A Bound on Expected Stock Returns*

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Abstract

We present a sufficient condition under which the prices of options written on a particular stock can be aggregated to calculate a lower bound on the expected returns of that stock. The sufficient condition imposes a restriction on a combination of the stock's systematic and idiosyncratic risk. The lower bound is forward-looking and can be calculated on a high-frequency basis for stocks with liquid option trading. We estimate the lower bound empirically for constituents of the S&P 500 index and study its cross-sectional properties. We find that the bound increases with beta and book-to-market ratio and decreases with size and momentum. The bound also provides an economically meaningful signal on future realized stock returns.

1 Introduction

What is the expected return of traded securities? Can option prices provide useful information on the expected return of their underlying stocks? These questions are of paramount importance for both theoretical and practical purposes. A traditional point of view starting with Black and Scholes (1973) is that option prices can teach us a great deal about the volatility of underlying assets. More generally, beginning with the work of Breeden and Litzenberger (1978), it has been well understood that option prices can be used to extract forward-looking risk-neutral probabilities, allowing one to price a variety of assets. What remains a puzzle to this day is whether

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option prices can be used to elicit any useful information on forward-looking expected returns. Such expectations would be derived from the physical (true) distribution of the returns of the underlying asset. It is for this reason that the Recovery theorem, proposed by Ross (2015), ignited much interest, as it suggests that given a regularity condition on the pricing kernel, option prices can in fact be used to recover the physical probability distribution of asset returns, and, in particular their implied expected returns (see also Martin and Ross (2013)).[1] Ross's result, however, has been criticized by Borovička, Hansen, and Scheinkman (2016), who argue that it restricts the dynamics of the stochastic discount factor in an unrealistic manner, and thereby, the recovered probability distribution differs from the true one.

Martin (2017) takes a different approach to address this important issue. Under mild assumptions on risk aversion he derives a simple and useful lower bound for the market premium. While not obtaining "full-recovery," Martin is still able to elicit useful forward-looking information on expected market returns from option prices. In this paper we follow Martin's approach by deriving a new condition under which his lower bound can be applied to individual assets. This gives rise to a forward-looking lower bound on the expected returns of individual stocks, which can be readily calculated from option prices at high frequency. We then estimate the lower bound for S&P 500 constituents, study its cross-sectional properties, and analyze the extent to which it is correlated with realized future returns.

Martin (2017) begins his analysis by offering an identity describing the expected return of any asset j, relying only on no-arbitrage. His identity shows that the expected excess return of asset j equals the risk-neutral variance of the asset's returns discounted at the risk free rate, less a covariance term calculated at the physical (true) probability measure. Martin then restricts attention to the case in which asset j is the market, and he explores whether the covariance term in his identity is weakly negative. He terms this the *Negative Correlation Condition* (NCC). The idea is that if the covariance term is negative, then the deflated risk-neutral variance can be used as a lower bound on the expected market premium. Moreover, Martin (2017) shows that this lower bound can be relatively easily calculated from the prices of put and call options written on the S&P 500 index with different strike prices.[2] Martin then

[1] The conventional approach to estimating pricing kernels from option prices relies on using the risk-neutral probability and then supplementing it with a physical probability derived from historical returns. See Jackwerth (2000) and Ait-Sahalia and Lo (2000).

[2] Martin's calculations of the lower bound are similar to prior results establishing expressions for

goes on to show that the NCC indeed holds theoretically under mild conditions in a variety of asset pricing settings. Essentially, what is needed to justify the NCC is that relative risk aversion in the economy be no less than 1. Martin also shows that the NCC holds empirically if one assumes typical factor structures for the stochastic discount factor.

Martin's approach is appealing. His assumptions are mild, leading to a credible forward-looking estimate of a lower bound on the market premium. The question then is whether the NCC holds for individual assets as well. If it does, then one can use an analogous approach to calculate a lower bound on expected stock returns cross-sectionally using options written on individual stocks. Intuitively, since the market is a weighted average of individual assets, and since the NCC holds for the market as a whole, one would expect the NCC to hold for a significant portion of individual assets. However, Martin does not provide us with a procedure to test whether a particular stock satisfies the NCC. In our main theoretical contribution, we provide such a tool by deriving a sufficient condition for the NCC to hold for individual stocks. Our condition, which relies on a first-order approximation, establishes that the NCC holds for stocks with a relatively low combination of systematic and idiosyncratic risk compared to the relative risk-aversion in the market.

Our sufficient condition is easily checked empirically provided that one makes some plausible assumption on the level of relative risk aversion in the economy. We show using both non-parametric and parametric approaches that given relative risk aversion levels commonly used in asset pricing calibrations, the NCC holds for about 90% of S&P 500 constituents. It follows that for a large cross-section of stocks we obtain a valid forward-looking lower bound on the expected returns of individual stocks, which can be estimated empirically from options with different strike prices using the procedure developed in Martin (2017).[3]

The average lower bound in our sample of S&P 500 stocks between 2003 and 2016 ranges from 8.7% to 20.7% depending on the level of conservatism one exerts when

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risk-neutral moments from option prices (see Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003)). Martin (2013) shows that the risk-neutral variance is closely related to an index of simple variance swaps (SVIX).

[3]Martin's (2017) approach to estimating the lower bound requires the use of European style options. However, options written on individual stocks in the U.S. are of the American style. Our estimates are mostly based on out-of-the money options for which the difference in price between American and European options is small. Moreover, our estimation approach offsets this potential bias. See Section 4 for details.

applying the NCC.[4] The average lower bound drops to 7.3%-19% when we exclude the crisis period of 2008-2009. Indeed, during the crisis period we observe a dramatic spike in expected stock returns consistent with the findings of Martin (2017) on the market premium. Cross-sectionally we find that the lower bound decreases with firm size and increases with book-to-market ratio, in line with the classic Fama and French (1992) findings, which are based on realized rather than forward-looking estimates. Strikingly, and in contrast to the Fama-French findings, the lower bound is strongly correlated with the CAPM beta. Thus, while beta does not seem to explain cross-sectional variations in realized stock returns, it appears to be a strong determinant of investors' expectations as reflected in option prices. Additionally, momentum works in the opposite direction from the one explored in Jegadeesh and Titman (1993). Namely, being a recent winner induces investors to lower rather than increase their expectations regarding a given stock.

In our last analysis we ask whether the forward looking lower bounds we obtain are correlated with future realized returns. Our lower-bound estimates appear to provide a statistically significant and economically meaningful signal on future stock returns, especially over horizons of 6-12 months. For example, stocks in the lowest decile of the lower bound on expected returns have average realized returns of 1.8% in the next six-months period, while stocks in the highest decile have average realized returns of 15%. The annualized difference of 26.4% is very large economically.

Our paper offers a forward-looking bound for expected stock returns relying on option prices. It thus complements the bound introduced by Martin (2017), which applies to the market premium as a whole. In a recent paper, Martin and Wagner (2016) take a different path toward a similar goal, also relying on Martin (2017). The difference between our two approaches is that we rely on the most robust prediction of Martin (2017) in the form of a lower bound, leading us to a lower bound on expected stock returns. Martin and Wagner (2016) instead rely on a more aggressive claim that the inequality in Martin (2017) actually holds as an equality. This would be the case, for example, if relative risk aversion in the market is exactly equal to 1 or, more generally, if the product of the stochastic discount factor and the market return would be uncorrelated with the market return. This stricter assumption leads Martin

[4] We rely on data from OptionMetrics, which starts in 1996. We begin our empirical analysis in 2003 because the data until that year is limited in terms of the cross section of stocks it offers. See Figure 2 and the discussion in Section 3.1 for details on the choice of the sample period.

and Wagner to a sharper prediction on expected returns in the form of an equality rather than an inequality. Both approaches are complementary and each has its own merit.

The paper also contributes to a broader literature focusing on forward looking expected returns. In particular, our findings regrading the importance of beta in determining expected returns are consistent with Berk and van Binsbergen (2015) who use capital flows to/from mutual funds to test asset pricing models. They find that the CAPM is the closest to the model investors use to make capital allocation decisions. Similarly, Brav, Lehavy, and Michaely (2005) calculate forward looking expected returns using analyst expectations and find them to be positively correlated with firm beta and negatively correlated with size. In contrast to our high-frequency lower bound, these alternative approaches rely on low frequency data such as mutual fund flows and analysts' outputs.

Our paper also adds to the literature showing the relation between option prices and future stock returns. An, Ang, Bali, and Cakici (2014) argue that implied option volatility incorporates incoming news about the underlying asset. For example, an informed trader with positive news can trade by buying a call option on the stock, increasing its implied volatility. Symmetrically, bad news about a stock may be incorporated in higher implied volatility of put options. In line with this view, they show empirically that increases in implied volatility of call (put) options are associated with higher (lower) future returns. Our focus in this paper is different. We provide a theoretical analysis showing that the deflated risk-neutral volatility serves as a lower bound for the expected excess returns of stocks. Empirically, we estimate this risk-neutral volatility from the cross-section of both call and put options and find that it has a predictive value for future returns. Thus, the rationale for our analysis and the quantitative implications we obtain are different from those of An et al (2014). Similarly, our empirical results are different from theirs as we focus on risk-neutral volatility being a lower bound, not associated specifically with either put or call options.[5]

Finally, our paper is also related to the literature on the role of idiosyncratic risk in the determination of expected returns. The "idiosyncratic volatility puzzle" asserts that stocks with low idiosyncratic risk are expected to yield high returns (Ang,

[5]Under the Black-Scholes (1973) framework risk-neutral and physical volatilities coincide. We do not confine ourself to this framework in this paper.

Hodrick, Xing, and Zhang (2006)). By contrast Fu (2009) argues that idiosyncratic risk is time varying and shows a positive relation between conditional idiosyncratic risk and expected returns. Our lower bound on expected returns is roughly equal to the implied risk-neutral volatility, which is a forward-looking proxy for idiosyncratic risk accounting for market risk aversion. Thus, our theoretical approach provides a justification for the relation between volatility and expected returns to the extent that our constraint is binding.

The paper proceeds as follows. In Section 2 we derive the lower bound for individual stocks and explain how to estimate it from option prices. In Section 3 we test the validity of the NCC and present our cross-sectional empirical analysis. In Section 4 we discuss and evaluate the precision of the approximations we are using. We conclude in Section 5.

2 Derivation of the Lower Bound

2.1 General Approach

Our setup follows that of Martin (2017) with the difference being that Martin focuses on the market as a whole, while we focus on the cross-section of individual assets. Consider a standard dynamic asset pricing model with uncertainty at time t about the realization of asset returns at time $T > t$. Assume no arbitrage so that a stochastic discount factor, M_T, exists satisfying $\mathrm{E}_t\left(M_T R_{j,T}\right) = 1$ for the gross returns $R_{j,T}$ from time t to T of any asset j. Denote $R_{f,t} = 1/\mathrm{E}_t\left(M_T\right)$ the gross risk free rate between times t and T. Then, it is immediate that

$$\frac{Var_t^*\left(R_{j,T}\right)}{R_{f,t}} = \mathrm{E}_t(M_T R_{j,T}^2) - R_{f,t}, \tag{1}$$

for all assets j, where $Var_t^*\left(\cdot\right)$ denotes variance taken under the risk-neutral measure. As Martin (2017) shows, the expected excess return of any asset j is given by

$$\begin{aligned} \mathrm{E}_t R_{j,T} - R_{f,t} &= \mathrm{E}_t R_{j,T} - R_{f,t} + \mathrm{E}_t(M_T R_{j,T}^2) - \mathrm{E}_t(M_T R_{j,T}^2) \qquad (2) \\ &= \frac{Var_t^*\left(R_{j,T}\right)}{R_{f,t}} - \mathrm{E}_t(M_T R_{j,T}^2) + \mathrm{E}_t R_{j,T} \quad \text{(using (1))} \\ &= \frac{Var_t^*\left(R_{j,T}\right)}{R_{f,t}} - \left[\mathrm{E}_t(M_T R_{j,T}^2) - \mathrm{E}_t R_{j,T}\mathrm{E}_t\left(M_T R_{j,T}\right)\right] \quad \text{(using that } \mathrm{E}_t\left(M_T R_{j,T}\right) = 1) \\ &= \frac{Var_t^*\left(R_{j,T}\right)}{R_{f,t}} - Cov_t\left(M_T R_{j,T}, R_{j,T}\right), \end{aligned}$$

where $Cov_t(\cdot,\cdot)$ denotes the covariance operator as of time t. Note that in (2) both the expectation and covariance are taken under the physical probability measure whereas the variance is taken under the risk-neutral measure.

Definition 1 *(Martin (2017)). We say that the Negative Correlation Condition (NCC) holds for asset j if $Cov_t(M_T R_{j,T}, R_{j,T}) \leq 0$.*

Note that (2) implies that if the NCC holds for asset j, then

$$\mathrm{E}_t R_{j,T} - R_{f,t} \geq \frac{Var_t^*(R_{j,T})}{R_{f,t}}. \tag{3}$$

Thus, whenever the NCC holds for asset j, the risk neutral return variance scaled by the risk free return serves as a lower bound for the expected excess return of asset j. On the other hand, if the NCC fails, then the inequality in (3) is reversed, and we obtain an upper bound on expected asset returns.

Martin (2017) restricts attention to the case in which j is the market portfolio. He shows that in this case, the NCC holds in a variety of standard asset pricing models subject to relative risk aversion being at least 1, allowing him to obtain a lower bound on the expected market premium. Our goal here is to show that the NCC holds for many individual assets satisfying a somewhat stricter (but still quite standard) condition. Thus, we can use (3) to obtain cross-sectional bounds on expected stock returns.

