318)	(0.200)
Pooled adj-R^2 (%)	0.096	0.767	3.218	4.423	5.989
$H_0 : \alpha = 0, \beta = 1, \gamma = 0.5$	0.267	0.242	0.169	0.184	0.015
$H_0 : \beta = \gamma = 0$	0.770	0.553	0.071	0.092	0.036
$H_0 : \gamma = 0.5$	0.333	0.497	0.908	0.971	0.377
$H_0 : \gamma = 0$	0.470	0.287	0.148	0.108	0.105
Theory adj-R^2 (%)	-0.107	0.227	1.491	1.979	1.660

Expected returns in excess of the market

Horizon	30 days	91 days	182 days	365 days	730 days
			Firm fixed-effects regressions		
$\sum_i w_i \alpha_i$	0.036	0.034	0.033	0.033	0.033
	(0.008)	(0.007)	(0.008)	(0.008)	(0.008)
γ	0.560	0.730	0.949	0.917	0.637
	(0.313)	(0.313)	(0.319)	(0.291)	(0.199)
Panel adj-R^2 (%)	0.398	3.015	7.320	12.637	17.479
$H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$	0.000	0.000	0.000	0.000	0.000
$H_0 : \gamma = 0.5$	0.848	0.461	0.160	0.152	0.491
$H_0 : \gamma = 0$	0.073	0.019	0.003	0.002	0.001

Expected returns in excess of the market

Horizon	30 days	91 days	182 days	365 days	730 days
			Pooled regressions		
α	0.016	0.016	0.013	0.014	0.019
	(0.015)	(0.015)	(0.016)	(0.019)	(0.019)
γ	0.301	0.414	0.551	0.553	0.354
	(0.285)	(0.273)	(0.306)	(0.302)	(0.200)
Pooled adj-R^2 (%)	0.135	0.617	1.755	2.892	1.901
$H_0 : \alpha = 0, \gamma = 0.5$	0.489	0.560	0.630	0.600	0.596
$H_0 : \gamma = 0.5$	0.486	0.752	0.869	0.862	0.467
$H_0 : \gamma = 0$	0.291	0.129	0.072	0.068	0.077
Theory adj-R^2 (%)	0.068	0.547	1.648	2.667	1.235

Conclusions so far

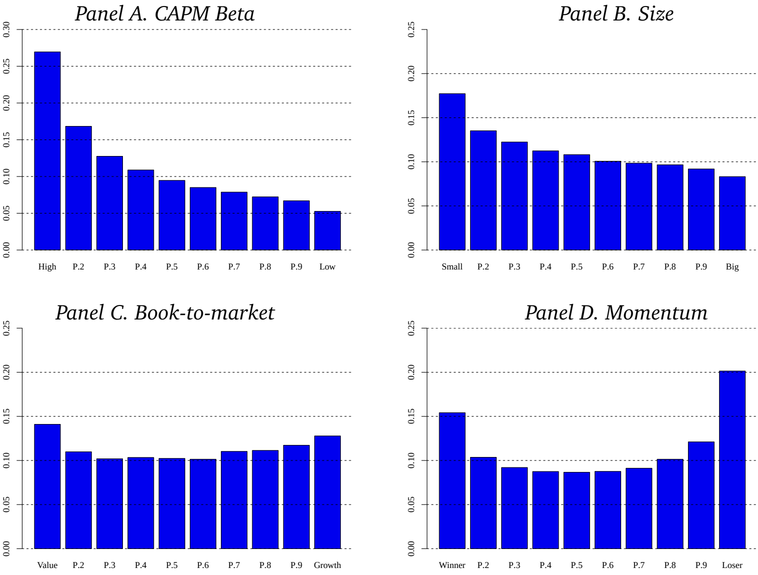
- Do not reject our model in most specifications
- At 6- , 12-, and 24-month horizons, we reject $\beta = \gamma = 0$ for excess returns (ER) and $\gamma = 0$ for excess market returns (EMR)

