Worcester Polytechnic Institute

Report of MA 575

Market and Credit Risk Models and Management

Final Project

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Table of Contents

- **Introduction 3**
- **Tail Model 4**
  - **Estimate of a 4**
    - Based on Regression Estimator Model 4
    - Based on Hill Estimator Model 5
  - **Calculation of $VaR\alpha$ and $CVaR\alpha$ 6**
  - **Comparison of $VaR\alpha$ and $CVaR\alpha$ 8**
- **GARCH Model 10**
  - **Estimation of Models 10**
    - Model Fitting 10
    - Conclusions and Comparisons between Two Models 11
  - **$VaR\alpha$ and $CVaR\alpha$ Values for the Conditional Loss Distribution 12**
  - **Comparison of Conditional and Unconditional Risk Management 13**
  - **Advantages and Disadvantages 14**
  - **Comparison of Risk Analysis 14**
- **IB Paper Trading 15**
  - **Risk Estimation 15**
  - **Stress Test 17**
    - Perform Stress Test 17
    - Risk Analysis Report 18
    - Comparison 20
  - **Risk reduction 21**
  - **Conclusion 21**

[Reference] 22

• Introduction

In modern financial risk management, we usually use VaR_α to measure portfolio's risk. VaR_α is defined as

Given confidence level $\alpha \epsilon (0,1)$*, the value-at-risk of portfolio at level* α *is the smallest number* ℓ *such that the probability that the loss* $\mathcal{L}$ *exceeds* ℓ *is no larger than* $1-\alpha$*.*

$$VaR_\alpha = inf\{\ell \epsilon R: P(\mathcal{L} > \ell) \leq 1-\alpha\} = inf\ \{\ell \epsilon R: F_{\mathcal{L}}(\ell) \geq \alpha\}$$

In this report we mainly focused on the analysis of market risk. For a given time horizon Δ, we define the loss of the portfolio over time period [s, s + Δ] as

$$L_{[s,s+\Delta]} = -[V_{(s+\Delta)} - V_{(s)}]$$ [1]

We tried to use two methods, Tail Model and GARCH Model, to simulate loss distribution of our portfolio. Then by focusing on VaR_α and $CVaR_\alpha$, we analyzed our portfolio's risk.

For Tail Model and GARCH Model analysis, we constructed our portfolio by choosing fifteen stocks traded on American exchanges. These assets are AAL, AAPL, AMZN, HSBC, C, HMC, GOOGL, JPM, MS, MSFT, GE, RY, KO, V and WFC. We mainly used each asset's historical daily price from January 1 2012 through December 31 2017, downloaded from *Yahoo Finance.*

Later in Interactive Broker account, we traded our portfolio with market price and held it for about eight weeks. Then, we would like to analyze our portfolio's performance in this report.

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[1] Lecture Notes from Professor Marcel Blais

• Tail Model

In this chapter, we assumed that our loss density $f(x)$ had a polynomial right tail of the form

$$f(x) = A * y^{-(a+1)} \ for\ all\ y \geq c\ for\ some\ c > 0\ and\ A, a > 0 \ ^{[2]} \tag{1.1}$$

Then we planned to use semi-parametric model for our loss distribution to calculate the VaR_α and $CVaR_\alpha$ for $0.90 \leq \alpha \leq 1$. In our semi-parametric model, we first used a non-parametric estimate of VaR_α for some small α_0 (e.g. $\alpha_0 = 0.9$), then used a parametric model for VaR_α and $CVaR_\alpha$ for $\alpha > \alpha_0$. Thus, we can get the estimate of VaR_α and $CVaR_\alpha$ of the forms:

$$VaR_\alpha = VaR_{\alpha 0} * \frac{(1-\alpha_0)^{1/a}}{(1-\alpha)} \ ^{[3]} \tag{1.2}$$

$$ES_\alpha = \frac{a}{a-1} * VaR_\alpha \ ^{[4]} \tag{1.3}$$

In order to compute VaR_α and $CVaR_\alpha$ here, one thing we have to do first is to estimate the a in our formulas. Thus, in our project, we tried to use Regression Estimator Model and Hill Estimate Model respectively to estimate the parameter a by constructing a historical time series of returns for your portfolio.

• Estimate of a

We trained our model on 2-year of rolling data and estimated 'a' using two methods: Regression Estimator Model and Hill Estimate Model. Since the number of data based on monthly losses is not enough, we tried to estimate 'a' based on weekly losses. After computing 'a' for each week, we selected the 'a' for the last week of each month.

• Based on Regression Estimator Model

Given the n loss samples of our portfolio with the upper order statistics, L_k is one of the losses and there are n-k losses that are larger than L_k. By calculating, we can get that

$$\ln(L_k) \approx \frac{1}{a} * \ln\left(\frac{A}{a}\right) - \frac{1}{a} * \ln\left(\frac{n-k}{n}\right) \ ^{[5]} \tag{1.4}$$

which is almost a regression function. Thus, we performed the regression function, plotted points $(ln^{\left(\frac{n-k}{n}\right)}, \ln^{(L_k)})$ and fitted the scatter plot with this regression line with the slope, which is an estimator of $-\frac{1}{a}$.

Here show some samples of estimated 'a' for the last week of each month below.

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[2] Lecture Notes from Professor Marcel Blais

[3] Lecture Notes from Professor Marcel Blais

[4] Lecture Notes from Professor Marcel Blais

[5] Lecture Notes from Professor Marcel Blais

Table 1 Samples of Estimated 'a' for the Last Week of Each Month Based on Regression Model

Date	Jan.2014	Feb.2014	Mar.2014	Apr.2014	May.2014	June.2014
Estimated 'a'	2.87584	2.87584	2.73742	2.62045	2.49	2.61503
Date	July.2014	Aug.2014	Sep.2014	Oct.2014	Nov.2014	Dec.2014
Estimated 'a'	2.61503	2.37849	2.37849	2.37619	2.25818	2.05991

• Based on Hill Estimator Model

In this model, the estimated 'a' can be calculated by

$$\hat{a} = \frac{n(c)}{\sum_{t=1}^{n(c)} \ln(\frac{L_{t,n}}{c})} \text{ [6]} \qquad (1.5)$$

where $n(c)$ is the number of losses L_i larger than c.

By forming a Hill plot, we plotted $\hat{a}(c)$ versus $n(c)$, then looked for a range of values for $n(c)$ where $\hat{a}(c)$ is relatively constant.

Our hill plots for each rolling window mainly had four kinds of patterns. But we found that the 30th point was within the flat range for all of our hill plots, thus we chose the 30th $\hat{a}(c)$ in our list, obtained from the rolling window.