2.2 Martin's Argument for the Validity of the NCC

To motivate our analysis we begin by reviewing Martin's (2017) basic argument for why the NCC is satisfied when j is the market portfolio. We then show that Martin's argument does not directly apply for individual assets, leading to our own analysis in the next subsection.

Martin shows that under a mild condition on risk aversion, when the asset being considered is the market the NCC holds for standard work-horse models in asset pricing including a one-period investment problem, a dynamic consumption/investment model with separable utility, and a dynamic consumption/investment model with recursive utility. Martin also shows that the NCC holds empirically when applying a Fama-French factor structure for the stochastic discount factor. For brevity we only review here his one period theoretical analysis and refer the reader to his paper for the other justifications of the NCC in his setting.

Assume the existence of a representative agent maximizing expected utility from consumption. The utility function $u(\cdot)$ is assumed to be twice differentiable with $u' > 0$ and $u'' < 0$. At time t the agent needs to construct an optimal portfolio out of N assets whose random returns will be realized at time $T > t$ and are denoted by $R_{i,T}$, $i = 1, 2, ...N$. The agent consumes only once at time T. The optimal portfolio is the market portfolio and we denote its return by $R_{m,T}$.

The agent's problem is thus to choose portfolio weights $\{w_i\}$ to solve

$$\max_{\{w_i\}} \mathrm{E}_t u(\sum w_i R_{i,T})$$
$$s.t. \sum w_i = 1.$$

At optimum, we have the first-order condition:[6]

$$\mathrm{E}_t[u'(R_{m,T})R_{i,T}] = \lambda, \forall i, \tag{4}$$

where λ is a positive Lagrange multiplier and the market portfolio is

$$R_{m,T} \equiv \sum_{i=1}^{N} w_i R_{i,T}. \tag{5}$$

Dividing both sides of (4) by λ, we have that

$$\mathrm{E}_t[\frac{u'(R_{m,T})}{\lambda} R_{i,T}] = 1, \forall i.$$

Therefore, $\frac{u'(R_{m,T})}{\lambda}$ is a stochastic discount factor in this economy,[7] and we conclude that M_T is proportional to $u'(R_{m,T})$. Thus, for the NCC to hold in this setup for some specific asset j we need

$$Cov_t\left(u'\left(R_{m,T}\right)R_{j,T}, R_{j,T}\right) \leq 0. \tag{6}$$

In particular, if j is the market portfolio we need

$$Cov_t\left(u'\left(R_{m,T}\right)R_{m,T}, R_{m,T}\right) \leq 0.$$

Denote

$$\gamma\left(w\right) \equiv -w\frac{u''\left(w\right)}{u'\left(w\right)},$$

[6] To justify this one needs to introduce assumptions on the validity of differentiation of an expectation. We skip these standard technical details for brevity.

[7] It is easy to see that $\lambda = E[R_{m,T}u'(R_{m,T})]$.

the coefficient of relative risk aversion at wealth level w. Note that we do not need to assume in this setup that $\gamma(w)$ is constant or monotone in w. If $\gamma(R_{m,T}) \geq 1$, then $u'(R_{m,T}) R_{m,T}$ is a decreasing function of $R_{m,T}$ and therefore $Cov_t(u'(R_{m,T}) R_{m,T}, R_{m,T}) \leq 0$.[8] We thus have

Proposition 1 *(Martin (2017)). Suppose $\gamma(R_{m,T}) \geq 1$, then the NCC holds for the market portfolio. In this case, $\frac{Var_t^*(R_{m,T})}{R_{f,t}}$ serves as a lower bound on the expected market premium between time t and T.*

Thus, as long as relative risk aversion in the economy is at least 1, $\frac{var_t^*(R_{m,T})}{R_{f,t}}$ is a lower bound on the expected market premium. As shown by Martin, this lower bound can be calculated from option prices on a high frequency basis.

Martin's argument is a powerful statement on the market portfolio, but it cannot be directly applied to individual assets. To see this note first that the sign of (6) depends on the covariance between $R_{j,T}$ and $u'(R_{m,T})$ and on the covariance between $R_{j,T}$ and itself, i.e., the variance of $R_{j,T}$. The former is related to the systematic risk of the asset and is likely to be negative since u' is decreasing. The latter is related to the idiosyncratic risk of the asset and is always positive. Thus, we have two conflicting effects driving the sign of the covariance in (6). The curvature of u', which is driven by risk aversion, will determine which one of the two effects dominates. Thus, intuitively, determining whether the NCC holds for a particular asset should depend on the asset's systematic risk, idiosyncratic risk, and on the prevailing level of risk aversion.

It is also instructive to note that the monotonicity argument used by Martin to sign the covariance does not extend to the case of individual assets. Indeed, for any given asset j, (6) can be written as

$$Cov_t\left(u'(\sum_{i=1}^{N} w_i R_{i,T}) R_{j,T}, R_{j,T}\right) \leq 0. \tag{7}$$

But now, asking that $u'(\sum_{i=1}^{N} w_i R_{i,T}) R_{j,T}$ be monotone decreasing in $R_{j,T}$ is no longer sufficient to guarantee that the covariance is negative.[9] Rather, the sign of the covariance depends on the entire correlation structure between $R_{j,T}$ and all other assets,

[8] This argument relies on the fact that if g is a decreasing function then $Cov(g(x), x) \leq 0$.

[9] As noted above, Martin (2017) uses the fact that if g is a decreasing function then $Cov(g(x), x) \leq 0$. This result, however, does not extend to multivariate functions. For example, even if $g(x, y)$ is decreasing in x, one cannot generally conclude that $Cov(g(x, y), x) \leq 0$. Instead, this depends also on the covariance between x and y.

which is being ignored if one only takes partial derivatives. We now present an alternative approach to signing this covariance.

2.3 The NCC for Individual Assets

To overcome the difficulty of calculating the covariance in (7) we use a first-order approximation leading to sufficient conditions under which the NCC holds for individual assets. As in Martin (2017), our approach is valid for both static and dynamic standard asset pricing models. We present the one-period model here and relegate the dynamic models to Appendix I.

Our goal is to find a condition under which (7) is satisfied. Define $f : \mathbb{R}^N \to \mathbb{R}$ by

$$f(R_{1,T}, R_{2,T}, ...R_{N,T}) = R_{j,T} u'(\sum_{i=1}^{N} w_i R_{i,T}). \tag{8}$$

The first order multivariate Taylor expansion of f around $(\mathrm{E}_t R_{m,T}, \mathrm{E}_t R_{m,T}, ...\mathrm{E}_t R_{m,T})$ gives

$$\begin{aligned} f(R_{1,T}, R_{2,T}, ...R_{N,T}) \quad \approx \quad & f(\mathrm{E}_t R_{m,T}, \mathrm{E}_t R_{m,T}, ...\mathrm{E}_t R_{m,T}) \\ & + \sum_{i=1}^{N} f_i(\mathrm{E}_t R_{m,T}, \mathrm{E}_t R_{m,T}, ...\mathrm{E}_t R_{m,T})(R_{i,T} - \mathrm{E}_t R_{m,T}), \end{aligned} \tag{9}$$

where f_i is the partial derivative of f with respect to its ith argument.

Partially differentiating (8) gives for $i \neq j$

$$f_i(R_{1,T}, R_{2,T}, ...R_{N,T}) = w_i R_{j,T} u''(\sum_{i=1}^{N} w_i R_{i,T}) \tag{10}$$

and for $i = j$

$$f_j(R_{1,T}, R_{2,T}, ...R_{N,T}) = u'(\sum_{i=1}^{N} w_i R_{i,T}) + w_j R_{j,T} u''(\sum_{i=1}^{N} w_i R_{i,T}). \tag{11}$$

Using (10) and (11), we can rewrite (9) as

$$\begin{aligned}
f(R_{1,T}, R_{2,T}, ...R_{N,T}) &\approx f(\mathrm{E}_t R_{m,T}, \mathrm{E}_t R_{m,T}, ...\mathrm{E}_t R_{m,T}) \\
&\quad + u'(\sum_{i=1}^{N} w_i \mathrm{E}_t R_{m,T})(R_{j,T} - \mathrm{E}_t R_{m,T}) \\
&\quad + \sum_{i=1}^{N} w_i \mathrm{E}_t R_{m,T} u''(\sum_{i=1}^{N} w_i \mathrm{E}_t R_{m,T})(R_{i,T} - \mathrm{E}_t R_{m,T}) \\
&= f(\mathrm{E}_t R_{m,T}, \mathrm{E}_t R_{m,T}, ...\mathrm{E}_t R_{m,T}) \\
&\quad + u'(\mathrm{E}_t R_{m,T})(R_{j,T} - \mathrm{E}_t R_{m,T}) + \sum_{i=1}^{N} w_i \mathrm{E}_t R_{m,T} u''(\mathrm{E}_t R_{m,T})(R_{i,T} - \mathrm{E}_t R_{m,T}).
\end{aligned}$$

We can now plug this result into the left hand side of (7) obtaining

$$\begin{aligned}
Cov_t\left(u'(\sum_{i=1}^{N} w_i R_{i,T}) R_{j,T}, R_{j,T}\right) &= Cov_t\left(f(R_{1,T}, R_{2,T}, ...R_{N,T}), R_{j,T}\right) \\
&\approx Cov_t\left(u'(\mathrm{E}_t R_{m,T})(R_{j,T} - \mathrm{E}_t R_{m,T}), R_{j,T}\right) \\
&\quad + Cov_t\left(\sum_{i=1}^{N} w_i \mathrm{E}_t R_{m,T} u''(\mathrm{E}_t R_{m,T})(R_{i,T} - \mathrm{E}_t R_{m,T}), R_{j,T}\right) \\
&= u'(\mathrm{E}_t R_{m,T}) Var_t\left(R_{j,T}\right) \\
&\quad + \mathrm{E}_t R_{m,T} u''(\mathrm{E}_t R_{m,T}) \sum_{i=1}^{N} w_i Cov_t\left(R_{i,T}, R_{j,T}\right) \\
&= u'(\mathrm{E}_t R_{m,T}) Var_t\left(R_{j,T}\right) + \mathrm{E}_t R_{m,T} u''(\mathrm{E}_t R_{m,T}) Cov_t\left(R_{m,T}, R_{j,T}\right),
\end{aligned}$$

where the penultimate equality follows from the fact that $\mathrm{E}_t R_{m,T}$ is constant as of time t, and the last equality follows from (5). Equivalently,

$$Cov_t\left(u'\left(R_{m,T}\right) R_{j,T}, R_{j,T}\right) \approx u'(\mathrm{E}_t R_{m,T}) \left[Var_t\left(R_{j,T}\right) + \mathrm{E}_t R_{m,T} \frac{u''(\mathrm{E}_t R_{m,T})}{u'(\mathrm{E}_t R_{m,T})} Cov_t\left(R_{m,T}, R_{j,T}\right)\right]. \tag{12}$$

Then, (12) can be written as

$$\begin{aligned}
Cov_t\left(u'\left(R_{m,T}\right) R_{j,T}, R_{j,T}\right) &\approx u'(\mathrm{E}_t R_{m,T}) \left[Var_t\left(R_{j,T}\right) - \gamma\left(\mathrm{E}_t R_{m,T}\right) Cov_t\left(R_{m,T}, R_{j,T}\right)\right] \\
&= u'(\mathrm{E}_t R_{m,T}) Var_t\left(R_{j,T}\right) \gamma\left(\mathrm{E}_t R_{m,T}\right) \left[\frac{1}{\gamma\left(\mathrm{E}_t R_{m,T}\right)} - \frac{Cov_t\left(R_{m,T}, R_{j,T}\right)}{Var_t\left(R_{j,T}\right)}\right].
\end{aligned}$$

We conclude that, up to a first order approximation, the NCC holds for asset j if

$$\frac{1}{\gamma\left(\mathrm{E}_t R_{m,T}\right)} \leq \frac{Cov_t\left(R_{m,T}, R_{j,T}\right)}{Var_t\left(R_{j,T}\right)}. \tag{13}$$

Note that when $Cov_t(R_{m,T}, R_{j,T}) \leq 0$, (13) implies that the NCC must fail (since risk aversion is positive). On the other hand, for "positive beta" assets where $Cov_t(R_{m,T}, R_{j,T}) > 0$, (13) can be written as

$$\gamma(\mathrm{E}_t R_{m,T}) \geq \frac{Var_t(R_{j,T})}{Cov_t(R_{m,T}, R_{j,T})}. \tag{14}$$

Intuitively, this means that the NCC holds for asset j with a positive beta if relative risk aversion is high enough to make the covariance between $R_{j,T}$ and $u'(R_{m,T})$ outweigh the covariance between $R_{j,T}$ and itself, i.e., the variance of $R_{j,T}$. As (14) shows, the ratio of the asset return's variance to its covariance with the market return is fundamental for evaluating whether the NCC holds for asset j. It will be convenient to denote this ratio by

$$\delta_{j,t} \equiv \frac{Var_t(R_{j,T})}{Cov_t(R_{m,T}, R_{j,T})}. \tag{15}$$

Thus, (14) is equivalent to $\gamma(\mathrm{E}_t R_{m,T}) \geq \delta_{j,t}$. The next proposition summarizes the discussion thus far, establishing a sufficient condition for the NCC and for the validity of the lower bound.

