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Applying Scenario Optimization to Portfolio Credit Risk

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In addition to measuring and monitoring risk, it is important for risk managers to understand the sources of portfolio risk, and how portfolios can be restructured effectively to reduce risk and maximize returns. However, while much academic and industrial effort has been devoted to developing methodologies to measure portfolio credit risk, the development of tools to manage and, more specifically, to optimize credit risk have lagged behind.

Risk management tools for market risk are largely based on modern portfolio theory (Markowitz [1952]; Sharpe [1964]) and assume that the profit and loss or the return distributions are normal. In a normal world, the standard deviation of the portfolio returns is a good measure of risk, and optimal portfolios are those that generate the best mean-variance profiles. However, distributions of credit losses are generally far from normal; they are heavily skewed with a long, fat tail. Most of the time, an obligor neither defaults nor changes credit rating, but when default occurs losses are generally substantial. While standard risk management and optimization tools originally developed for market risk (for example, Litterman [1996a] and [1996b]) cannot be effective in this setting, scenario-based tools are ideal for these problems (see Dembo [1991, 1999]; Dembo and Rosen [1998]; Mausser and Rosen [1998]).

This article develops scenario optimization tools that can restructure portfolios effectively to reduce credit risk and optimize the

risk/return trade-off. The inputs to these models are the mark-to-future tables of portfolio credit risk simulations.

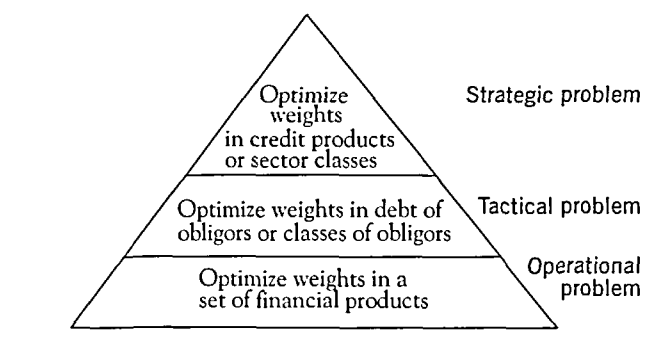
We present three optimization models. The first one minimizes portfolio risk by modifying the position of a single asset or obligor. This is achieved by constructing the trade risk profile (TRP) of a single position (or basket) as shown in Mausser and Rosen [1998]; the best hedge is the position that gives the minimum risk on this profile. The second optimization model minimizes the credit risk of a portfolio by simultaneously adjusting multiple positions subject to general (linear) restrictions. Finally, a credit risk-return efficient frontier is constructed using parametric linear programming. We further demonstrate the application of these models by extending the portfolio credit risk case study by Bucay and Rosen [1999], in which the CreditMetrics methodology is used to calculate the credit risk of a portfolio of long-dated corporate and sovereign bonds issued in emerging markets.

Credit risk scenario optimization is difficult due to the sheer size of the problems; because credit events are relatively rare, it is generally necessary to sample a large number of scenarios on joint obligor credit migration and default in order to obtain accurate estimates of the loss distribution. For practical purposes, therefore, we limit our attention to linear programming models and demonstrate their utility.

It is important to note that there are very few articles in the risk management

EXHIBIT 1

A Hierarchy of Credit Risk Optimization Problems



As in the CreditMetrics framework, we consider a one-step model. Credit losses are measured as the losses at a fixed time horizon, due exclusively to credit events, which include both default and credit migration of a set of obligors in a portfolio. It is straightforward to use the optimization models presented here in more general settings that include multi-period and joint market and credit events.

The credit risk optimization problem, namely the rebalancing of a portfolio to achieve a better risk profile, can be formulated at various levels. At a strategic level, the risk manager may wish to restructure the concentrations in various credit products or sector classes. In this case, the optimization model solves for the weights in each class. At a second, or tactical level, the manager might be interested in risk capital allocations for each obligor or type of obligor. The portfolio weights associated with the obligors are then the unknowns of the corresponding model. Finally, at an operational level, the manager may choose which positions to take in a set of financial instruments in order to hedge portfolio risk optimally or obtain a better risk/return trade-off. In this case the unknowns are the size of the positions to take in those tradeable instruments. These alternate formulations are summarized in Exhibit 1.

For ease of exposition, we formulate the problem at the second level, in terms of obligor weights, but the models presented apply to all levels. Suppose that the portfolio is simulated over a large set of scenarios (i.e., 10,000 to 100,000). Each scenario describes the joint credit states of all obligors at the specified time horizon. The result of this simulation is a mark-to-future table containing the scenario-dependent values of the holdings for each obligor. The optimization problem, then, is to choose the obligor weights that minimize the risk or maximize some trade-off between risk and return over the given scenarios, subject to several constraints on the weights. Models will differ with respect to the risk measure used in the objective function, and the limitations imposed by the constraints.

In general, the obligor weights can be expressed in monetary terms, as changes in the current positions or as a fraction of the portfolio value or exposure. In this article, the existing (i.e., non-optimized) portfolio is assumed to consist of one unit of each obligor. The obligor weights then represent multiples of the original positions. For example, a weight of zero implies liquidating a position, while a weight of two denotes doubling the position size.

In the following, we present three portfolio credit risk optimization models in progressive order of sophistication.

literature that deal with optimizing credit risk. One reason for this is the fact that the advances in credit risk measurement methodologies are quite recent. More importantly, the problem of minimizing credit risk is less tractable than that of minimizing market risk. Kealhofer [1995, 1998] applies standard mean-variance optimization to portfolio credit risk. More recently, Arvanitis, Browne, Gregory, and Martin [1998] briefly discuss applying simulation-based tools to estimate the efficient frontier in the space of expected returns and unexpected percentile losses (CreditVaR). They solve the optimization problem by a brute force random search method, and only briefly mention the possibility of using more advanced numerical tools. The authors also demonstrate in a simple example how the mean-variance efficient portfolio might be far from the efficient frontier, which is expected given the non-normality of the problem.

