Quantitative Macroeconomics

Problem Set 5
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Simple Wealth Model

We solve for the sequential formulation of the wealth model given by,

$$\max_{c_t} E_0 \left[\sum_{t=0}^{T} \beta_t u(c_t) \right]$$

Where, $\beta = \frac{1}{1+\rho} \in (0,1)$. The Budget Constraint is given by $c_t + a_{t+1} = w_t y_t + (1+r_t)a_t$. The individual faces a stochastic endowment process of efficiency units of labour $\{y_t\}_t^T$, with $y_t \in Y = \{y_1, .., y_N\}$. The endowmnet process is Markov wih $\pi(y'|y)$ denoting that tomorrow's endowmment is y' if today's endowment is y.

The two utility functions are,

$$u(c_t) = -\frac{1}{2}(c_t - \bar{c})^2$$
$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

Let us write the plug the budget constraint into the maximisation problem of the household and take first order conditions with respect to a_{t+1} and get the following Euler equation,

$$u'(c) \geq \beta(1+r)\frac{\sum_{y'} \pi(y'|y)}{\pi(y)} u'(c')$$

We are unable to programme the transition probabilities correctly and therefore find these graphs below in the case of certainty. Nonetheless, the inequality can arise from whether the borrowing limit is reached or not. The borrowing limit is given by the following,

$$a_{t+1} \geq -\bar{y}_{min} \sum_{s=0}^{T-(t+1)} (1+r)^{-s}$$

This condition implies that the agent cannot die in debt. That is, $a_{T+1} \geq 0$.

The other condition we have is $a_{t+1} \geq 0$, which prevents borrowing altogether.

We find the following policy functions for the quadratic utility and the CRRA utility function.

Figure 1: Quadratic Utility

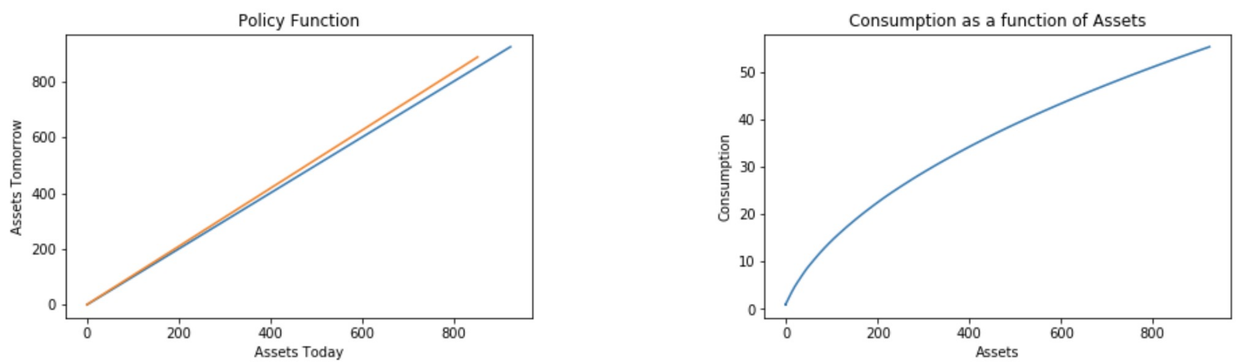
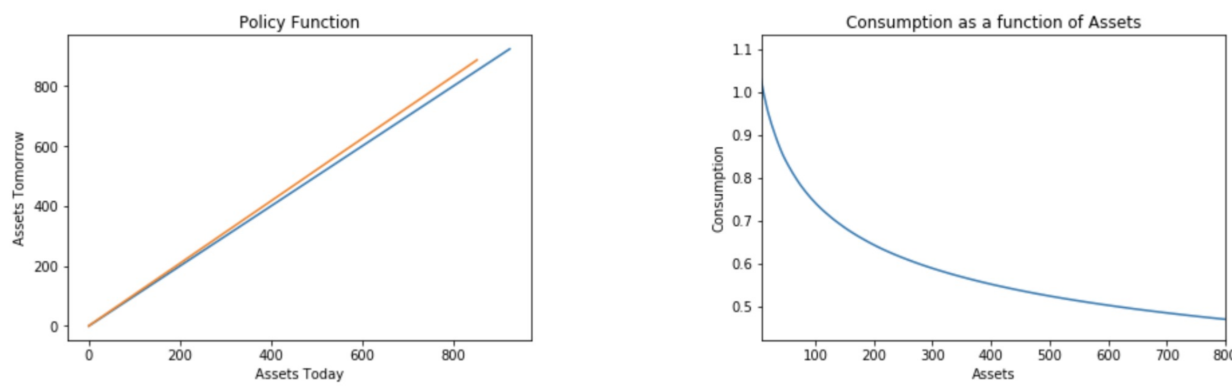


Figure 2: CRRA Utility



Recrsive Formulation of the Wealth Model

The recursive formuation of the simple wealth economy is given by,

$$v_t(a, y) = \max_{-\bar{A} \leq a' \leq (1+r)(a+y)} u(wy + (1+r)a - a') + \beta \sum_{y'} \pi(y'|y) v_{t+1}(a', y')$$

Which we get by replacing the value of consumption from the budget constraint into the utility function.