Proposition 2 *Up to a first order approximation, the NCC holds for asset j whenever $Cov_t(R_{m,T}, R_{j,T}) > 0$ and $\gamma(\mathrm{E}_t R_{m,T}) \geq \delta_{j,t}$. For such assets, $\frac{Var_t^*(R_{j,T})}{R_{f,t}}$ serves as a lower bound on the asset's expected excess return between time t and T. The NCC fails when $Cov_t(R_{m,T}, R_{j,T}) \leq 0$.*

To understand this statement note that from (2) and (12) we have

$$\mathrm{E}_t R_{j,T} - R_{f,t} = \frac{Var_t^*(R_{j,T})}{R_{f,t}} - u'(\mathrm{E}_t R_{m,T}) \left[Var_t(R_{j,T}) + \mathrm{E}_t R_{m,T} \frac{u''(\mathrm{E}_t R_{m,T})}{u'(\mathrm{E}_t R_{m,T})} Cov_t(R_{m,T}, R_{j,T}) \right] + error. \tag{16}$$

The sufficient condition in Proposition 2 insures that the bracketed term in (16) is non-positive, but does not apply to the error term. However, given that the error is of smaller order of magnitude, restricting attention to the bracketed term provides for an approximate way to test for the NCC. In Section 4 we evaluate the precision of this approximation and show that the errors it entails are small.

Note that when j is the market, $\delta_{j,t} = \delta_{m,t} = \frac{Var_t(R_{m,T})}{Cov_t(R_{m,T}, R_{m,T})} = 1$. Thus, in this case, Proposition 2 agrees with Martin's Proposition 1. More generally, Proposition 2 provides us with a sufficient condition to check whether any given asset j satisfies the NCC, and so, whether $\frac{Var_t^*(R_j)}{R_f}$ is a valid lower bound on its expected excess returns. We next study this condition and explain its economic meaning.

2.4 Combined Risk: The Economic Meaning of δ_j

The condition in Proposition 2 says that for the NCC to hold for asset j we need relative risk aversion in the economy to be greater than $\delta_{j,t}$. Thus, the NCC is more likely to hold for assets with low idiosyncratic risk and high covariance with the market. We call $\delta_{j,t}$ the *combined risk* of asset j, as it accounts for both idiosyncratic and systematic risk of the asset. This is in contrast to Martin's result which asked that risk aversion be uniformly greater than 1 for the lower bound on the market premium to hold. To obtain additional intuition for our condition, how it compares to Martin's condition, and the type of assets for which it is likely to hold, we now provide two different economic interpretations of $\delta_{j,t}$.

2.4.1 Relation to CAPM Beta and Idiosyncratic Risk

While $\delta_{j,t}$ is time varying, for estimation we will follow the standard approach (e.g., for estimating betas) and assume it is constant for some (short) period of time and denote it by δ_j. Then, δ_j could be easily estimated from time-series data of asset returns and the returns on the market. We denote by $\hat{\delta}_j$ the estimator of δ_j,

$$\hat{\delta}_j = \frac{Var\left(R_{j,t}\right)}{Cov\left(R_{j,t}, R_{m,t}\right)}. \tag{17}$$

To gain further intuition consider the following standard one-factor model

$$R_{j,t} = \alpha_j + \beta_j R_{m,t} + \varepsilon_{j,t}, \tag{18}$$

in which asset j's returns are being regressed over the market returns and $Cov\left(\varepsilon_{j,t}, R_{m,t}\right) = 0$. Considering the OLS estimation of this regression model, the slope is given by

$$\hat{\beta}_j = \frac{Cov\left(R_{j,t}, R_{m,t}\right)}{Var\left(R_{m,t}\right)},$$

and the R-squared is given by

$$\begin{aligned}
\rho_j^2 &= \frac{\hat{\beta}_j^2 Var\left(R_{m,t}\right)}{Var\left(R_{j,t}\right)} \\
&= \frac{\hat{\beta}_j Cov\left(R_{j,t}, R_{m,t}\right)}{Var\left(R_{j,t}\right)} \\
&= \frac{\hat{\beta}_j}{\hat{\delta}_j}.
\end{aligned}$$

We conclude that

$$\hat{\delta}_j = \frac{\hat{\beta}_j}{\rho_j^2}. \tag{19}$$

In words, δ_j is estimated as the ratio of the asset's beta and the R-squared from a regression of the asset returns on the market returns. Since $\frac{1}{\rho_j^2}$ is a measure of the asset's idiosyncratic risk, we obtain that δ_j is large for assets with either large beta and/or large idiosyncratic risk. Still, it is important to note that the relation between the lower bound and expected returns is not automatically driven in this setting by either beta or idiosyncratic risk. These are just building blocks in the validation of the lower bound, while the bound itself is calculated from options and is not mechanically related to either the CAPM beta or the R-squared.

One extreme case is when an asset has idiosyncratic risk of zero, i.e., $\rho_j^2 = 1$. In this case $\hat{\delta}_j = \hat{\beta}_j$. Another extreme case would be if idiosyncratic risk is very large and so ρ_j^2 would be (approximately) zero. In this case $\hat{\delta}_j$ diverges to infinity. The typical cases would be somewhere in between, in which $0 < \rho_j^2 < 1$ and hence the asset's combined risk is strictly higher than $\hat{\beta}_j$ as it is inflated by a factor of $\frac{1}{\rho_j^2}$.

Corollary 1 *For any asset j, $\hat{\delta}_j \geq \hat{\beta}_j$ with equality occurring only for assets with zero idiosyncratic risk.*

Since the weighted average of asset betas in the market equals 1 (the market beta), it follows from Corollary 1 that the weighted average of all combined risks will be strictly larger than 1.

According to Proposition 2, for the NCC to hold for asset j we need relative risk aversion to be greater than δ_j. Corollary 1 implies that the NCC for individual assets is typically more demanding than Martin's NCC for the market. Indeed, his condition only requires that risk aversion be greater than 1, while our condition requires that risk aversion be greater than δ_j, which is, on average, strictly greater than 1.

2.4.2 Relation to the Reverse CAPM Regression and Estimation Bias

More insight into the meaning of δ_j can be obtained by considering the reverse CAPM regression in which market returns are regressed on stock returns

$$R_{m,t} = \omega_j + \nu_j R_{j,t} + \eta_{j,t}. \tag{20}$$

Then, from (17),

$$\hat{\delta}_j = \frac{1}{\hat{v}_j}. \tag{21}$$

Thus, another interpretation of δ_j is as the reciprocal of the slope coefficient from a reverse CAPM regression. This interpretation is important for practical estimation since it has implications for the bias of the estimator.[10] To see this note that

$$\mathrm{E}\left(\hat{\delta}_j\right) = \mathrm{E}\left(\frac{1}{\hat{v}_j}\right) \geq \frac{1}{\mathrm{E}\left(\hat{v}_j\right)} = \frac{1}{v_j} = \delta_j,$$

where the first equality follows from (21), the inequality follows from Jensen's inequality, the second equality follows from the fact that OLS coefficients are unbiased, and the last equality follows by taking expectations on both sides of (18) and (20) and then comparing coefficients. We have obtained the following corollary.

Corollary 2 *For any asset j, $\mathrm{E}\left(\hat{\delta}_j\right) \geq \delta_j$. That is, the estimator for δ_j is upward biassed.*

This result is important since it implies that by using $\hat{\delta}_j$ as an estimator for δ_j we are actually being conservative. Namely, we require risk aversion to be larger than $\hat{\delta}_j$ while it actually can be somewhat lower.

2.5 Estimating the Lower Bound

To estimate the lower bound on the return of asset j from option prices, we follow the approach in Martin (2017), which is similar to prior results by Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003). The idea is that the risk-neutral variance of a stock return can be calculated by integrating option prices with different strike prices. Formally, let $S_{j,t}$ denote the price of asset j at time t and let $d_{j,t}$ denote the present value of dividends paid between times t and T. Consider a set of call and put European options on asset j with strike prices K ranging from 0 to infinity with the same maturity, T. Denote the prices of these options by $call_{j,t}(K)$ and $put_{j,t}(K)$, respectively. Martin shows that

$$\frac{1}{R_{f,t}} Var_t^* R_{j,T} = \frac{2}{S_{j,t}^2}\left[\int_0^{F_{j,t}} put_{j,t}(K)\, dK + \int_{F_{j,t}}^{\infty} call_{j,t}(K)\, dK\right] \tag{22}$$

[10] We thank Kerry Back for highlighting this point to us.

where

$$F_{j,t} = R_{f,t}\left(S_{j,t} - d_{j,t}\right),$$

is the forward price of asset j as of time t for delivery at time T. Note that as long as $d_{j,t}$ is not too large, the integration in (22) is performed using options that are primarily out the money for both the put and the call options.

To obtain a numerical estimate of (22) we consider all put options with strike prices less than or equal to $F_{j,t}$ and call options with strike prices larger than $F_{j,t}$ on asset j. Assume there are N_P such put options and N_C such call options available at a given point in time and maturity for which we would like to estimate (22) with corresponding strike prices $K_1^P < ... < K_{N_P}^P < K_1^C < ... < K_{N_P}^C$. Denote the prices of these options by $put_1, ..., put_{N_p}$ and $call_1, ..., call_{N_C}$ respectively. Then, our numerical estimate for the bracketed integrals in (22) is

$$\sum_{i=1}^{N_P-1} put_i\left(K_{i+1}^P - K_i^P\right) + \sum_{i=1}^{N_C-1} call_{i+1}\left(K_{i+1}^C - K_i^C\right) + \left(K_1^C - K_{N_P}^P\right)\min\left(put_{N_P}, call_1\right). \tag{23}$$

Figure 1 illustrates this numerical integration. Note that our numerical approach estimates the integral from below by using the minimum price at each interval, which is consistent with our goal to obtain a lower bound. This approach is somewhat different from Martin's, who uses the price at the mid-point of each interval and then relies on the convexity of the option prices to justify the validity of the lower bound. Our more conservative approach is mandated by the option data for individual stocks, which often does not offer equally spaced strike prices, and thus does not allow us to rely on a similar convexity argument. Note also that dropping the left and right tails from the integral in (22) (i.e., starting the integration at K_1^P and ending at $K_{N_C}^C$) again has the effect of lowering the integral, in line with obtaining a valid lower bound.

A limitation unique to our setting of dealing with individual assets is the fact that options for individual stocks in U.S. markets are all of the American style, while the correct implementation of (22) requires European style options. Since the prices of the American options include an early exercise premium, using American options instead of European options tends to overestimate the lower bound. It is known, however, that the value of the early exercise premium in out-of-the-money options is small (see Barone-Adesi and Whaley, 1987 and Shu, 2011). In Section 4 we provide estimates for the positive bias introduced by using American instead of European options and show that this bias is indeed typically small. Moreover, we show that this bias is more than offset by the slack we create through estimating the integral in (22) by rectangles from below (Figure 1).

3 Empirical Analysis

3.1 Data and Main Calculations

To calculate (23) we obtain option price data from OptionMetrics. An observation in this database consists of the closing bid and ask prices for an option on a given date for a given stock. Typically, a stock will have multiple options traded on it with different strike prices and maturities.

It is well known that options for individual stocks may be quite illiquid. Thus, we limit our attention to a subset of stocks which are known to be highly traded and liquid. In particular, we focus our attention on options written on stocks which were included in the S&P 500 index starting from 1996 (the beginning of OptionMetrics data). We use the following criteria to further screen illiquid options (similar or related filters have been used in Figlewski, 2008 and Benzoni, Collin-Dufresne, and Goldstein, 2011):

- We drop options with maturity horizon of less than 7 days, as these options are often thinly traded.

- We drop observations that have missing values in their bid and/or ask prices.

- We do not estimate the integrals in (22) for a given day and stock if the total volume of options with the same maturity is less than 20 contracts as the prices of such thinly traded options are not reliable.[11]

- We do not calculate the integrals in (22) for a given day and stock if the lowest strike to closing price ratio is greater than 0.8 or the highest strike to closing price ratio is less than 1.2. In these cases the left and/or right tails are very large and we are not likely to obtain a good estimate.

[11] We have repeated all the analysis with a screen of at least 50 contracts traded per day. This results in about 10% drop in the number of daily observations but no material effect on any of our results.

- We do not calculate the integrals for a given day and stock if the number of distinct options for a given maturity is less than 20, as the grid would become too coarse to obtain a viable estimate.

- We do not calculate the integrals for a given day and stock if the maximum distance between two adjacent strike prices is greater than the maximum of 20% of the closing stock price and 10. Again, this would imply a very coarse grid.

- We drop all options with a non-standard settlement.

Figure 2 plots the number of stocks which have options passing the above screens on each day starting January 1996. The number of stocks increases gradually over time. At the beginning of 1996, there are around 20 stocks which have available option data to calculate the lower bound. The number increases to above 150 at the beginning of year 2003 and up to about 500 in 2016. In order to have a rich cross-section for the empirical analysis, we choose to begin our empirical analysis at the year 2003. Thus, we set the sample period to January 2003-April 2016 and consider a sample of S&P 500 constituents during that period.

After applying the above screens we are left with 214,441,029 observations which yield 1,238,669 stock/day combinations for which we can estimate (22) to obtain the lower bounds on expected excess returns of 696 distinct stocks. For each stock/day combination in this sample we estimate the lower bound (22) using (23) for each available maturity separately and then annualize it. For these calculations we take $S_{j,t}$, as the closing price of the underlying security, and the put and call prices used for the integration as the average of the closing bid and ask prices of these options. We then estimate the lower bound for a given day as the average of the lower bounds across different maturities for that day.[12]

Besides the OptionMetrics data we also draw data from CRSP and Compustat to calculate δ_j as well as firm characteristics such as beta, size, book-to-market ratio and momentum. These characteristics are used in our cross-sectional Fama-MacBeth regressions.

[12] An alternative approach would be to take the maximum lower bound across different maturities (as this is the most binding of the lower bounds). We have tried this approach as well and it yields similar results.

