Quantitative Macroeconomics

Problem Set 8
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[1]Using the code provided by Luis Rojas.

Model

Asssumptions

- Small open economy with stochastic endowment y_t, which follows a Markov Process.
- The government preferences are given by $E_0\{\sum \beta^t u(c^t)\}$.
- Government trades one period bondsith risk neutral investors.
- Timing of the Model
    1. Outstanding debt b_t and the GDP realisation y_t.
    2. Decision to repay debt or default.
    3. If there is no default, the government issues new debt at $q_(b_{t+1}, y_t)$
    4. If there is default, the government is excluded from the financial markets and can return with a probability λ
    5. The economy suffers a loss of τ fraction of the endowment during default.

Government's Problem - Value Functions

Value function in the case where government repays the debt.

$$V^{ND}(b,y) = \max_{b'}\{u(y + q(b',y)' - b) + \beta E\{V(b',y'|y)\}\} \tag{1}$$

The value of default is,

$$V^D(y) = u((1-\tau)y) + (1-\lambda)\beta E\{V^D(y'|y)\} + \lambda\beta E\{V(0,y'|y)\} \tag{2}$$

The value function before taking the default decision is given by,

$$V(b,y) = \max_{D\in[0,1]}\{(1-D)V^{ND}(b,y) + DV^D(y)\} \tag{3}$$

Creditors

The Break even condition is given by,

$$q(b',y) = \frac{E\{(1-D(b',y')|y)\}}{R} \tag{4}$$

Where R is the opportunity cost and the creditors know the function $D(b',y')$.

Equilibrium

A recursive equilibrium is a price schedule $q(b',y)$, value fuctions $V(b,y)$, $V^{ND}(b,y)$, $V^D(y)$ and policy functions $b'(b,y)$ and $D(b,y)$ such that,

1. Given the price schedule, the value functions and policy functions correspond to the solution of the government problem.

2. Creditors break even in expectation.

3. In equilibrium the following is satisfied,

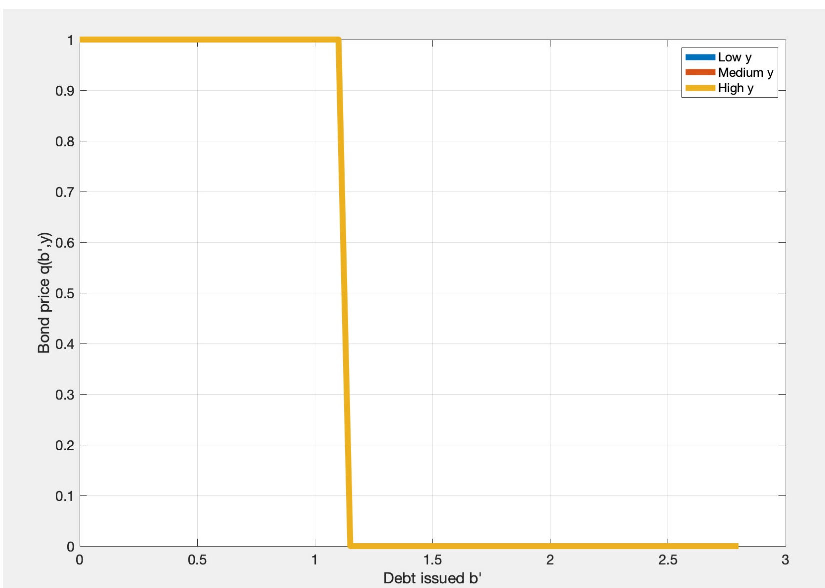
$$q(b', y) = \frac{E\{(1 - I\{V^{ND}(b', y', q) \leq V^{D}(y', q)\}|y)\}}{R} \tag{5}$$

Where, I is an indicator function which is one if there is default.