Taking first order conditions with respect to assets tomororow we get,

$$\begin{aligned} u'(wy + (1+r)a - a') =& \beta \sum_{y'} \pi(y'|y) v'_{t+1}(a', y') \\ u'(c) =& \beta \sum_{y'} \pi(y'|y) v'_{t+1}(a', y') \end{aligned}$$

By the envelope theorem we know that,

$$\begin{aligned} v'_t =& u'(c) \\ \implies v'_{t+1} =& u'(c') \end{aligned}$$

Thus, the Euler equation is given by,

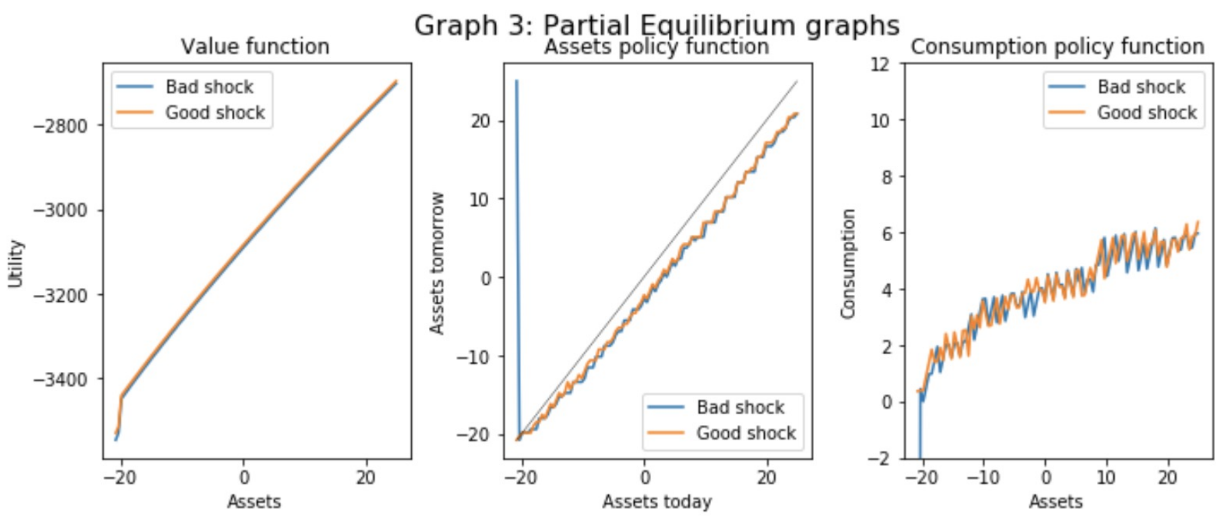
$$u'(wy + (1+r)a - a') = \beta \sum_{y'} \pi(y'|y) u'(wy' + (1+r)a' - a'') \tag{1}$$

Partial Equilibrium

We compute the partial equilibrium for infinite periods, guessing an interest rate and solving for the value function using the value function iteration method. Further, we find that the policy functions associated to the partial equilibrum.

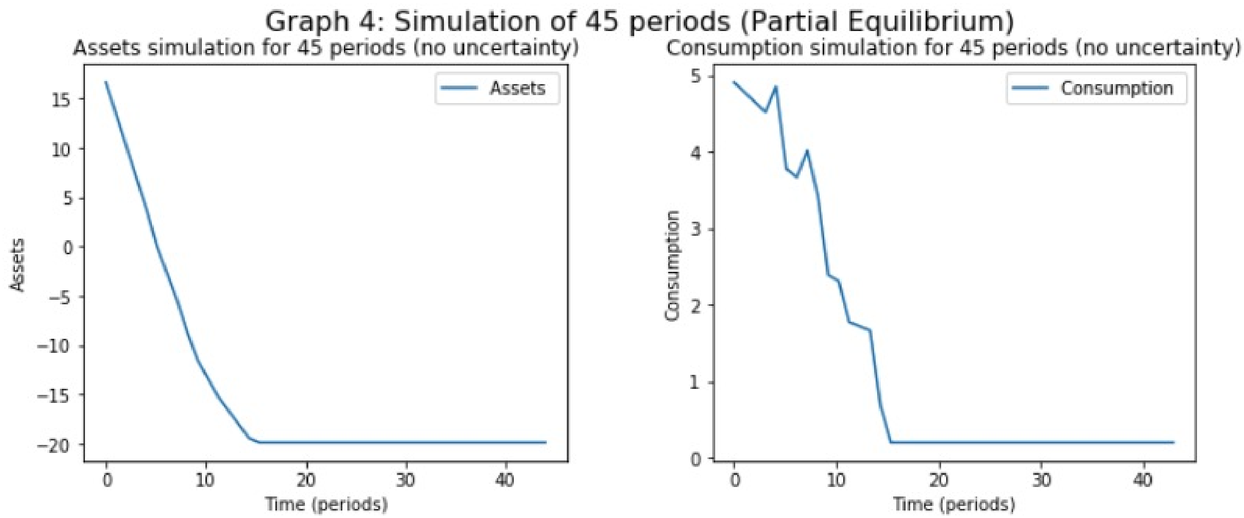
The graphs below show the value function, and the policy function for the assets and consumption.

