Chapter 6

Advanced Analytics for Index-Linked Bonds

Chapter Contents

Bonds that have part or all of their cash flows linked to an inflation index form an important segment of several government bond markets. In the United Kingdom, the first index-linked bonds were issued in 1981 and at the end of 2012 they accounted for approximately 25% of outstanding nominal value in the gilt market. Index-linked bonds were also introduced in the United States Treasury market but are more established in Australia, Canada, the Netherlands, New Zealand and Sweden. There is no uniformity in market structure and as such there are significant differences between the index-linked markets in these countries. There is also a wide variation in the depth and liquidity of these markets.

Index-linked or inflation-indexed bonds present additional issues in their analysis, due to the nature of their cash flows. Measuring the return on index-linked bonds is less straightforward than with conventional bonds, and in certain cases there are peculiar market structures that must be taken into account as well. For example, in the United States market for index-linked treasuries (known as 'TIPS' from Treasury Inflation-Indexed Securities) there is no significant lag between the inflation link and the cash flow payment date. In the United Kingdom, there is an 8-month lag between the inflation adjustment of the cash flow and the cash flow payment date itself, while in New Zealand there is a 3-month lag. The existence of a lag means that inflation protection is not available in the lag period, and that the return in this period is exposed to inflation risk; it also must be taken into account when analysing the bond.

From market observation we know that index-linked bonds can experience considerable volatility in prices, similar to conventional bonds, and therefore, there is an element of volatility in the real yield return of these bonds. Traditional economic theory states that the level of real interest rates is constant; however, in practice they do vary over time. In addition, there are liquidity and supply and demand factors that affect the market prices of index-linked bonds. In this chapter, we present analytical techniques that can be applied to index-linked bonds, the duration and volatility of index-linked bonds and the concept of the real interest rate term structure. Moreover, we show the valuation of inflation-linked bonds with different cash flow structures and embedded options.

6.1 INDEX-LINKED BONDS AND REAL YIELDS

The real return generated by an index-linked bond, or its real yield, is usually defined as yield on risk-free index-linked bonds, or in other words the long-term interest rate on risk-free funds minus the effect of inflation. There may also be other factors involved, such as the impact of taxation. Therefore, the return on an index-linked bond should in theory move in line with the real cost of capital. This will be influenced by the long-term growth in the level of real gross domestic product in the economy. This is because in an economy experiencing rapid growth, real interest rates are pushed upwards as the demand for capital increases, and investors, therefore, expect higher real yields. Returns are also influenced by the demand for the bonds themselves.

The effect of general economic conditions and the change in these over time results in real yields on index-linked bonds fluctuating over time, in the same way nominal yields fluctuate for conventional bonds. This means that the price behaviour of indexed bonds can also be fairly volatile.

The yields on indexed bonds can be used to imply market expectations about the level of inflation. For analysts and policy makers to use indexed bond yields in this way, it is important that a liquid secondary market exists in the bonds themselves. For example, the market in Australian index-linked bonds is relatively illiquid, so attempting to extract an information content from their yields may not be valid. Generally, though, the real yields on indexed bonds reflect investors' demand for an inflation premium, or rather a premium for the uncertainty regarding future inflation levels. This is because holders of indexed bonds are not exposed to inflation-eroded returns; therefore, if future inflation was expected to be 0, or known with certainty (whatever its level) there would be no requirement for an inflation premium, because there would be no uncertainty. In the same way, the (nominal) yields on conventional bonds reflect market expectations on inflation levels. Therefore, higher volatility of the expected inflation rate will lead to a higher inflation risk premium on conventional bonds, and a lower real yield on indexed bonds relative to nominal yields. It is the uncertainty regarding future inflation levels that creates a demand for an inflation risk yield premium, rather than past experience of inflation levels. However, investor sentiment may well demand a higher inflation premium in a country with a poor record in combating inflation.

Therefore, the inflation expectation could be assumed comparing the yield of a conventional bond to the yield of an inflation-linked bond with similar maturity.[1] This average inflation expectation is known as 'break-even inflation rate' and is given by Fisher's equation (6.1):

$$\mathrm{BE} = \frac{(1+r)}{\left(1+r_y\right)} - 1 \tag{6.1}$$

where BE is the break-even inflation rate; r is the yield on the conventional bond and r_y is the yield on the inflation-linked bond. The yield breakdown is shown in Figure 6.1.

Therefore, the break-even analysis allows to determine the *spread* that equals the price of a conventional bond to the one of an inflation-linked bond. This approach assumes a risk-neutral pricing by which an investor treats conventional and inflation-linked bonds the same. Under break-even hypothesis, both bonds have the same nominal yield. Note if the inflation breakeven is greater than expected inflation, for an investor is favorable to buy a conventional bond. Conversely, the inflation-linked bond is more attractive. If inflation breakeven and expectations are equal, the investor bond's choice will be then indifferent. Figure 6.2 shows the trend of UKGGBE10 and UKGGBE20 Index

1. See Section 6.3.2.

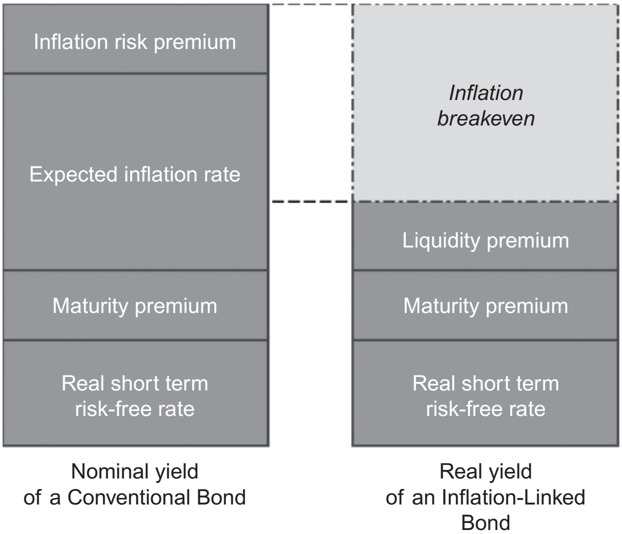

FIGURE 6.1 Inflation Breakeven. *(Source Reproduced from J.P. Morgan 2013.)*

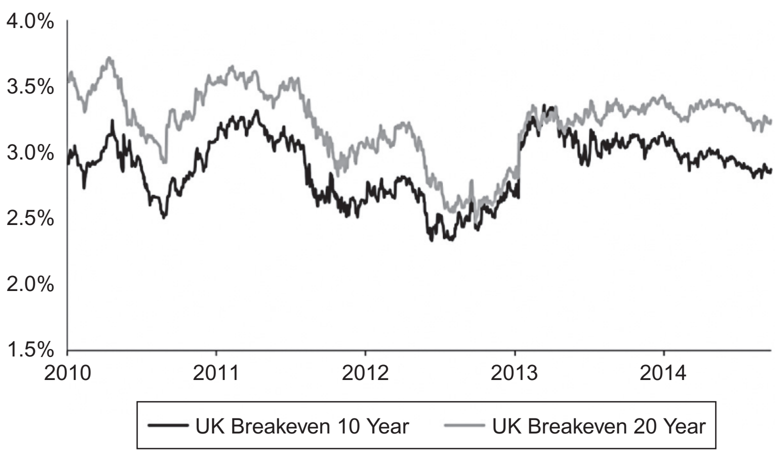

FIGURE 6.2 Ten years Inflation Breakeven, UKGGBE10 Index and UKGGBE20 Index. *(Data source: Bloomberg.)*

(2010-2014) that represent the break-even rates between nominal and real United Kingdom Treasury bonds with maturity of 10 and 20 years.

