Krusell-Smith with Taxes

Qauntitative Macroeconomics
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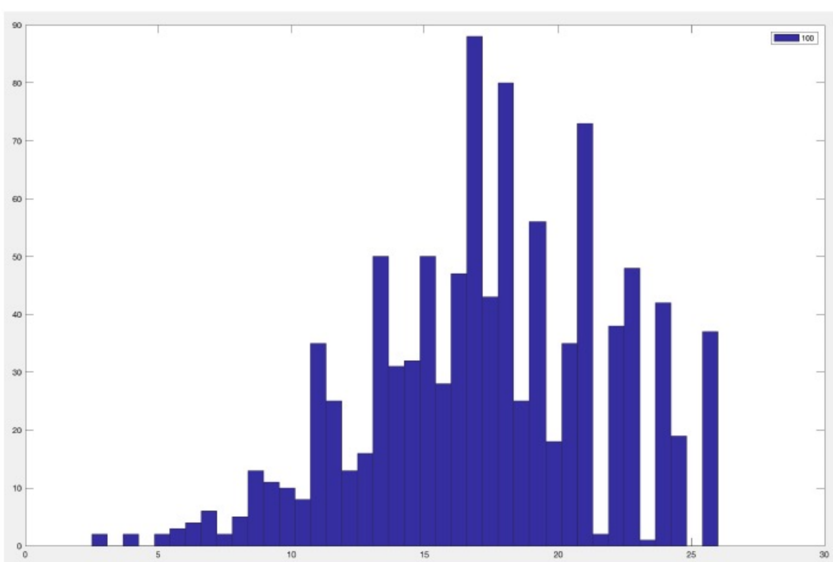
Introduction

Representative Agent (RA) models have long dominated the Macroeconomics literature. However, this assumption makes it difficult to reconcile the presence of idiosyncratic shocks and complete insurance markets. Krusell and Smith (1998) extend this literature by including substantial heterogeniety across income and wealth. They do so by introducting idosyncratic income (emloyment) shocks. Moreover, they assume that consumers cannot insure against these shocks. However, they can trade assets subject to an extrernal lower bound or a borrowing constraint.

Krusell and Smith (1998) follow the framework by Aiyagari (1994) where the asset is aggregate capital. Thus, savings can be precautionary and allow for partial insurance against the idiosyncratic shocks. Given the lack of full insurance, the model generates an endogenous distribution of wealth. Furthermore, they characterise the equilibria of the model numerically and use *approximate aggregation* to account for heterogeneity. They detail that using this methods allows us to descibe all aggregate variables, in equilibrium as a function of the mean of the wealth distribution and the aggregate productvity shock.

Krusell and Smith (1998) find that the aggregate wealth is mainly in the hands of the rich. They argue that the marginal propensity to save is very similar across agents except for the very poor. Additionally, since the number of poor is very small, any redistributions will have no effect on aggregate savings and thus, no effect on market prices. Figure 1[1] shows us the distribution computed using their parameter values for the benchmark model.

Figure 1: Simulated Asset Distribution for the Benchmark Krusell and Smith (1998) model.



While the benchmark model does well in emulating the variation in income due to employment, it does not provide sufficient variation for the wealth distribution as witnessed in the real world.

[1]This graph is take from my solution to Problem Set 6. Solving the Krusell-Smith economy using without elastic labour suppy.

That is, there are very few agent who have lowflevels of wealth and the concentration of wealth among the rich is considerably low. One way to circumvent the problem of "too few" poor is to introduce taxes and subsidies.

This paper follows the work of Den Haan and Rendahl (2010), Maliar et al. (2010) and Algan et al. (2014) in introducing taxes and unemployment benefits to the basic Krusell-Smith model and algorithm. The main aim of these papers is to solve the Krusell-Smith economy using different numerical and simulation methods, with little focus on the actual results achieved. We use the model, to introduce a government with a balanced budget.

Model

As aforementioned, this model extends the Krusell-Smith economy by introducing uneployment benefits and taxes.