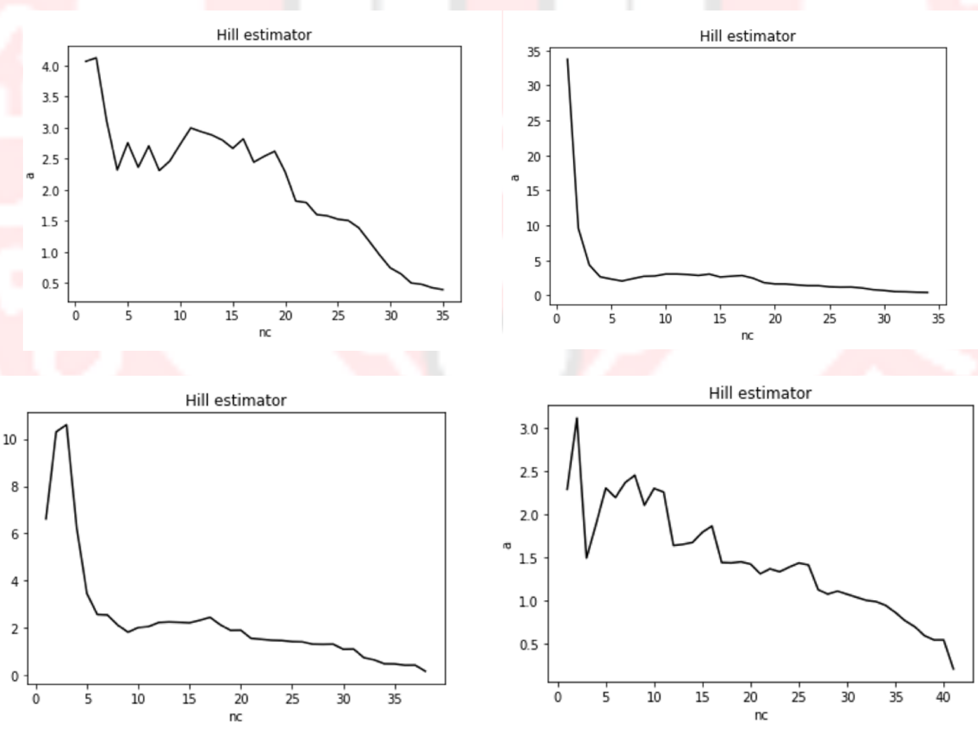

Figure 1 Four Patterns of Hill Plots

Besides, from our Hill Plots, we found that the four plots of every month almost showed the same pattern, this means that both the amount and the distribution of losses for each week within

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[6] Lecture Notes from Professor Marcel Blais

a month are similar. As a result, we decided to choose the last week's estimated 'a' to predict VaR_α and $CVaR_\alpha$ for each month.

Table 2 Samples of Estimated 'a' for the Last Week of Each Month Based on Hill Model

Date	Jan.2014	Feb.2014	Mar.2014	Apr.2014	May.2014	June.2014
Estimated 'a'	2.77467	2.4393	2.75852	2.24574	2.31513	2.63272
Date	July.2014	Aug.2014	Sep.2014	Oct.2014	Nov.2014	Dec.2014
Estimated 'a'	2.04058	2.1483	1.88828	2.34965	2.12403	3.45704

The graph below shows that there are some differences in estimated 'a' based on two different models and there are large fluctuations in estimated 'a' based on Hill Model. The reason why this kind of situation happened we thought was that based on the Hill Model, we had a very strong subjectivity to choose 'a'.

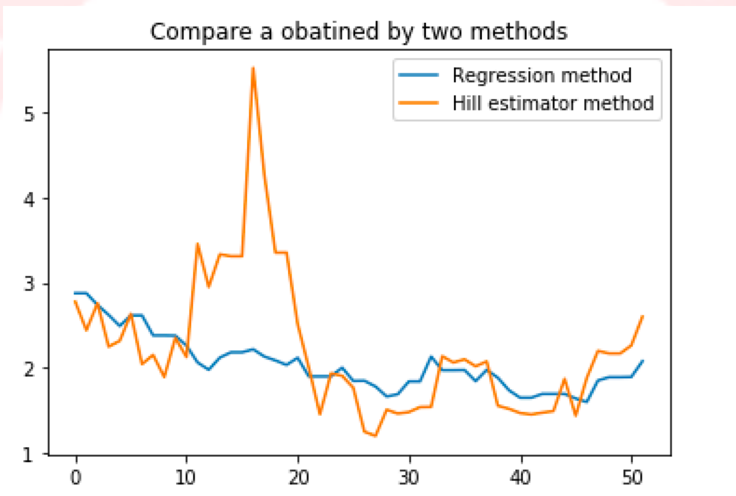


Figure 2 Comparison of Estimated 'a' Based on Two Models

• Calculation of VaR_α and $CVaR_\alpha$

Used the semi-parametric model for our loss distribution, we calculated VaR_α and $CVaR_\alpha$ for $0.90 \leqslant \alpha \leqslant 1$ for each week. The plot below shows a sample of VaR_α and $CVaR_\alpha$ for $0.90 \leqslant \alpha \leqslant 1$ for the first week of January, 2014.

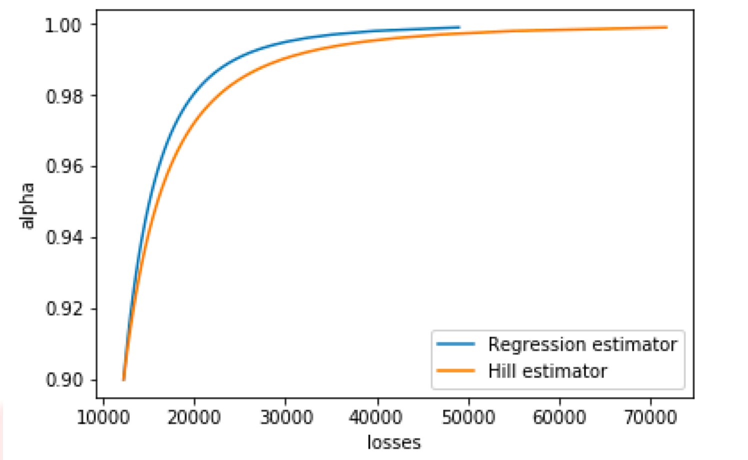


Figure 3 VaR(alpha) and ES(alpha) for the First Week of Jan, 2014

In further analysis, we mainly focused on the VaR_α and $CVaR_\alpha$ with α equal to 0.95 and 0.99.

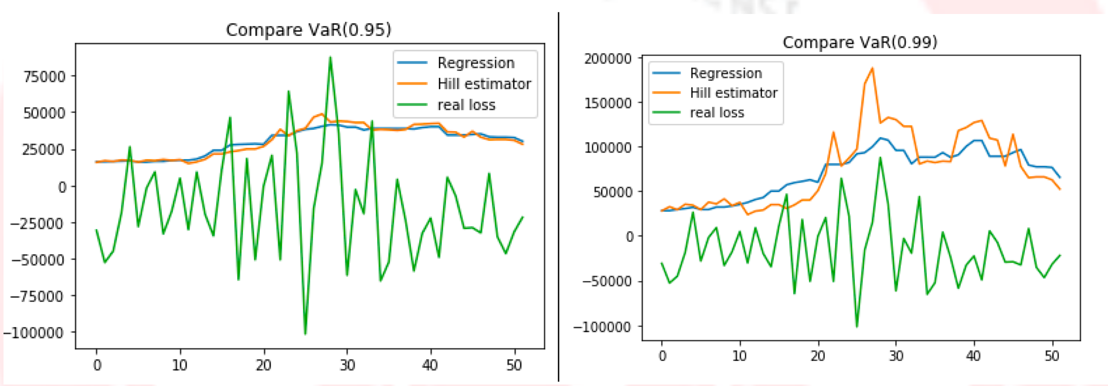
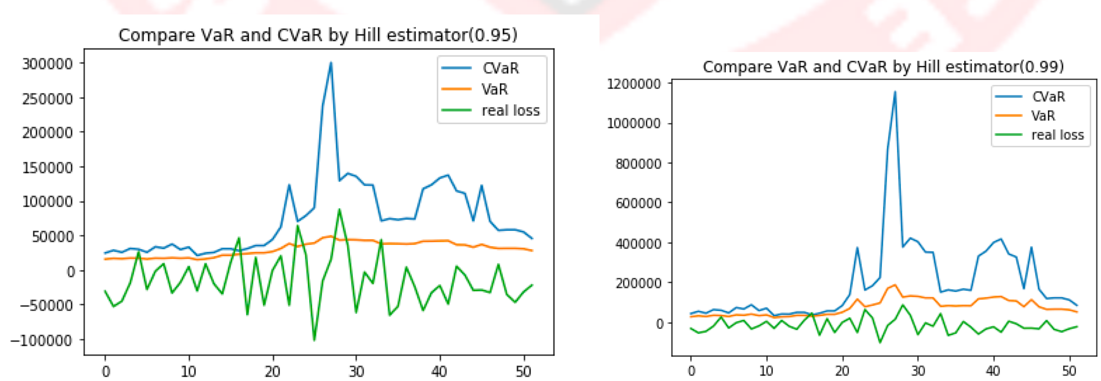