	S&P100 6mo	S&P100 12mo	S&P100 24mo	S&P500 6mo	S&P500 12mo	S&P500 24mo
ER, pooled	*	**	**	*	*	**
ER, FE	***	***	***	**	***	***
EMR, pooled	**	**	***	*	*	*
EMR, FE	***	***	***	***	***	***

* = p-value < 0.1, ** = p-value < 0.05, *** = p-value < 0.01

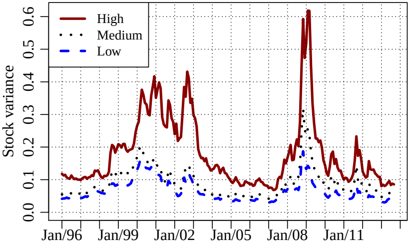
- In FE regression for excess-of-market returns, avg FE $\neq 0$. But the economic magnitude is small and we will see that the model performs well out-of-sample when we drop FEs entirely

Characteristics and SVIX_i^2

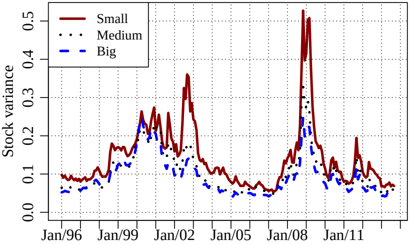


Characteristics and SVIX_i^2

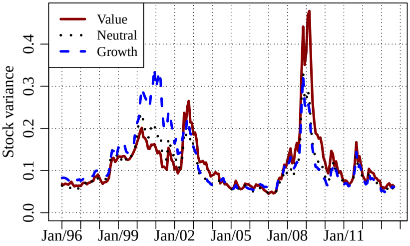
Panel A. CAPM Beta



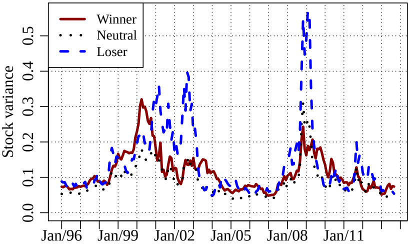
Panel B. Size



Panel C. Book-to-market



Panel D. Momentum



SVIX variables drive out firm characteristics

1-year horizon, excess returns

	Realized returns		Expected returns		Unexpected returns	
			estimated	theory	estimated	theory
const	0.721	0.452	0.259	0.164	0.462	0.557
	(0.341)	(0.320)	(0.133)	(0.035)	(0.332)	(0.331)
$\text{Beta}_{i,t}$	0.038	-0.048	0.082	0.097	-0.044	-0.059
	(0.068)	(0.068)	(0.064)	(0.018)	(0.046)	(0.072)
$\log(\text{Size}_{i,t})$	-0.030	-0.019	-0.010	-0.009	-0.019	-0.021
	(0.014)	(0.013)	(0.007)	(0.002)	(0.013)	(0.013)
$\text{B/M}_{i,t}$	0.071	0.068	0.003	0.001	0.068	0.069
	(0.034)	(0.038)	(0.010)	(0.006)	(0.038)	(0.037)
$\text{Ret}_{i,t}^{(12,1)}$	-0.049	-0.005	-0.046	-0.026	-0.003	-0.023
	(0.063)	(0.054)	(0.042)	(0.015)	(0.050)	(0.058)
SVIX_t^2		2.792				
		(1.472)				
$\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$		0.511				
		(0.357)				
adj-R^2 (%)	1.924	5.265	17.277	30.482	0.973	1.197
$H_0 : b_i = 0$	0.003	0.201	0.702	0.000	0.187	0.092
$H_0 : b_i = 0, c_0 = 1, c_1 = 0.5$		0.143				
$H_0 : b_i = 0, c_0 = 0, c_1 = 0$		0.001				