Households

The economy consists of a unit mass of ex-ante identical households. Each period, agents face an idiosyncratic shock ϵ that determines whether they are employed $\epsilon = 1$ or unemployed $\epsilon = 0$. An employed agent ears w_t and after-tax wage is given by $(1 - \tau)w_t$. An unemployed agent receives unemployment benefits μw_t. Houhseholds can partially insure by accumulating capital. The maximisation problem that agent i faces, is given by.

$$\max_{\{c_t^i, k_{t+1}^i\}} E \sum_{t=0}^{\infty} \beta^t \frac{(c_t^i)^{1-\nu} - 1}{1 - \nu} \tag{1}$$

$$\text{s.t.} \;\; c_t^i + k_{t+1}^i = r_t k_t^i + [(1 - \tau_t \bar{l} \epsilon_l^i) + \mu(1 - \epsilon_l^i)]w_t + (1 - \delta)k_t^i \tag{2}$$

$$k_{t+1}^i \geq 0 \tag{3}$$

Firms

Production technology is characterised by a Cobb-Douglas production function. The per capita inputs are capital K_t and labour L_t.

$$y = zK_t^{\alpha} \bar{l} L_t^{1-\alpha} \tag{4}$$

$$w_t = (1 - \alpha)z\Big(\frac{K_t}{\bar{l}L_t}\Big)^{\alpha} \tag{5}$$

$$r_t = \alpha z\Big(\frac{K_t}{\bar{l}L_t}\Big)^{1-\alpha} \tag{6}$$

Government

Since we assume a balanced budget for the governemnt, the only role the government plays is to tax the workers and redistribute wealth to the unemployed. The implied tax rate of this proble is given by,

$$\tau_t = \frac{\mu u_t}{\bar{l} L_t} \tag{7}$$

Exogenous Shocks

There are two stochastic processes. The first is the aggregate productivity shock and the second is the employment shock. We let $\pi_{ss'ee'}$ denote the probability that $a_{t+1} = a'$ and $\epsilon^i_{t+1} = \epsilon'$ when $a_t = a$ and $\epsilon_t = \epsilon$.

Parameter Values

We use the Krusell-Smith parameter values for the all relevant variables, except unemployment benefits. They are the following,

Table 1: Paramter Values for the economy

Parameter	Parameter Value
β	0.99
δ	0.025
σ	1
α	0.36

The productivity shock z_t has two vaues namely, $z_g = 1.01$ and $z_b = 0.99$. The unemployment rates are given by $u_g = 0.04$ and $u_b = 0.1$. The process for (z, ϵ) is chosen such that the average duration of the good and bad times is eight quarters. Moreover, the transition matrix is given by the following,

$$\Pi = \begin{bmatrix} 0.6950 & 0.1800 & 0.0809 & 0.0441 \\ 0.0200 & 0.8550 & 0.0049 & 0.1200 \\ 0.1241 & 0.0009 & 0.7550 & 0.1200 \\ 0.0078 & 0.1171 & 0.0050 & 0.8700 \end{bmatrix}$$

We consier three different levels, $9\%, 15\%, 30\%$ of unemployment benefits for our analysis. The time endowment is chosen to normalise totale labour supply in a recession to one. Therefore, $\bar{l} = 1/0.9$, since the average duration of unemployment spell is 1.5 quarters in the good times and 2.5 in the bad times.

Value functions

With this set up in mind, let us now write a guess for the value functions.

$$v(k,\epsilon;\bar{k},z) = \max_{\{c,k'\}} \frac{c^{1-\gamma}-1}{1-\gamma} + \beta\mathbb{E}\big[v(k',\epsilon';\bar{k}',z')\big] \tag{8}$$

The individul budget constraints are given by the following,

If employed,

$$c + k' = (1+r)k + wl(1-\tau) \tag{9}$$

If unemployed,

$$c + k' = (1+r)k + wl\mu \tag{10}$$

Plugging, (5) and (6) in (9) and (10), we get the following.