3.2 Testing for the NCC – Non-Parametric Approach

By Proposition 2, for the NCC to hold we need that $\gamma \geq \delta_j$, where $\delta_j = \frac{Var(R_j)}{Cov(R_j,R_m)}$ is asset $j's$ combined risk and γ is the relative risk aversion in the economy. While δ_j can be easily estimated from data, γ is not directly observable. The finance literature has provided a wide range of reasonable values for relative risk aversion. For example, Bliss and Panigirtzoglou (2004, Table 7) gather estimates of relative risk aversion from several prior studies. Their table shows estimates ranging anywhere between 0 and 55. Recent studies in asset pricing typically consider relative risk aversion levels between 1 and 10 as being "reasonable." For example, Mehra and Prescott (1985) argue that relative risk aversion should be lower than 10, and Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2011), and Epstein, Farhi, and Strzalecki (2014) use a relative risk aversion coefficient of either 7.5 or 10 for their calibrations. Such levels are supported by recent estimates such as in Vissing-Jorgensen and Attanasio (2003), who estimate relative risk aversion between 5 to 10 under realistic assumptions for Epstein-Zin Euler equations. Similarly, Bliss and Panigirtzoglou (2004), when considering power utility, estimate relative risk aversion between 3 to 10 as implied from option prices.

To see whether the condition $\gamma \geq \delta_j$ is empirically reasonable, and so whether the NCC may hold for typical stocks, we begin by calculating δ_j from historical stock returns. We estimate δ_j as the variance of the stock returns divided by the covariance of its returns with the CRSP value-weighted returns. We perform these calculation using both daily and monthly data over our sample period.

Panel A of Table 1 reports summary statistics for δ_j based on daily returns and with a 12-month rolling window and updated on a monthly basis (i.e., we use daily data but calculate δ_j for each month). We estimate δ_j for a particular month provided that there are at least 200 return observations available for stock j in the most recent 12 months. The mean of δ_j is 4.3, the median is 3.3, and the 75th percentile is 5.1. Thus, with risk aversion of 5, about 75% of S&P 500 constituents would satisfy the NCC. If one assumes risk aversion of between 7.5 and 10 (as in Bansal and Yaron (2004), and the literature that follows), then at least about 90% of S&P 500 stocks satisfy the NCC.

Panel B reports parallel results using monthly return data with δ_j being calculated based on a 60-month rolling window. The δ_j estimates here are a bit higher with mean of 6 and median of 4.3. Recall, however, that our estimates of δ_j are biased upward, and so the proportion of stocks satisfying the NCC is likely higher than we report.

This discussion demonstrates that checking whether the NCC holds for a particular stock ultimately depends on one's views regarding relative risk aversion in the economy. To fix ideas we divide all S&P 500 constituents in each month τ starting from January 2003 into four groups denoted by *conservative, moderate, liberal, and very liberal* based on their $\hat{\delta}_{j,\tau}$ (calculated using daily returns) estimated over a rolling window of 12 months from $\tau - 12$ to $\tau - 1$ as follows

$$\text{stock } j \text{ in month } \tau \text{ is} \begin{cases} \textit{conservative} & \text{if} \quad 1 \leq \hat{\delta}_{j,\tau} < 4 \\ \textit{moderate} & \text{if} \quad 4 \leq \hat{\delta}_{j,\tau} < 7 \\ \textit{liberal} & \text{if} \quad 7 \leq \hat{\delta}_{j,\tau} \leq 10 \\ \textit{very liberal} & \text{if} \quad \hat{\delta}_{j,\tau} > 10 \end{cases}. \tag{24}$$

Our main analysis will be centered around the conservative, moderate, and liberal stocks, which collectively cover about 95% of all S&P 500 constituents (see Table 1). For these stocks, risk aversion in the ranges of 1-4, 4-7, or 7-10 respectively, guarantees the validity of the lower bound. These risk-aversion levels are quite standard in the literature as discussed above. For very liberal stocks (consisting of about 5% of S&P 500 stocks) the lower bound is unlikely to be valid, and the sign of the covariance in (7) is likely positive. Thus, instead of a lower bound we will likely be getting an upper bound in this case.

Table 2 reports summary statistics for the combined risk, $\hat{\delta}_{j,\tau}$, for the four groups defined in (24). The conservative group is the largest with 371 stocks on average, followed by the moderate and liberal groups with 147 and 46 stocks. The very liberal group is the smallest with just 32 stocks on average. As expected, when $\hat{\delta}_{j,\tau}$ becomes larger, both the systematic risk (reflected in beta) and idiosyncratic risk (reflected in the inverse of R-squared) become larger. Simultaneously, firms that belong to the more liberal groups are also smaller in size.

3.3 Testing for the NCC – Parametric Approach

An important advantage of Martin's approach is that it does not rely on a particular model for the stochastic discount factor. Accordingly, in the previous subsection we did not take a stance on the structure of M_t when testing for the validity of the NCC. Still, it is useful to consider standard parametrizations of M_t and test whether the NCC applies under reasonable levels of risk aversion. Specifically, for a given structure of M_t we use the same sample as in the previous subsection and calculate $Cov\left(M_t R_{j,t}, R_{j,t}\right)$ and the corresponding Pearson correlation $\hat{\rho} = corr\left(M_t R_{j,t}, R_{j,t}\right)$

directly using both a 12-month rolling window with daily returns and a 60-month rolling window with monthly returns. We then test for the null hypothesis $H_0 : \rho \leq 0$ (NCC is satisfied) against the alternative $H_1 : \rho > 0$ using a standard t-test for correlations.

We consider four standard models for the stochastic discount factor: CRRA utility, Epstein-Zin preferences, one-factor CAPM model, and the Fama-French three-factor model. The first two models allow for direct calculations of M_t, whereas the factor models require also the estimation of relevant coefficient parameters, for which we apply the Generalized Method of Moments (GMM). We describe the details of the construction of the models and the estimation approach to each one of them in Appendix II.

Table 3 reports the proportion of stocks for which we cannot reject the null (NCC is satisfied) at the 5% significance level. For the first three models (CRRA, Epstein-Zin, and CAPM) the proportion of stocks for which the NCC is satisfied grows monotonically with relative risk aversion. In these cases, when risk aversion is at least 5 this proportion ranges between 84% and 100% for both the 12-months rolling window and the 60-month rolling window specifications. And, when risk aversion is at least 6, the proportion of NCC-satisfying stocks is above 90%. The picture for the three-factor model is a bit different. Here we impose a more involved requirement on the correlation structure between a stock return and the returns of three different factors, as opposed to just one factor in the previous models. As a result, the fraction of stocks for which the NCC is satisfied ceases to be monotone in relative risk aversion. In fact, in this specification the fraction of stocks satisfying the NCC appears to follow a U-shape: it is high (70%-90%) for low and high levels of risk aversion, and low (40%-60%) for intermediate levels.

Overall, the parametric results reinforce the non-parametric results, showing that for reasonable levels of risk aversion the proportion of stocks for which the NCC holds is large, supporting our approach of exploiting the NCC to bound the expected returns.

3.4 Lower Bound Example: Microsoft, P&G, and Apple

Before moving to a full-scale empirical analysis, we begin by illustrating the lower bound for three household names: Microsoft, P&G, and Apple. The medians of $\delta_{j,t}$ for these three stocks are 2.45, 1.80 and 3.46, respectively (based on daily returns).

Thus they typically belong to the conservative group and a risk aversion of 4 or above would validate the NCC for them.

Figure 3 presents the time series of the lower bound for these three stocks between January 2003 and April 2016. For Microsoft, the average lower bound is 6%. A clear pattern is that the lower bound spiked very significantly during the years of the financial crisis. Excluding the years 2008-2009, the average lower bound on expected excess returns for Microsoft is 5.2%. Considering P&G, the average lower bound is 3.1% and it also exhibits a sharp increase during the crisis period. Excluding this period the average lower bound is 2.5%. For Apple, the average lower bound on expected excess returns is much larger at 13.1%. It decreases to 11.6% once the crisis period is excluded.

3.5 Summary Statistics for the Lower Bound

Figure 4 and Table 4 provide summary statistics for the lower bound. Consider first the stocks in the conservative group. The average lower bound for these stocks is 8.7% and the median is 6.2%. Figure 4 also clearly shows that the expected return for these stocks spiked significantly during the crisis years of 2008-2009, in line with the pattern observed for the three household names discussed above. Excluding these two years shows that during "normal times" the average lower bound on these stocks is 7.3% and the median is 5.7%. Note that the distribution of lower bounds is right skewed, which is driven by some very large occasional values. Turning to the moderate group, the average lower bound is 15.8% and the median is 10.5% for the whole sample period. Excluding 2008-2009 we have an average lower bound of 13.3% and a median of 9.6%. Evidently, the lower bounds of the expected excess returns in this group are higher than in the conservative group. Considering the liberal group we obtain an even higher lower bound, which spikes occasionally not only during the financial crisis but also in later years. This volatility may be attributed to the fact that this group is very small with only 37.7 stocks on average. The average lower bound in this group is 18.9% and the median is 11.8%. Excluding the crisis period we have a mean of 16.0% and a median of 9.7%.

Thus, as $\delta_{j,\tau}$ becomes larger, the lower bound we obtain rises as well. This, however, is not a mechanical result of sorting stocks by $\delta_{j,\tau}$. Indeed, $\delta_{j,\tau}$ is based on information calculated from historical stock returns, whereas the lower bound is calculated from option prices, reflecting forward looking expectations. Instead, this result can stem from two different sources. First, as $\delta_{j,\tau}$ becomes larger the lower bound becomes tighter and (if the risk aversion is sufficiently low) may even switch to becoming an upper bound. Second, a higher $\delta_{j,\tau}$ corresponds to higher systematic and/or idiosyncratic risk. And, as shown in Panel B of Table 2, higher $\delta_{j,\tau}$ corresponds to higher beta and smaller size. To the extent that these two risks are compensated for in expected returns, we should expect a higher lower bound.

Considering the very liberal group we again see a rather volatile pattern of lower bounds, which may be attributed to the small number of stocks in this group (28.4 on average). Recall that for this group it is quite likely that what we are calculating is an upper bound rather than a lower bound on expected excess returns. In line with this view, the average bound we obtain is 20.7% and the median is 13.5%. Excluding 2008-2009 we still have a quite high mean of 19.0% and median of 12.2%.

It is interesting to compare the average lower bound over the entire sample (Panel E of Figure 4) to the lower bound for the S&P 500 (Panel F of Figure 4). The latter is simply our version of Martin's (2017) calculations obtained using options on the S&P 500 index (compared this to Figure IV in Martin (2017)) . The former is the average of the lower bounds for all stocks in our sample considering all the four groups lumped together. The average lower bound for the market premium we obtain is 3.9% and the median is 2.9%. These numbers are comparable to those obtained by Martin (2017) over a larger sample period. The average lower bound across all stocks in our sample is 13.0% and the median is 8.2%. The fact that the average lower bound is higher than the lower bound for the market is not a coincidence. The reason for this is that our lower bounds are convex functions of returns, and thus the lower bound for a portfolio is strictly lower than the weighted average of lower bounds. To see this point formally note that by the convexity of the variance operator we have

$$Var^*\left(\sum_{j=1}^{N} w_j R_j\right) < \sum_{j=1}^{N} w_j Var^*\left(R_j\right),$$

implying that the lower bound for the market is strictly lower than the weighted average of lower bounds for individual stocks. Thus, it would be inappropriate to estimate Martin's lower bound for the market premium by averaging our lower bounds for individual stocks.

3.6 Cross-Sectional Analysis of the Bound

Having documented the time-series summary statistics of the lower bounds we now turn to study how they vary in the cross-section of stocks. We hypothesize that firm characteristics that have been documented to affect average realized returns will be reflected also in the forward-looking lower bounds on expected returns we study. The usual suspects are of course beta, size, and book-to-market (Fama and French (1992)) as well as momentum (Jegadeesh and Titman (1993)).

Our goal is to perform a standard Fama-MacBeth (1973) cross-sectional analysis on a monthly basis in which the estimated bound on expected excess returns serves as a left-hand side variable. Thus, we perform a standard analysis replacing realized returns with forward looking lower bounds of expected returns. To this end, we first calculate for each stock in our sample an average monthly lower bound by averaging the daily estimates of the lower bound for each month during our sample period. We then run cross-sectional regressions of the monthly lower bound against beta, size, book-to-market ratio, and momentum.[13] We perform this analysis for the entire sample as well as separately for each of the four groups defined in (24).

Table 5 reports the time-series averages of the cross-sectional coefficient estimates along with Newey-West adjusted standard errors. These adjustments are needed to account for possible autocorrelations in the time series of the lower bound. The results are striking. To begin, consider the entire sample. First, beta has a highly significant and positive coefficient. That is, firms with high beta are associated with a high lower bound on their expected returns in line with the predictions of the classic CAPM. Thus, when considering forward-looking expected returns it appears that beta is getting its life back after being "announced dead" in Fama and French (1992). Second, and just as important, size obtains its expected signs as in Fama and French (1992). Indeed, the coefficient of size is negative and highly significant. Similarly, the book-to-market ratio is also positively significant like in Fama and French (1992). Finally and somewhat surprisingly, the coefficient on momentum is negative and significant, suggesting that stocks that experienced a run-up in their

[13] We estimate beta by regressing the daily stock realized excess returns on the daily market excess return over a rolling window of 12 months prior to the month of interest. We estimate firm size as the log of the year-end market-cap for the fiscal year that preceded the month of interest and book-to-market is the log of the ratio of the book-value of equity to market-cap as of the end of the preceding fiscal year. Finally, momentum is estimated as the stock return during the 12 months preceding the month of interest.

price in the past 12 months are associated with lower forward looking expectations.