SVIX variables drive out firm characteristics

1-year horizon, excess-of-market returns

	Realized returns		Expected returns		Unexpected returns	
			estimated	theory	estimated	theory
const	0.429	0.277	0.131	0.107	0.298	0.321
	(0.371)	(0.377)	(0.073)	(0.027)	(0.365)	(0.359)
$\text{Beta}_{i,t}$	0.016	-0.131	0.113	0.105	-0.097	-0.088
	(0.075)	(0.062)	(0.066)	(0.016)	(0.046)	(0.078)
$\log(\text{Size}_{i,t})$	-0.018	-0.006	-0.009	-0.009	-0.009	-0.010
	(0.014)	(0.015)	(0.006)	(0.002)	(0.015)	(0.013)
$\text{B/M}_{i,t}$	0.032	0.031	0.001	0.001	0.032	0.032
	(0.025)	(0.027)	(0.006)	(0.005)	(0.026)	(0.026)
$\text{Ret}_{i,t}^{(12,1)}$	-0.051	-0.029	-0.017	-0.015	-0.034	-0.035
	(0.041)	(0.041)	(0.018)	(0.010)	(0.039)	(0.040)
$\text{SVIX}_{i,t}^2 - \overline{\text{SVIX}}_t^2$		0.705				
		(0.308)				
adj-R^2 (%)	1.031	3.969	37.766	37.766	1.051	0.974
$H_0 : b_i = 0$	0.347	0.153	0.435	0.000	0.157	0.619
$H_0 : b_i = 0, c = 0.5$		0.234				
$H_0 : b_i = 0, c = 0$		0.018				

Risk premia and firm characteristics

- Our predictor variables drive out stock characteristics
- Characteristics relate to expected returns but not to unexpected (by our model) returns
- The model also performs well on portfolios sorted on characteristics

Expected excess returns

5x5 [*characteristic*]-SVIX$_{i,t}$ double-sorted portfolios, 1-yr horizon

	Beta	Size	B/M	Mom
	Portfolio fixed-effects regressions			
$\sum w_i \alpha_i$	-0.014	-0.019	-0.019	-0.009
	(0.068)	(0.072)	(0.069)	(0.067)
β	2.790	2.723	2.908	2.756
	(1.502)	(1.503)	(1.563)	(1.525)
γ	0.688	0.826	0.593	0.772
	(0.554)	(0.599)	(0.563)	(0.542)
Panel adj-R^2 (%)	21.174	22.404	21.481	21.908
$\sum w_i \alpha_i$, β, γ	0.250	0.314	0.232	0.249
$\beta = \gamma = 0$	0.153	0.132	0.152	0.133
$\gamma = 0.5$	0.734	0.586	0.868	0.616
$\gamma = 0$	0.214	0.168	0.292	0.154

Expected excess returns

5x5 [*characteristic*]-SVIX$_{i,t}$ double-sorted portfolios, 1-yr horizon

	Beta	Size	B/M	Mom
	Pooled regressions			
α	-0.020	-0.021	-0.021	-0.021
	(0.071)	(0.071)	(0.071)	(0.071)
β	2.974	2.963	3.024	2.960
	(1.603)	(1.600)	(1.619)	(1.606)
γ	0.450	0.520	0.446	0.511
	(0.326)	(0.340)	(0.342)	(0.335)
Pooled adj-R^2 (%)	9.184	9.879	9.178	10.036
α, β, γ	0.170	0.203	0.159	0.185
$\beta = \gamma = 0$	0.119	0.107	0.127	0.116
$\gamma = 0.5$	0.877	0.954	0.874	0.983
$\gamma = 0$	0.168	0.126	0.193	0.141
Theory adj-R^2 (%)	3.468	4.237	3.152	3.943