$$c + k' = \Big(1 + \alpha z\Big(\frac{K_t}{\bar{l}L_t}\Big)^{1-\alpha}\Big)k + (1-\alpha)z\Big(\frac{K_t}{\bar{l}L_t}\Big)^{\alpha}\bar{l}(1-\tau)\epsilon + (1-\alpha)z\Big(\frac{K_t}{\bar{l}L_t}\Big)^{\alpha}\mu(1-\epsilon) \tag{11}$$

Finally, we have that $c \geq 0$ and $k' \geq b$ where, b is the borrowing constraint. We will consider two cases, one where the agents have $b = 0$ that is, they cannot borrow and the other when agents can borrow. Moreover, we follow the parameterised expectations approach to solve for the equilibrium. We suggest that the agents learn about capital tomorrow using the following functional form.

$$log(\bar{k}') = \begin{cases} \beta_{0g} + \beta_{1g}log(\bar{k}), & \text{if } z = z_g \\ \beta_{0b} + \beta_{1b}log(\bar{k}), & \text{if } z = z_g \end{cases}$$

Where, $\beta_{0g} = 0.2067$, $\beta_{1g} = 0.9292$, $\beta_{0b} = 0.2696$ and $\beta_{1b} = 0.8936$

Results

We have introduced the taxes in accordance to the budget constraint described above. The table below summarises the results for the different parameter values and access to debt.

Table 2: Paramter Values for the economy

Measure	No Tax	Unemployment Benefit			
		9%		15 %	30 %
		NB	B	NB	NB
Mean	18.28	16.36	17.25	18.09	6.28
St. Dev.	4.75	4.9	5.32	4.19	2.60
Skewness	0.01	-0.67	-0.51	-0.11	0.05

NB - No Borrowing, B - Borrowing

We find that the highest capital is accumulated in the absence of taxes. This is not a surprising result, since introducing a labour tax introduces frictions in the economy and thus reduces the marginal propensity to save at higher wealth levels. Nontheless, a 15% unemployment benefit (without borrowing) comes close to the produce the wealth level found with no taxes. This could be because agents at the lower tail of the distribution might have enough support to consume and save, resulting in higher average savings.

No Borrowing

We begin by setting the unemployment benefit, $\mu = 0.09$, which is the value that Krusell and Smith (1998) suggest. Compared to the baseline distribution, we find that there are more poor close to the borrowing limit (which is zero in this case that is, all agents must save). Moreover, the distribution suggests that there are agents who also save a large amount of wealth, relative to the average wealth which is, 16.136. Thus, it seems that including unemployment benefits makes agents less afraid of lower asset holdings. Yet, the tax is not high enough to prevent high savings.