This article is organized as follows. First, we formally introduce the credit risk optimization problem and then describe the basic ideas behind the optimization tools; the full mathematical formulations are given in the appendix. The utility of these models is illustrated by applying them to a portfolio comprising long-dated corporate and sovereign bonds issued in emerging markets. Following a discussion of the case study, we present our conclusions and offer suggestions for further study.

FORMULATING PORTFOLIO CREDIT RISK OPTIMIZATION PROBLEMS

In this section we present the basic principles behind the credit risk scenario optimization models. The mathematical details of the first model can be found in Mausser and Rosen [1998] and the linear programming formulations of the second and third models are given in the appendix.

Optimizing the Position of a Single Obligor

As a first step to rebalancing a portfolio, we consider trading the debt of only a single obligor in order to minimize a selected risk measure. This is achieved by constructing the simulation-based, or non-parametric, trade risk profile (nTRP) for the obligor and finding the best hedge position, as described in Mausser and Rosen [1998]. The nTRP plots the portfolio risk as a function of the size of the exposure in the selected obligor. The best hedge is simply the position, which yields the minimum value on the nTRP.

From the mark-to-future table, it is possible to construct a nTRP for any desired risk measure. Quantile-based measures, such as the maximum losses at the 99th percentile level, for example, result in a piecewise-linear nTRP. Fitting a smooth approximation to the nTRP may provide more robust estimates of the relevant risk analytics, such as the best hedge position, in some cases.

Optimizing the Positions of Multiple Obligors

A more general model simultaneously adjusts the position in each obligor to minimize the risk of the portfolio subject to a set of constraints, such as a fixed budget or limits on the obligor weights.

The objective is typically to reshape the loss distribution in order to balance the expected credit losses, which define the credit reserves, and the unexpected losses, which determine the amount of capital required to support the portfolio. Unexpected losses are commonly measured by CreditVaR, the difference between the maximum percentile losses and the expected losses.

In practice, the definition of the risk measures is the key to these optimization models. These measures must be chosen to meet two criteria: relevance and tractability. Relevant measures are directly linked to the management process and capture the main properties of the distribution. Expected losses, maximum percentile losses, CreditVaR and average shortfall are some examples of relevant measures. Note that when distributions are not normal, minimizing variance does not guarantee an effective minimization of the capital required to cover unexpected credit losses. In such circumstances, risk measures such as standard deviation or variance are not relevant.

Tractable measures can be optimized using methods that are computationally efficient. Though relevant, quantile measures such as maximum percentile losses, CreditVaR and average shortfall, lead to optimization problems

that are conceptually simple, but very difficult to solve in practice. Simply stated, calculating any of these risk measures requires identifying the k -th largest loss, over all scenarios, for some k corresponding to the selected quantile. For example, in a set of 1,000 (equally likely) scenarios, the maximum losses (99%) correspond to $k = 10$. However, in an optimization framework, identifying the k -th largest loss requires the use of a discrete, or integer variable (i.e., one that can assume only the value zero or one) for each scenario. Given the number of scenarios involved in a credit risk simulation, the optimization problem can potentially contain tens of thousands of integer variables. Methods for solving problems that include integer variables, collectively known as mixed integer programming, are far more computationally demanding than linear programming, which deals only with continuous variables.

Fortunately, risk measures exist that are both relevant and tractable. Optimizing such measures effectively reshapes the loss distribution and therefore, also provides the means to improve quantile-based measures, albeit in a somewhat indirect manner. We now present two risk minimization models based on a measure that is not only relevant, but that can be optimized using linear, rather than integer, programming.

Minimizing Expected Regret

Regret (Dembo [1991, 1999]), which measures the difference between a scenario outcome and a benchmark, exhibits both tractability and relevance. We define the expected regret as the expectation of losses that exceed some fixed threshold K . While this definition is similar to that of expected shortfall, the latter involves conditional probabilities and a quantile-based, rather than a prespecified, threshold. It is important to note that, in general, the benchmark need not be a fixed amount; it can be a scenario-dependent value, such as an index.

Regret is relevant since it is a general measure that can effectively capture both the mean and the tail of the distribution, without the need for any distributional assumptions. Furthermore, it leads to linear programming optimization models, which can be solved very quickly. Specifying an appropriate threshold in advance eliminates the need to solve an integer program, since it is not necessary to identify the particular scenario that corresponds to a given quantile.

One cannot use regret to minimize quantile-based measures directly, since there is no guarantee that the specified threshold will in fact correspond to the desired

EXHIBIT 2

Statistics for One-Year Loss Distribution in Millions USD

Expected losses	95
Standard deviation	232
Maximum losses (99%)	1,026
CreditVaR (99%)	931
Expected shortfall (99%)	1,320
Maximum losses (99.9%)	1,782
CreditVaR (99.9%)	1,687
Expected shortfall (99.9%)	1,998

quantile of the optimized loss distribution. Nevertheless, it is possible to select a reasonable threshold based on the risk characteristics of an existing portfolio. For example, by setting the threshold K to one million USD, the optimization problem finds the weights that yield the smallest expected losses in excess of this amount. Thus K acts as a parameter of the model that can be chosen by the risk manager and be used to shape the distribution. A small (or even negative) K leads to a reduction in the expected losses (reserves), perhaps at the expense of a longer distribution tail (capital). On the other hand, a large value of K reduces the unexpected losses in the tail (capital), at the expense of higher expected losses (reserves). Thus, the manager has the flexibility to create a portfolio that best fits the liquidity and capital structure of the firm at a given time.

Minimizing Maximum Regret

Another relevant, solvable measure is MaxRegret, the largest loss in excess of a threshold, K . This is a more conservative measure than expected regret that also leads to linear programming problems. Again, K is a parameter of the optimization model, but the optimal solution is independent of values of K that are less than the smallest possible maximum loss.

Optimizing Risk-Return and the Efficient Frontier

Investors generally take credit positions to obtain higher yields that compensate them for their extra risk. The achievement of any risk reduction from a risk minimization model likely comes at the expense of returns. Since the seminal work by Markowitz [1952], tools that perform mean-variance optimization and compute efficient frontiers have been well known to investment professionals.