Figure 3: Quadratc Utility



We find that consumption is increasing in assets but with many kinks. Moreover, even with the good shocks savings tomorrow are not higher than savings today. We use the quadratice utility function to solve for the value function. We further simulate the economy for 45 periods and find the following,

Figure 4: Quadratc Utility Simulation



General Equilibrium

This section has been cmpleted adapting the code from *Quant Econ*. For the general equilibrium we find the following,

Figure 5: Assets under General Equilibrium

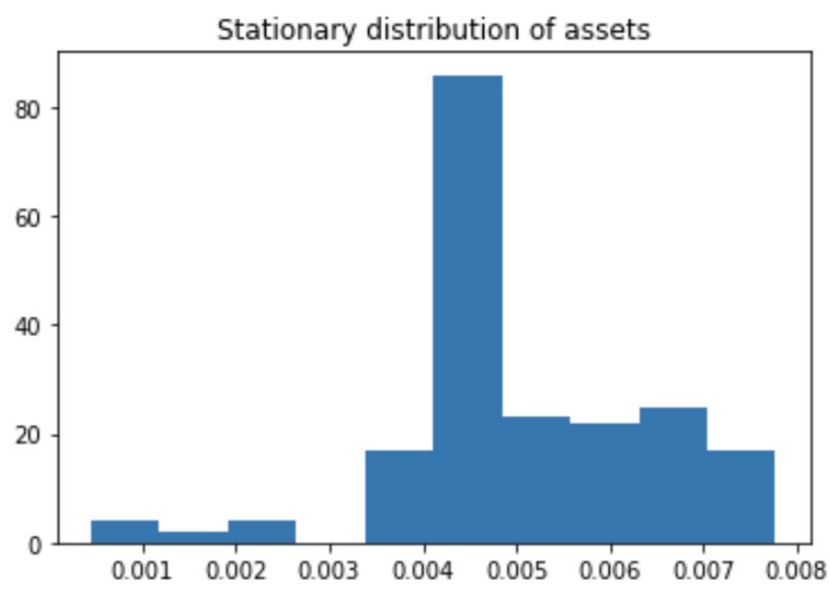
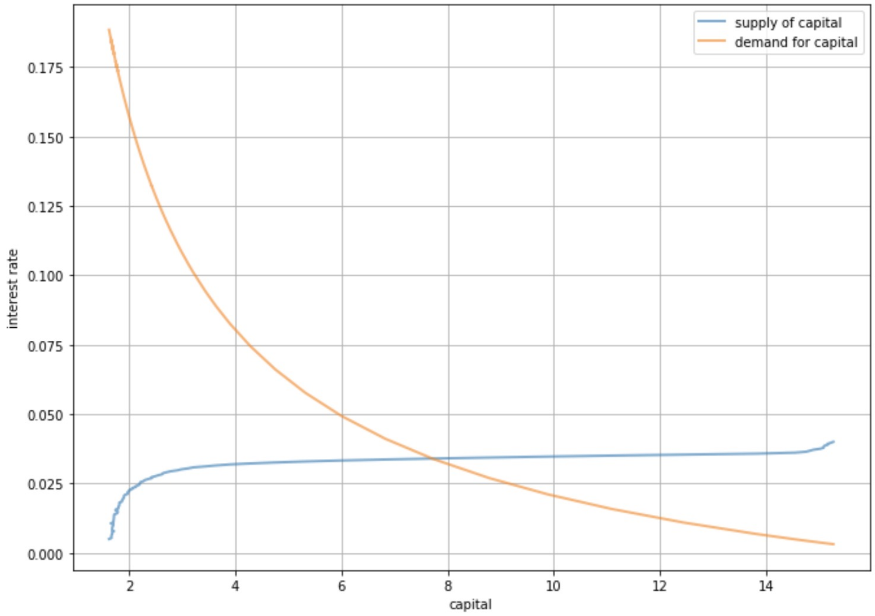


Figure 6: Interest Rate under General Equilibrium



The equilibrium interest is such that the asset market clears. We find that the equilibrium interest rate is close to 3% for the economy under generl equilibrium. Moreover, the supply for assets is limited, increasing very slowly across time. Suggesting that the rate of time preference and the interest rate are at odds with each other in many periods. The model is simulated using seven states and an equivalent dimension transition matrix.