Figure 2: Simulated Asset Distribution for $\mu = 0.09$

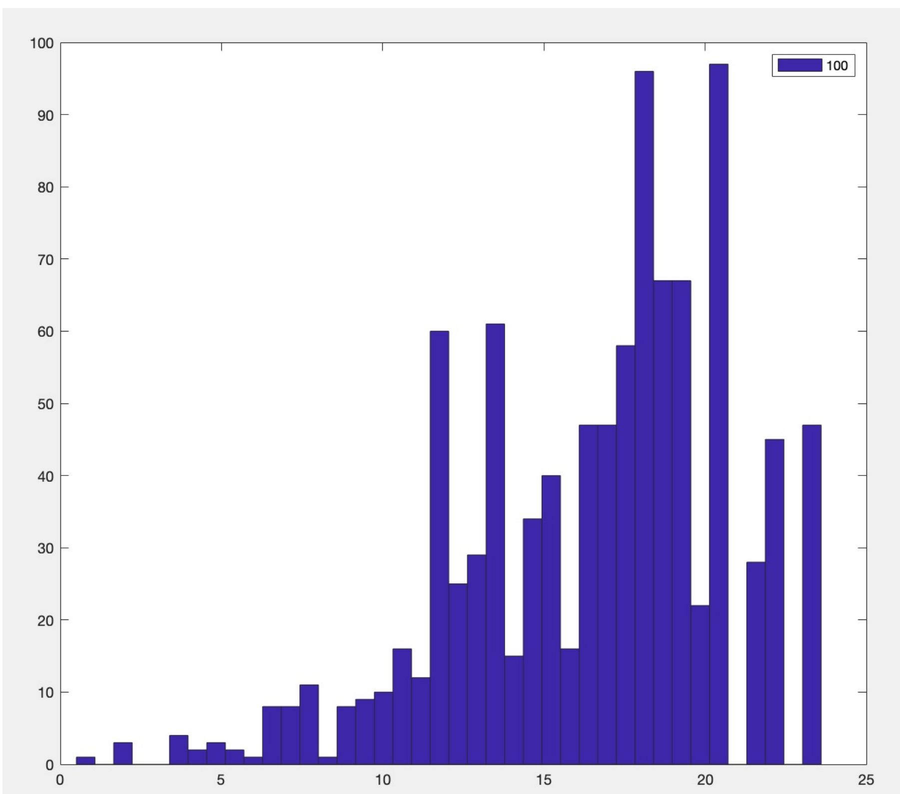
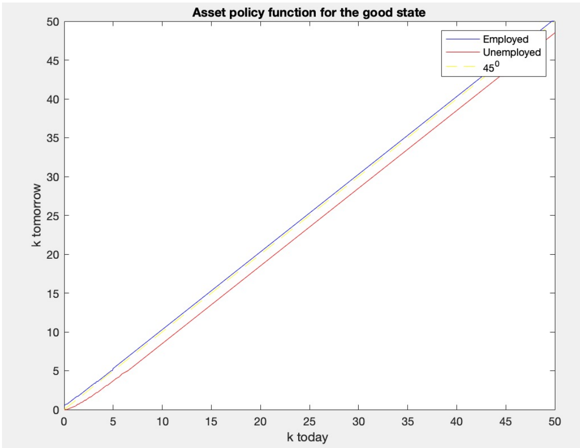
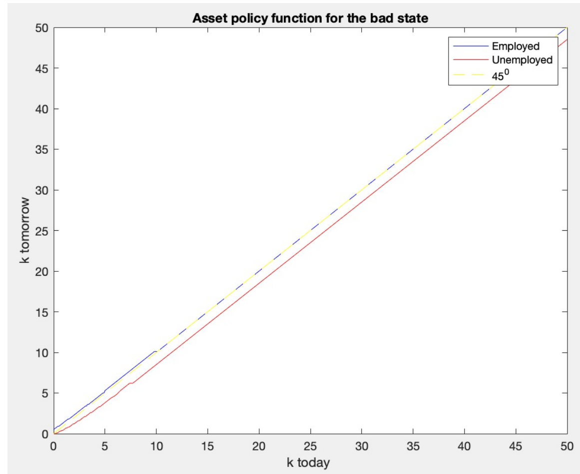