Also the simple difference between the yield of conventional and inflation-linked bonds or *yield spread* is an indicator for expected inflation. For example, on 18 September 2014 the 10-year UK 0 1/8% inflation-linked 2024 had a money yield of 2.125% and a real yield of –0.410%, assuming an inflation of 2.571%. Differently, the 10-year benchmark, the UK 2¼% 2023 had a gross

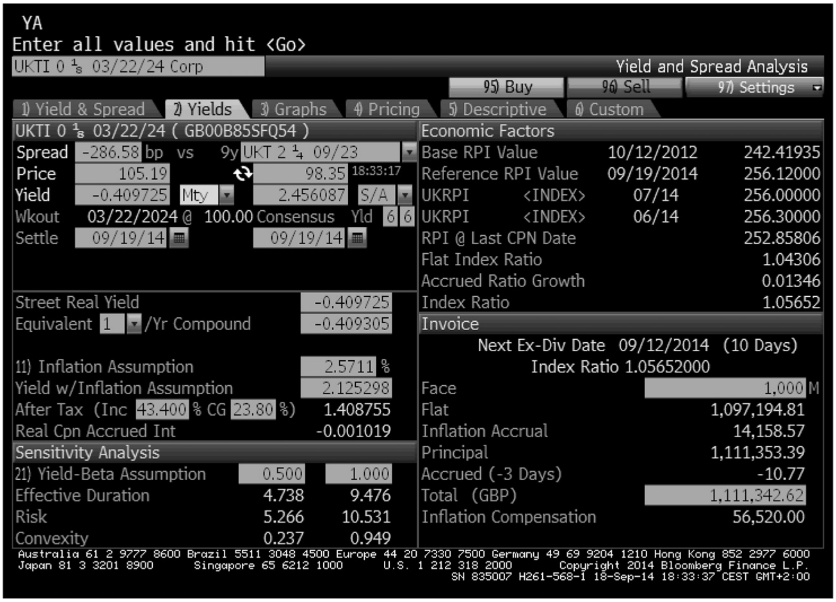

FIGURE 6.3 Example of index-linked yield analysis, UK 0 1/8% Treasury 2024 (assumed annual inflation rate 2.57%, base inflation index 242.4, reference RPI value 256.1), showing real yield and money yield, 18 September 2014. *(©Bloomberg L.P. Reproduced and used with permission.)*

redemption yield of 2.456%. Figure 6.3 shows the Bloomberg YA page for the UK 0 1/8% inflation-linked 2024.

Therefore, the yield spread is around 2% reflecting the expected inflation during the life of the bond. A higher inflation expectation will mean a greater spread between inflation-linked and conventional bonds.

This obviously approximates the expected inflation in which the yield spread cannot be attributed to the inflation only.

Traditionally, information on inflation expectations has been obtained by survey methods or theoretical methods. These have not proved reliable however, and were followed only because of the absence of an inflation-indexed futures market.[2] Certain methods for assessing market inflation expectations are not analytically valid; for example, the suggestion that the spread between short- and long-term bond yields cannot be taken to be a measure of inflation expectation, because there are other factors that drive this yield spread, and not just inflation risk premium.

Equally, the spread between the very short-term (overnight or 1 week) interest rate and the 2-year bond yield cannot be viewed as purely driven by inflation expectations. Using such approaches to glean information on inflation

2. The New York Coffee, Sugar and Cocoa Exchange traded a futures contract on the United States consumer prices index (CPI) in the 1980s.

expectations is logically unsound. One approach that is valid, as far as it goes, would be to analyse the spread between historical real and nominal yields, although this is not a forward-looking method. It would, however, indicate the market's inflation premium over a period of time. The best approach though is to use the indexed bond market; given a liquid market in conventional and index-linked bonds it is possible to derive estimates of inflation expectations from the yields of both sets of bonds. This is reviewed later in the chapter.

6.2 DURATION AND INDEX-LINKED BONDS

In earlier chapters, we reviewed the basic features of index-linked bonds and their main uses. We also discussed the techniques used to measure the yield on these bonds. The largest investors in indexed bonds are long-dated institutions such as pension fund managers, who use them to match long-dated liabilities that are also index linked; for example, a pension contract that has payments linked to the inflation index. It is common though for investors to hold a mixture of indexed and conventional bonds in their overall portfolio.

The duration of a bond is used as a measure of its sensitivity to changes in interest rate. The traditional measure, if applied to indexed bonds, will result in high values due to the low coupon on these bonds and the low real yield. In fact, the longest duration bonds in most markets are long-dated indexed bonds. The measure, if used in this way however, is not directly comparable to the duration measure for a conventional bond. Remember that the duration of a conventional bond measures its sensitivity to changes in (nominal) yields, or put another way to changes in the combined effect of inflation expectations and real yields. The duration measure of an indexed bond on the other hand, would be a measure of its sensitivity to changes in real yields only, that is, to changes in real interest-rate expectations. Conventionally, the price of an inflation-linked bond is less volatile than a conventional bond because the real yields of the former are less volatile than nominal yield of the second. Therefore, it is not valid to compare traditional duration measures between conventional and indexed bonds, because one would not be comparing like for like. If this analysis is made, the main assumption is that the implied inflation rate is 0 or constant for the overall observed time horizon. The current approach could be adjusted introducing the concept of *beta* or *inflation beta*. In corporate finance and portfolio theory, beta measures the sensitivity between an asset to the market portfolio or an index. In inflation-linked bonds, this analysis is known as 'inflation beta-adjusted duration'. The advantage of this adjustment is that it mitigates the protection for inflation included in this asset class. In practice, the beta is calculated as the sensitivity between real and nominal yields, given the same maturity or also between real yields and inflation rates. The choice of the time period influences the value of beta, in which a shorter time period determines higher volatility of beta and vice versa.

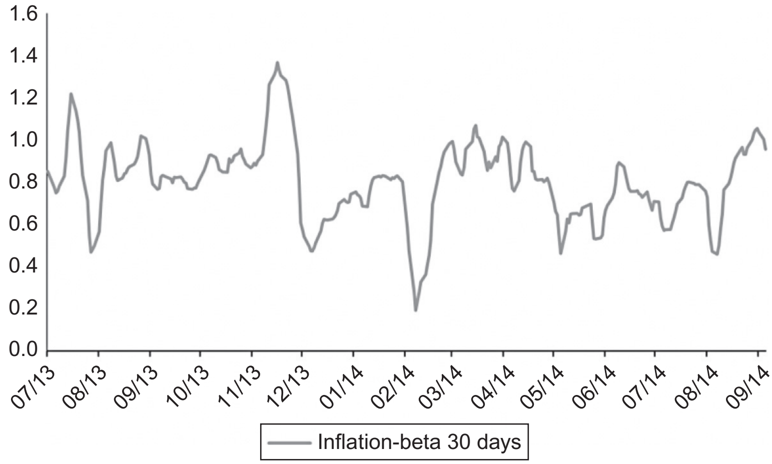

FIGURE 6.4 Inflation-beta calculated over 30 days. *(Data source: Bloomberg.)*

Figure 6.4 shows the inflation beta calculated as the correlation between nominal and real yield of the United Kingdom gilts, based to 30 days. How can be seen the inflation beta is unstable over time.

Moreover, the disadvantage of this analysis is that beta is calculated from historical data and not looking forward. Mathematically, the duration for an inflation-linked instrument is corrected as follows Equation (6.2)[3]:

$$D_{\text{adj}} = \beta_{\text{ILB}} \times D \tag{6.2}$$

where D_{adj} is the adjusted duration; β_{ILB} is the inflation beta and D is the unadjusted duration.