Expected returns in excess of the market

5x5 [*characteristic*]-SVIX$_{i,t}$ double-sorted portfolios, 1-yr horizon

	Beta	Size	B/M	Mom
	Portfolio fixed-effects regressions			
$\sum_i w_i \alpha_i$	0.015	0.008	0.014	0.019
	(0.017)	(0.005)	(0.016)	(0.017)
γ	0.794	0.941	0.711	0.864
	(0.490)	(0.529)	(0.507)	(0.491)
Panel adj-R^2 (%)	13.010	16.419	12.679	15.020
$H_0 : \sum_i w_i \alpha_i = 0, \gamma = 0.5$	0.439	0.070	0.479	0.212
$H_0 : \gamma = 0.5$	0.549	0.405	0.677	0.459
$H_0 : \gamma = 0$	0.106	0.075	0.161	0.079

Expected returns in excess of the market

5x5 [*characteristic*]-SVIX$_{i,t}$ double-sorted portfolios, 1-yr horizon

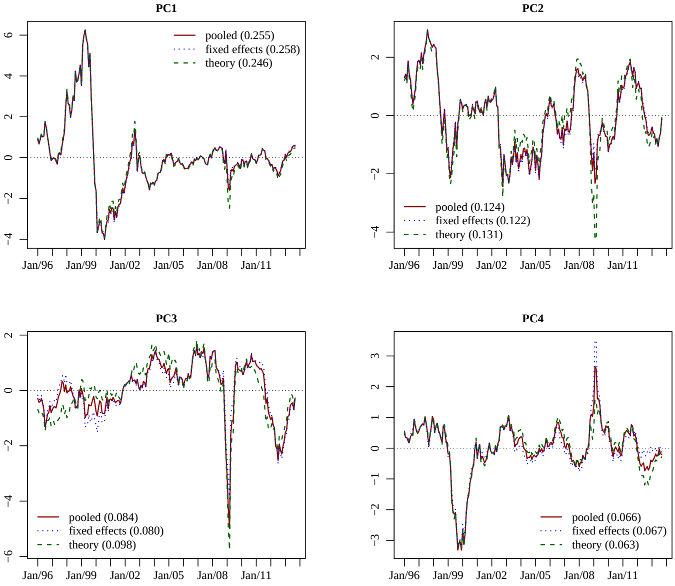
	Beta	Size	B/M	Mom
	Pooled regressions			
α	0.015	0.013	0.015	0.014
	(0.019)	(0.020)	(0.020)	(0.019)
γ	0.495	0.572	0.502	0.559
	(0.311)	(0.323)	(0.327)	(0.319)
Pooled adj-R^2 (%)	8.391	9.908	8.098	10.245
$H_0 : \alpha = 0, \gamma = 0.5$	0.635	0.593	0.635	0.613
$H_0 : \gamma = 0.5$	0.987	0.823	0.996	0.890
$H_0 : \gamma = 0$	0.112	0.076	0.125	0.088
Theory adj-R^2 (%)	7.598	8.995	7.232	8.555

The factor structure of unexpected stock returns

What do we miss, relative to an oracle with perfect foresight?

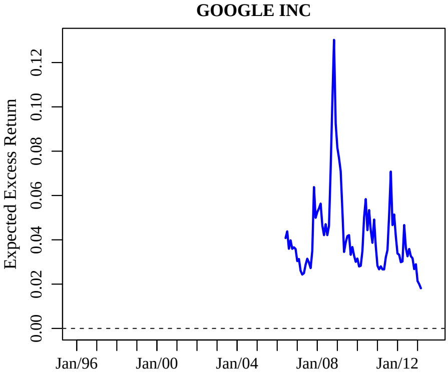
- Run PCA on unexpected returns-in-excess-of-market, 1yr horizon, S&P 500 firms with complete time-series coverage
  - Residuals from pooled regression
  - Residuals from fixed-effects regression
  - Residuals when coefficients constrained to theoretical values
- PC1: 25% of variance. PC2: 12%. PC3: 8%. PC4: 7%
- PC loadings do not show any relationship with market cap, B/M, SVIX_i, beta, or industry