Figure 3: Policy Functions for the $z = z_g$ and $z = z_b$



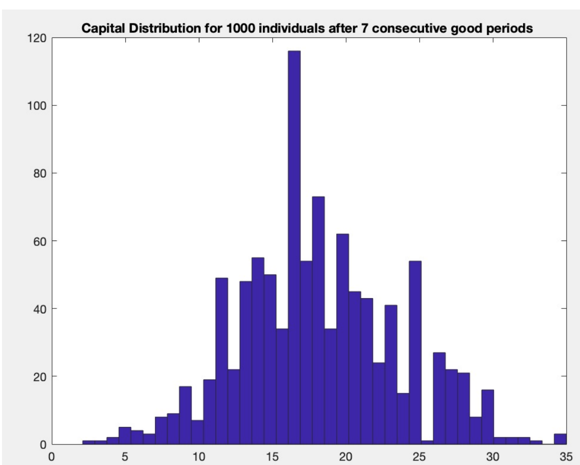
(a) Policy function for the Good State



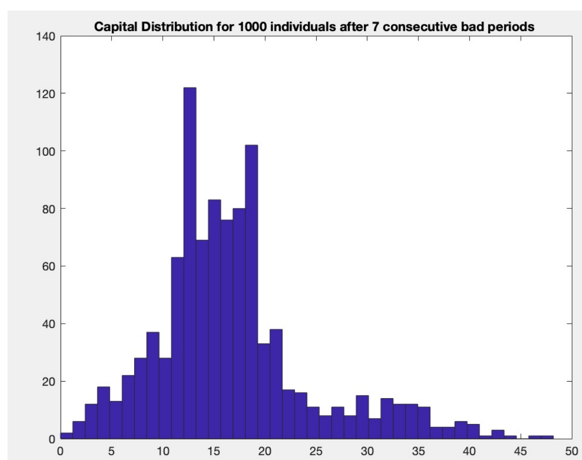
(b) Policy function for the Bad State

We further analyse the policy functions for the agents. We find that in the good state employed and unemployed agents alike, accumulate assets. However, in the bad state uneployment agents continue to save assets. Whereas, the employed agents accumulate caoital if they are below 10 units of capital. Above this level, they start to deplete assets onlyaccumulating, intermittently.

Figure 4: Asset Distribution for 7 consecutive $z = z_g$ and $z = z_b$



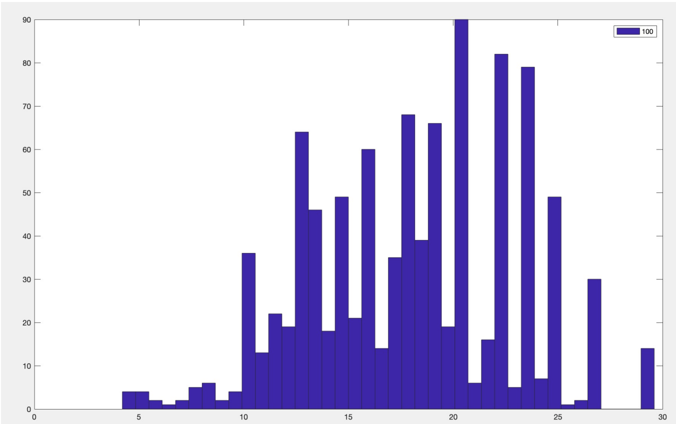
(a) Asset Distribution for the Good State



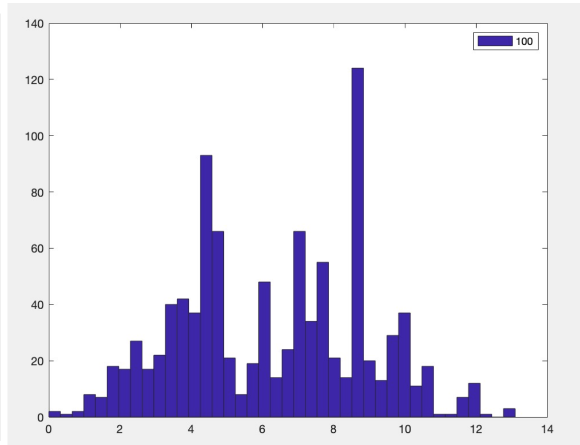
(b) Asset Distribution for the Bad State

We further check the asset distribution according to the probability of receiving 7 consecutive good and bad poductitivty shocks. This simulation leads to some interesting and unexpected results. While the average wealth in the gooo period is the same as in the bad state, the standard deviation is high in the bad state. Furthermore, in the bad state there are a a lot of agents with wwealth at the borrowing limit. However, the maximum wealth accumulated in the bad state is significantly higher than the wealth accumulated in the good state. Nonetheless, this distribution emulates the policy functions presented above (Figure 3).