For instance, considering again the example of UK 0 1/8% inflation-linked 2024 shown in Figure 6.3. If we assume the analysis without the beta adjustment, the duration is equal to 9.48. Assuming a beta of 0.5, the adjusted duration becomes equal to 4.74.

The particular duration feature for inflation-linked bonds has important implications for the portfolio level. If a portfolio is composed of both conventional and indexed bonds, how does one measure its combined duration? The traditional approach of combining the duration values of individual bonds would have no meaning in this context, because the duration measure for each type of bond is measuring something different. For example, consider a situation where there are two portfolios with the same duration measure. If one portfolio was composed of a greater amount by weighting of index-linked bonds, it would have a different response to changes in market yields, especially so if investors' economic expectations shifted significantly, compared to the

3. The inflation adjustment can be made on both, Macaulay and Modified duration.

portfolio with a lower weighting in indexed bonds. Therefore, a duration-based approach to market risk would no longer be adequate as a means of controlling portfolio market risk.

Therefore, the key focus of fund managers that run combined portfolios of conventional and indexed bonds is to manage the duration of the conventional and indexed bonds on a separate basis, and to be aware of the relative weighting of the portfolio in terms of the two bond types. A common approach is to report two separate duration values for the portfolio, which would measure two separate types of risk exposure. One measure would be the *portfolio real yield duration*, which is the value of the combined durations of both the conventional and indexed bonds. This measure is an indication of how the portfolio value will be affected by a change in market real yields, which would impact both indexed and conventional bond yields. The other measure would be the *portfolio inflation duration*, which is a duration measure for the conventional bonds only. This duration measure indicates the sensitivity of the portfolio to a change in market inflation expectations, which have an impact on nominal yields, but not real yields. Portfolio managers also follow a similar approach with regard to interest-rate volatility scenarios. Therefore if carrying out a parallel yield curve shift simulation, which in terms of a combined portfolio would actually correspond to a real-yield simulation, the portfolio manager would also need to undertake a simulation that mirrored the effect of a change in inflation expectations, which would have an impact on nominal yields only.

The traditional duration approach can be used with care in other areas. For instance, the Bank of England monetary policy committee is tasked with keeping inflation at a level of 2.5%. If, therefore, the 10-year benchmark gilt is trading at a yield of 6.00% while the 10-year index-linked gilt is trading at a real yield of 3.00%, this implies that the market expectation of average inflation rates during the next 10 years is 3.00%. This would suggest that the benchmark gilt is undervalued relative to the indexed gilt. To effect a trade that matched the market maker's view, one would short the 10-year index-linked gilt and buy the conventional gilt. If the view turned out to be correct and market inflation expectations declined, the trade would generate a profit. If on the other hand real interest-rate expectations changed, thus altering real yields, there would be no effect. The other use of the traditional duration approach is with regard to hedging. Indexed bonds are sometimes difficult to hedge because of the lack of suitable hedging instruments. The most common hedging instrument is another indexed bond, and the market maker would use a duration weighting approach to calculate the nominal value of the hedging bond.

In the traditional approach the duration value is calculated using nominal cash flows, discounted at the nominal yield. A more common approach is to assume a constant average rate of inflation, and adjust cash flows using this inflation rate. The real yield is then used to discount the assumed future cash flows. There are a number of other techniques that can be used to calculate a duration value, all requiring the forecasting of the level of future cash flows and discounting using the nominal yield. These include:

- As above, assuming a constant average inflation rate, which is then used to calculate the value of the bond's coupon and redemption payments. The duration of the cash flow is then calculated by observing the effect of a parallel shift in the zero-coupon yield curve. By assuming a constant inflation rate and constant increase in the cash flow stream, a further assumption is made that the parallel shift in the yield curve is as a result of changes in real yields, not because of changes in inflation expectations. Therefore, this duration measure becomes in effect a real yield duration;
- A repeat of the above procedure, with the additional step, after the shift in the yield curve, of recalculating the bond cash flows based on a new inflation forecast. This produces a duration measure that is a function of the level of nominal yields. This measure is in effect an inflation duration, or the sensitivity to changes in market inflation expectations, which is a different measure to the real yield duration;
- An assumption that the inflation scenario will change by an amount based on the historical relationship between nominal yields and the market expectation of inflation. This is in effect a calculation of nominal yield duration, and would be a measure of sensitivity to changes in nominal yields.

Possibly the most important duration measure is the real yield duration, which is more significant in markets where there is a lag between the indexation and cash flow dates, due to the inflation risk exposure that is in place during the lag period. This is the case in both the United Kingdom and Australia, although as we noted the lag is not significant in the United States market. It is worth noting that index-linked bonds do not have stable nominal duration values, that is, they do not exhibit a perfectly predictable response to changes in nominal yields. If they did, there would be no advantage in holding them, as their behaviour could be replicated by conventional bonds. For this reason, index-linked bonds cannot be hedged perfectly with conventional bonds, although this does happen in practice on occasions when no other hedging instrument is available.

One final point regarding duration is that it is possible to calculate a *tax-adjusted duration* for an index-linked bond in markets where there is a different tax treatment to indexed bonds compared to conventional bonds. In the United States market, the returns on indexed and conventional bonds are taxed in essentially the same manner, so that in similar fashion to Treasury strips, the inflation adjustment to the indexed bond's principal is taxable as it occurs, and not only on the maturity date. Therefore, in the US-indexed bonds do not offer protection against any impact of after-tax effects of high inflation. That is, Tips real yields reflect a premium for only pretax inflation risk. In the United Kingdom market however, index-linked gilts receive preferential tax treatment, so their yields also reflect a premium for after-tax inflation risk. In practice, this means that the majority of indexed gilt investors are those with high marginal tax rates.[4] This factor also introduces another element in analysis; if the demand for indexed or conventional bonds were to be a function of expected after-tax returns, this would imply that pretax real yields should rise as expected inflation rates rise, in order to maintain a constant after-tax real yield. This has not been observed explicitly in practice, but is a further factor of uncertainty about the behaviour of real yields on index-linked bonds.[5]

6.3 ESTIMATING THE REAL TERM STRUCTURE OF INTEREST RATES

In Chapter 11 of the author's book *The Bond and Money Markets*, we show some approaches used to measure inflation expectations, with reference to the United Kingdom index-linked gilts. To recap, these measures include:

- The 'simple' approach, where the average expected inflation rate is calculated using the Fisher identity, so that the inflation estimate is regarded as the straight difference between the real yield on an index-linked bond, at an assumed average rate of inflation, and the yield on a conventional bond of similar maturity;
- The 'break-even' inflation expectation, where average inflation expectations are estimated by comparing the return on a conventional bond against that on an indexed bond of similar maturity, but including an application of the compound form of the Fisher identity. This has the effect of decomposing the nominal rate of return on the bond into components of real yield and inflation;
- A variation of the break-even approach, but matching stocks by duration rather than by maturity.

The drawbacks of each of these approaches are apparent. A rather more valid and sound approach is to construct a term structure of the real interest rates, which would indicate, in exactly the same way that the conventional forward rate curve does for nominal rates, the market's expectations on future inflation rates. In countries where there are liquid markets in both conventional and inflation-indexed bonds, we can observe a nominal and a real yield curve. It then becomes possible to estimate both a conventional and a real term structure; using these allows us to create pairs of hypothetical conventional and indexed bonds that have identical maturity dates, for any point on the term structure.[6] We could then apply the break-even approach to any pair of bonds

4. For example, see Brown and Schaefer (1996).
5. For further detail on this phenomenon, see Roll (1996).
6. We are restricted, however, to the longest dated maturity of either of the two types of bonds.

to obtain a continuous curve for both the average and the forward inflation expectations. To maximise use of the available information, we can use all the conventional and indexed bonds that have reasonable liquidity in the secondary market.