The factor structure of unexpected stock returns



Out-of-sample analysis

- No need for historical data or to estimate any parameters
  - Google: First IPO on August 19, 2004
  - OptionMetrics data from August 27, 2004
  - Included in the S&P 500 from 31, March 2006



The formula performs well out-of-sample

- Out-of-sample R^2 of the model-implied expected excess returns relative to competing forecasts

$$R^2_{OS} = 1 - \mathrm{SSE}_{\mathrm{model}} / \mathrm{SSE}_{\mathrm{competitor}}$$

Horizon	30 days	91 days	182 days	365 days	730 days
SVIX^2_t	0.09	0.57	1.77	3.08	2.77
$\overline{\mathrm{S\&P500}}_t$	0.09	0.79	2.56	3.82	4.46
$\overline{\mathrm{CRSP}}_t$	-0.09	0.24	1.43	1.70	0.88
6% *p.a.*	-0.01	0.46	1.84	2.54	2.06
$\mathrm{SVIX}^2_{i,t}$	0.95	1.87	1.55	2.17	7.64
$\overline{\mathrm{RX}}_{i,t}$	1.40	4.97	11.79	27.10	56.67
$\widehat{\beta}_{i,t} \times \overline{\mathrm{S\&P500}}_t$	0.09	0.79	2.54	3.76	4.72
$\widehat{\beta}_{i,t} \times \overline{\mathrm{CRSP}}_t$	-0.06	0.28	1.46	1.68	1.61
$\widehat{\beta}_{i,t} \times \mathrm{SVIX}^2_t$	0.04	0.46	1.58	2.87	2.91
$\widehat{\beta}_{i,t} \times$ 6% *p.a.*	0.00	0.47	1.84	2.48	2.58

...even against in-sample predictions

- Out-of-sample R^2 of the model-implied expected excess returns relative to competing forecasts

$$R^2_{OS} = 1 - \mathrm{SSE}_{\mathrm{model}}/\mathrm{SSE}_{\mathrm{competitor}}$$

Horizon	30 days	91 days	182 days	365 days	730 days
in-sample avg mkt	-0.05	0.31	1.52	1.90	1.42
in-sample avg all stocks	-0.09	0.17	1.26	1.42	0.56
$\widehat{\beta}_{i,t}\times$ in-sample avg mkt	-0.03	0.34	1.54	1.87	2.04
$\mathrm{Beta}_{i,t}$	-0.09	0.16	1.22	1.30	0.56
$\log(\mathrm{Size}_{i,t})$	-0.19	-0.17	0.62	0.21	-1.34
$\mathrm{B/M}_{i,t}$	-0.18	-0.03	0.89	0.77	0.00
$\mathrm{Ret}_{i,t}^{(12,1)}$	-0.10	0.15	1.09	1.05	-0.76
All	-0.25	-0.30	0.26	-0.53	-2.71

The formula performs well out-of-sample

- Out-of-sample R^2 of the model-implied expected returns in excess of the market relative to competing forecasts

Horizon	30 days	91 days	182 days	365 days	730 days
Random walk	0.16	0.76	1.92	3.07	1.99
$(\widehat{\beta}_{i,t} - 1) \times \overline{\text{S\&P500}}_t$	0.18	0.80	1.98	3.10	2.17
$(\widehat{\beta}_{i,t} - 1) \times \overline{\text{CRSP}}_t$	0.21	0.89	2.14	3.35	2.83
$(\widehat{\beta}_{i,t} - 1) \times \text{SVIX}_t^2$	0.11	0.62	1.68	2.80	2.01
$(\widehat{\beta}_{i,t} - 1) \times$ 6% *p.a.*	0.19	0.83	2.04	3.19	2.49

. . . even against in-sample predictions

- Out-of-sample R^2 of the model-implied expected returns in excess of the market relative to competing forecasts