Portfolios on the frontier are called efficient since they provide the best returns for the level of risk they pose, or conversely, they pose the least risk for their level of returns.

Mean-variance optimization tools have also been applied recently to credit risk (Kealhofer [1995, 1998]). Although analytically and computationally tractable, it is quite clear that these tools are generally ineffective in credit risk applications, given the pronounced non-normality of the distributions. An example of this is given in Arvanitis, Browne, Gregory, and Martin [1998].

Scenario risk minimization models can be naturally extended to construct risk-return efficient frontiers or to maximize risk-return utility functions (Dembo [1999]; Dembo and Rosen [1998]). The solution to the regret minimization problem represents the point on the efficient frontier having the lowest risk (and the lowest return). The entire efficient frontier is defined by the solutions of a linear parametric program that minimizes regret as a function of the expected excess returns over the risk-free rate or any other benchmark (equivalently, the problem can be formulated as a maximization of the expected excess returns subject to the portfolio regret not exceeding a given level).

CASE STUDY

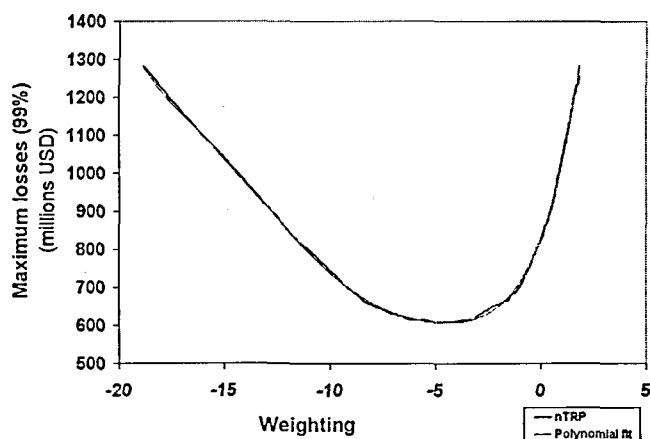
Bucay and Rosen [1999] apply the CreditMetrics methodology to calculate the credit risk of a portfolio of long-dated corporate and sovereign bonds issued in emerging markets. Credit events include both default and credit migration. We apply the optimization models introduced in the previous section to this emerging market bond portfolio.

The date of the analysis is October 13, 1998 and the time frame for estimating credit risk is one year. The portfolio consists of 197 emerging markets bonds, issued by 86 obligors in 29 countries. The mark-to-market value of the portfolio is 8.8 billion USD. Most instruments are denominated in USD, except for 11 fixed-rate bonds, which are denominated in seven other currencies. Bond maturities range from a few months to 98 years with a portfolio duration of approximately five years.

Credit migration probabilities are obtained from Standard & Poor's transition matrix as of July 1998 (Standard & Poor's [1998]). Recovery rates, in the event of default, are assumed to be constant and equal to 30% of the risk-free value for all obligors except two, which have lower rates. Asset correlations are driven by a multi-factor model through a set of country and industry indexes chosen from the CreditMetrics dataset, and through a specific volatility

EXHIBIT 3

Trade Risk Profiles for Brazil



component. The mappings of obligors to the indexes and the assumptions are described in Bucay and Rosen [1999]. The portfolio's loss distribution is created from a Monte Carlo simulation of 20,000 scenarios on joint credit states.

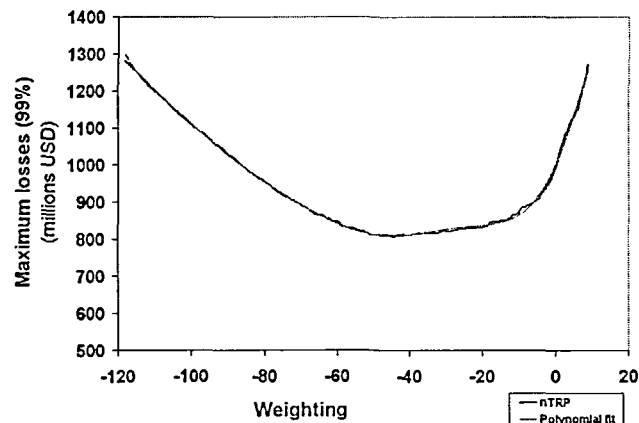
Exhibit 2 summarizes the relevant statistics from the one-year credit loss distribution of the portfolio. In addition to the expected losses and standard deviation, we also report maximum percentile losses, CreditVaR and expected shortfall, at the 99% and 99.9% percentiles.

Best Hedges

As a first step, let us focus on the risk reductions that can be achieved by modifying the position of a single obligor. Exhibit 3 shows the nTRP and its polynomial approximation for Brazil, currently the largest contributor to risk. The horizontal axis gives the weight (multiple of the current holdings) in Brazilian debt, while the vertical axis gives the corresponding maximum losses (99%) of the portfolio (all other positions remain unchanged). The nTRP indicates that the maximum losses (99%) can be reduced to 607 million USD when Brazil is given a weight of -5.02 (i.e., a short position of five times the current holdings). For comparison, the polynomial approximation suggests that the maximum losses (99%) can be reduced to 606 million USD when Brazil is weighted -4.56 . Similarly, Exhibit 4 shows the nTRP and a polynomial approximation for Colombia. In this case, a weight of -44.49 reduces the maximum losses (99%) to 807 million USD, as determined by the nTRP (the respective values for the smooth approximation are -45.07 and 809 million USD). Note that the large num-

EXHIBIT 4

Trade Risk Profiles for Colombia



ber of scenarios used in the simulation results in a good fit between the nTRP and the smooth approximation.