Figure 4 Comparison of VaR Based on Two Models for Alpha 0.95 and 0.99

When alpha equals 0.95, the VaR_α calculated from two models are similar. In both plots, the VaR_α calculated based on Hill Estimator Model show more fluctuations compared with the ones based on Regression Model.



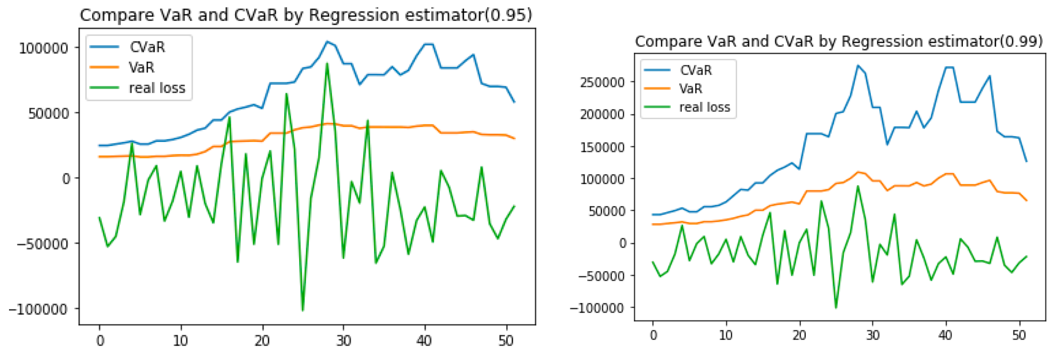


Figure 5 VaR(alpha) and CVaR(alpha) Based on Two Models and Two Alphas

$CVaR_\alpha$ is more influenced by ‘a’ than VaR_α is, according to their formulas, and we had a very strong subjectivity of choosing ‘a’. This is the reason why there are large fluctuations, when using the estimated ‘a’ based on Hill Model to compute $CVaR_\alpha$.

Based on both models, almost all VaR_α and $CVaR_\alpha$ for each month are larger than real losses as alpha equals to 0.99. Thus, it is more conservative with alpha equal to 0.99 when we predict the losses.

Overall, we think it will be more suitable to use regression model for estimate of a and choose alpha with 0.99 to compute both VaR_α and $CVaR_\alpha$.

• Comparison of VaR_α and $CVaR_\alpha$

In this part, we compared the VaR_α and $CVaR_\alpha$ computed from different models.

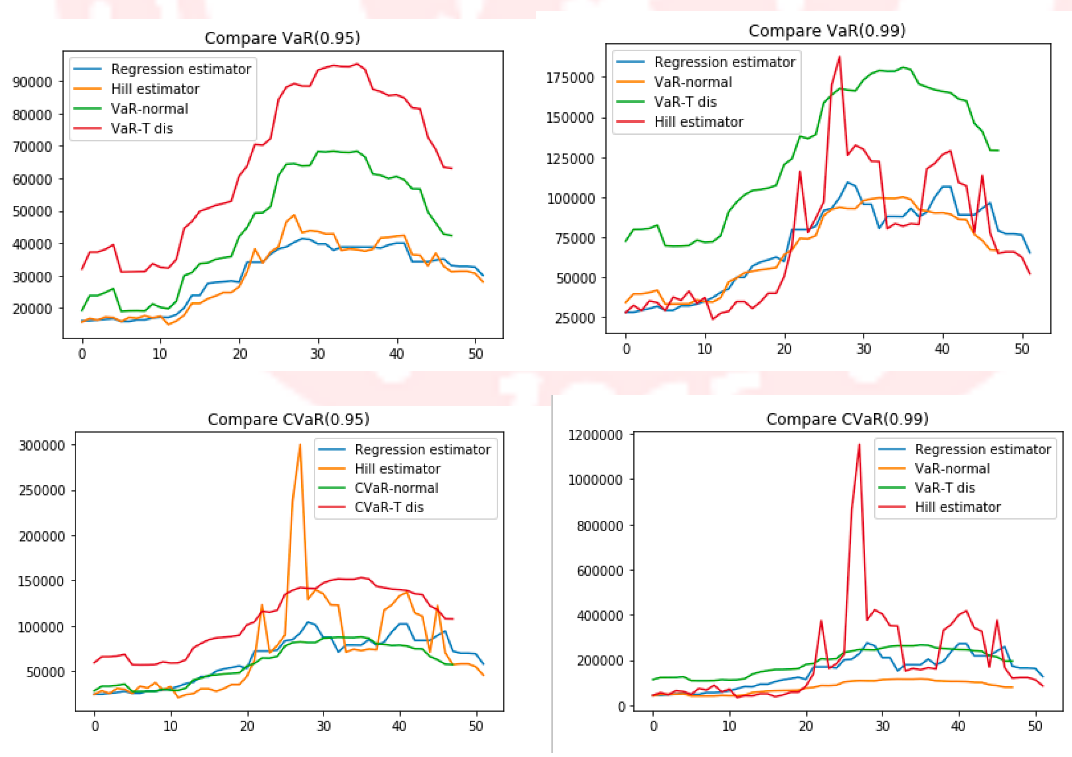


Figure 6 VaR(alpha) and CVaR(alpha) Based on Different Models

Compared with other three models, the results from Hill Estimator Models show greater fluctuations, it will be riskier to choose this model for loss estimation.

Compared with other three models, the results from t distribution show relatively high loss estimation, it will be more conservative to choose this model for loss estimation.

• GARCH Model

In previous chapter, we mainly focused on unconditional loss distribution $F_{L_{(t+1)}}$, which is defined as the distribution of $L_{(t)}(\chi)$ under the stationary distribution F_x of the risk-factor changes.[7] However, motivated by the empirical properties of financial risk-factor, we tried to univariate time-series models to estimate our loss distribution.[8]

In this chapter, we again considered our portfolio of asset positions from Project 1, and assumed that the weekly portfolio loss followed an ARMA(1,1)-GARCH(1,1) model of the form

$$L_t = \mu_t + \sigma_t * Z_t \tag{2.1}$$

where

$$\mu_t = \mu + \varphi * (X_{t-1} - \mu) + \theta * (X_{t-1} - \mu_{t-1}) \tag{2.2}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 * (X_{t-1} - \mu_{t-1})^2 + \beta * \sigma_{t-1}^2 \text{ [9]} \tag{2.3}$$

We first estimated our model, then used this model to calculate VaR_α and $CVaR_\alpha$ for confidence levels $\alpha \geq 0.9$.