Horizon	30 days	91 days	182 days	365 days	730 days
in-sample avg all stocks	0.11	0.58	1.60	2.48	0.95
$(\widehat{\beta}_{i,t} - 1)\times$ in-sample avg mkt	0.20	0.86	2.11	3.29	2.63
$\text{Beta}_{i,t}$	0.11	0.58	1.60	2.45	0.95
$\log(\text{Size}_{i,t})$	0.05	0.39	1.27	1.90	0.12
$\text{B/M}_{i,t}$	0.07	0.50	1.47	2.31	0.88
$\text{Ret}_{i,t}^{(12,1)}$	0.10	0.56	1.47	2.05	0.03
All	0.03	0.34	1.11	1.46	-0.64

- We even beat the model that knows the **multivariate in-sample** relationship between returns and beta, size, B/M, lagged return

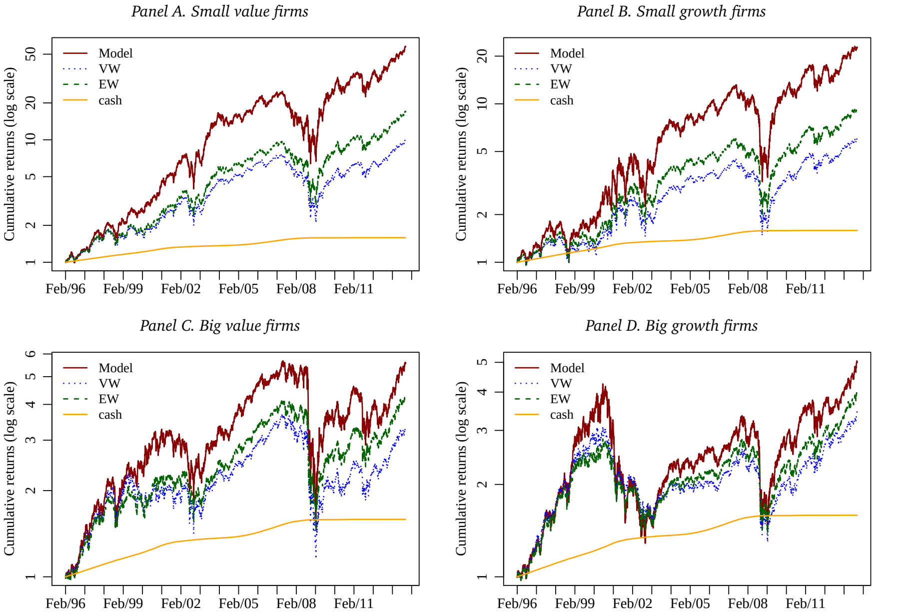
Cross-sectional trading strategies (1)

- Use $\mathbb{E}_t R_{i,t+1}$ to construct rank portfolios (reduces impact of outliers, following Asness, Moskowitz and Pedersen, 2013, *Journal of Finance*)
- Daily rebalancing. Neglect transaction costs
- Portfolio weight of firm i for the period from t to $t+1$ is

$$w_{i,t}^{XS} = \frac{\text{rank}[\mathbb{E}_t R_{i,t+1}]^\theta}{\sum_i \text{rank}[\mathbb{E}_t R_{i,t+1}]^\theta}$$

  - No short positions; θ controls aggressiveness (today, $\theta = 2$)
- Calculate fee an investor with risk aversion ρ would pay for our strategy (Fleming, Kirby and Ostdiek, 2001, *Journal of Finance*)
  - $\sim 5\%$ relative to value-weighted portfolio: natural benchmark
  - $\sim 2\%$ relative to equally-weighted portfolio: tougher benchmark

Cross-sectional trading strategies (2)



Summary

- We derive a formula for the expected return on a stock
- Computable in real time
- Requires observation of option prices but no estimation
- Performs well in and out of sample
- Risk premia vary a lot in the time-series and cross-section
- Many potential applications!