In this section, we review one method that can be used to estimate and fit a real term structure.

6.3.1 The Term Structure of Implied Forward Inflation Rates

In previous chapters, we reviewed the different approaches to yield curve modelling used to derive a nominal term structure of interest rates. We saw that the choice of yield curve model can have a significant effect on the resulting term structure; in the same way, the choice of model will impact the resulting real rate term structure as well. One approach has been described by McCulloch (1975), while in the United Kingdom market the Bank of England uses a modified version of the approach posited by Waggoner (1997) which we discussed in the previous chapter. McCulloch's approach involves estimating a discount function by imposing a constraint on the price of bonds in the sample to equal the sum of the discounted values of the bonds' cash flows. The Waggoner approach uses a cubic spline-based method, like McCulloch, with a roughness penalty that imposes a trade-off between the smoothness of the curve and its level of forward rate oscillation. The difference between the two approaches is that with McCulloch it is the discount function that is specified by the spline function, whereas in the Waggoner model it is the zero-coupon curve. Both approaches are valid, in fact due to the relationship between the discount function, zero-coupon rate and forward rate, both methods will derive similar curves under most conditions.

Using the prices of index-linked bonds, it is possible to estimate a term structure of real interest rates. The estimation of such a curve provides a real interest counterpart to the nominal term structure that was discussed in the previous chapters. More important it enables us to derive a real forward rate curve. This enables the real yield curve to be used as a source of information on the market's view of expected future inflation. In the United Kingdom market, there are two factors that present problems for the estimation of the real term structure; the first is the 8-month lag between the indexation uplift and the cash flow date, and the second is the fact that there are fewer index-linked bonds in issue, compared to the number of conventional bonds. The indexation lag means that in the absence of a measure of expected inflation, real bond yields are dependent to some extent on the assumed rate of future inflation. The second factor presents practical problems in curve estimation; in December 1999 there were only 11 index-linked gilts in existence, and this is not sufficient for most models. Neither of these factors presents an insurmountable problem however, and it is still possible to estimate a real term structure.

6.3.2 Estimating the Real Term Structure[7]

There are a number of techniques that can be applied in estimating the real term structure. One method was described by Schaefer (1981). The method we describe here is a modified version of the cubic spline technique described by Schaefer. This is a relatively straightforward approach. The adjustment involves simplifying the model, ignoring tax effects and fitting the yield-to-maturity structure. A reduced number of nodes defining the cubic spline is specified compared with the conventional term structure, because of the fewer number of index-linked bonds available, and usually only three node points are used. Our approach, therefore, estimates three parameters, defining a spline consisting of two cubic functions, using 11 data points. The approach is defined below.

In the first instance, we require the real redemption yield for each of the indexed bonds. This is the yield that is calculated by assuming a constant average rate of inflation, applying this to the cash flows for each bond, and computing the redemption yield in the normal manner. The yield is, therefore, the market-observed yield, using the price quoted for each bond. These yields are used to define an initial estimate of the real yield curve, as they form the initial values of the parameters that represent the real yield at each node point. The second step is to use a nonlinear technique to estimate the values of the parameters that will minimise the sum of the squared residuals between the observed and fitted real yields. The fitted yield curve is viewed as the real par yield curve; from this curve we calculate the term structure of real interest rates and the implied forward rate curve, using the technique described in Chapter 5. In estimating the real term structure in this way, we need to be aware of any tax effects. In the United Kingdom market, there is a potentially favourable tax effect, which may not apply in say, the United States Tips market. Generally for UK-indexed gilts, high marginal taxpayers are the biggest holders of index-linked bonds because of the ratio of capital gain to income, and their preference is to hold shorter dated indexed bonds. On the other hand pension funds, which are exempt from income tax, prefer to hold longer dated indexed gilts. The approach we have summarised here ignores any tax effects, but to be completely accurate any tax impact must be accounted for in the real term structure.

6.3.3 Fitting the Discount Function

The term structure method described by McCulloch (1971) involved fitting a discount function, rather than a spot curve, using the market prices of a sample

7. This section follows the approach (with permission) from Deacon and Derry (1994), a highly accessible account. This is their Bank of England working paper, 'Deriving Estimates of Inflation Expectations from the Prices of UK Government Bonds'.

of bonds. This approach can be used with only minor modifications to produce a real term structure. Given the bond price Equation (6.3):

$$P_i = C_i \int_0^{T_i} \mathrm{d}f(\mu)\mathrm{d}\mu + M_i \mathrm{d}f(T_i) \tag{6.3}$$

where P_i, C_i, T_i and M_i are the price, coupon, maturity and principal payment of the i-th bond, we set the set of discrete discount factors as the discount function df, defined as a linear combination of a set of k linearly independent underlying basis functions, given by Equation (6.4):

$$\mathrm{d}f(T) = 1 + \sum_{j=1}^{k} a_j f_j(T) \tag{6.4}$$

where $f_j(T)$ is the j-th basis function and a_j is the corresponding coefficient, with $j=1, 2, \ldots, k$. It can be shown (see Deacon and Deny (1994)) that for index-linked bonds equation (6.4) can be adapted by a scaling factor Δ_i that is known for each bond, once an assumption has been made about the future average inflation rate, to fit a discount function for indexed bonds. We estimate the coefficients a_j from:

$$y_i = \sum_{j=1}^{k} a_j x_{ij}$$

where

$$y_i = P_i - \Delta_i C_i T_i - \Delta_i M_i$$

$$x_{ij} = \Delta_i C_i \int_0^{T_i} f_j \mu \,\mathrm{d}\mu + \Delta_i M_i f_j(T_i)$$

$$u = (1+\pi^e)^{-1/2}$$

$$\Delta_i = \begin{cases} \left[u^{t\,\mathrm{d}j}\right]_i \cdot \dfrac{\mathrm{RPID}_j}{\mathrm{RPIB}_i} & \text{if } \ \mathrm{RPID}_j \text{ is known} \\ \left[u^{t\,\mathrm{d}l - L/6}\right]_i \cdot \dfrac{\mathrm{RPIL}}{\mathrm{RPIB}_i} & \text{otherwise} \end{cases}$$

where P_i, C_i, T_i, M_i are as before, but this time representing the index-linked bond. The scaling factor Δ_i is that for the i-th bond, and depends on the ratio of the retail price index (RPI) at the time compared to the RPI level in place at the time the bond was issued, known as the *base* RPI.[8] If in fact the RPI that is used to index any particular cash flow is not known, it must be estimated using the latest available RPI figure, in conjunction with an assumption about the path of future inflation, using π^e.

8. Due to the lag in the United Kingdom gilt market, for index-linked gilts the base RPI is actually the level recorded for the 8 months before the issue date.