• Estimation of Models

• Model Fitting

In this part, we performed two versions of analysis, one with Gaussian innovations and the other one with student-t innovations. For each model, we used 200 weeks as our rolling window. Here we mainly used Matlab toolboxes, Econometrics, for the parameters of the ARMA-GARCH model. Here we showed two results of our estimated models of two versions of innovations below.

```
ARIMA(1,0,1) Model:
--------------------
Conditional Probability Distribution: Gaussian

                                Standard          t
 Parameter       Value           Error        Statistic
-----------   -----------   ------------   -----------
 Constant        -4470.3        2347.98       -1.90389
    AR{1}      -0.895218       0.121322       -7.37884
    MA{1}       0.835037       0.151581        5.50884


GARCH(1,1) Conditional Variance Model:
----------------------------------------
Conditional Probability Distribution: Gaussian

                                Standard          t
 Parameter       Value           Error        Statistic
-----------   -----------   ------------   -----------
 Constant     1.8209e+07      0.0637432    2.85661e+08
 GARCH{1}       0.874451        0.03256        26.8566
  ARCH{1}      0.0792035      0.0380758        2.08016
```

Figure 7 Sample of ARMA(1,1)-GARCH(1,1) Model Based on Gaussian Innovations

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7 Lecture Notes from Professor Marcel Blais

8 Alexander J. McNeil & Rudiger Frey & Paul Embrechts. Quantitative Risk Management: Concepts Techniques and Tools.

9 Lecture Notes from Professor Marcel Blais

```
ARIMA(1,0,1) Model:
--------------------
Conditional Probability Distribution: t

                                 Standard          t
 Parameter       Value            Error        Statistic
-----------   -----------   ------------   -----------
 Constant         -4470.3        2101.95       -2.12674
    AR{1}      -0.826839       0.192083       -4.30459
    MA{1}        0.76859       0.224486        3.42377
      DoF        6.40206        2.96668        2.15799

GARCH(1,1) Conditional Variance Model:
----------------------------------------
Conditional Probability Distribution: t

                                 Standard          t
 Parameter       Value            Error        Statistic
-----------   -----------   ------------   -----------
 Constant     1.8209e+07      0.0290489    6.26837e+08
 GARCH{1}       0.876042      0.0457556        19.1461
  ARCH{1}      0.0798286      0.0530073        1.50599
      DoF        6.40206        2.96668        2.15799
```

Figure 8 Sample of ARMA(1,1)-GARCH(1,1) Model Based on Student-t Innovations

• Conclusions and Comparisons between Two Models

In order to compare the similarity and difference between the two versions of our GARCH models, we mainly focused on the plots of σ_t and μ_t for the two models.

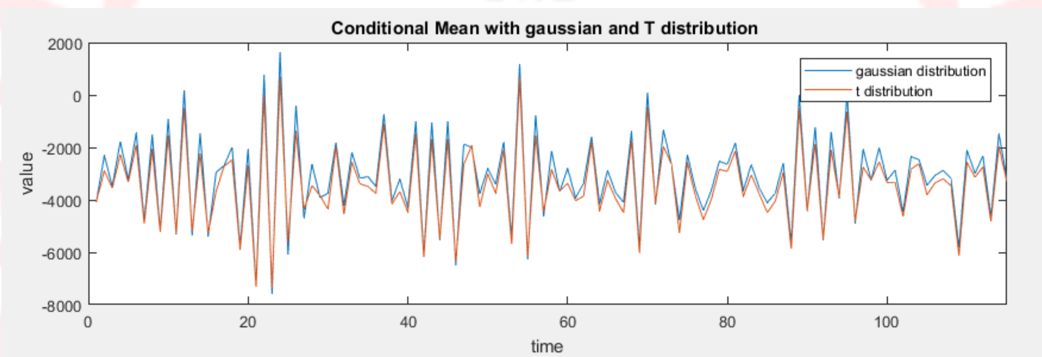

Figure 9 Conditional Mean of Models with Gaussian and Student-t Innovations

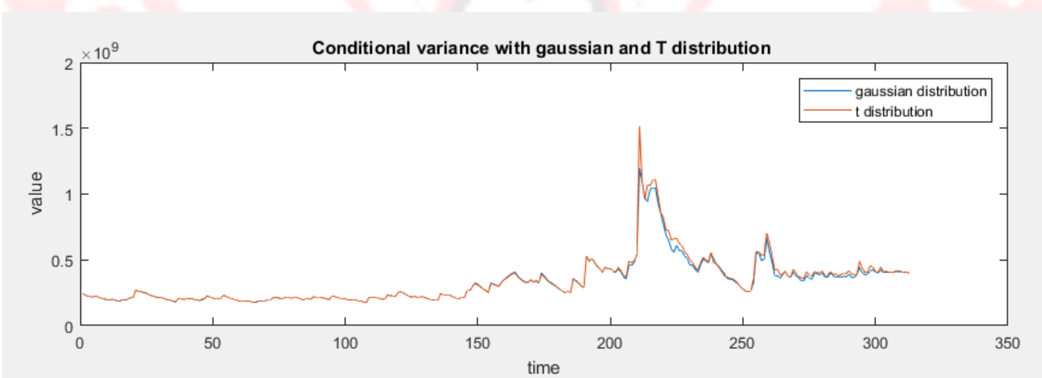

Figure 10 Conditional Variance of Models with Gaussian and Student-t Innovations

From the plots above, we can see that there isn't much large difference between the models with Gaussian innovations and student-t innovations in both of their conditional mean and conditional variance. This means that there was not much tail in the loss distribution of our portfolio.

From the conditional variance plot, the conditional variance suddenly raised a lot at about the 210th week, so our portfolio in these days must have large changes in its values.

• VaR_α and $CVaR_\alpha$ Values for the Conditional Loss Distribution

After got the conditional mean and conditional variance, we tried to estimate loss distribution of our portfolio and then plotted the VaR_α and $CVaR_\alpha$ values for the conditional loss distribution versus time.