When considering the conservative, moderate, and liberal groups separately we obtain quite similar results. The coefficient of beta is positive and significant, the coefficient of size is negative and significant, the coefficient of book-to-market ratio is positive and significant, and the coefficient of momentum is negative and significant for all three groups. Finally, when considering the very liberal group – the coefficient of book-to-market ratio is not significant, perhaps reflecting the noise in the estimation of the bound for this group and its small size.

It is important to note that the relation between beta and the lower bound is not mechanical. The fact that $\delta_j = \frac{\beta_j}{\rho_j^2}$ has no bearing on these results since δ_j is not playing any role in this analysis. The only role of δ_j is to determine whether the NCC holds or not, but this is not related to the estimation of the lower bound from option prices and to its relation to firm characteristics. Furthermore, given that these results hold for each of the conservative, moderate, and liberal groups (and within each group δ_j is similar) it is unlikely that they are driven by δ_j itself.

Overall, the cross-sectional results suggest that beta, size, book-to-market, and momentum are reflected in the lower bounds for the expected returns of individual assets. The result for beta is particularly important since it comes up as a major determinant of expected stock returns unlike in the studies using realized returns. This is consistent with previous studies using other approaches for estimating forward looking returns as in Berk and van Binsbergen (2015) and Brav, Lehavy, and Michaely (2005). As for recent returns, they still play a significant role, but rather than expecting continuation, it appears that investors are expecting reversals in the short term.

3.7 Correlation of the Bound with Future Returns

In the next set of analyses we ask if the lower bound on expected excess returns provides a valuable signal on realized future returns. Finding such a predictive value is not obvious for two reasons. First, it may be that the market expectations reflected in option prices are systematically wrong or irrational, and so they (or lower bounds thereof) are not providing a valuable signal about future returns. Second, it may well be that the lower bound we obtain is far from being binding. Thus, variations in the lower bound may not tell us much about variations in realized future returns. Despite these challenges, we do find statistically significant and economically meaningful evidence of predictability as we describe below, especially in the 6-12 months investment horizons.

To evaluate whether the lower bound delivers an informative signal about future returns, in each month t we sort all stocks in our sample based on their monthly average lower bound during that month. We then divide the stocks into ten deciles based on their average bound in month t. Decile 1 consists of the stocks with the lowest estimates and Decile 10 with the highest estimates. We calculate the equal weighted average of stock realized returns for each decile over three different horizons starting from the end of month t: one month, six months, and 12 months. If the lower bound provides an informative signal, we expect stocks in lower deciles to exhibit lower average realized returns compared to stocks in the higher deciles.

The results for this analysis are documented in Table 6 for the entire sample and for each of the four groups separately. Consider first Panel B, which reports the time-series averages of the six-month returns for each decile as well as the returns of a portfolio which is long in Decile 10 and short in Decile 1. Considering the entire sample (first column) we see a quite monotone pattern. The average six-month returns in the low decile is 1.84% as compared to 14.96% in the top decile. The difference of 13.12% is very large economically and significant at the 1% level. Similar results are obtained for each of the sub-groups. For example, the moderate group (column 3) shows an increase in average realized returns of 4.52% in the low decile to 11.14% in the top decile, a difference of 6.61% over a six months period.[14]

In Panel C we consider a 12-month investment horizon. The entire sample (column 1) shows an almost perfect monotone trend. Indeed, average returns for deciles 1 through 6 are all below 10% while for deciles 7 through 10 they are all above 10%. The average return in the low decile equals 3.91% and it rises to 26.74% in the top decile – a difference of 22.84% over 12 months (significant at the 1% level). A similar pattern exists in each of the individual groups. For example, in the liberal group, the average return in the low decile is 10.74% compared to 41.00% in the top decile. This difference is very large economically and is significant at the 1% level.

In Panel A, which reports the one-month returns, we see economically similar but statistically weaker results. The trends in realized returns are still monotone and the differences between the two extreme deciles are still always positive and economically

[14]We use Newey-West standard errors with 4/10/16 lags in this analysis to account for the potential autocorrelations resulting from the overlapping windows of return estimation.

large. For example, for the conservative group the average difference between the top and bottom deciles is 0.9% per month, which (after annualizing) is similar to the corresponding difference obtained over the 12 -month horizon reported in Panel C. For the entire sample we have a difference of 1.8% per month, which translates to about 21.6% annually – in line with the corresponding result in Panel C. However, only the entire sample and the very liberal group exhibit statistically significant differences. Apparently, the signal-to-noise ratio in the one-month horizon of the lower bound is lower than in the longer-term horizons, reflected in weaker statistical significance.

To summarize, the results in Table 6 provide evidence in support of a predictive signal incorporated in the lower bound on expected stock returns. The evidence is economically large and statistically significant for an investment horizon of 6-12 months. The evidence of predictability applies to all four groups of stocks.

4 Precision of Approximations

Our analysis relies on two types of approximations. First, to study whether the NCC holds for a particular asset we rely on a first-order Taylor approximation. Second, when estimating the lower bound we do not account for the fact that the options being used are American. In this section we estimate the precision of these approximations. Our overall conclusion is that in most cases our approximations are quite precise and that our conservative approach to estimating the bound more than offsets the potential biases.

4.1 First Order Approximation for the NCC

A key advantage of our approach is that the form of the utility function need not be known in order to check whether the NCC holds or to estimate the lower bound for a particular asset. Instead, all that is needed is an assumption on the relative risk aversion (which may not be constant) in the economy and its magnitude relative to the combined risk of the asset. However, in order to obtain this "utility irrelevance" we replaced $u'(R_{m,t})\, R_{j,t}$ with its first order Taylor approximation in the calculation of $Cov\left(u'(R_{m,t})\, R_{j,t}, R_{j,t}\right)$. Similar approximations are rather common in asset pricing models attempting to linearize or log-linearize non-linear expressions. In order to assess the precision of this approximation we consider standard utility functions, which allow us to estimate $Cov\left(u'(R_{m,t})\, R_{j,t}, R_{j,t}\right)$ directly without resorting to an approximation. We can then test how often the first order approximation leads us to an incorrect inference about the NCC. Specifically, we assume that u takes the CRRA form $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$, with γ varying between 2 to 10. We then repeat the analysis in Section 3.2, but instead of using the condition $\gamma \geq \delta_{j,\tau}$ (relying on the approximation) we check directly whether $cov(u'(R_{m,t}) R_{j,t}, R_{j,t}) \leq 0$ holds true.

Table 7 reports the type I (false positive) and type II (false negative) errors associated with using the approximation. The table shows that for all risk aversion levels between 2 and 10, the probability of concluding that the NCC holds while in fact it does not is less than 2.2%. At the same time, the probability of concluding that the NCC fails while it actually holds is less than 1%. Thus, both Type I and II errors are very rare, indicating a precise approximation. Note that as γ grows larger, the probability of type I errors increases while that of type II errors declines.

4.2 Using American Options to Calculate the Lower Bound

The estimation of the lower bound in (22) relies on the prices of European options. However, all options on individual stocks in the U.S. are of the American style, introducing a potential upward bias in our lower bound estimation due to the early exercise premium (EEP). It is important to note that the options we are using are mostly out-of-the-money, a case in which the EEP is known to be relatively small. Still, in this section we evaluate the magnitude of this potential bias and demonstrate that it is, in fact, being offset through our conservative estimation approach.

A key advantage of (22) is that it makes no assumptions regarding the underlying framework such as the underlying process driving stock prices. To estimate the EEP we need to make additional assumptions on the dynamics of the underlying security prices. To this end, we follow the framework in MacMillan (1986) and Barone-Adesi and Whaley (1987), who offer an analytic approximation for the EEP of American options in the Black-Scholes framework. Specifically, we calculate the lower bound in (22) by obtaining option prices from OptionMetrics and subtracting from them the estimated EEP to obtain a synthetic price of a corresponding European option.[15]

The results are reported in Table 8, which is analogous to Table 4, with the only difference being the use of the synthetic European option prices instead of the

[15] An alternative approach to evaluating the role of the EEP in our estimation would be to restrict attention to stocks that do not pay dividends. For such stocks, the EEP is zero for call options. However, given that we use both call and put options for the estimation of the lower bound, this approach offers only a partial remedy to this issue.

American options prices. As expected, the lower bounds in Table 8 are all lower than in Table 4, but the differences are typically small. To illustrate, the median lower bound for the entire sample in Panel A of Table 4 is 8.16% as opposed to 7.85% in Panel A of Table 8, a difference of 31 basis points annually. When considering the conservative group, the median is 6.16% as opposed to 5.97%, a difference of 21 basis points. Similar differences apply to the three other groups. The differences are even smaller when considering Panel B of Tables 4 and 8, which exclude the crisis period.

When considering the mean of the lower bound distribution, the differences for the conservative groups are also small at 39 basis points. The differences in means become larger for the other three groups. The deviation between the mean and the median in that regard is a result of what looks like outliers in the right tail, specifying lower bounds close to or higher than 100%.

To further evaluate the potential bias in using American options we take a different approach to evaluating the integral in (22). Instead of approximating the integral using rectangles, we approximate the curve using a cubic spline interpolation. This estimation approach is not conservative as the one based on (23) since it attempts to approximate the curve using polynomials, but not necessarily from below. Due to the long computing time required by this procedure we perform this analysis for the 30 Dow Jones constituents only. Consistent with this approach being less conservative, the median lower bound we obtain for this group of stocks is 5.79% annually as opposed to 5.13% relying on (23). The median difference in lower bound calculation due to the EEP estimated for this group stocks is just 0.2%. This suggests that our conservative approach for estimation of the lower bound using (23) more than compensates for the potential bias in using American options.

Finally, as a robustness test we also repeat the empirical analysis discussed in Sections 3.6 and 3.7, replacing the original lower bounds by the modified lower bounds obtained using the synthetic European options described above. None of the conclusions is affected.

5 Conclusion

In this paper we rely on the approach introduced in Martin (2017) to offer a forward looking high-frequency lower bound on the expected excess returns of individual stocks. The lower bound is valid as long as risk aversion is greater than the combined risk of the stock, reflecting both idiosyncratic and systematic risk. We estimate the lower bound and find that it is positively correlated with beta and book-to-market ratio and negatively correlated with firm size and momentum. Moreover, the lower bound appears to provide a strong signal on future realized returns, especially at horizons of 6-12 months.

This paper adds to the growing literature focusing on forward-looking estimates of expected returns rather than relying on a backward-looking approach. The high frequency at which we can calculate our lower bound opens the door to many potential applications and situations in which expected returns are likely to react to specific events. For example, one can use the bound in event studies to test how expected returns react to earnings announcements, dividend changes, analyst recommendation changes, and management projections. Additionally, a lower bound on expected returns translates naturally into an upper bound on valuation. Thus, our approach leads to a natural upper bound of the value of different securities. We leave the exploration of such applications to future research.

Appendix I

In this appendix we derive approximations and conditions for when an asset satisfies the NCC in the context of two dynamic models. First is a standard consumption/investment model with separable utility in which risk aversion may not be constant. Second is a dynamic model with a recursive Epstein-Zin utility.

A Dynamic Model with Separable Utility

Consider a representative investor with time-separable utility $u(\cdot)$, which is increasing and concave and a subjective discount factor $\beta \in (0,1)$. The investor faces $N > 1$ assets with random return $R_{i,t+1}$ between time t and $t+1$. In each period $t = 0, 1, 2, ...$ the investor needs to allocate his initial wealth W_t among consumption C_t and investment in each asset i, $w_{i,t}$.

The value function $J(\cdot)$ describing the expected lifetime utility of the investor is defined recursively as a function of her wealth W_t as follows

$$\begin{aligned} J(W_t) &= \max_{C_t,\{w_{i,t}\}} \left[u(C_t) + \beta \mathrm{E}_t J\left((W_t - C_t) \sum_{i=1}^{N} w_{i,t} R_{i,t+1} \right) \right] \\ s.t. \sum_{i=1}^{N} w_{i,t} &= 1. \end{aligned}$$

The first-order condition for $w_{i,t}$ is

$$\beta \mathrm{E}_t J' \left((W_t - C_t) \sum_{i=1}^{N} w_{i,t} R_{i,t+1} \right) (W_t - C_t) R_{i,t+1} = \lambda,$$

where λ is a positive multiplier. It follows that

$$M_{t+1} \equiv \frac{\beta J' \left((W_t - C_t) \sum_{i=1}^{N} w_{i,t} R_{i,t+1} \right) (W_t - C_t)}{\lambda}$$

is a time-t stochastic discount factor (pricing claims between time t and $t+1$). Since $W_{t+1} = (W_t - C_t) \sum w_i R_{i,t+1}$ we have

$$M_{t+1} = \frac{\beta J'(W_{t+1})(W_t - C_t)}{\lambda}.$$

Given that the representative investor holds the market, the portfolio $(w_{1,t}, ..., w_{N,t})$ is the market portfolio as of time t, and we denote the return on this portfolio between times t and $t+1$ by

$$R_{m,t+1} = \sum_i w_{i,t} R_{i,t+1},$$

implying that $W_{t+1} = (W_t - C_t) R_{m,t+1}$. This shows that M_{t+1} is proportional to $J'((W_t - C_t) R_{m,t+1})$. Thus, the NCC holds for asset j at time t if and only if

$$Cov_t \left(J'((W_t - C_t) R_{m,t+1}) R_{j,t+1}, R_{j,t+1} \right) \leq 0.$$

Define $f : \mathbb{R}^N \to \mathbb{R}$ by

$$f(R_{1,t+1}, R_{2,t+1}, ...R_{N,t+1}) = R_{j,t+1} J'((W_t - C_t) \sum_{i=1}^{N} w_{i,t} R_{i,t+1}). \tag{25}$$

An analysis parallel to that in Section 2.3 yields the following first order approximation

$$Cov_t\left(J'\left(\left(W_t - C_t\right)R_{m,t+1}\right)R_{j,t+1}, R_{j,t+1}\right) \approx J'(\mathrm{E}_t W_{t+1})\left(Var_t\left(R_{j,t+1}\right) - \Gamma(\mathrm{E}_t W_{t+1})Cov_t\left(R_{m,t+1}, R_{j,t+1}\right)\right)$$

where $\Gamma(\mathrm{E}_t W_{t+1}) = -\mathrm{E}_t W_{t+1}\frac{J''(\mathrm{E}_t W_{t+1})}{J'(\mathrm{E}_t W_{t+1})}$ is the investor's relative risk aversion with respect to her lifetime utility evaluated at $\mathrm{E}_t W_{t+1}$.