6.3.4 Deriving the Term Structure of Inflation Expectations

Using any of the methods described in Chapter 5 or the discount function approach summarised above, we can construct curves for both the nominal and the real implied forward rates. These two curves can then be used to infer market expectations of future inflation rates. The term structure of forward inflation rates is obtained from both these curves by applying the Fisher identity:

$$1+\frac{f}{2}=(1+i)^{1/2}\left(1+\frac{r}{2}\right) \tag{6.5}$$

where f is the implied nominal forward rate; r is the implied real forward rate and i is the implied forward inflation rate. As with the term structure of real spot rates, the real implied forward rate curve is dependent on an assumed rate of inflation. To make this assumption consistent with the inflation term structure that is calculated, we can use an iterative procedure for the assumed inflation rate. Essentially this means that the real yield curve is reestimated until the assumed inflation term structure and the estimated inflation term structure are consistent. Real yields are usually calculated using either a 3% or a 5% flat inflation rate. This enables us to estimate the real yield curve, from which the real forward rate curve is derived. Using (6.5) we can then obtain an initial estimate of the inflation term structure. This forward inflation curve is then converted into an average inflation curve, using Equation (6.6):

$$i_i=\prod_{j}^{k}(1+if_i)^{-1/k}-1 \tag{6.6}$$

where if_i is the forward inflation rate at maturity i and i_i is the average inflation rate at maturity i.

From this average inflation curve, we can select specific inflation rates for each index-linked bond in our sample. The real yields on each indexed bond are then recalculated using these new inflation assumptions. From these yields the real forward curve is calculated, enabling us to produce a new estimate of the inflation term structure. This process is repeated until there is consistency between the inflation term structure used to estimate the real yields and that produced by Equation (6.5).

Using the modified Waggoner method described in Chapter 5, the nominal spot yield curve for the gilt market in July 1999 is shown in Figure 6.5. The real term structure is also shown, which enables us to draw the implied forward inflation expectation curve, which is simply the difference between the first two curves.

6.3.5 Application

Real yield curves are of some use to investors, for a number of reasons. These include applications that arise in insurance investment management and corporate finance, such as the following:

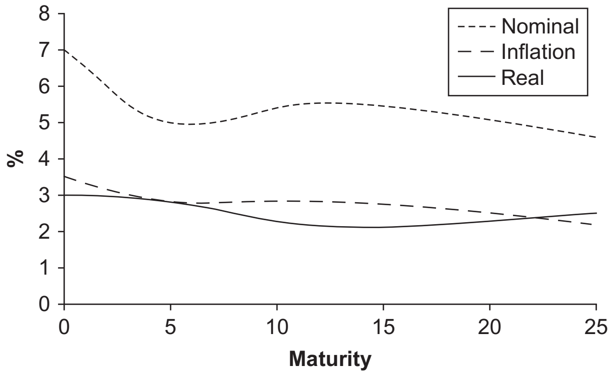

FIGURE 6.5 United Kingdom market nominal and real term structure of interest rates, July 1999. *(Yield source: BoE)*

- They can be used to value inflation-linked liabilities, such as index-linked annuity contracts;
- They can be used to value inflation-linked revenue streams, such as taxes that are raised in line with inflation, or for returns generated in corporate finance projects; this makes it possible to assess the real returns of project finance or government revenue;
- They can be used to estimate the present value of a company's future staff costs, which are broadly linked to inflation.

Traditionally, valuation methods for such purposes would use nominal discount rates and an inflation forecast, which would be constant. Although the real term structure also includes an assumption element, using estimated market real yields is equivalent to using a nominal rate together with an implied market inflation forecast, which need not be constant. This is a more valid approach; a project financier in the United Kingdom in July 1999 can obtain more meaningful estimates on the effects of inflation using the rates implied in Figure 6.2, rather than an arbitrary, constant inflation rate. The inflation term structure can be used in other ways as well; for example, an investor in mortgage-backed bonds, who uses a prepayment model to assess the prepayment risk associated with the bonds, will make certain assumptions about the level of prepayment of the mortgage pool backing the bond. This prepayment rate is a function of a number of factors, including the level of interest rates, house prices and the general health of the economy. Rather than use an arbitrary assumed prepayment rate, the rate can be derived from market inflation forecasts.

In essence, the real yield curve can and should be used for all the purposes for which the nominal yield curve is used. Provided that there are enough liquid index-linked bonds in the market, the real term structure can be estimated using standard models, and the result is more valid as a measure of market inflation expectations than any of the other methods that have been used in the past.

6.4 THE VALUATION OF INFLATION-LINKED BONDS

To obtain the price of an inflation-linked bond, it is necessary to determine the value of coupon payments and principal repayment. Inflation-linked bonds can be structured with a different cash flow indexation. As noted above, duration, tax treatment and reinvestment risk, are the main factors that affect the instrument design. For instance, index-annuity bonds that give to the investor a fixed annuity payment and a variable element to compensate the inflation have the shortest duration and the highest reinvestment risk of all inflation-linked bonds. Conversely, inflation-linked zero-coupon bonds have the highest duration of all inflation-linked bonds and do not have reinvestment risk. In addition, also the tax treatment affects the cash flow structure. In some bond markets, the inflation adjustment on the principal is treated as current income for tax purpose, while in other markets it is not.

6.4.1 Forecasting the Inflation-Index Level

The first step for estimating future streams is to know the expected inflation. To do this, the procedure needs the future trend of index in which the inflation expectation is built. The inflation expectation determined by countries is based on a different basket of products and services. For instance, inflation-linked bonds issued in the United Kingdom or *UK index-linked gilts*, are linked to the Retail Price Index (RPI); inflation-linked bonds issued in the United States or *TIPS* are linked to the Consumer Price Index (CPI). Table 6.1 summarizes the key global inflation indices used by the major issuers of inflation-linked bonds.

Therefore, in order to obtain the future index level we consider the actual index level as at bond issue and we assume a constant inflation rate. The expected 1 year index level is given by Equation (6.7):

$$I_1 = I_0 \times (1 + \tau) \tag{6.7}$$

where I_1 is the expected index level for 1 year; I_0 is the actual index level and τ is the annual inflation rate.

6.4.2 Valuing the Inflation-Linked Bonds

The price of an inflation-linked bond is determined as the present value of future coupon payments and principal at maturity. Like a conventional bond, the valuation depends on the cash flow structure. We can have three main cash flow structures of index-linked bonds.

6.4.2.1 Inflation-Linked Bonds with Zero-Coupon Indexation

Zero-coupon bonds linked to the inflation do not pay coupons. Therefore, the unique adjustment is made to the principal. These types of bonds offer no

TABLE 6.1 Key Global Inflation Indices

Key global inflation indices		
Country	*Issue*	*Inflation Index*
United States	Treasury Inflation-Protected Securities (TIPS)	US Consumer Price Index (NSA)
United Kingdom	Inflation-Linked Gilt	Retail Price Index
Japan	JGBI	Japan CPI
Germany	Bund Index and BO Index	EU HICP exTobacco
France	OATI and OATeI	France CPI ex-tobacco (OATI), EU HICP (OATeI)
Canada	Real Return Bond	Canada All-items CPI
Australia	Capital Indexed Bonds	ACPI
Sweden	Index-Linked Treasury Bonds	Swedish CPI
Italy	BTP€I	EU HICP exTobacco

Reproduced from Standard Life Investments, 2013.

reinvestment risk due to the absence of coupon payments and have the longest duration than other inflation-linked bonds. The value is given by Equation (6.8):

$$P_{\mathrm{IL}} = \frac{M_{\mathrm{IL}}}{(1+r)^{\mathrm{N}}} \tag{6.8}$$

where P_{IL} is the fair price of an inflation-linked bond; M_{IL} is the indexed principal repayment and r is the money or nominal yield.

The inflation adjustment on the redemption value can be derived as follows Equation (6.9):

$$M_{\mathrm{IL}} = M \times \frac{I_{\mathrm{M}}}{I_{\mathrm{i}}} \tag{6.9}$$

where M is the redemption value; I_M is the expected index level as at maturity date and I_i is the actual index level as at issue date.

Table 6.2 illustrates the cash flow structure of an inflation-linked bond with zero-coupon indexation.