Exhibit 5 presents a "Hot Spots™ and Best Hedges" report of the 10 obligors making the largest contributions to portfolio risk. It shows first the percent contribution of each obligor to mean losses and maximum losses (99%). In addition, it presents for each obligor the best hedge position, the maximum losses (99%) at that position and the corresponding percent reductions in the maximum losses (99%) as given by the nTRP (the results are virtually the same for the polynomial approximation). Note from Exhibit 5 the substantial additional risk reductions that can be achieved by taking short positions in these obligors. For example, while roughly 12% of the risk can be eliminated if the positions on Venezuelan debt are closed, the risk can be reduced by 34% by entering into a short position of about three times its current holdings.

It is important to understand the limitations of this best hedge analysis. First, the trade risk profile of a single obligor assumes that the remainder of the portfolio remains fixed; hence, multiple positions cannot be traded concurrently. Second, no consideration is given to factors that may limit the size of the trade, or of the best hedge position itself. For example, the best hedge position in Brazilian government bonds is obtained by taking a leveraged short position. In practice, it may not be feasible to enter directly into such a position. However, the hedge may be achieved by entering into a credit derivative contract, such as a total return swap. The best hedge position suggests the size of the optimal contract. Similarly, simulation can be used to construct the trade risk profile of the credit derivative contract.

EXHIBIT 5

"Hot Spots and Best Hedges" Report

Obligor	Percentage Contribution		Best Hedge		
	Expected Losses (%)	Maximum Losses (99%) (%)	Position Size	Maximum Losses (99%) (millions USD)	Percentage Reduction in Maximum Losses (99%) (%)
Brazil	14.55	20.27	-5.02	607	41
Russia	9.81	14.31	-8.71	666	35
Venezuela	6.16	12.25	-3.32	675	34
Argentina	9.87	10.47	-7.81	737	28
Peru	10.33	8.30	-6.65	739	28
Colombia	2.30	3.26	-44.49	807	21
Russia CCC	1.29	1.80	-18.29	751	27
Mexico	9.20	1.69	-2.83	994	3
Morocco	1.58	0.82	-77.92	784	24
Philippines	6.67	0.26	-4.67	1,008	2

Minimizing Expected Regret

We turn our attention now to the linear programming formulations of the risk minimization problem. First, we minimize expected regret for a fixed threshold K , then we examine the impact of the choice of the threshold on the optimal regret. Finally, we investigate the impact of the alternate, MaxRegret, objective function. Minimizing regret has the effect of reducing the tail of the distribution and thus it also affects the quantile-based risk measures.

From Exhibit 2, the original portfolio has Maximum losses (99%) of 1,026 million USD and an expected shortfall (99%) of 1,320 million USD. Thus, when setting the parameters of the optimization, it is reasonable for a risk manager to consider thresholds below these values. Let us first fix the threshold at $K = 750$ million USD, so that losses less than this amount are effectively ignored and the

optimization focuses only on the tail of the loss distribution beyond 750 million USD.

We impose the following constraints. First, the rebalanced portfolio maintains the same expected future value, in the absence of any credit migration, as the original portfolio; this is a simple normalization constraint. Second, to avoid unrealistically large long or short positions in any one counterparty, the size of each position is bounded. The bounds are expressed as a multiple of the current long position of one unit, for each obligor. Two cases are considered:

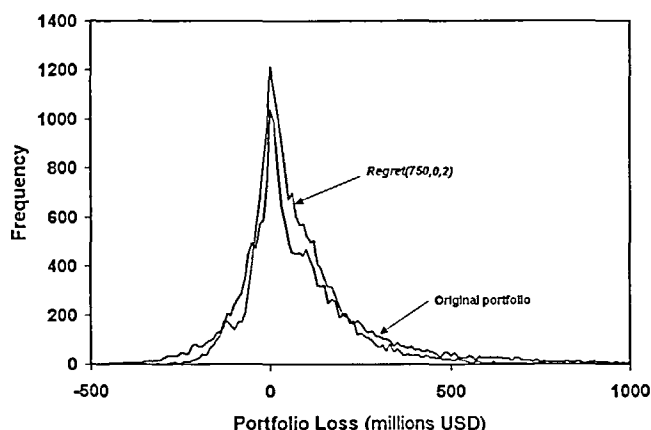
- No short positions, and the existing long positions can be, at most, doubled in size;
- Positions, whether short or long, can be, at most, double the current size.

EXHIBIT 6

Comparison of Portfolio Risk Measures in Millions USD (% reduction)

Case	Expected Losses	Standard Deviation	Maximum Losses (99%)	Expected Shortfall (99%)	Maximum Losses (99.9%)	Expected Shortfall (99.9%)
Original	95	232	1,026	1,320	1,782	1,998
Regret(750,0,2)	65 (32)	128 (45)	511 (50)	604 (54)	750 (58)	772 (61)
Regret(750, -2,2)	63 (35)	128 (45)	505 (51)	601 (54)	738 (59)	759 (62)

EXHIBIT 7 Loss Distributions



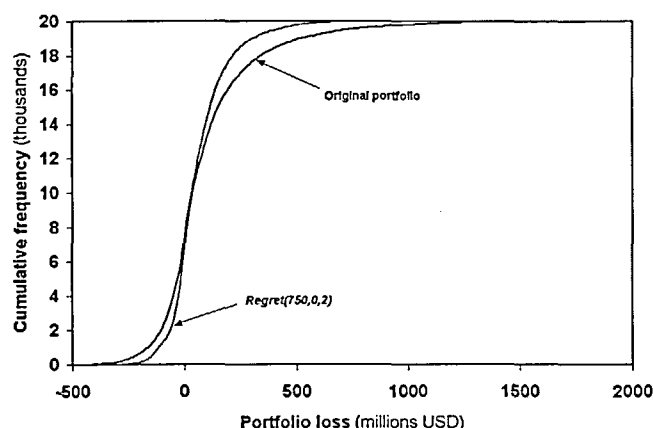
The optimal solution to the problem posed in the first case is identified as $\text{Regret}(750,0,2)$ and as $\text{Regret}(750,-2,2)$ in the second case. Exhibit 6 compares the original and optimized portfolios with respect to various risk measures. It is evident that the optimized portfolios significantly improve all of the risk measures. However, relaxing the trading limits to allow for short positions results only in a slightly better performance overall.