Thus, similar to the conclusion in Section 2.3 we have that the NCC holds for asset j at time t if $Cov_t\left(R_{m,t+1}, R_{j,t+1}\right) > 0$ and relative risk aversion is greater than $\delta_{j,t} = \frac{Var_t(R_{j,t+1})}{Cov_t(R_{m,t+1}, R_{j,t+1})}$.

Proposition 3 *Up to a first order approximation, the NCC holds for asset j whenever $Cov_t\left(R_{m,t+1}, R_{j,t+1}\right) > 0$ and $\Gamma\left(\mathrm{E}_t W_{t+1}\right) \geq \delta_{j,t}$. For such assets, $\frac{var_t^*(R_{j,t+1})}{R_{f,t}}$ serves as a lower bound on the asset's expected excess return between times t and $t+1$. The NCC fails when $Cov_t\left(R_{m,t+1}, R_{j,t+1}\right) \leq 0$.*

Note that we have obtained this result in a traditional consumption/investment framework in which returns are exogenous while consumption and investment are endogenous. A similar result can be obtained in an endowment economy, in which consumption is assumed exogenous and prices are determined endogenously.

A Dynamic Consumption/Investment Model with Recursive Utility

Consider an infinitely lived representative investor with recursive value function

$$V_t = J\left(C_t, \mu\left(V_{t+1}\right)\right),$$

where the function J is an aggregator mapping current consumption C_t, and the certainty equivalent of future lifetime value, $\mu\left(V_{t+1}\right)$, to current value, V_t. We follow Epstein and Zin (1989) and consider the following functional form

$$J\left(C, \mu\right) = \left[\left(1-\beta\right)C^{1-\rho} + \beta\mu^{1-\rho}\right]^{\frac{1}{1-\rho}}, \qquad \rho \geq 0$$

and the certainty equivalent function is

$$\mu\left(V_{t+1}\right) = \left[E_t\left(V_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}, \qquad \gamma > 0.$$

In the above expressions, $\rho = \frac{1}{\psi}$, where ψ is the inter-temporal elasticity of substitution (IES), γ is the relative risk aversion, and β is a subjective discount factor.

The investor faces $N > 1$ assets with random return $R_{i,t+1}$ between time t and $t+1$. In each period the investor needs to allocate her initial wealth W_t among consumption C_t and investment weight in each asset i, $w_{i,t}$ to maximize her lifetime value subject to the budget constraint

$$(W_t - C_t)\, R_{m,t+1} = W_{t+1},$$

where $R_{m,t+1} = \sum_1^N w_{i,t} R_{i,t+1}$. We further assume that market returns are i.i.d.

The stochastic discount factor in this economy takes the form

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{\frac{-\theta}{\psi}} R_{m,t+1}^{\theta-1}, \tag{26}$$

where

$$\theta \equiv \frac{1-\gamma}{1-\rho}.$$

Since C_t is known as of time t, we have that M_{t+1} is proportional to $C_{t+1}^{-\theta/\psi} R_{m,t+1}^{\theta-1}$. It follows that $M_{t+1}R_{j,t+1}$ is proportional to

$$C_{t+1}^{-\theta/\psi} R_{m,t+1}^{\theta-1} R_{j,t+1} = \left(\frac{C_{t+1}}{W_{t+1}} \right)^{-\theta/\psi} W_{t+1}^{-\theta/\psi} R_{m,t+1}^{\theta-1} R_{j,t+1}.$$

Since market returns are i.i.d., the consumption to wealth ratio is constant. It follows that $M_{t+1}R_{j,t+1}$ is proportional to $W_{t+1}^{-\theta/\psi} R_{m,t+1}^{\theta-1} R_{j,t+1}$. Moreover, from the budget constraint

$$W_{t+1} = (W_t - C_t) R_{m,t+1} = W_t (1 - \frac{C_t}{W_t}) R_{m,t+1},$$

and since both W_t and $\frac{C_t}{W_t}$ are known by time t we have that $M_{t+1}R_{j,t+1}$ is proportional to

$$\left(\sum_i w_i R_{i,t+1} \right)^{\theta - 1 - \theta/\psi} R_{j,t+1} = \left(\sum_i w_i R_{i,t+1} \right)^{-\gamma} R_{j,t+1}.$$

Thus, $Cov_t(M_{t+1}R_{j,t+1}, R_{j,t+1})$ has the same sign as

$$Cov_t \left(\left(\sum_i w_i R_{i,t+1} \right)^{-\gamma} R_{j,t+1}, R_{j,t+1} \right).$$

Define $f : \mathbb{R}^N \to \mathbb{R}$ by

$$f(R_{1,t+1}, R_{2,t+1}, ...R_{N,t+1}) = \left(\sum_i w_i R_{i,t+1} \right)^{-\gamma} R_{j,t+1}.$$

An analysis parallel to that in Section 2.3 yields the following first order approximation

$$Cov_t(\left(\sum_i w_i R_{i,t+1}\right)^{-\gamma} R_{j,t+1}, R_{j,t+1}) \approx (\mathrm{E}_t R_{m,t+1})^{-\gamma} \left(Var_t(R_{j,t+1}) - \gamma Cov_t(R_{m,t+1}, R_{j,t+1})\right).$$

Thus, up to a first order approximation, $Cov_t(M_{t+1}R_{j,t+1}, R_{j,t+1})$ is non-positive if and only if $Cov_t(R_{m,t+1}, R_{j,t+1}) > 0$ and $\gamma \geq \delta_{j,t}$ as before.

Proposition 4 *Up to a first order approximation, the NCC holds for asset j whenever $Cov_t(R_{m,t+1}, R_{j,t+1}) > 0$ and $\gamma \geq \delta_{j,t}$. For such assets, $\frac{var_t^*(R_{j,t+1})}{R_{f,t}}$ serves as a lower bound on the asset's expected excess return between times t and $t+1$. The NCC fails when $Cov_t(R_{m,t+1}, R_{j,t+1}) \leq 0$.*

Appendix II

In this appendix we provide details for the parametric approach to testing for the NCC in Section 3.3. We consider four different parametrizations of the stochastic discount factor: CRRA, Epstein-Zin, CAPM, and the Fama-French three-factor model. Below we discuss the estimation procedure for each case.

CRRA Model

Let $R_{m,t}$ denote the market return at time t. In the CRRA model the stochastic discount factor is proportional to $R_{m,t}^{-\gamma}$, where γ is relative risk aversion. It follows that the correlation of interest to test the NCC is

$$corr(M_t R_{j,t}, R_{j,t}) = corr\left(R_{m,t}^{-\gamma} R_{j,t}, R_{j,t}\right),$$

which is estimated for each stock using the daily returns with a 12-month rolling window or monthly returns with a 60-month window (see Table 3). In this calculation $R_{m,t}$ is taken as the CRSP value weighted return portfolio for month t and we allow γ to change between 2 and 10.

Epstein-Zin Model

In the Epstein-Zin model, the stochastic discount factor takes the form in (26). In Table 3, we set $\beta = 0.998$ and $\psi = 1.5$ as in Bansal and Yaron (2004). $R_{m,t}$ is taken as the CRSP value weighted return portfolio and we let γ change between 2 and 10. The consumption growth rate is calculated from the Real Personal Consumption Expenditures, Percent Change from Preceding Period, Monthly, Seasonally Adjusted, taken from the Federal Reserve Economic Data.

One-Factor Model (CAPM)

Assume a representative agent is holding the market, earning utility $u\left(R_{m,t}\right)$. Then, the stochastic discount factor is proportional to $u'\left(R_{m,t}\right)$. Denote $mktrf_t = R_{m,t} - R_{f,t}$ as the market excess return. If the stochastic discount factor is taking a one-factor structure then it is given by

$$M_t = \beta_0 + \beta_1 mktrf_t = \beta_0 - \beta_1 R_{f,t} + \beta_1 R_{m,t}.$$

Namely, $u'\left(R_{m,t}\right)$ is proportional to

$$\beta_0 - \beta_1 R_{f,t} + \beta_1 R_{m,t}.$$

It follows that relative risk aversion, $-R_{m,t}\frac{u''(R_{m,t})}{u'(R_{m,t})}$, is given by

$$\gamma = -R_{m,t}\frac{\beta_1}{M_t} = -\frac{\beta_1\left(mktrf_t + R_{f,t}\right)}{\beta_0 + \beta_1 mktrf_t}.$$

This condition should hold for all t, and thus also on average:

$$E\left[\gamma\left(\beta_0 + \beta_1 mktrf_t\right) + \beta_1\left(mktrf_t + R_{f,t}\right)\right] = 0. \tag{27}$$

In addition, we have

$$E\left(M_t R_{f,t}\right) = 1,$$

or equivalently

$$E\left(\left(\beta_0 + \beta_1 mktrf_t\right) R_{f,t}\right) = 1. \tag{28}$$

We now let γ vary between 2 and 10 and estimate β_0 and β_1 using GMM from the moment conditions (27) and (28). For this purpose we let $mktrf_t$ be the CRSP value weighted excess return portfolio and $R_{f,t}$ is taken from Kenneth French's website.

Fama-French Three-Factor Model

In any factor model the assumption is that the stochastic discount factor is a linear combination of the different factors. Moreover, the representative agent holds a combination of the factors with the weight assigned to each factor being proportional to its weight in the stochastic discount factor. Thus, in the Fama-French three-factor model we have

$$\begin{aligned} M_t &= \beta_0 + \beta_1 smb_t + \beta_2 hml_t + \beta_3 mktrf_t \\ &= \beta_0 - \beta_1 - \beta_2 - \beta_3 R_{f,t} + \beta_1 R_{smb,t} + \beta_2 R_{hml,t} + \beta_3 R_{m,t}, \end{aligned}$$

where smb_t, hml_t and $mktrf_t$ are three factors constructed by Fama and French (1993), and $R_{smb,t}$, $R_{hml,t}$ and $R_{m,t}$ are their corresponding gross returns. In this model, the representative agent holds a portfolio with return $k\left(\beta_1 R_{smb,t} + \beta_2 R_{hml,t} + \beta_3 R_{m,t}\right)$, where k is a constant. It follows that $u'\left(k\left(\beta_1 R_{smb,t} + \beta_2 R_{hml,t} + \beta_3 R_{m,t}\right)\right)$ is proportional to $\beta_0 - \beta_1 - \beta_2 - \beta_3 R_{f,t} + \beta_1 R_{smb,t} + \beta_2 R_{hml,t} + \beta_3 R_{m,t}$. We conclude that the relative risk aversion calculated at the portfolio held by the representative agent is

$$\gamma = -k\left(\beta_1 R_{smb,t} + \beta_2 R_{hml,t} + \beta_3 R_{m,t}\right) \frac{\frac{1}{k}}{m} = -\frac{\beta_1 + \beta_2 + \beta_3 R_{f,t} + \beta_1 smb_t + \beta_2 hml_t + \beta_3 mktrf_t}{\beta_0 + \beta_1 smb_t + \beta_2 hml_t + \beta_3 mktrf_t}.$$

As before, this condition must also hold on average yielding the following moment condition

$$E\left(\gamma\left(\beta_0 + \beta_1 smb_t + \beta_2 hml_t + \beta_3 mktrf_t\right) + \beta_1 + \beta_2 + \beta_3 R_{f,t} + \beta_1 smb_t + \beta_2 hml_t + \beta_3 mktrf_t\right) = 0.$$

We need to estimate four parameters: β_i, $i = 0, ..., 3$, and thus we need at least three more moment conditions. We obtain them by using three test assets and applying the standard pricing equation $E\left(M_t R_{j,t}\right) = 1$. The test assets we choose are the risk-free rate $R_{f,t}$ and two of the Fama-French six portfolios sorted by size and book to market ratio (downloaded from Kenneth French's website). The results reported in Table 3 are based on the two portfolios with medium level of book to market ratio. We have also used other combinations of the Fama-French portfolios as test assets with no material effect on the estimation results.

Table 1 Summary Statistics for Delta and Non-Parametric Tests of the NCC

This table reports summary statistics for $\delta_j = \frac{Var(R_j)}{Cov(R_j,R_m)}$ calculated using return data of CRSP common stocks (share code 10 and 11) included in the S&P index starting from the year 2003 until April 2016. In Panel A, δ_j is calculated based on daily stock returns and a 12-month rolling window between January 2003 and April 2016. Beta is the regression coefficient from time-series regressions of the stock excess return on CRSP value weighted excess return. Size is the log market-cap at the end of previous calendar year. B/M is the log of the ratio of equity book value to market value at the end of the previous fiscal year obtained from Compustat. In Panel B, δ_j and beta are calculated using a 60-month rolling window and monthly returns. P-values are reported in parentheses below correlation estimates.