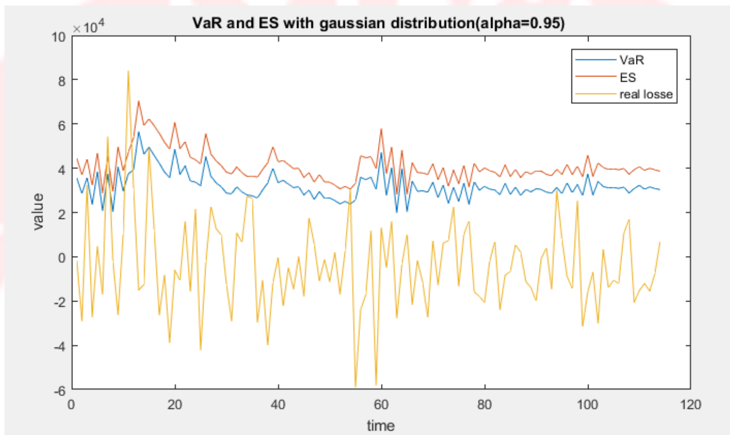

Figure 11 Comparison of VaR and ES Based on Gaussian Innovations with alpha 0.95

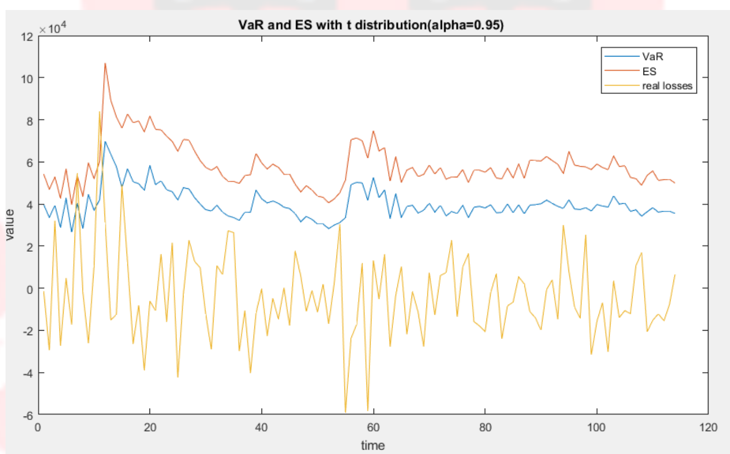

Figure 12 Comparison of VaR and ES Based on Student-t Innovation with alpha 0.95

The VaR_α and $CVaR_\alpha$ values with alpha 0.95 estimated based on both Gaussian innovations and student t innovations are almost higher than real losses.

Using the Gaussian-innovations in the model, we observed that actual losses were underestimated in the 210th week, whereas the T-innovation model gave better estimation when compared to VaR_α and $CVaR_\alpha$ values at 95% confidence.

• Comparison of Conditional and Unconditional Risk Management

In this part, we compared the conditional risk management and unconditional risk management. Here show two plots below.

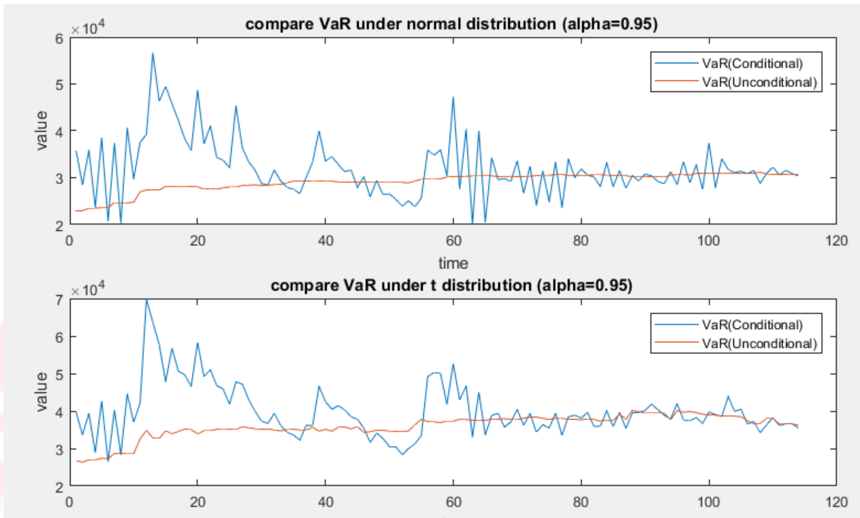

Figure 13 Comparison of VaR based on Gaussian Distribution

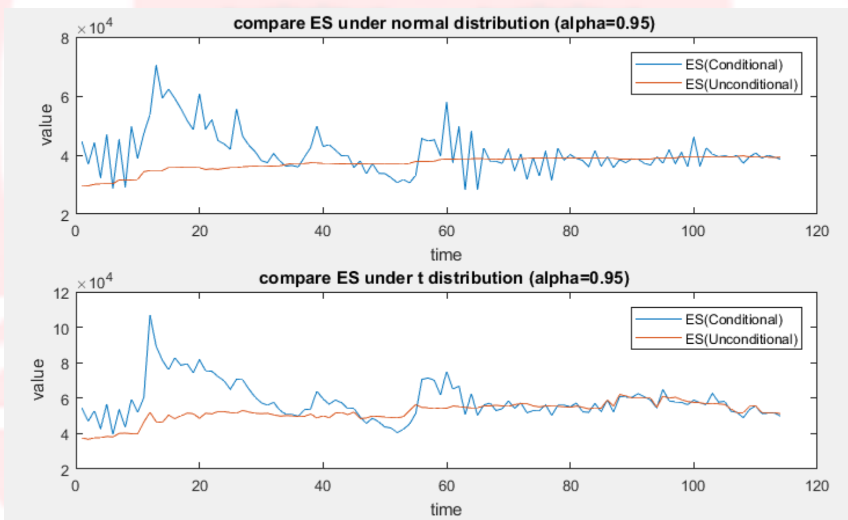

Figure 14 Comparison of VaR based on Student-t Distribution

The VaR_α and $CVaR_\alpha$ values computed under conditional loss distribution show much greater fluctuation than the values under unconditional loss distribution, which are quite stable in time series. We observe huge differences between VaR_α and $CVaR_\alpha$ estimations of conditional and unconditional distribution.

We can also observe from the plots above that conditional variance under both model is coinciding with unconditional variance if we forecast 'H' days ahead.

• Advantages and Disadvantages

One big disadvantage of using Unconditional Risk Management is the assumption that the same distribution of the training data will be applicable to the forecasts.

Another disadvantage of Unconditional risk management for semi-parametric method is that VaR_α and $CVaR_\alpha$ will be poorly estimated if the tail is very small, like when alpha equals to 99%, since we will have too few data points for the estimation of VaR_α and $CVaR_\alpha$.

Advantage of using conditional risk management, like ARMA – GARCH, is that these models are able to accurately forecast the variance and mean if time-series is exhibiting non-constant variance and mean. Here the forecasts are done presuming that they depend on more recent information.[10]

One issue we encountered while ARMA-GARCH that we require at-least 200 weeks of data for implementation of GARCH.

• Comparison of Risk Analysis

Table 3 VaR and ES Based on Different Models

Model	VaR_α	$CVaR_\alpha$	Actual Loss
Parametric	~ 50000	~ 75000	84220
Semi-Parametric	~ 45000	~ 125000	84220
ARMA-GARCH	~ 70000	~ 110000	84220

These are estimated results from previous project methods. Here we have picked up the VaR_α and $CVaR_\alpha$ values estimated in 210th week, where there is biggest loss of 80000 in portfolio value. We have used the Hill Estimator result for Semi-Parametric Method and T-Linearized Loss model from parametric methods project.

[10] Lecture Notes from Professor Marcel Blais

• IB Paper Trading

In this chapter, we mainly focused on the portfolio that we have managed in our Interactive Brokers paper trading account. We used about $1,000,000 initial capitals to form a portfolio consisting of the following positions on February 26. 2018.

• Formed a $450,000 position in our portfolio we used in our Project 1. This portfolio consisted of fifteen stocks traded on the American exchanges, including AAL, AAPL, AMZN, HSBC, C, HMC, GOOGL, JPM, MS, MSFT, GE, RY, KO, V and WFC. For each security, they had equal weight $\omega_i = \frac{1}{15}$ for $i = 1, \cdots, 15$.

• Formed five covered call options on AAL, five covered call options on WFC and bought seven European put options on KO. They all had maturity time of April 20. 2018.

• With our leftover capitals, we formed a position in two US Treasury-based bonds, United States Treasury Strip Coupon with maturity time May 15. 2019 and United Stated Treasury Strip Principal with maturity time August 15. 2028, with equal amount of value.

Our work was to perform risk management on our portfolio.