The largest reduction in maximum losses (99%) achievable by trading in the debt of a single obligor is 41% (Exhibit 5). By trading simultaneously in the debt of various obligors, we can achieve a reduction of 50% in this measure, of 30%–35% in expected losses, and more generally, 40%–60% in standard deviation and all quantile-based risk measures.

To understand the effect of the optimization model better, it is instructive to look at Exhibits 7 and 8, which compare the frequency and cumulative distributions of credit losses, respectively, for the original portfolio and for the optimized portfolio with no short positions. The optimization effectively reduces the tails, resulting in a more peaked distribution. The shortening of the right tail indicates reductions of the loss measures; this is achieved through shortening the left tail, that is, at the expense of some potential gains.

As a further comparison, Exhibits 9 and 10 plot the marginal risk (marginal standard deviation as a percent of mean exposure) versus mean exposure for the original and optimized portfolios when no short positions are allowed. These plots illustrate the differences in obligor weights between the two portfolios. For example, while the original portfolio has the largest positions in Brazil, Russia, and Argentina, the optimal portfolio reduces these holdings and roughly doubles the original positions in Poland, the

EXHIBIT 8 Cumulative Loss Distributions



Philippines, and Israel, which become the largest positions in the optimal portfolio.

Some intuitively appealing properties of the optimal portfolio are apparent from these figures. First, the optimal portfolio reduces substantially the marginal risk contributions. For example, the maximum risk contribution is reduced from 12% (Russia CCC) to about 8% (Vietnam); the rest of the contributions remain below 4%. Second, while the largest positions in the original portfolio (Brazil, Russia, and Argentina) have marginal risks of about 4%, those in the optimal portfolio are much lower (Philippines about 2%, Poland and Israel less than 1%). Finally, the optimal portfolio eliminates outliers on the plot (i.e., obligors with a large position and large marginal risk). In the original portfolio we observe three groups of outliers: Russia CCC with a marginal risk of 12% and an exposure of 50 million USD; Peru and Venezuela with marginal risk of about 8% and exposures of between 300 and 400 million USD; and Argentina, Russia, and Brazil with marginal risk of about 4% and exposures of between 600 and 900 million USD. In contrast, the main outlier, if any, from the optimal portfolio is Bulgaria with a marginal risk of about 4% and an exposure of 300 million USD.

Next, we examine the effect of the threshold K on the optimization results. The trading limits prevent short positions and allow the current long positions for each obligor to be at most doubled. The optimal solutions to the problem described here are identified as $\text{Regret}(K,0,2)$, where $K = 0, 250, 500$ and $1,000$ (millions USD).

Exhibit 11 presents the statistics of the various optimal portfolios and compares them to those of the original portfolio. It is interesting to note that a significant improvement in all risk measures is obtained even with $K = 1,000$ million USD. Different thresholds generally yield more favorable

EXHIBIT 9

Marginal Risk Versus Exposure for Original Portfolio

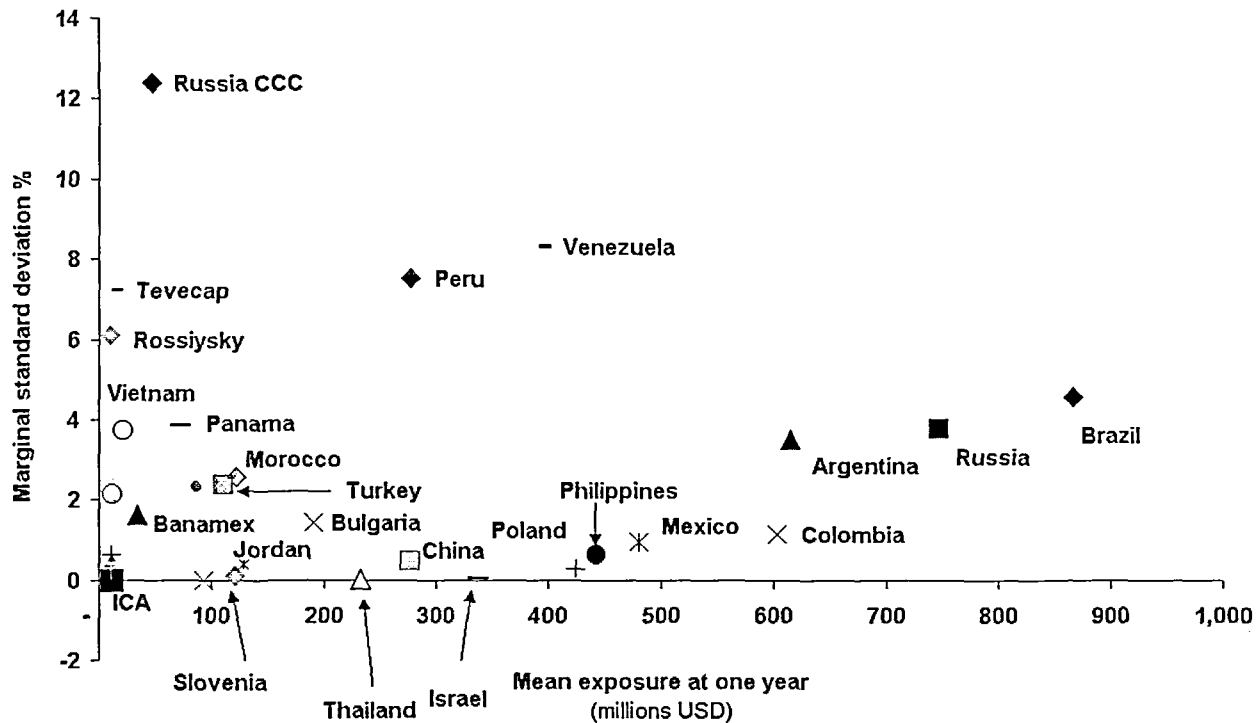


EXHIBIT 10

Marginal Risk Versus Exposure for Regret (750,0,2)