Bond price schedule

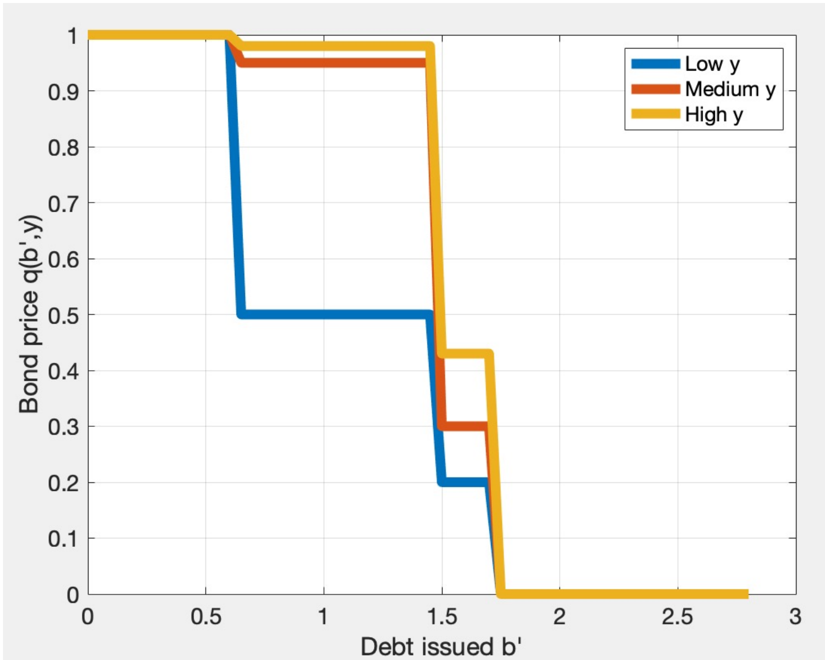
The bond-price schedule is telling us the set of quantity-price combinations for the public debt given the GDP endowments. The graph 1 shows the bond-price menu for $\tau = 0.2$ and $\lambda = 0.5$, where τ is the fraction of the GDP lost if the government defaults and λ is the probability of returning to the financial markets once you've been excluded.

Figure 1: Bond-price menu for $\tau = 0.2$



Now, we change the value of τ, assigning different τ to different levels of GDP endowments. In other words, we assume that the output cost of default depends on the level of output. Thus, introducing state-contingent losses makes default more responsive to GDP shocks and then, less responsive to outstanding debt.

Figure 2: Bond-price menu for different $\tau = [0.1, 0.4, 0.5]$ and y_t.



Equilibrium default

Adjusting the parameters to have that each country is 3% in default, we get a graph with the probability of default at t+1 given the debt-to-gdp ratio.

Figure 3: Probability of default for different $\tau = 0.2$.

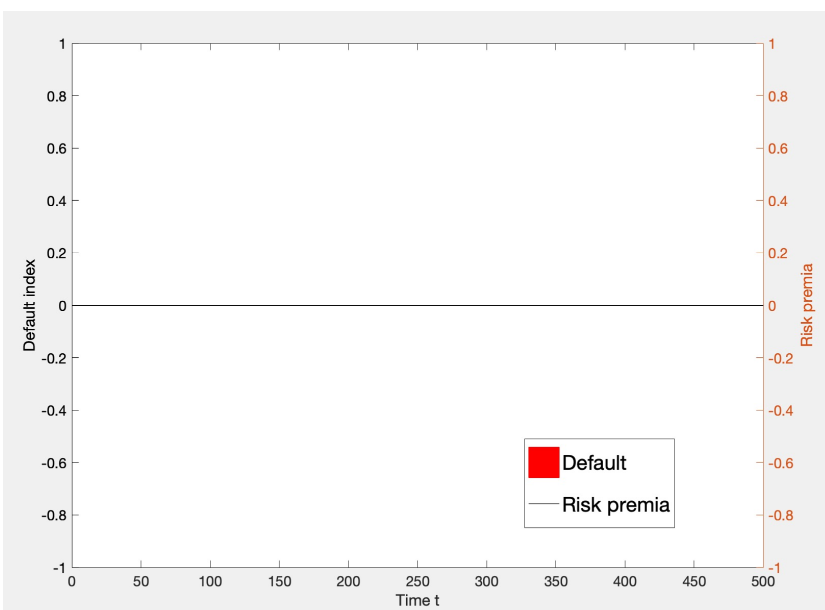


Figure 4: Probability of default for different $\tau = [0.1, 0.4, 0.5]$.