Aiyagari Model

We now use the calibaration for the Aiyagri model, especially the fact that no one can borrow in this economy. We find the wealth distribution below. As well as the equilibrium interest rate. While simulating this economy, we find that the Gini coefficient for the model matches the Gini coefficient achieved by the original paper of 0.77. Because the code is an adaptation of the code from Quant Econ, we are unable to extract the income and consumtion distribuions. However, we can anticipate that given the asset distrbution, more people would prefer consumption today compared to tommorow and therefore decline their asset position over time. Moreover, the equilibrium interest rate is not high enough to nudge individuals to save more with the income shocks that have been used.

Figure 7: Interest Rate under Aiyagari

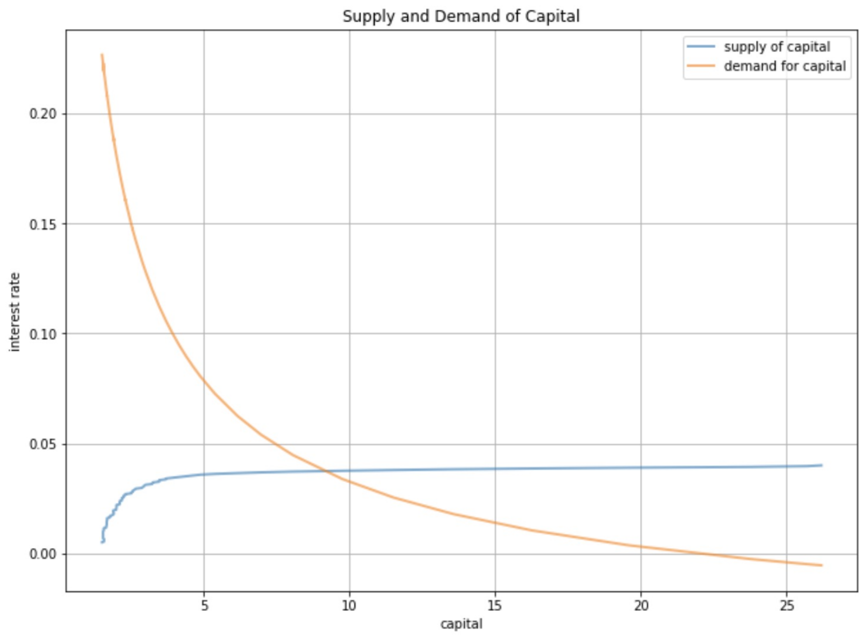
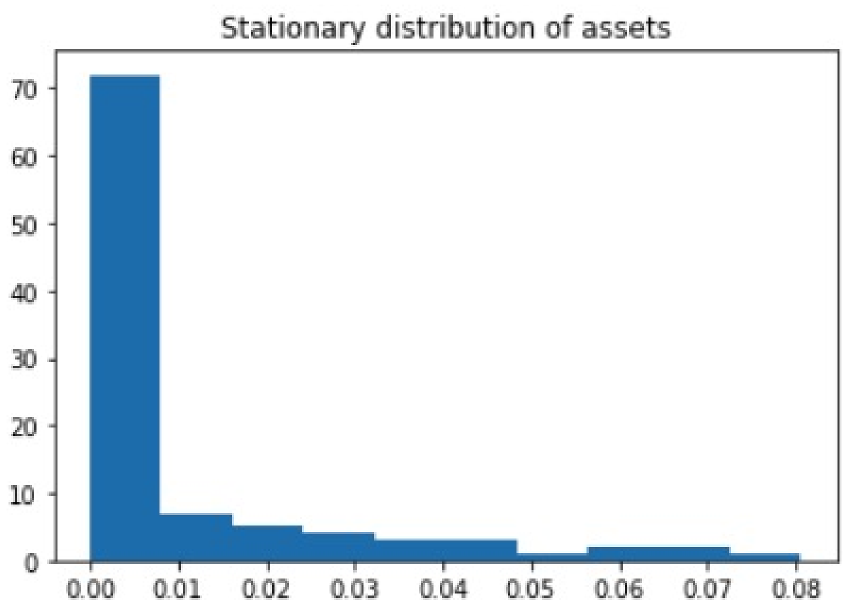


Figure 8: Asset Distribution under Aiyagari



One drawback of the code is that when, trying to calibarate the model according to Aiyagari(1994) we are unable to introduce negative income shocks of the kind of $-(3\sigma, 3\sigma)$, where σ is the shock to income.