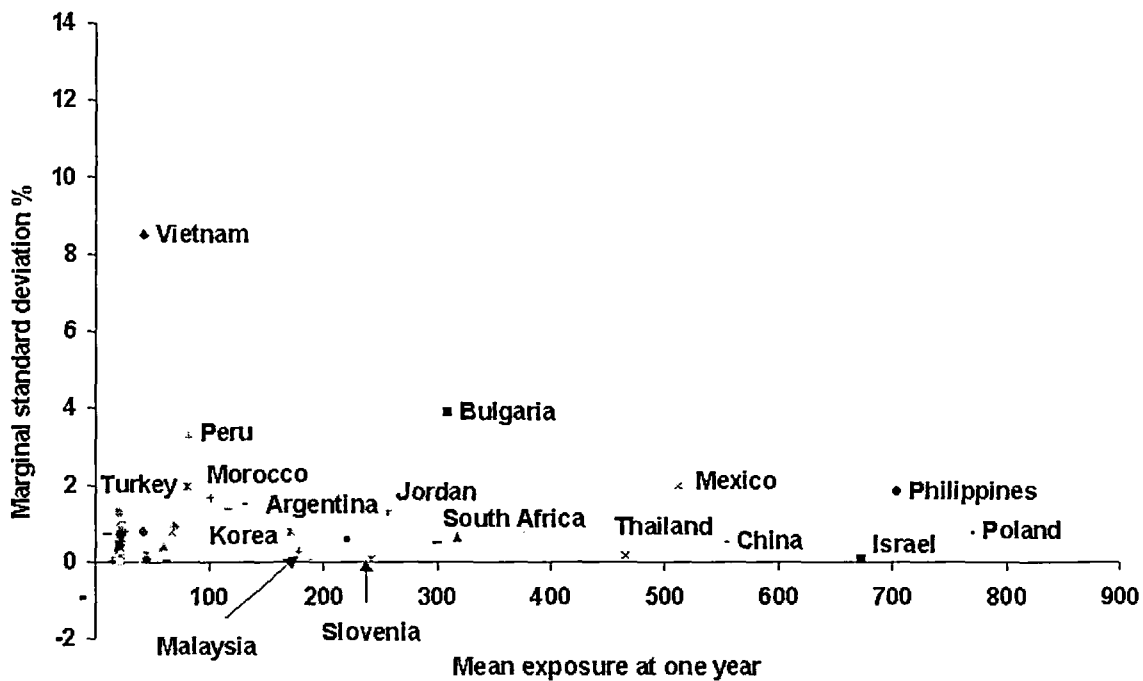


EXHIBIT 11

Comparison of Portfolio Risk Measures in Millions USD (% reduction)

Case (99.9%)	Expected Losses	Standard Deviation	Maximum losses (99%)	Expected Shortfall (99%)	Maximum Losses (99.9%)	Expected Shortfall (99.9%)
Original	95	232	1,026	1,320	1,782	1,998
Regret(0,0,2)	47	114	495	727	1,074	1,370
	(51)	(51)	(52)	(45)	(40)	(31)
Regret(250,0,2)	52	109	408	598	999	1,152
	(45)	(53)	(60)	(55)	(44)	(42)
Regret(500,0,2)	60	121	461	561	696	791
	(37)	(48)	(55)	(57)	(61)	(60)
Regret(750,0,2)	65	128	511	604	750	772
	(32)	(45)	(50)	(54)	(58)	(61)
Regret(1000,0,2)	70	142	650	735	876	931
	(26)	(39)	(37)	(44)	(51)	(53)

results for different risk measures. For instance, expected losses and standard deviation benefit from a low K , the 99% level measures are lower for $K = 250$ million and 500 million USD, while the 99.9% level measures benefit from a higher K in the range 500 to 750 million USD.

To explain these patterns further, we graph the loss distributions of four of these portfolios in Exhibit 12. This demonstrates how the various choices can be used to "shape" the loss distribution to effect changes in various risk measures such as expected losses, maximum percentile losses and expected shortfall. While the objective functions do not explicitly optimize on the quantile-based risk measures, the ability to solve the resulting linear programs quickly allows a risk manager to perform several analyses in order to obtain the desired distribution characteristics.

For example, the time required to solve the problem

on a Sun Ultra 1 workstation, using the CPLEX LP solver (CPLEX [1995]), ranges from three seconds ($K = 1,000$ million) to almost 40 minutes ($K = 0$). This pattern is expected, since at lower values of K , more scenario losses exceed the threshold, and thus contribute to regret. This increases the computational effort required to solve the problem. Note that significant reductions in computation time can be achieved by using specialized techniques to solve multiple instances of the problem under various thresholds.

Minimizing Maximum Regret

Finally, we examine the effect that the objective function has on the loss distribution by solving the MaxRegret optimization problem. We specify $K = 0$ to minimize the maximum loss over all scenarios. Again, the trading limits prevent short positions and allow the current long positions for each obligor to be at most doubled.

Exhibit 13 presents the statistics of the MaxRegret optimal portfolio and compares them to those of the original portfolio. Again, there is a significant improvement in all risk measures. Comparing the results of Exhibits 11 and 13, we note that the loss measures associated with MaxRegret lie between those of Regret(750,0,2) and Regret(1000,0,2) for each measure other than expected losses.