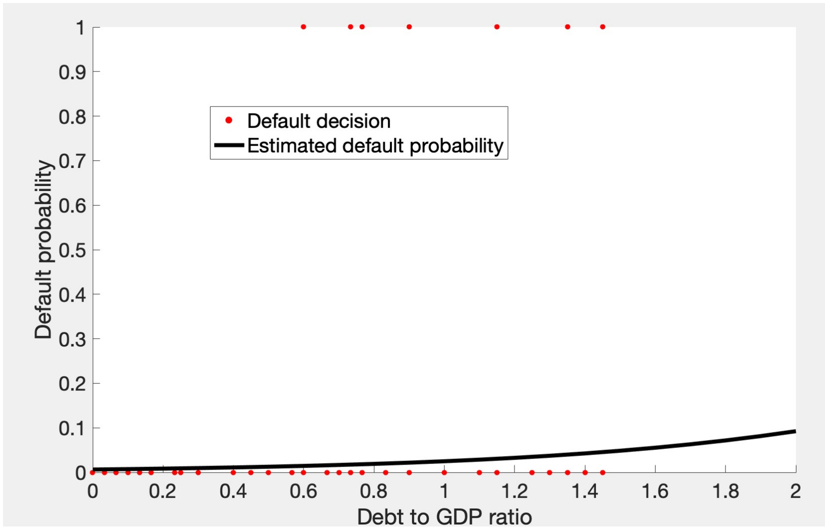
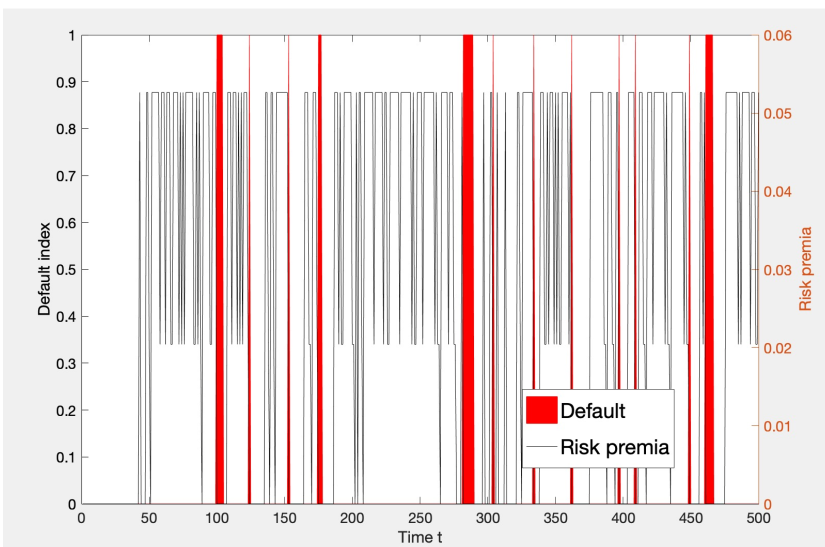


Figure 5: Probability of default.



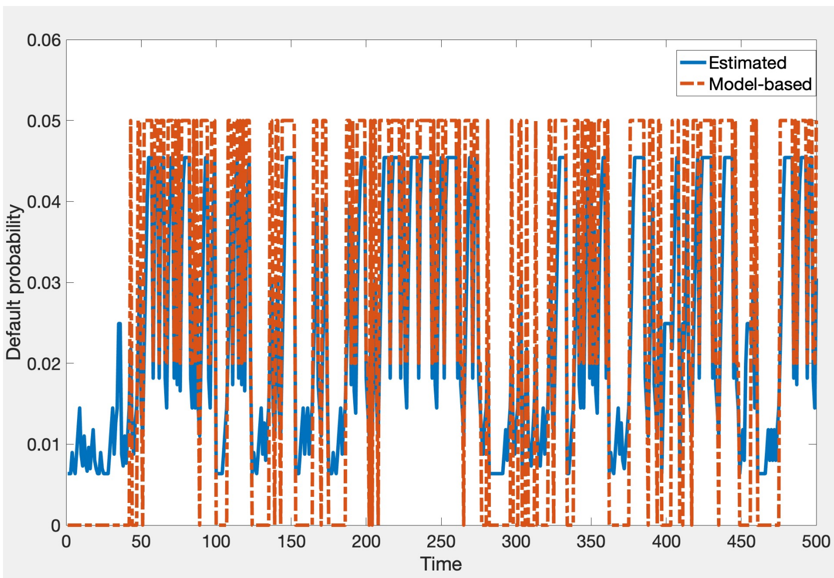
We find that with a fixed cost of default, the country does not default. Whereas with a different cost of default, there is a hgher probability of default. This could be due to the fact that now the cost it too small relative to the GDP endowment. Thus, allowing for greater felxibility when make a decision about default. As discussed before, the more variable the cost of default the higher the elasticity of default.