Panel A: Delta Calculation Based on a 12-Month Rolling Window and Daily Returns

Obs	Mean	Std.Dev	1%	5%	10%	25%	50%	75%	90%	95%	99%
95344	4.3341	3.8204	1.1169	1.5071	1.7505	2.3002	3.2585	5.0995	7.7727	10.2033	18.8651
Corr with		Beta			Size			B/M			
		-0.1979 (0.0000)***			-0.3092 (0.0000)***			-0.0029 (0.4092)			

Panel B: Delta Calculation Based on a 60-Month Rolling Window and Monthly Returns

Obs	Mean	Std.Dev	1%	5%	10%	25%	50%	75%	90%	95%	99%
96431	5.9562	6.3226	1.5603	1.9214	2.2231	2.8965	4.2539	6.6671	10.5323	14.8901	33.2529
Corr with		Beta			Size			B/M			
		-0.0727 (0.0000)***			-0.2161 (0.0000)***			-0.0280 (0.4476)			

Table 2 Summary Statistics for δ_j by Different Groups

This table reports summary statistics for $\delta_j = \frac{Var(R_j)}{Cov(R_j,R_m)}$ for the conservative, moderate, liberal, and very liberal groups of stocks defined in (24). The calculation of δ_j is based on a 12-month rolling window of daily stock returns from January 2003 to April 2016, and the market return is taken as the CRSP value weighted return. Obs is the average number of monthly observations in each group. We also report the mean and standard deviations of the firm's beta, size, and book-to-market ratio. Beta is the regression coefficient from 12-month rolling window time-series regressions of the stock excess return on CRSP value weighted excess return. Size is the average of market-cap over the sample period measured in billions of dollars. B/M is the average over the sample period of the ratio of equity book value to market value at the end of the fiscal year obtained from Compustat. ρ^2 is the R-squared from 12-month rolling window time-series regressions of the stock excess return over CRSP value weighted excess return.

Panel A: Summary Statistics for δ_j

	Obs	Mean	Std.Dev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Conservative	370.7875	2.5468	0.7293	1.1469	1.4039	1.5753	1.9796	2.5112	3.1063	3.5933	3.7875	3.9548
Moderate	147.1687	5.2035	0.8373	4.0158	4.0853	4.1794	4.4745	5.0701	5.8448	6.4801	6.7354	6.9401
Liberal	45.5938	8.2135	0.8407	7.0161	7.0929	7.1823	7.4768	8.0794	8.8798	9.4718	9.7321	9.9476
Very Liberal	31.5750	17.6544	54.7425	10.0458	10.1972	10.4328	11.1896	13.1179	17.0727	24.3836	31.2439	65.7616

Panel B: Firm Characteristics

	Beta	Size($ billions)	B/M	ρ^2
	Mean (Std. Dev)	Mean (Std. Dev)	Mean (Std. Dev)	Mean (Std. Dev)
Conservative	0.9874	26.3813	0.4498	0.2188
	(0.5450)	(47.9161)	(0.3135)	(0.1438)
Moderate	1.2742	13.7496	0.4622	0.1661
	(0.8149)	(27.4139)	(0.4513)	(0.1013)
Liberal	1.5329	8.3868	0.4659	0.1611
	(1.0532)	(21.9937)	(0.4157)	(0.0892)
Very Liberal	1.6585	6.1908	0.5880	0.1729
	(1.0864)	(16.6788)	(0.7171)	(0.0957)

Table 3 Parametric Tests of the NCC

This table reports the proportion of S&P 500 stocks that satisfy the NCC under different models. The sample consists of S&P 500 constituents during January 2003 to April 2016 with the same sample period. We calculate stochastic discount factors under four models: CRRA utility, Epstein-Zin utility, CAMP one-factor model, and the Fama-French three-factor model, and for levels of relative risk aversion (γ) between 2 and 10. Then for each stock j in the sample, we calculate the correlation $\rho = corr\left(MR_j, R_j\right)$ using both a 12-month rolling window (with daily returns) and 60-month rolling window (with monthly returns). For each estimated correlation, we consider the null $H_0 : \rho \leq 0$ against the alternative $H_1 : \rho > 0$. The table reports the proportion of stocks for which we are not able to reject the null using a t-test for correlations under different models and at different risk-aversion levels.

	12-Month Rolling Window				60-Month Rolling Window			
γ	CRRA	Epstein-Zin	1-Factor	3-Factor	CRRA	Epstein-Zin	1-Factor	3-Factor
2	0.2595	0.7454	0.2581	0.8822	0.2990	0.7250	0.2620	0.7179
3	0.5719	0.9398	0.5704	0.7833	0.6504	0.9492	0.6099	0.6226
4	0.7448	0.9801	0.7436	0.5886	0.8272	0.9842	0.7970	0.4935
5	0.8446	0.9917	0.8434	0.4204	0.9169	0.9932	0.8932	0.5395
6	0.9035	0.9959	0.9029	0.5801	0.9616	0.9953	0.9456	0.7005
7	0.9388	0.9977	0.9380	0.7188	0.9801	0.9957	0.9710	0.7925
8	0.9592	0.9985	0.9585	0.7940	0.9871	0.9962	0.9822	0.8431
9	0.9718	0.9990	0.9715	0.8362	0.9910	0.9966	0.9879	0.8709
10	0.9799	0.9993	0.9795	0.8634	0.9943	0.9967	0.9918	0.8893
	Observations=62366				Observations=59755			

Table 4 Summary Statistics for Estimated Lower Bound

This table reports summary statistics for the estimated lower bound obtained from (23). The calculation is based on options with underlying securities included in the S&P 500 index starting from the year 2003 and subject to the filters described in Section 3. We estimate the lower bound for each stock and each day during the sample period of January 2003 to April 2016. The summary statistics are reported for the entire sample and for each of the four groups defined in (24) separately. The table also reports an estimated lower bound for the market premium using option prices on the S&P 500 (SPX). All reported lower bound estimates are annualized.

Panel A: Entire Sample Period

	Obs	Mean	Std.Dev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Entire Sample	1,154,317	0.1292	0.2131	0.0180	0.0263	0.0330	0.0497	0.0816	0.1417	0.2481	0.3678	0.8168
Conservative	594,456	0.0869	0.1207	0.0173	0.0239	0.0289	0.0408	0.0616	0.0984	0.1590	0.2187	0.4600
Moderate	344,231	0.1584	0.2151	0.0192	0.0339	0.0445	0.0669	0.1054	0.1728	0.3066	0.4459	0.9399
Liberal	123,822	0.1889	0.3028	0.0177	0.0293	0.0399	0.0657	0.1180	0.2070	0.3523	0.5177	1.2244
Very Liberal	91,808	0.2066	0.3912	0.0220	0.0372	0.0494	0.0809	0.1348	0.2297	0.4016	0.5862	1.2897
Market Premium	3341	0.0387	0.0320	0.0149	0.0164	0.0176	0.0213	0.0289	0.0435	0.0662	0.0956	0.1928

Panel B: Sample Period Without Years 2008 and 2009

	Obs	Mean	Std.Dev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Entire Sample	963,720	0.1087	0.1964	0.0174	0.0250	0.0311	0.0460	0.0728	0.1200	0.1977	0.2792	0.6290
Conservative	517,862	0.0732	0.0978	0.0168	0.0232	0.0278	0.0387	0.0570	0.0868	0.1297	0.1688	0.2944
Moderate	278,632	0.1333	0.1836	0.0182	0.0314	0.0416	0.0623	0.0957	0.1484	0.2369	0.3377	0.7310
Liberal	97,507	0.1598	0.3061	0.0166	0.0267	0.0353	0.0575	0.0967	0.1687	0.2776	0.4059	1.0059
Very Liberal	69,719	0.1903	0.4157	0.0206	0.0345	0.0463	0.0740	0.1220	0.2094	0.3593	0.5248	1.2508
Market Premium	2839	0.0308	0.0158	0.0148	0.0162	0.1727	0.0203	0.0259	0.0345	0.0514	0.0654	0.0933

Table 5: Fama MacBeth Analysis of the Bounds

This table presents the results of monthly cross-sectional Fama-MacBeth regressions. The sample period is January 2003 to April 2016. We first estimate a monthly lower bound for each stock by averaging the daily lower bounds reported in Table 4 over the month. For each month we then run cross-sectional regressions in which the dependent variable is the average monthly lower bound and the independent variables are beta, size, book-to-market ratio, and momentum. Obs is the time-series average of the number of firms in the cross-sectional regressions. Beta is calculated as the coefficient estimate from regressing each stock's monthly excess returns on the market excess returns during a 60-month rolling window. Firm size is the log of the firm's market-cap at the end of previous year. B/M is the log of the ratio of equity book value to market cap at the end of the preceding fiscal year. Momentum is given by the cumulative return over the 12 months period prior the current month. The table reports the time-series average of the cross-sectional coefficient estimates as well as Newey-West standard errors with 4 lags. The analysis is performed both for the entire sample and for each of the four groups defined in (24) separately. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (*) level.

	Obs	β	Size	B/M Ratio	Momentum
Entire Sample	352.90	0.0493	-0.0276	0.0156	-0.0570
		(0.0077)***	(0.0025)***	(0.0045)***	(0.0188)***
Conservative ($1 \leq \delta < 4$)	180.93	0.0401	-0.0105	0.0077	-0.0328
		(0.0097)***	(0.0020)***	(0.0017)***	(0.0107)***
Moderate ($4 \leq \delta < 7$)	105.86	0.0471	-0.0224	0.0137	-0.0891
		(0.0082)***	(0.0038)***	(0.0047)***	(0.0206)***
Liberal ($7 \leq \delta \leq 10$)	37.70	0.0753	-0.0232	0.0393	-0.0660
		(0.0234)***	(0.0086)***	(0.0124)***	(0.0226)***
Very Liberal ($\delta > 10$)	28.41	0.1171	-0.0440	-0.0153	-0.0721
		(0.0223)***	(0.0147)***	(0.0179)	(0.0345)**

Table 6: Predictive Value of the Bound

This table reports average realized returns sorted by the estimated lower bound on expected excess returns. Each month t during our sample period we classify stocks into 10 deciles according to the monthly average of their estimated lower bound. The table reports the average of the realized returns in the next month/6 months/12 months for each decile for the entire sample and for each of the four groups defined in (24) separately. The bottom row reports the average returns of a portfolio which is long in Decile 10 and short in Decile 1 along with Newey-West standard errors with 4, 10, and 16 lags in parenthesis. Asterisks denote statistical significance at the 1% (***), 5% (**) and 10% (*) level.

Panel A: Average Future Return of Stocks Grouped by Lower Bound Estimates — One-Month Horizon

Decile	Entire Sample	Conservative ($1 \leq \delta < 4$)	Moderate ($4 \leq \delta < 7$)	Liberal ($7 \leq \delta \leq 10$)	Very Liberal ($\delta > 10$)
1	0.0030	0.0002	0.0058	0.0076	0.0041
2	0.0079	0.0018	0.0068	0.0135	0.0030
3	0.0087	0.0022	0.0063	0.0033	0.0115
4	0.0092	0.0071	0.0081	0.0122	0.0102
5	0.0099	0.0053	0.0085	0.0221	0.0159
6	0.0098	0.0030	0.0130	0.0252	0.0263
7	0.0105	0.0074	0.0129	0.0193	0.0174
8	0.0146	0.0082	0.0109	0.0268	0.0322
9	0.0149	0.0083	0.0137	0.0142	0.0472
10	0.0209	0.0094	0.0139	0.0212	0.0272
10-1	0.0179	0.0092	0.0081	0.0110	0.0240
	(0.0056)***	(0.0077)	(0.0080)	(0.0145)	(0.0117)**

Panel B: Average Future Return of Stocks Grouped by Lower Bound Estimates — Six-Month Horizon

Decile	Entire Sample	Conservative ($1 \leq \delta < 4$)	Moderate ($4 \leq \delta < 7$)	Liberal ($7 \leq \delta \leq 10$)	Very Liberal ($\delta > 10$)
1	0.0184	-0.0069	0.0452	0.0465	0.0755
2	0.0412	0.0153	0.0627	0.1159	0.0365
3	0.0463	0.0387	0.0709	0.1027	0.0531
4	0.0387	0.0428	0.0665	0.1251	0.0812
5	0.0476	0.0385	0.0828	0.1312	0.1270
6	0.0450	0.0381	0.0712	0.1308	0.1542
7	0.0584	0.0316	0.0701	0.1512	0.0936
8	0.0681	0.0220	0.1050	0.2356	0.1959
9	0.0831	0.0394	0.0913	0.1666	0.2269
10	0.1496	0.0577	0.1114	0.2261	0.2899
10-1	0.1312	0.0646	0.0661	0.1796	0.2234
	(0.0323)***	(0.0287)**	(0.0300)**	(0.0962)*	(0.0705)***

Panel C: Average Future Return of Stocks Grouped by Lower Bound Estimates — Twelve-Month Horizon

Decile	Entire Sample	Conservative ($1 \leq \delta < 4$)	Moderate ($4 \leq \delta < 7$)	Liberal ($7 \leq \delta \leq 10$)	Very Liberal ($\delta > 10$)
1	0.0391	0.0112	0.1158	0.1074	0.0944
2	0.0723	0.0324	0.1219	0.2129	0.0987
3	0.0821	0.0647	0.1323	0.2163	0.1887
4	0.0666	0.0770	0.1860	0.2920	0.2052
5	0.0797	0.0802	0.1497	0.5031	0.2487
6	0.0801	0.0666	0.1277	0.2525	0.2988
7	0.1039	0.0867	0.1774	0.2889	0.2637
8	0.1138	0.0912	0.1420	0.3519	0.3336
9	0.1311	0.1009	0.2174	0.3179	0.3587
10	0.2674	0.1309	0.2517	0.4100	0.5276
10-1	0.2284	0.1196	0.1358	0.3027	0.4239
	(0.0596)***	(0.0301)***	(0.0666)**	(0.0814)***	(0.1440)***

Table 7: Type I and Type II Errors for Taylor Approximations

This table reports the type I (false positive) and type II (false negative) error probabilities resulting from using the first order approximation to test the validity of the NCC. We assume a utility function of the form $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$, where γ ranges between 2 and 10. The NCC holds if $cov(u'(R_m) R_j, R_j) \leq 0$ and the first order approximation holds if $\gamma \geq \delta_j$, where $\delta_j = \frac{Var(R_j)}{Cov(R_j,R_m)}$ is calculated using a 12-month rolling window of daily returns. Our sample period is January 2003 to April 2016. For the market return R_m we use daily CRSP value weighted returns. The analysis is restricted to S&P 500 constituents starting from the year 2003.