Risk-Return Analysis

The previous examples focus exclusively on credit risk reductions, without considering the expected portfolio returns. We now compute the risk-return efficient frontier with the following specifications:

EXHIBIT 12

Loss Distributions for Different Thresholds

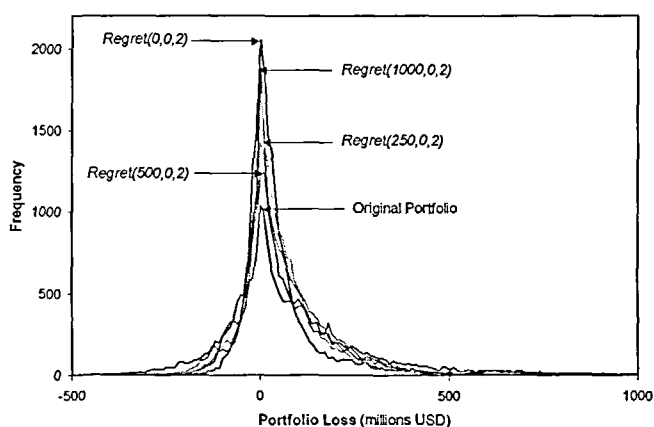


EXHIBIT 13

Comparison of Portfolio Risk Measures in Millions USD (% reduction)

Case (99.9%)	Maximum Expected Losses	Expected Standard Deviation	Maximum losses (99%)	Expected Shortfall (99%)	Losses (99.9%)	Shortfall (99.9%)
Original	95	232	1,026	1,320	1,782	1,998
MaxRegret(0, 0, 2)	63 (34)	129 (44)	527 (49)	619 (53)	757 (58)	790 (40)

- The risk measure used is regret with a threshold $K = 250$ million USD;
- The current mark-to-market value of the portfolio must be maintained;
- No short positions are allowed;
- The long position in the debt of an individual counterparty cannot exceed 20% of the (current) portfolio value.

Note that the trading limits are not as restrictive as those imposed in the previous minimization models.

For simplicity, the expected returns for each obligor are given by the one-year forward returns of their holdings, assuming they do not change rating. The original portfolio has an expected one-year return of 7.26%, which exceeds the one-year risk-free rate (5.86%) by 1.40%.

Exhibit 14 shows the efficient frontier and the relative position of the original portfolio. The portfolio with the minimum regret (93,283) attains a return of 6.84%.

Note that the original portfolio is clearly inefficient in this case: Exhibit 14 compares the original portfolio with its "radial projections," portfolio A and portfolio B, on the efficient frontier. At this level of return, the

minimum regret incurred is substantially smaller than the regret in the current portfolio, though portfolio A is not risk free. Alternatively, at this level of regret, the maximum return is significantly higher (portfolio B). The various measures of portfolio risk for the portfolios indicated in Exhibit 14 are summarized in Exhibit 15.

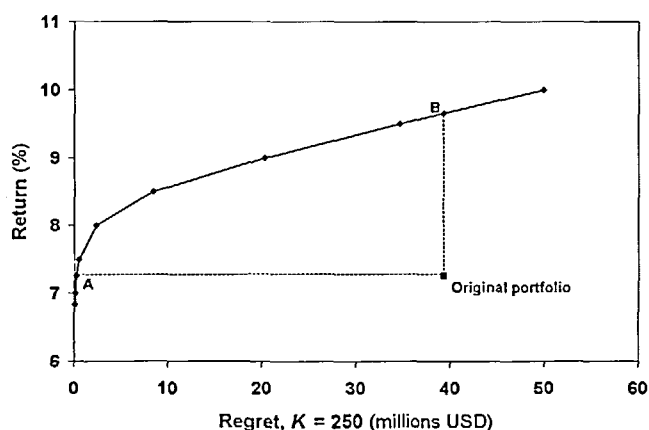
Portfolio A achieves the same level of returns based on about one-fifth of the total capital. The quantile-based risk measures are reduced significantly, by about 80%, and expected losses by over 90%. Note that these reductions are considerably larger than the 40%-60% reductions achieved with regret minimization (Exhibit 11) as a result of the relaxed trading limits. Exhibit 16 shows that the optimization not only reduces the right tail (extreme losses) of the distribution, but also offers greater potential gains (left tail) in this case.

On the other hand, portfolio B achieves an extra 240 basis points of return for the same regret. Furthermore, there are corresponding reductions in the other measures of portfolio risk. The expected losses are negative, suggesting expected gains from credit events and the unexpected losses are reduced by approximately one third.

Note that once again standard deviation is a deceiving measure of risk, suggesting that the risk more than doubles for portfolio B, when in fact risk reductions are achieved.

EXHIBIT 14

Efficient Frontier



CONCLUDING REMARKS

Currently, the practice of managing, and particularly optimizing, credit risk presents a major challenge for risk managers. Techniques developed exclusively for market risk, based on assumptions of normality, are ineffective in this case. However, scenario-based techniques, such as constructing trade risk profiles, finding best hedges and scenario optimization, extend naturally to credit risk. Scenario optimization of credit risk is complicated by the fact that, given the large number of scenarios required to model credit events, many relevant risk measures are not tractable. This article demonstrates that regret is an

EXHIBIT 15

Performance Comparison of Original and Efficient Portfolios in Millions USD (% reduction)

Portfolio	Return (%)	Regret (K = 250)	Expected Losses	Standard Deviation	Maximum Losses (99%)	Expected Shortfall (99%)	Maximum Losses (99.9%)	Expected Shortfall (99.9%)
Original	7.26	39.3	95	232	1,026	1,320	1,782	1,998
A	7.26	0.2	8 (92)	137 (41)	234 (77)	270 (80)	326 (82)	365 (82)
B	9.66	39.3	-73 (-)	593 (-156)	692 (33)	868 (34)	1,174 (34)	1,330 (33)

attractive risk measure, exhibiting both relevance and tractability. The fact that optimizing regret involves only linear, rather than integer, programming allows regret-based models to be used to reshape the loss distributions of portfolios that are exposed to credit risk. In doing so, risk managers can also obtain substantial improvements in intractable (i.e., quantile-based) measures such as maximum percentile losses and expected shortfall.

We have considered models for minimizing risk and optimally trading off risk and return. These results can be extended in several ways. For instance, it is straightforward to use multi-attribute optimization to implement a risk measure that is the weighted combination of expected regret and MaxRegret. Also, a more thorough investigation of the effects of the threshold K on reshaping the loss distribution may provide guidance for risk managers who need to balance the allocation of reserves and capital. While we have observed improvements in quantile-based risk measures as a result of optimizing regret, the development of heuristic or other specialized

methods for explicitly optimizing such measures remains an interesting possibility. Using the solution found by the regret model as an initial starting point for such techniques may prove beneficial. Finally, a more sophisticated model that incorporates both market and credit factors over multiple horizons, while more demanding in terms of both scenario generation and optimization, will significantly enhance an organization's ability to manage risk on an enterprisewide basis.