Figure 5: Asset Distribution for $z = z_g$ and $z = z_b$ and $\mu = 15\%$ and $mu = 30\%$



(a) Asset Distribution for $\mu = 15\%$



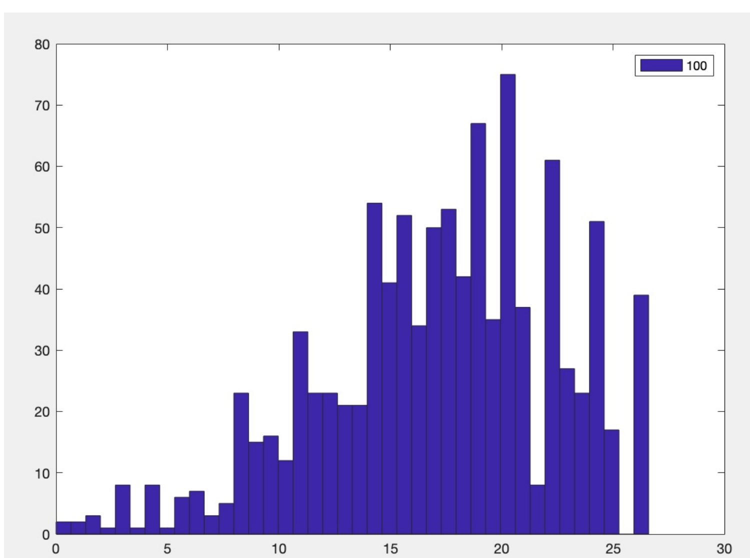
(b) Asset Distribution for $\mu = 30\%$

We now perform the simulation with two more values of μ. The results are presented above (Figure 5). We find that introducing an unemployment benefit of 30% significantly reduces the average wealth level. That is, more agents are willing to hold lower levels of wealth. However, now no gent wants to hold a very high level of assets since a high unemployment benefit implies an increased tax rate. However, the policy functions induced by these high taxes do not change significantly compared to the case with unemployment benefit at 9%.

Borrowing

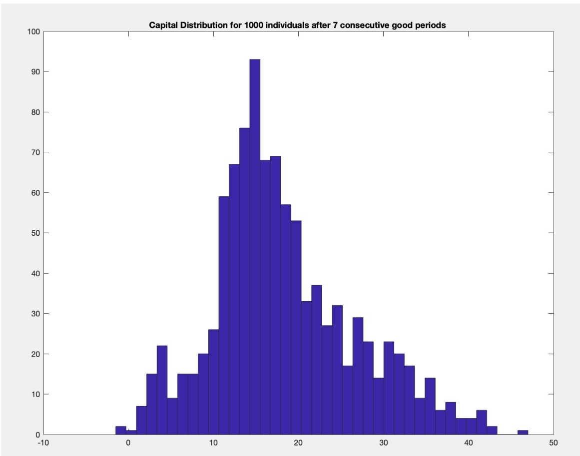
Finally, we allow the agents to borrow by increasing the size of the grid for individual grid to include negative asset holdings. The borrowing limit was set at $b = -2$.

Figure 6: Simulated Asset Distribution for $\mu = 0.09$ with borrowing

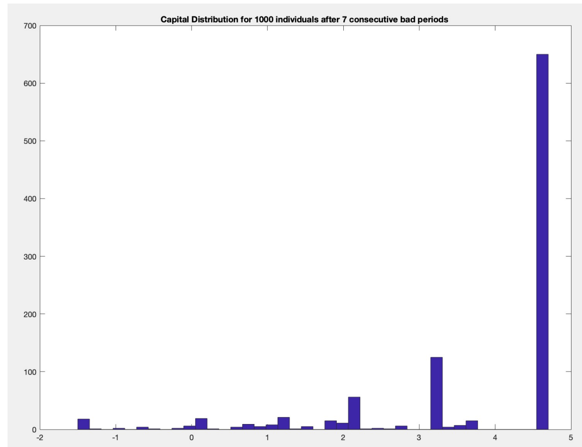


While the number of agents with very few assets, increases. We find that no agent in the economy borrows. It is possible that a 9% unemployment benefit agents are able to able to consume without requiring additional reserves.

Figure 7: Asset Distribution for 7 consecutive $z = z_g$ and $z = z_b$



(a) Asset Distribution for the Good State



(b) Asset Distribution for the Bad State

When computing the distribution for consectuive good and bad TFP shocks we find that wealth is concentrated at a very low level. Moreover, we now have agents who need to borrow for consumption.

References

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