• Risk Estimation

We first assumed the stationarity of weekly losses for the assets in our portfolio, then used past data and Greeks for our options to estimate the rolling weekly loss distribution for our portfolio and reported both the VaR_α and $CVaR_\alpha$ with α equal to 0.95.

In our risk model, our portfolio was regarded as consisting of three components, Equities, Bonds and Options. For equities, we used the linearized loss to estimate the loss of equity position.

$$L_{t+1}^{\Delta} = -V_t * \sum_{i=1}^{15} \omega_{t,i} * X_{t+1,i} \tag{3.1}$$

where $\omega_{t,i}$ is defined as the weight for each stock and X_{t+1} is the log-return for each stock.

For bonds, we also used the linearized loss to estimate the loss of bond positions.

$$L_{t+1}^{\Delta} = -\sum_{i=1}^{2} \lambda_i * p(t_i, T_i) * (y(t_i, T_i) * \Delta t - (T_i - t_i) * x_i) \tag{3.2}$$

where x_i represents the change in yield of the ith bond.

For options, we used both the linearized and quadratic loss to estimate the loss of options.

$$L_{t+1}^{\Delta} = -[C_t^{BS} * \Delta + C_S^{BS} * S_t * X_{t+1,1} + C_r^{BS} * X_{t+1,2} + C_\sigma^{BS} * X_{t+1,3}] \tag{3.3}$$

$$L_{t+1}^{\Delta\Gamma} = -[C_t^{BS} * \Delta + C_S^{BS} * S_t * X_{t+1,1} + C_r^{BS} * X_{t+1,2} + C_\sigma^{BS} * X_{t+1,3} + \frac{1}{2} * (C_{SS}^{BS} * S_t^2 * X_1^2 + 2 * C_{S\sigma}^{BS} S_t * X_1 * X_3 + C_{\sigma\sigma}^{BS} * X_3^2)] \tag{3.4}$$

where C_t^{BS} is the theta, which is known as the first order derivative with respect to time; C_S^{BS} is the delta, which is known as the first order derivative with respect to underlying price; C_r^{BS} is the rho, which is known as the first order derivative with respect to short rate; C_{σ}^{BS} is the Vega, which is known as the first order derivative with respect to volatility C_{SS}^{BS}; is the second order derivative with respect to stock price; $C_{\sigma\sigma}^{BS}$ is the second order derivative with respect to volatility; $C_{S\sigma}^{BS}$ is the second order derivative of underlying price and volatility; Δ is the time difference. $X_{t+1,1}$ is the Log-Return. $X_{t+1,2}$ is the change of interest rate and $X_{t+1,3}$ is the change of implied volatility. Also, notice that here we omit the second order partial derivative with respect to r.

For Greeks, Gamma, Vega, Delta, Theta and Rho, we found them from Bloomberg. But for Vanna and Vomma, we used the following formulas to calculate.

$$vanna = \frac{vega}{S}\left(1 - \frac{d_1}{\sigma * \sqrt{\tau}}\right) \tag{3.5}$$

$$vomma = vega * \frac{d_1 * d_2}{\sigma} \tag{3.6}$$

In our project, we modelled our loss distribution based on both Normal Distribution and T Distribution. Here show our VaR_{α} and $CVaR_{\alpha}$ for α equal to 0.95 below.

Table 4 Comparison of VaR and CVaR (Dollar)

	$VaR_{0.95}$ (N)		$CVaR_{0.95}$ (N)		$VaR_{0.95}$ (t)		$CVaR_{0.95}$ (t)		
	Linearized	Quadratic	Linearized	Quadratic	Linearized	Quadratic	Linearized	Quadratic	Actual Losses
1st Week	-2640	-16948	61	-14239	1941	-12355	11775	-2497	-30270
2nd Week	20086	2761	22886	15575	24834	17532	35026	27773	16752
3rd Week	8617	-4565	11522	-1644	13543	388	24117	11021	-259
4th Week	9234	-4382	12181	-1446	14231	596	24958	11281	-16062
5th Week	10614	9042	14050	12472	16439	14858	28945	27342	7042
6th Week	13943	1816	17398	5281	19801	7690	32376	20300	57290
7th Week	12269	-55762	15734	-52244	18144	-49797	30755	-36990	-36990

Table 5 Comparison of VaR and CVaR (Percentage)

	$VaR_{0.95}$ (N)		$CVaR_{0.95}$ (N)		$VaR_{0.95}$ (t)		$CVaR_{0.95}$ (t)		
	Linearized	Quadratic	Linearized	Quadratic	Linearized	Quadratic	Linearized	Quadratic	Actual Losses
1st Week	-0.25%	-1.63%	0.01%	-1.37%	0.19%	-1.19%	1.13%	-0.24%	-2.91%
2nd Week	1.96%	0.27%	2.23%	1.52%	2.42%	1.71%	3.42%	2.71%	1.63%
3rd Week	0.83%	-0.44%	1.11%	-0.16%	1.30%	0.04%	2.32%	1.06%	-0.02%
4th Week	0.89%	-0.42%	1.18%	-0.14%	1.38%	0.06%	2.42%	1.09%	-1.55%
5th Week	1.05%	0.90%	1.40%	1.24%	1.63%	1.48%	2.88%	2.72%	0.70%
6th Week	1.37%	0.18%	1.71%	0.52%	1.95%	0.76%	3.19%	2.00%	5.64%
7th Week	1.21%	-5.52%	1.56%	-5.17%	1.80%	-4.93%	3.04%	-3.66%	-3.66%

• Stress Test

• Perform Stress Test

In this part, we implemented a $SVaR_{\alpha}$ -based risk Analysis for our portfolio by using three historical scenarios. Then, scenarios we chose were Greece Financial crisis in 2015, Japan Earthquake crisis of 2011 and the Financial crisis of 2008. And we chose $VaR_{0.95}$ for each scenarios as our $SVaR_{0.95}$.

First, as for the stocks part, because VISA was not listed in 2008. So, we decided to use America Express to replace the VISA, which both have the same property. We thought that the stock of America Express would reflect variation of the stock of VISA in 2008. Second, considering that we couldn't buy the same options, which were in our portfolio, during these three scenarios, we calculated the number of share of underlying stock that each option covered and hedged back to the underlying stock. Then, we bought this new shares of underlying stock to form our new portfolio at three scenarios. Finally, for the bond part, we decided not to take bonds into consider to build the portfolio of different scenarios. Because compared with the stock and option part, the influence of bonds was too small to consider, which had the much small yield.