APPENDIX

We present the formulation of the expected regret and MaxRegret minimization models, and their extension to a risk-return model. The formulations are based on the notation summarized in Exhibit 1A. Suppose the problem involves n counterparties, indexed by i , and m scenarios, indexed by j .

Let V denote the $(n \times m)$ matrix of losses due to credit migration, where

$$v_{ij} = b_i - d_{ij}$$

is the loss incurred by the existing holding (i.e., weight equal to one) of counterparty i in scenario j . Note that an upward migration (i.e., a move to a more favorable credit state) implies a negative loss. The loss of the entire portfolio in scenario j is

$$\sum_{i=1}^n v_{ij} x_i$$

EXPECTED REGRET MINIMIZATION MODEL

Let us restrict our attention to those losses that exceed some threshold K (i.e., we focus only on the tail of the loss distribution extending beyond the point K). The following model adjusts the counterparty weights to minimize the expectation of all losses that exceed K :

EXHIBIT 16

Loss Distributions of Original Portfolio and of Portfolio A

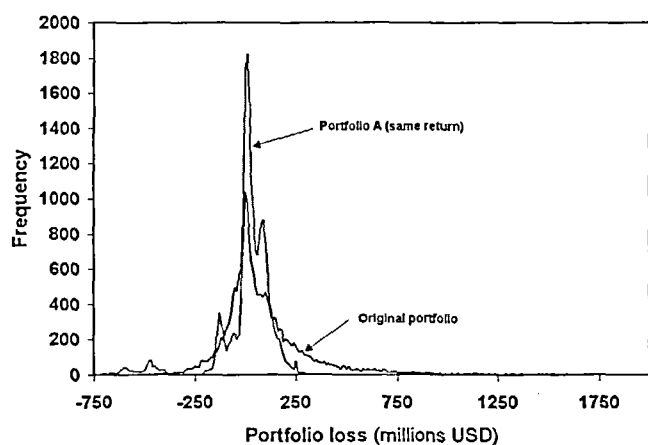


EXHIBIT 1A

Variables and Parameters of the Models

Definition	Elements	Dimension
x counterparty weights	x_i	$n \times 1$
y losses exceeding the threshold	y_j	$m \times 1$
q current mark-to-market counterparty values	q_i	$n \times 1$
b future counterparty values with no credit migration	b_i	$n \times 1$
D future (scenario-dependent) counterparty values with credit migration	d_{ij}	$n \times m$
l lower trading limits, as multiple of current weighting	l_i	$n \times 1$
u upper trading limits, as multiple of current weighting	u_i	$n \times 1$
p scenario probabilities	p_j	$m \times 1$
V matrix of losses due to credit migration	v_{ij}	$n \times m$

$$\min \sum_{j=1}^m p_j y_j \quad (A-1)$$

$$\text{s.t.} \quad \sum_{i=1}^n v_{ij} x_i - y_j \leq K \quad j = 1, \dots, m \quad (A-2)$$

$$\sum_{i=1}^n b_i x_i = \sum_{i=1}^n b_i \quad (A-3)$$

$$l_i \leq x_i \leq u_i \quad i = 1, \dots, n \quad (A-4)$$

$$y_j \geq 0 \quad j = 1, \dots, m \quad (A-5)$$

The objective function, Equation (A-1), minimizes the probability-weighted sum of the amounts by which losses exceed the threshold K , as defined in Equation (A-2). The normalization constraint, Equation (A-3), ensures that the future values of the original and the rebalanced portfolios are equal in the absence of any credit migration. Equation (A-4) specifies the upper and lower limits for each obligor weight, while Equation (A-5) stipulates that regret must be non-negative.

Note that, alternatively, one can choose to maintain the current value of the portfolio, in which case Equation (A-3) is replaced by

$$\sum_{i=1}^n q_i x_i = \sum_{i=1}^n q_i$$

MAXREGRET MINIMIZATION MODEL

An alternative model, that minimizes MaxRegret, the maximum loss exceeding K , is

$$\min z \quad (A-6)$$

$$\text{s.t.} \quad \sum_{i=1}^n v_{ij} x_i - z \leq K \quad j = 1, \dots, m \quad (A-7)$$

$$\sum_{i=1}^n b_i x_i = \sum_{i=1}^n b_i \quad (A-8)$$

$$l_i \leq x_i \leq u_i \quad i = 1, \dots, n \quad (A-9)$$

$$z \geq 0 \quad (A-10)$$

The objective function, Equation (A-6), minimizes the maximum amount by which losses exceed the threshold K , as defined in Equation (A-7). The normalization and trading limit constraints, (Equations A-8 and A-9, respectively) are exactly as described in the previous model. Finally, Equation (A-10) stipulates that the maximum excess loss, z , must be non-negative.

Note that, mathematically, while expected regret is given by the 1-Norm of the excess losses, MaxRegret is given by the infinity-Norm.

RISK-RETURN MODEL

Suppose that we know the expected return, r_i , for counterparty i in the absence of credit migration. The expected portfolio return, r_p , given counterparty weights x , is

$$r_p = \frac{\sum_{i=1}^n (q_i x_i) r_i}{\sum_{i=1}^n q_i x_i}$$

Thus, the requirement that the portfolio achieves an expected return of at least R when there is no credit migration, can be modeled by the constraint

$$\sum_{i=1}^n (q_i x_i) r_i \geq R \sum_{i=1}^n q_i x_i$$

or, more simply,

$$\sum_{i=1}^n q_i(r_i - R)x_i \geq 0 \quad (\text{A-11})$$

Equation (A-11) can be added to any of the above models. Varying the required return, R , produces an efficient frontier for trading off risk, as measured by the objective function in the respective model, and return.

ENDNOTE

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