Indirect inference

First, we have estimated a logit on the probability of default using the debt-to-gdp as the only regressor and controlling for country fixed effects. By doing so, we got that the coefficient of the *debt_to_gdp* is equal to 1.15.

We then simulate the model and run the same logit regression over the simulated time series. In that case, the coefficient is 1.96. To make them closer, we change β to 0.91; as a result, the coefficient became 1.12. Graph 5 shows the probability of default out of the simulation.

Figure 6: Probability of default of the simulation.



STATA CODE

```
xtset Year Ind
xtlogit default debt_to_gdp, fe nolog
```

MATLAB Code

```
%% Code that solves the quantitative sovereign debt model
% Problem Set 8
rng(13);
%% Loading parameters
% Discount factor $\beta=0.95$

betta=0.91;
%%
% Possible values for GDP $y\in\{0.9,1,1.05\}$
```

```
y_grid=[0.6, 1, 1.5];
%%
% Transition matrix for GDP
%
% $$\pi_{yy'}=\left[\begin{array}{ccc}0.5 & 0.3 & 0.2\\0.1 & 0.6 & 0.3\\0.2
% & 0.4 & 0.4\end{array}\right]$$

piy=[0.5, 0.3, 0.2;...
     0.05, 0.65, 0.3;...
     0.02, 0.55, 0.43];
%%
% Risk aversion parameter $\sigma=2$

sig=1.5;
%%
% Utility function $u(c)=\frac{c^{1-\sigma}}{1-\sigma}$

u = @(c) c.^(1-sig)./(1-sig);
%%
% Risk-free interest rate $R=1$

R=1;
%%
% Probability of regaining access to capital markets next period
% $\lambda=0.3$ We want that the probability of regaining access is 1 - the
% probability of remaining in default (0.03).
lamda=0.5;
%%
% GDP loss during default. Varies with the level of GDP.
%tau = [0.2]
tau=[0.1,0.4,0.5];
%%
%
%% Initial values and the discretized state space
% Possible levesl of debt issuance $b\in B= \{0,0.05,0.1,...,0.5\}$

B=0:0.05:2.8;

max_iter=10000;
max_iterq=10000;
%%
```

```
% Initial values for the value functions:
%
% We set the inital guess by assuming that GDP and the level of debt do not
% change over time.
%
% Value function of no default
%
% $$V_{0}(b,y)=\frac{u(y)}{1-\beta}$$

V= ones(size(B,2),1)*u(y_grid)/(1-betta);
%%
% Value function of default
%
% $${V}_{0}(y)=\frac{u(y)}{1-\beta}$$

Vd = u((1-tau).*y_grid)/(1-betta);
%%
%
%% Equilibrium computation
% Our Initial guess for the bond-price schedule is constructed assuming the
% government never defaults $q(b',y)=\frac{1}{R}$

q=ones(size(B,2),size(y_grid,2));
%% Iterations on the price schedule

for iter_q=1:max_iterq

%
%% Value function iterations (taking  as given)


for iter=1:max_iter

    for iy=1:size(y_grid,2)

%%
% $$V_{i+1}(y)=u\left((1-\tau)y\right)+(1-\lambda)\beta E\left\{ V_i(y')\mid
% y\right\} +\lambda\beta E\left\{ V_{i}(0,y')\mid y\right\} $$

    Vd(iy)=u((1-tau(iy))*y_grid(iy))+(1-lamda)*betta*piy(iy,:)*Vd'+lamda*betta*V(1,:)*piy(iy,:)';
```

```
        for ib=1:size(B,2)

    V_old=V;
    Vd_old=Vd;
        V(ib,iy)=max(max(u(max(y_grid(iy)+q(:,iy).*B'-B(ib),0))+betta
            *V*piy(iy,:)'),Vd(iy)) ;
        end
    end