Table 6 SVaR of three Scenarios

Scenarios	Financial crisis of 2008	Greece Financial of 2015	Japan Earthquake crisis of 2011
$SVaR_{0.95}$ (dollar)	23197.38	6844.6	9069
$SVaR_{0.95}$ (Percentage)	9.65%	2.08%	5.23%
$SVaR_{0.95}$ (Last week N)	1.27%	1.27%	1.27%
$SVaR_{0.95}$ (Last week T)	1.8%	1.8%	1.8%

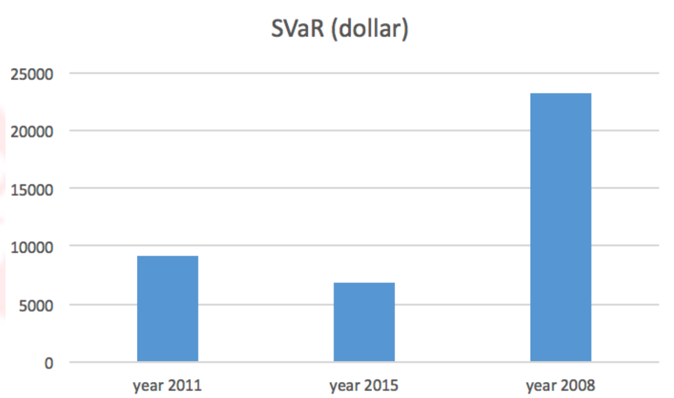

Figure 15 Comparison SVaR (Dollar)

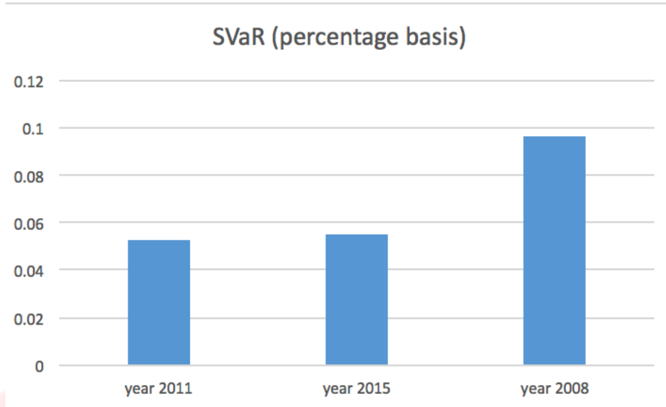

Figure 16 Comparison of SVaR (Percentage)

In order to calculate the Stressed VaR_{α} for each scenario, we found out the weekly losses, sorted them and then chose the 95% one as our Stressed VaR_{α}. From the plot above, we can notice that the $SVaR_{\alpha}$ was the largest in 2008. This means that our portfolio suffered the most during Financial Crises in 2008.

• Risk Analysis Report

In order to check $SVaR_{\alpha}$ we calculated before, we used the built-in tool "PORT" in Bloomberg Terminal. Based on Bloomberg, our assets are divided into five main parts. As following,

1. Consumer Discretionary: AMZN, HMC

2. Consumer Staples: KO

3. Financials: C, HSBC, JPM, MS, RY, WFC

4. Industrials: AAL, GE

5. Information Technology: AAPL, MSFT, VISA, GOOGL

Portfolio Name 575 NEW ID U22547535-2 Cur
Date |< < 04/19/18 > >| Display Curre
Rebalanced 02/26/18 Short Margin 0 Futures Margin 0

	Security	ID	Position	Price	PCS	FX Rate	Current Principal	Accrued	Market Val
	<Search>								
	Totals						428,932.22	0.00	428,932.22
	Cash		0.0000						
12)	AAL US 04/20/18 C55	AAL 4 C55	-5.0000	0.01	EXCH	1.00000	-5.00		-5.00
13)	AAPL US	AAPL	168.0000	172.80	EXCH	1.00000	29,030.40		29,030.40
14)	AMZN US	AMZN	20.0000	1556.91	EXCH	1.00000	31,138.20		31,138.20
15)	C US	C	387.0000	70.28	EXCH	1.00000	27,198.36		27,198.36
16)	GE US	GE	2,054.0000	13.99	EXCH	1.00000	28,735.46		28,735.46
17)	GOOGL US	GOOGL	26.0000	1089.45	EXCH	1.00000	28,325.70		28,325.70
18)	HMC US	US4381...	818.0000	34.22	EXCH	1.00000	27,991.96		27,991.96
19)	HSBC US	US4042...	588.0000	49.41	EXCH	1.00000	29,053.08		29,053.08
20)	JPM US	JPM	254.0000	111.72	EXCH	1.00000	28,376.88		28,376.88
21)	KO US	KO	682.0000	44.31	EXCH	1.00000	30,219.42		30,219.42
22)	KO US 04/20/18 P44	KO 4 P44	7.0000	0.09	EXCH	1.00000	63.00		63.00
23)	MS US	MS	527.0000	54.70	EXCH	1.00000	28,826.90		28,826.90
24)	MSFT US	MSFT	316.0000	96.11	EXCH	1.00000	30,370.76		30,370.76
25)	RY US	CA7800...	371.0000	76.32	EXCH	1.00000	28,314.72		28,314.72
26)	V US	V	241.0000	123.96	EXCH	1.00000	29,874.36		29,874.36
27)	WFC US	WFC	501.0000	51.54	EXCH	1.00000	25,821.54		25,821.54
28)	WFC US 04/20/18 C60	WFC 4 C60		0.01	EXCH	1.00000			
29)									

Figure 17 Portfolio set up in Bloomberg

• VaR_α Analysis

Table 7 VaR Analysis for Different Industries

	Consumer Discretionary	Consumer Staples	Financials	Industrials	Information Technology
$VaR_{0.95}$	780.72	332.47	2043.78	1002.7	1408
$CVaR_{0.95}$	1122.71	494.31	3051.24	1492.01	2082.82

From the table, we observed that the Financials part made more contribution to the loss of portfolios and the Consumer Staples part contributed less. It meant that buying assets belonged to the Financials part would make the portfolio more risker.

• Scenarios Analysis

After we set up our portfolio in Bloomberg, we could get $SVaR_{0.95}$ of these three scenarios based on Bloomberg Scenarios Analysis.

575 vs Default (None by GICS Sectors in USD As
Model Bloomberg Ri Scen Lehman Def

Name	P&L (Lehman Default - 2008)	P&L% (Lehman Default -	Stress MV (Lehman Default - 2008)	Mkt Val
575	-14,281,064.00	-15.19	79,726,104.00	94,007,166
Consumer Discretionary	-3,122,776.75	-24.91	9,411,512.00	12,534,288
Consumer Staples	-982,031.88	-15.67	5,285,112.50	6,267,144
Financials	-2,232,958.00	-5.94	35,369,912.00	37,602,866
Industrials	-3,491,654.50	-27.86	9,042,634.00	12,534,288
Information Technology	-4,451,642.50	-17.76	20,616,936.00	25,068,577

575 vs Default (None by GICS Sectors in USD A
Model Bloomberg Ri Scen Greece Fina

Name	P&L (Greece Financial Crisis - 2015)	P&L% (Greece Financial	Stress MV (Greece Financial Crisis - 2015)	Mkt Val
575	-4,319,518.00	-4.59	89,687,648.00	94,007,166
Consumer Discretionary	-372,270.66	-2.97	12,162,018.00	12,534,288
Consumer Staples	-22,507.20	-0.36	6,244,637.50	6,267,144
Financials	-1,992,440.75	-5.30	35,610,428.00	37,602,866
Industrials	-1,050,653.50	-8.38	11,483,636.00	12,534,288
Information Technology	-881,646.00	-3.52	24,186,932.00	25,068,577

575 vs Default (None by GICS Sectors in USD A
Model Bloomberg Ri Scen Japan Earth

Name	P&L (Japan Earthquake in Mar 2011)	P&L% (Japan Earthquake	Stress MV (Japan Earthquake in Mar 2011)	Mkt Val
575	-4,072,235.25	-4.33	89,934,936.00	94,007,166
Consumer Discretionary	-967,148.25	-7.72	11,567,141.00	12,534,288
Consumer Staples	-171,575.06	-2.74	6,095,569.50	6,267,144
Financials	-1,747,288.50	-4.65	35,855,580.00	37,602,866
Industrials	-313,330.59	-2.50	12,220,958.00	12,534,288
Information Technology	-872,892.94	-3.48	24,195,686.00	25,068,577

Figure 18 Different Scenarios

Table 8 SVaR During Three Scenarios

	Financial crisis of 2008	Greece Financial of 2015	Japan Earthquake crisis of 2011
$SVaR_\alpha$ (dollar)	65630.92	19211.98	18595.47
$SVaR_\alpha$ (percentage)	18%	4.69%	4.5%
Worst position	Information Technology	Financials	Financials

From the table above, we found that under the scenarios of Financial crisis our portfolio had the largest $SVaR_{\alpha}$, and had the smallest during the Japan Earthquake crisis in 2011. Also, in different Scenarios, the worst position was different. So, we need to choose assets to form portfolios by considering different risks.