γ	Type I Error	Type II Error
2	0.0009	0.0085
3	0.0022	0.0060
4	0.0035	0.0049
5	0.0060	0.0038
6	0.0067	0.0033
7	0.0111	0.0034
8	0.0116	0.0028
9	0.0163	0.0025
10	0.0221	0.0020

Table 8 Summary Statistics for Estimated Lower Bound Using Synthetic European Option Prices

This table reports summary statistics for the estimated lower bound obtained from (23) using synthetic European option prices calculated following the approximation presented in Barone-Adesi and Whaley (1987). The estimation is based on options with underlying securities included in the S&P 500 index starting from the year 2003 and subject to the filters described in Section 3. We estimate the lower bound for each stock and each day during the sample period of January 2003 to April 2016. The summary statistics are reported for the entire sample and for each of the four groups defined in (24) separately. All reported lower bound estimates are annualized.

Panel A: Entire Sample Period

	Obs	Mean	Std.Dev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Entire Sample	1,154,317	0.1249	0.1500	0.0113	0.0233	0.0307	0.0447	0.0785	0.1378	0.2134	0.3452	0.4842
Conservative	594,456	0.0830	0.1219	0.0050	0.0123	0.0268	0.0373	0.0597	0.0903	0.1516	0.2007	0.3379
Moderate	344,231	0.1494	0.1834	0.0139	0.0311	0.0369	0.0639	0.1032	0.1693	0.2452	0.3156	0.6264
Liberal	123,822	0.1805	0.2937	0.0155	0.0204	0.0389	0.0626	0.1127	0.1907	0.3302	0.4441	0.9930
Very Liberal	91,808	0.2012	0.3725	0.0206	0.0325	0.0426	0.0736	0.1324	0.2133	0.3768	0.5393	1.0397

Panel B: Sample Period Without Years 2008 and 2009

	Obs	Mean	Std.Dev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Entire Sample	963,720	0.0948	0.1771	0.0144	0.0206	0.0278	0.0397	0.0705	0.1135	0.1805	0.2213	0.4188
Conservative	517,862	0.0698	0.0808	0.0104	0.0196	0.0260	0.0367	0.0504	0.0821	0.1211	0.1638	0.2245
Moderate	278,632	0.1296	0.1782	0.0131	0.0273	0.0381	0.0650	0.0867	0.1436	0.2036	0.2876	0.4741
Liberal	97,507	0.1546	0.2583	0.0122	0.0161	0.0259	0.0588	0.0950	0.1658	0.2448	0.3312	0.6139
Very Liberal	69,719	0.1868	0.3352	0.0160	0.0321	0.0405	0.0682	0.1189	0.1865	0.3179	0.3944	0.7890

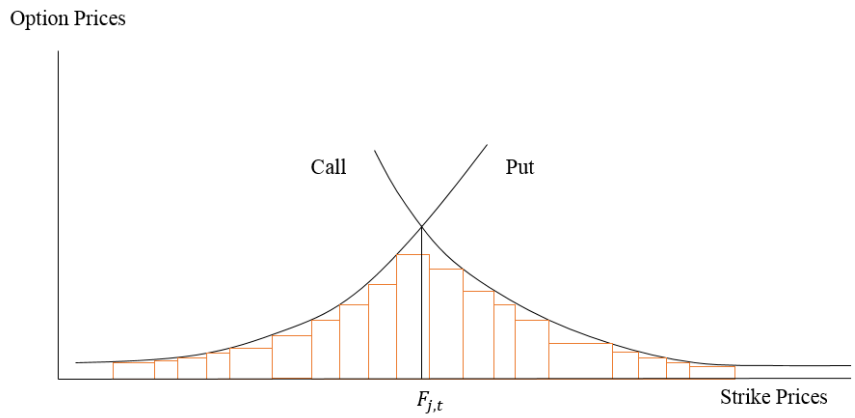


Figure 1: Illustration of the Numerical Estimation of the Lower Bound

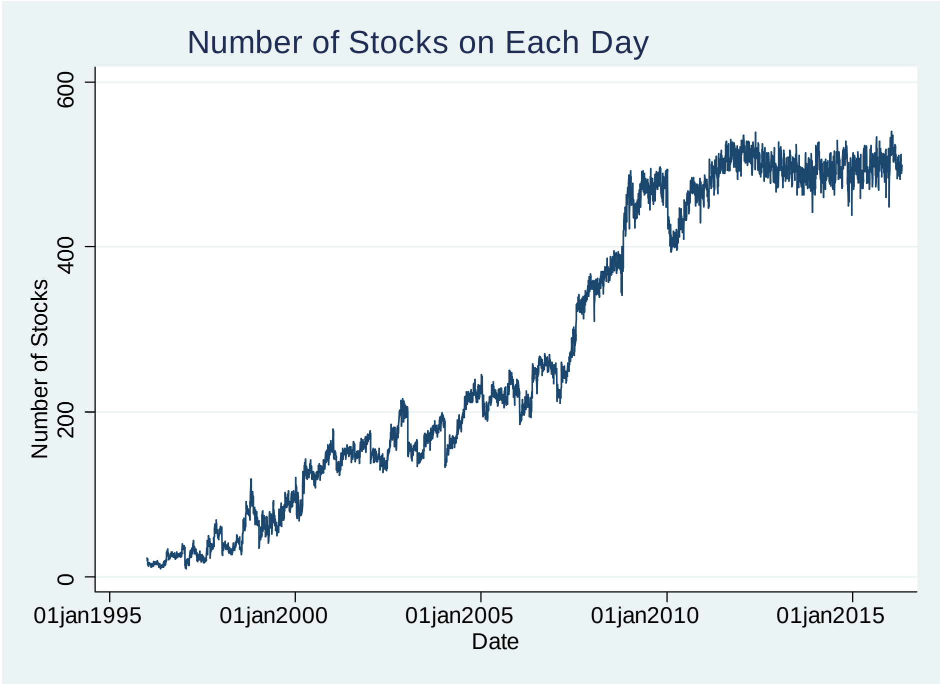
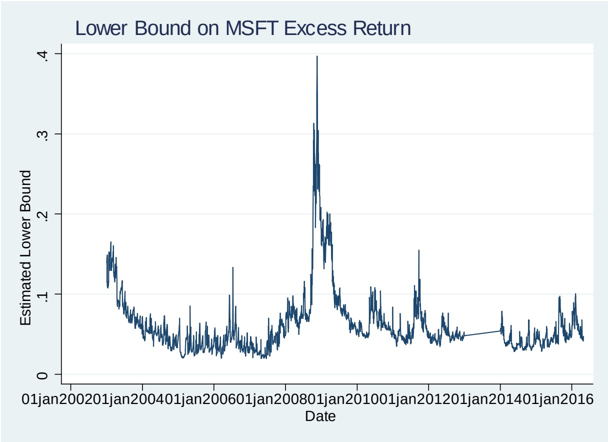
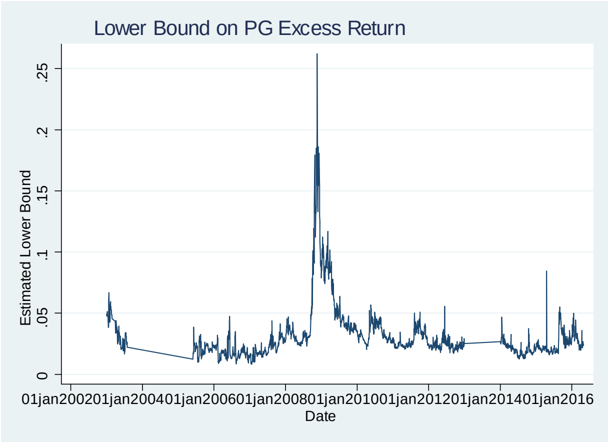


Figure 2: Daily Number of Stocks Passing the Screens

Panel A: Estimated Lower Bound for Microsoft



Panel B: Estimated Lower Bound for P&G



Panel C: Estimated Lower Bound for Apple

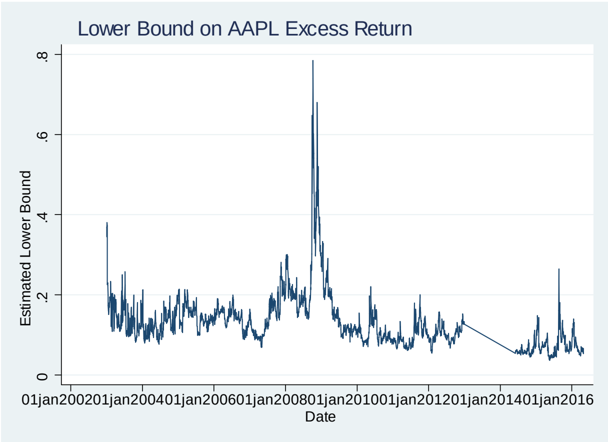
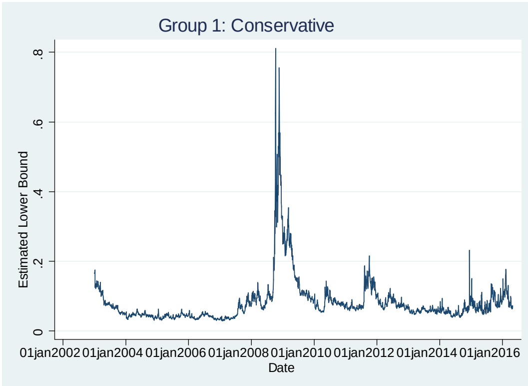
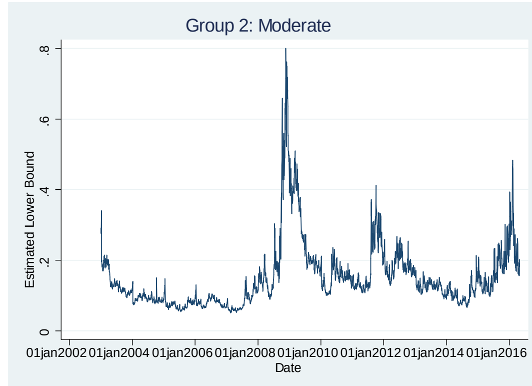


Figure 3: Time Series of Estimated Lower Bound for Microsoft, P&G and Apple

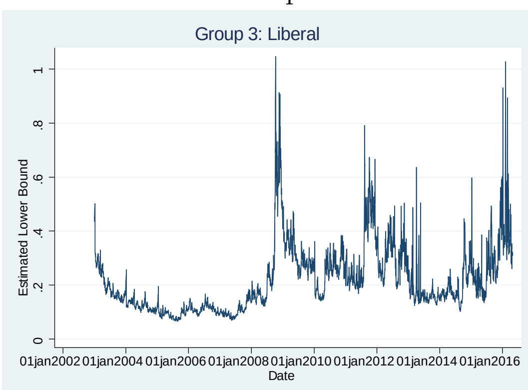
Panel A: Conservative Group



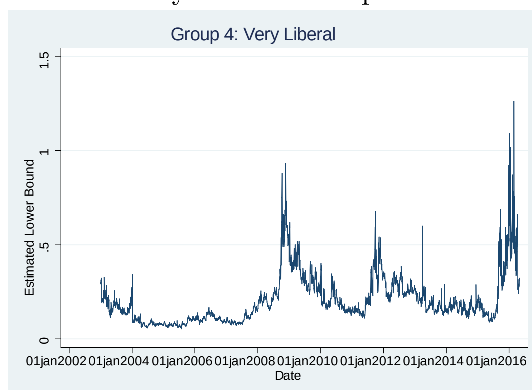
Panel B: Moderate Group



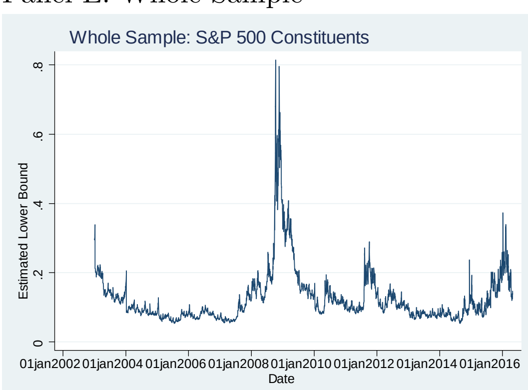
Panel C: Liberal Group



Panel D: Very Liberal Group



Panel E: Whole Sample



Panel F: Market Premium

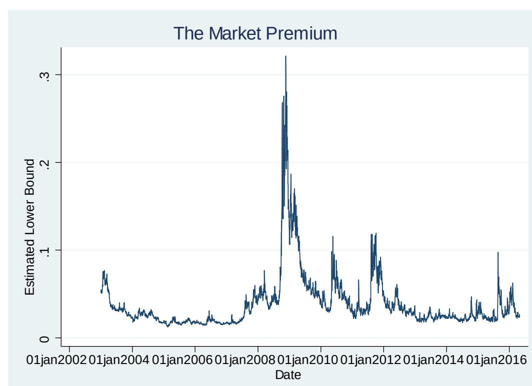


Figure 4: Time Series of Average Estimated Lower Bound

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