 dev= max(max(abs([V_old-V;Vd_old-Vd])));
if dev<=0.001
    break
end
end

%%
% Updating the bond price menu

 for iy=1:size(y_grid,2)
    for ib=1:size(B,2)

        q(ib,iy)=1-piy(iy,:)*(V(ib,:)<=Vd)';
    end
 end

end
%% Bond price menu
plot_q(B,q)
%% Simulation of the model
% recovering the policy function

 for iy=1:size(y_grid,2)
       for ib=1:size(B,2)
      % No default value and debt issuance (conditional on no deault
          )
        [V_ND(ib,iy),bp(ib,iy)]=max(u(max(y_grid(iy)+q(:,iy).*B'-B(ib
            ),0))+betta*V*piy(iy,:)')   ;
        % Default decision
        Dp(ib,iy)=(V_ND(ib,iy)<=Vd(iy)) ;

    end
 end
%%
```

```
% Simuated sequence of GDP
%
%  starting value (index)

yt=1;

for t=2:500
    draw_t=rand;
    yt(t)=1+(draw_t>=piy(yt(t-1),1))+(draw_t>=sum(piy(yt(t-1),1:2)));
end

%%
% initial level of debt
%
% index in B

bt=ones(500,1);
%%
% index for the default decision =1 and the default state

Def_b=nan(1,500);
Def_state(500)=0;

for t=2:500

   if Def_state(t-1)==0

%%
%      default decision (decided at t)

   Def_b(t)=Dp(bt(t-1),yt(t));

   if Def_b(t)==0

%%
%     Debt issuance decision (decided at t)


   bt(t)=bp(bt(t-1),yt(t));

        Def_state(t)=0;
   else
        bt(t)=1;
        Def_state(t)=1;
```

```
    end

    elseif rand<=lamda

        Def_b(t)=Dp(bt(t-1),yt(t));

    if Def_b(t)==0

%%
%       Debt issuance decision (decided at t)

    bt(t)=bp(bt(t-1),yt(t));
    Def_state(t)=0;
    else
        bt(t)=1;
        Def_state(t)=1;
    end

    else

        Def_state(t)=1;
    bt(t)=1;
    end


%%
%     Observed risk spread (1/q-1)

    r_spread(t)=1/q(bt(t),yt(t))-1;
%%
%     Default probability

    p_model(t)=1-q(bt(t),yt(t));

end

%%
%  graph of default and the risk premia

risk_premia_graph(Def_state, r_spread)
%% Estimating a logit
%  Arranging the data.
%
%  We have to be sure that if a default spell lasts for more than one
```

```
    period ,
% we only include the first time default was declared (this is why we
%
%  distinguish between the default decision and the default state)
%
%
%
% % Number of observations

N=sum(1-(isnan(Def_b)));
% Regressors. Constant and debt/GDP
X=[B(bt)'./y_grid(yt)'];

% Default decision
Y=Def_b';

% Logit regression. Binomial outcome (0, 1)
[par_est,dev,stats]=glmfit(X(1:end-1),Y(2:end),'binomial');
disp('The Coefficient from the Logit')
disp(par_est)

% Estimated Default probability
X_grid=0:0.1:2;
p_est=1./(1+exp(-par_est(1)-par_est(2)*X_grid));
Mean_Def_P = mean(p_est);
disp('Probability of Default from the Model')
disp(Mean_Def_P)
Fit_model_graph(X(1:end-1), Y(2:end), X_grid, p_est)


% default probability in the simulation
p_est_sim=1./(1+exp(-par_est(1)-par_est(2)*X));
Mean_Def_Sim = mean(p_est_sim);
disp('Probability of Default from the Simulation')
disp(Mean_Def_Sim)

%Graph for the probability of default for the model and the Simualtion
sim_and_model_graph([p_est_sim,p_model'])
```