• Comparison

For comparing, first we chose to compare Linearized and Quadratic methods as following.

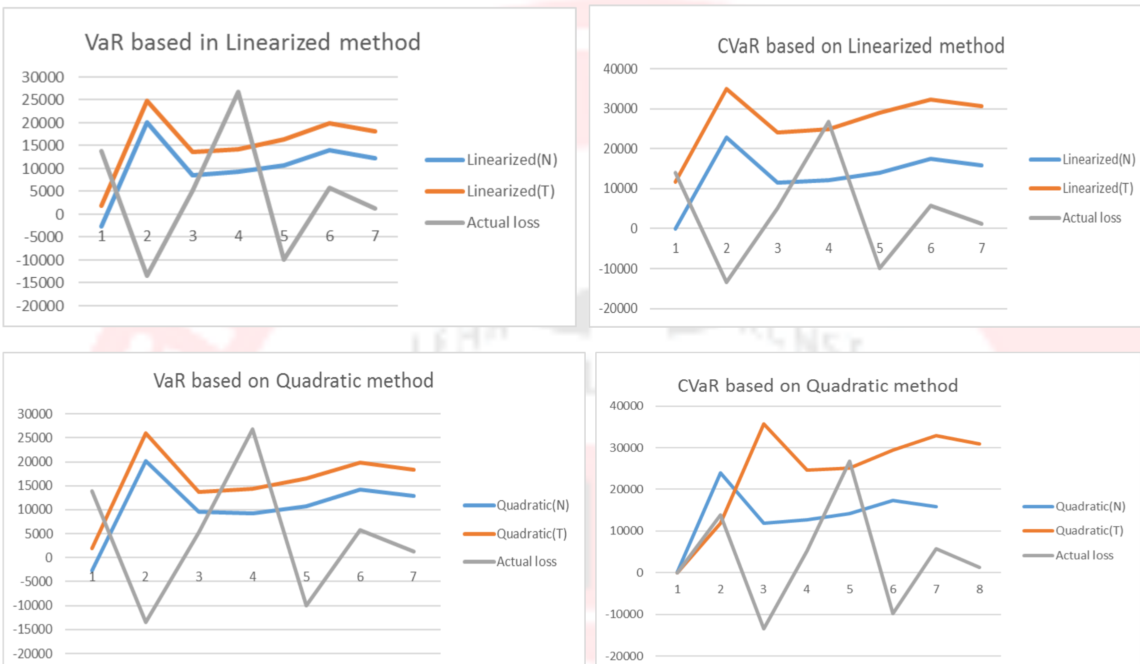

Figure 19 Comparison of different methods

From the figures above, we found that the actual loss during our holding period were not stable, it changed a lot from week to week. But we thought that the $CVaR_{\alpha}$ based on quadratic method was more conservative and VaR_{α} based on Linearized method was more risky. Since, $CVaR_{\alpha}$ calculated by Quadratic method based on T distribution almost covered all the actual loss.

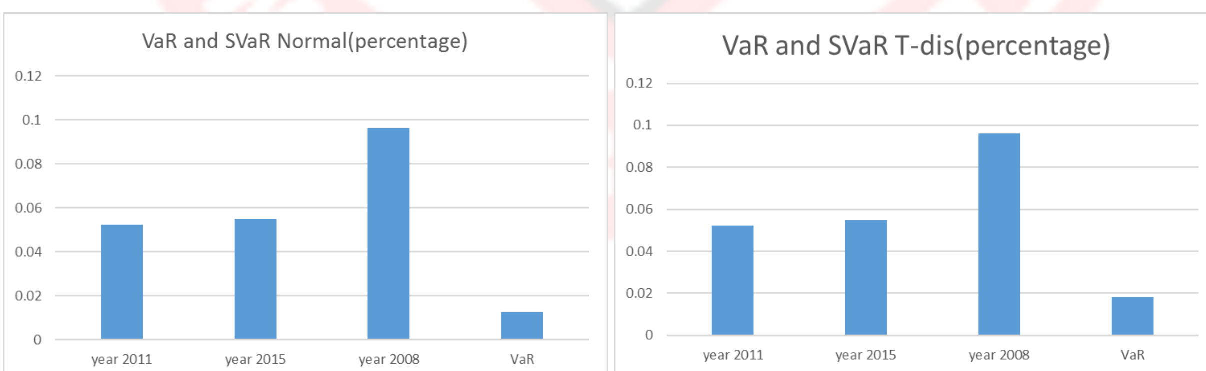

Figure 20 Comparison of VaR and SVaR

In this part, we use percentage basis of VaR_α and $SVaR_\alpha$ for comparing. Because in each scenario, the Portfolio values was different. As figures showed, we found that our portfolios suffered a huge loss in Financial crisis of 2008 and the worst position of 2008 was information technology. So, we thought our portfolios were mainly influenced by the information technology part.

• Risk reduction

In this part, we used the VaR_α as the measure to indicate that how many risks we are bearing in our portfolio. Thus, in order to reduce risk by 15%, we only needed to reduce VaR_α by 15%.

First, we fitted our weekly losses with T-distribution, and obtained $VaR_{0.95}$ and actual losses. We calculated the $VaR_{0.95}$ of our portfolio before as $13042.39, which means that we needed to reduce our $VaR_{0.95}$ by $1956.36. Then we found the America Airline (AAL) and the Wells Fargo Company (WFC) contributed the most losses to our portfolio. Thus, we chose to sell 53 positions of AAL Call Option expired on May 18.2018 and 32 positions of WFC Call Option expired on April 20. 2018. After closed our portfolio on April 20. 2018, we used our linearized weekly losses to fit a T-distribution, and found the $VaR_{0.95}$ as about $11768.00.

Table 9 Comparison of VaR_α

	Before	After
$VaR_{0.95}$ ($)	13042.39	11768.00
Reduced by 10%		

We reduced risk of our portfolio by about 10%. Even though our risk reduction was not obtained by 15%, we thought our method to reduce risk was an efficient way.

• Conclusion

From our results, we can see that there was not such large difference between the portfolio losses based on linearized and quadratic method. So, we thought options in our portfolio didn't influence our portfolio's performance a lot, because our options had much smaller positions than stocks and bonds. And after comparing $SVaR_{0.95}$ and $VaR_{0.95}$, we noticed that $SVaR_{0.95}$ was always much larger than $VaR_{0.95}$. So, the risk management based on $SVaR_{0.95}$ will be more conservative than just using $VaR_{0.95}$.

As for risk reduction part, our method didn't perform as we expected. It only reduced risk by 11%. Since for improvement, if we chose $CVaR_{0.95}$ for risk reduction, we would get more effective result than $VaR_{0.95}$.

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[4] Yahoo Finance (https://finance.yahoo.com/). Data Collection from January 1 2012 through December 31 2017