6.4.2.2 *Inflation-Linked Bonds with Coupon Indexation*

The pricing of this type of bond is similar to a straight bond, that is, the value is found as the present value of expected coupons and principal. The main

TABLE 6.2 The Cash Flow Structure with Zero-Coupon Bond Indexation

	0	**1**	**2**	**3**	**4**	**5**
Coupon payment		0	0	0	0	0
Expected inflation		2.1%	1.8%	2.0%	2.0%	1.9%
Compounded expected inflation		1.02	1.04	1.06	1.08	1.10
Inflation adjusted Coupon payment		0	0	0	0	0
Indexed Principal repayment						110.18

difference is that coupon payments are linked to the inflation. The value of bond is given by Equation (6.10):

$$P_{\mathrm{IL}} = \sum_{t=1}^{t=\mathrm{N}} \frac{C_{\mathrm{IL}t}}{(1+r)^t} + \frac{M}{(1+r)^{\mathrm{N}}} \tag{6.10}$$

where P_{IL} is the fair price of an inflation-linked bond; C_{ILt} is the inflation-adjusted coupon payment; M is the redemption value and r is the money or nominal yield.

As noted above for inflation-linked bonds with zero-coupon indexation, the coupon can be adjusted in a similar way:

$$C_{\mathrm{IL}} = C \times \frac{I_{\mathrm{t}}}{I_{\mathrm{i}}} \tag{6.11}$$

where C_{IL} is the inflation-adjusted coupon payment; C is the coupon payment; I_t is the expected index level as at next coupon date and I_i is the actual index level as at issue date.

Table 6.3 illustrates the cash flow structure of an inflation-linked bond with coupon indexation.

6.4.2.3 Inflation-Linked Bonds with Coupon and Principal Indexation

Another structure for inflation-linked bonds is when both, coupons and principal, are linked to the inflation.[9] The value is given by (6.12):

$$P_{\mathrm{IL}} = \sum_{t=1}^{t=\mathrm{N}} \frac{C_{\mathrm{IL}t}}{(1+r)^t} + \frac{M_{\mathrm{IL}}}{(1+r)^{\mathrm{N}}} \tag{6.12}$$

9. See in Appendix A the example of bond pricing.

TABLE 6.3 The Cash Flow Structure with Coupon Indexation

	0	1	2	3	4	5
Coupon payment		2	2	2	2	2
Expected inflation		2.1%	1.8%	2.0%	2.0%	1.9%
Compounded expected inflation		1.02	1.04	1.06	1.08	1.10
Inflation adjusted Coupon payment		2.04	2.08	2.12	2.16	2.20
Principal repayment						100

or

$$\begin{aligned} P_{\mathrm{IL}} &= \frac{C_1 \times (I_1/I_0)}{(1+r)} + \frac{C_2 \times (I_2/I_0)}{(1+r)^2} + \cdots + \frac{M \times (I_{\mathrm{N}}/I_0)}{(1+r)^{\mathrm{N}}} \\ &= \frac{C_{\mathrm{IL}1}}{(1+r)} + \frac{C_{\mathrm{IL}2}}{(1+r)^2} + \cdots + \frac{M_{\mathrm{IL}}}{(1+r)^{\mathrm{N}}} \end{aligned} \tag{6.13}$$

where P_{IL} is the fair price of an inflation-linked bond; C_{IL} is the inflation-adjusted coupon payment; M_{IL} is the indexed principal repayment and r is the money or nominal yield.

Table 6.4 illustrates the cash flow structure of an inflation-linked bond with coupon and principal indexation.

TABLE 6.4 The Cash Flow Structure with Coupon and Principal Indexation

	0	1	2	3	4	5
Coupon payment		2	2	2	2	2
Expected inflation		2.1%	1.8%	2.0%	2.0%	1.9%
Compounded expected inflation		1.02	1.04	1.06	1.08	1.10
Inflation adjusted Coupon payment		2.04	2.08	2.12	2.16	2.20
Indexed Principal repayment						110.18

EXAMPLE 6.1 Inflation premium

Consider the example in which a hypothetical conventional bond and inflation-linked bond pay an annual coupon of 2%, with a discount rate of 3%. The second one pays coupons and principal linked to the inflation. To understand the effect of the inflation, Figure 6.6 shows the loss of value of a conventional bond compared to an inflation-linked bond. The evidence is that maintaining a flat discount rate, a change of the inflation rate affects only to an inflation-linked bond, increasing its value, while the value of a conventional bond remains unchanged. In reality, over the bond's life, the value of the first one remains unchanged while the value of second one depreciates as the inflation increases.

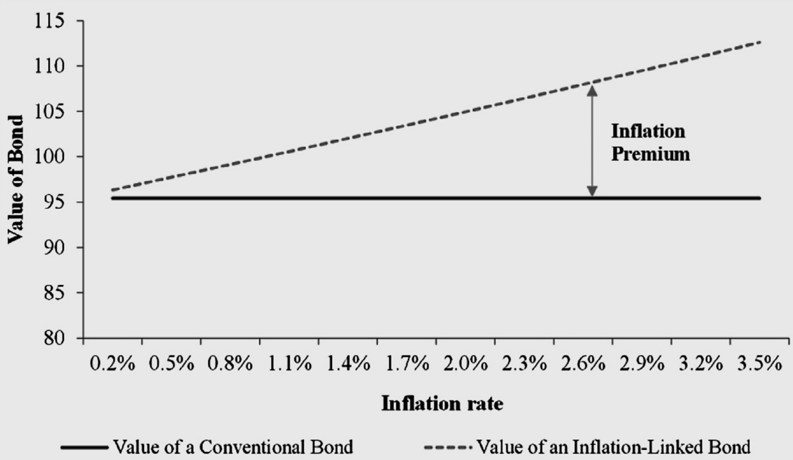

FIGURE 6.6 The effect of the inflation on bond's value.

If the price of an inflation-linked bond is 104.95 and the price of a conventional bond is 95.42, the difference of value represents the inflation premium, or 9.53.

6.4.3 Other Features of Inflation-Linked Securities

6.4.3.1 Bond valuation with real cash flows and yields

We currently illustrate the pricing of inflation-linked bonds adopting real cash flows. As noted earlier, according to Fisher's theory, the real yield is given by the following equation:

$$r_y = \frac{(1+r)}{(1+\tau)} - 1 \tag{6.14}$$

where r_y is the real yield; r is the money or nominal yield and τ is the annual inflation rate.

In hypothesis by which both coupons and redemption value are linked to the inflation, we can rearrange the Equations (6.12) and (6.13) as follows:

$$P_{\mathrm{IL}} = \sum_{t=1}^{t=\mathrm{N}} \frac{C}{(1+r_{\mathrm{y}})^{t}} + \frac{M}{(1+r_{\mathrm{y}})^{\mathrm{N}}} = \frac{C_1}{(1+r_{\mathrm{y}})} + \frac{C_2}{(1+r_{\mathrm{y}})^{2}} + \cdots + \frac{M}{(1+r_{\mathrm{y}})^{\mathrm{N}}} \quad (6.15)$$

Therefore, in Equations (6.12) and (6.13) we use both nominal cash flows and discount rates. Conversely, in Equation (6.15) the real yield is the appropriate discount rate for discounting real cash flows. This relationship is true only in the perfect indexation scenario without indexation lag.

6.4.3.2 The Deflation Floor

In a deflationary environment a conventional bond performs very well, while an inflation-linked bond gives negative returns. Some of inflation-linked bonds include deflation protection.[10] In practice, in the event of deflation, the bondholder will receive at maturity the par value although the redemption value is less than 100. Therefore, the bondholder will obtain at least the par value. Note that the deflation floor applies to the redemption value only leaving coupon payments exposed to the deflation risk.

This feature can be assimilated as an embedded put option by which the investor will receive the par value, in the case of deflation, or the redemption value linked to the inflation. The payoff is given by Equation (6.16):

$$\text{Redemption value} = M_{\mathrm{IL}} + \max(0, M - M_{\mathrm{IL}}) \quad (6.16)$$

or by Equation (6.17):

$$\text{Redemption value} = \max(M, M_{\mathrm{IL}}) \quad (6.17)$$

Therefore, we propose an example in which we price an inflation-linked bond by using a binomial tree. Conventionally, this type of pricing model is not implemented in the reality, but it allows to understand the impact of the embedded option on bond's value.

Considering the example in which a 5-year bond pays an annual coupon of 1%, with a discount rate of 2%. The bond price is given by the following steps:

- Determining the binomial inflation rate tree;
- Determining the value of coupon payments and principal repayment;
- Determining the value of a European put option;
- Determining the value of an inflation-linked bond.

The first step determines the binomial inflation rate tree according to the inflation expectations. The binomial tree is used for pricing a hypothetical annual

10. For example, inflation-linked bonds issue in the United States, Italy, France, Sweden and Germany. Note that inflation-linked bonds issue in the United Kingdom does not include deflation protection.

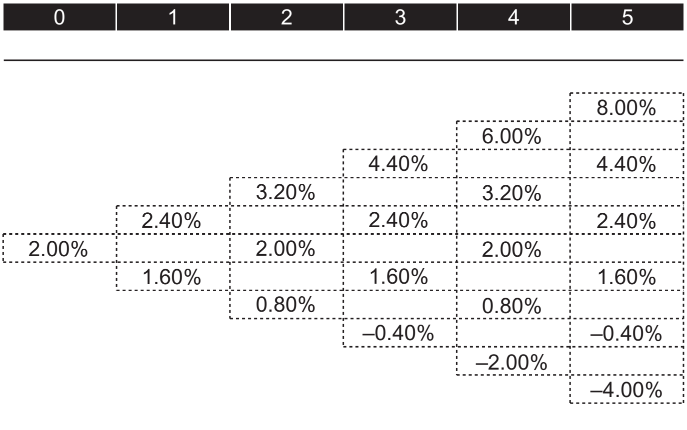

FIGURE 6.7 Binomial inflation rate tree.

inflation-linked bond. In order to predict the evolution of the inflation rate we assume a volatility of 0.4%, calculated as 30 days annualized standard deviation of the break-even inflation rate. Figure 6.7 shows the binomial inflation rate tree.

However, this is not the market inflation rate. Therefore, the binomial inflation rate tree is fitted with the market inflation rate curve because the inflation rate derived by the model differs from the one observed in the market or implied inflation rate. Thus, using the *drift* factor, the binomial inflation rate tree is adjusted with the numerical iteration process setting the *pricing error* (difference between model inflation rate curve and market inflation rate curve) equal to 0. Note that in an environment with higher expected inflation the pricing error will be greater and vice versa. Note also that in an environment with lower expected inflation the probability of deflation scenario rises, increasing the value of the embedded option. Figure 6.8 shows the binomial tree with the market inflation rate.

After having determined the binomial inflation rate tree, we calculate the value of coupon payments and principal repayment. Each coupon is linked to the inflation rate in the binomial inflation rate tree and the value is obtained as the present value of coupon payments.

In the same way, the value of the principal repayment is given at maturity as the par value linked to the inflation rate (Figure 6.9). Consequently, the principal is discounted at time 0. For instance, at higher node t_5, the pricing is given by (6.18):

$$P_{\mathrm{IL}} = \frac{(108.70 \times 0.5) + (105.10 \times 0.5)}{(1 + 0.02)} = 104.80 \qquad (6.18)$$

Note that when the binomial inflation rate tree is negative, therefore, with a deflation rate, the value of both coupons and principal decreases. For

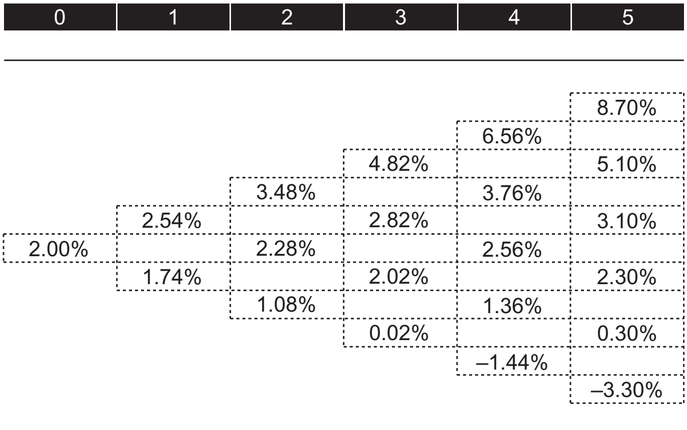

FIGURE 6.8 Binomial inflation rate tree with market expectations.

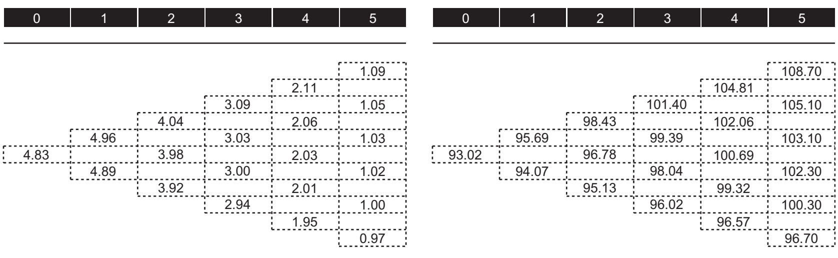

FIGURE 6.9 Binomial tree of coupon payments and principal repayment.

instance, in the last year and node, the value of the principal is lower than 100 (96.70).

Therefore, without considering the deflation floor, the value of an inflation-linked bond is equal to $4.83+93.02=97.85$.

The benefit of the deflation floor is assimilated to a European put option in which the payoff is given by Equation (6.19):

$$P_{\mathrm{T}}^{\text{put option}} = \max\left(0, M - M_{\mathrm{IL}}\right) \tag{6.19}$$

in which M is the strike price of the option or par value, in this case 100, and M_{IL} is the value of the principal at maturity T that follows the binomial inflation rate tree.

In other words, the option is valuable when the principal is lower than 100. When the inflation rate drops and the economy is in a hypothetical deflation scenario, the option is in the money. This increases the value of an inflation-linked bond. Note that the probability of a deflation scenario depends on the market sentiment and implied inflation rate between conventional and

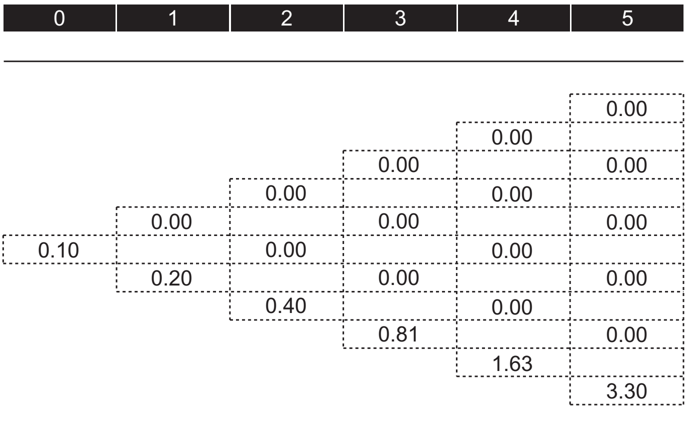

FIGURE 6.10 Binomial Tree of a European Put Option.

inflation-linked bonds.[11] For instance, at last node, the value of a European put option is given by Equation (6.20):

$$P_5^{\text{put option}} = \max(0, 100 - 96.70) = 3.30 \tag{6.20}$$

Then, the value of the option is discounted in each node with a discount rate assumed equal to 1% as follows:

$$P_{\text{t}}^{\text{put option}} = \frac{0.5 \times P_{\text{u}} + 0.5 \times P_{\text{d}}}{(1 + r)} \tag{6.21}$$

where it is the value of the option for a further period; P_u is the value of the option in the up state and P_d is the value of the option in the down state. Figure 6.10 shows the binomial tree of a put option.

Finally, the value of an inflation-linked bond is determined as the sum of the value of coupon payments, principal and put option at time 0. The value is given by Equation (6.22):

$$P_{\text{IL}} = C_{\text{IL}} + M_{\text{IL}} + P = 4.83 + 93.02 + 0.10 = 97.94 \tag{6.22}$$

6.4.3.3 Accrued Interest

In order to obtain the full or invoice price, we need to take into account the accrued interest. The accrued interest should be adjusted with the inflation for each period. The adjusted accrued interest is given by Equation (6.23):

$$\text{AI}_{\text{adj}} = C \times \frac{t}{T} \times I_{\text{R}} \tag{6.23}$$

11. See Section 6.3.

where AI_{adj} is the adjusted accrued interest; C is the coupon payment; t is the number of days accrued; T is the number of days in the coupon period and I_R is the index ratio.

Therefore, adding the accrued interest to the clean or flat price including the inflation adjustment we obtain the full or invoice price. The invoice price is given by Equation (6.24):

$$P_{\mathrm{i}} = (P_{\mathrm{f}} \times I_{\mathrm{R}}) + \mathrm{AI}_{\mathrm{adj}} \tag{6.24}$$

where P_i is the full or invoice price; P_f: is the flat or clean price and I_R is the index ratio.

6.4.3.4 Indexation Lag

As described above, inflation-linked bonds allow to save investors from changes in the general level of prices. However, the indexation is not perfect creating a lag between the index prices and the adjustment to the bond cash flows. According to Deacon and Derry (2004) at the end of a bond's life there is no inflation protection, matched with an equal period before the issue in which the inflation compensation is paid. Figure 6.11 shows an example of indexation lag according to Deacon and Derry (2004).

There are several ways in which the indexation lag could be reduced:

- First of all, choosing the index with the most frequent updating;
- Secondly, increasing the frequency of coupon payments;
- Finally, adopting the method proposed by Barro (1994), which the inflation for the previous coupon is assumed as a proxy of the actual inflation.

For the last point, we consider the following example. We assume to have an indexation lag of 1 year. Without indexation lag, at first payment date, the coupon should be adjusted with the inflation rate τ_1 between the issue date and first coupon date. The adjustment is given by Equation (6.25):

$$C_{\mathrm{IL}} = C \times (1 + \tau_1) \tag{6.25}$$

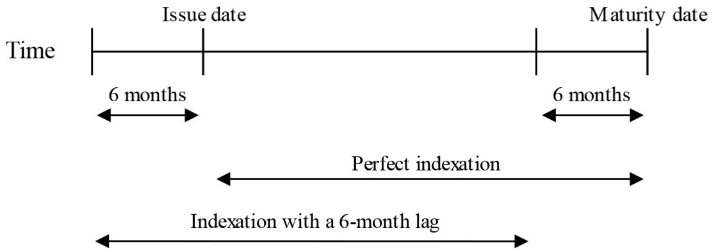

FIGURE 6.11 Indexation lag. *(Reproduced from Deacon and Derry (2004).)*

where C_{IL} is the inflation-adjusted coupon payment; C is the coupon payment and τ_1 is the inflation between issue date and first coupon payment.

Conversely, under Barro's method the inflation used for the first coupon is 1 year before the issue date. The Barro's adjustment is given by Equation (6.26):

$$C_{\mathrm{IL}} = C \times (1 + \tau_0) \tag{6.26}$$

where τ_0 is the inflation between 1 year before issue date and issue date.

After the first coupon payment, Barro attributes a double weight on the periodical inflation rate before the coupon payment. For instance, in the case of perfect indexation, the second coupon payment should be adjusted in the following way:

$$C_{\mathrm{IL}} = C \times (1 + \tau_1) \times (1 + \tau_2) \tag{6.27}$$

while under Barro's approach we give a double weight to the inflation τ_1 between the issue date and first coupon date as follows:

$$C_{\mathrm{IL}} = C \times (1 + \tau_1) \times (1 + \tau_1) \tag{6.28}$$

6.5 WEB SITE MODELS

The Web site associated with this book contains an Excel spreadsheet demonstrating the valuation of inflation-linked bonds, as described in this chapter. The reader may use the spreadsheet to value such bonds using his or her own parameter inputs.

Details of how to access the Web site are contained in the preface.

APPENDIX A INFLATION-LINKED BOND PRICING

Considering the example shown in Table A1 of a hypothetical bond with coupons and principal linked to the inflation. We assume a 5-year inflation-linked bond with a 2% annual coupon payment. The expected cash flows, coupons and principal, are discounted with a discount rate of 3%. The valuation is performed by the following steps.

Determining the Expected Inflation Rate

The inflation rate is determined by the inflation index expectations. For instance, at first coupon date, the inflation rate is 2.1% and is given by:

$$\tau_1 = \frac{I_1}{I_0} - 1 = \frac{102.3}{100.2} - 1 = 2.1\%$$

TABLE A1 The Valuation of An Inflation-Linked Bond

	0	1	2	3	4	5
Inflation Index	100.2	102.3	104.1	106.2	108.3	110.4
Coupon payment		2	2	2	2	2
Expected inflation		2.1%	1.8%	2.0%	2.0%	1.9%
Compounded expected inflation		1.02	1.04	1.06	1.08	1.10
Inflation adjusted Coupon payment		2.04	2.08	2.12	2.16	2.20
Indexed Principal repayment						110.18
Cash Flow		2.04	2.08	2.12	2.16	112.38
Discounted Cash flow		1.98	1.96	1.94	1.92	96.94
Price of an Inflation-Linked Bond	104.74					

Determining the Coupon Payment

The coupon payments are determined as percentage on the par value and they are adjusted to the expected inflation rate. For instance, at first payment date, the coupon is calculated as follows:

$$C_1 = C \times (1 + \tau_1) = 2 \times (1 + 0.021) = 2.04$$

The present value of the coupon at first payment date is equal to 1.98.

The second coupon payment is calculated as the compounded inflation rate at time t_2 as follows:

$$C_2 = C \times (1 + \tau_1) \times (1 + \tau_2) = 2 \times (1 + 0.021) \times (1 + 0.018) = 2.08$$

or

$$C_2 = 2 \times \frac{104.1}{100.2} = 2.08$$

The present value of the second coupon payment is 1.96.

Determining the Principal Repayment

As made for coupon payments, also the redemption value is adjusted for inflation. The nominal redemption value is equal to 110.18 and is given by:

$$M_{\mathrm{IL}} = M \times \frac{I_5}{I_0} = 100 \times \frac{110.4}{100.2} = 110.18$$

Determining the Value of an Inflation-Linked Bond

Finally, the value of an inflation-linked bond is calculated as the sum of the present value of coupons and principal, assuming a nominal discount rate of 3%. The bond value obtained is 104.74.

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