Model specification

A more thorough and in depth analysis of the methods used is available in the papers by Brownlees and Engle (2015) and Idier et al. (2014). Below we shall have a high level discussion sufficient for readers to have an understanding of the dynamics of our model.

Estimation procedure of the Marginal Expected Shortfall

MES is broadly defined as the short-run expected equity loss conditional on the market taking a loss greater than a specified threshold indicative of market distress. Formally,

$$MES_{i,t-1} = E_{t-1}\left(r_{i,t} | r_{m,t} < C\right),$$

where $r_{m,t}$ and $r_{i,t}$ are the returns on the market index and the equity of firm i respectively. The process for the estimation of this measure is laregely based on the work of Brownlees and Engle (2015) as described by Idier et al. (2014). The first step is to model the bivariate process of firm and market returns, which we specify as:

$$\begin{aligned} r_{m,t} &= \sigma_{m,t}\varepsilon_{m,t} \\ r_{i,t} &= \sigma_{i,t}\varepsilon_{i,t} \\ &= \sigma_{i,t}\rho_{i,t}\varepsilon_{m,t} + \sigma_{i,t}\sqrt{1-\rho_{i,t}^2}\xi_{i,t}, \end{aligned}$$

where $\sigma_{m,t}$ and $\sigma_{i,t}$ are the volatilities of the market and financial institution i at time t; $\rho_{i,t}$ the correlation at time t between $r_{m,t}$ and $r_{i,t}$. In this model, the disturbances $\varepsilon_{m,t}$ and $\xi_{i,t}$ are assumed to be independently and identically distributed over time and have zero mean, unit variance and zero covariance under a distribution F that is kept unspecified. The MES can therefore now be rewritten as a function of the above quantities:

$$\begin{aligned} MES_{i,t-1} &= E_{t-1}\left(r_{i,t} | r_{m,t} < C\right) \\ &= \sigma_{i,t}E_{t-1}\left(\varepsilon_{i,t} | \varepsilon_{m,t} < \frac{C}{\sigma_{m,t}}\right) \\ &= \sigma_{i,t}\rho_{i,t}E_{t-1}\left(\varepsilon_{i,t} | \varepsilon_{m,t} < \frac{C}{\sigma_{m,t}}\right) + \sigma_{i,t}\sqrt{1-\rho_{i,t}^2}E_{t-1}\left(\xi_{i,t} | \varepsilon_{m,t} < \frac{C}{\sigma_{m,t}}\right). \end{aligned} \tag{1}$$

The next step has to do with the modelling of stochastic volatilities and time varying correlations. Conditional volatilities of the equity returns are modelled using an asymmetric GJR-GARCH specification i.e. we assume

$$\sigma_{m,t}^2 = \omega_m + \alpha_m r_{m,t-1}^2 + \gamma_m r_{m,t-1}^2 \mathbb{I}_{r_{m,t}<0} + \beta_m \sigma_{m,t-1}^2$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i r_{i,t-1}^2 + \gamma_i r_{i,t-1}^2 \mathbb{I}_{r_{i,t}<0} + \beta_i \sigma_{i,t-1}^2,$$

where $\sigma_{m,t}^2$ and $\sigma_{i,t}^2$ are the conditional volatilities of the market and the firm respectively. The indicator variables $\mathbb{I}_{r_{m,t}<0}$ and $\mathbb{I}_{r_{i,t}<0}$ allow the model to capture the asymmetric effects of leverage on volatility as it has been observed empirically that negative shocks have a greater volatility impact than positive shocks (Alexander, 2008). This specification also accounts for the effects of volatility clusters as β_m and β_i measure the persistence of conditional volatility irrespective of anything happening in the market. When β_m and β_i are relatively large, this implies that volatility will take a long time to die out following a crisis in the market (Alexander, 2008).

Time varying conditional correlation is modelled using a modified DCC approach developed by Cappiello et al. (2006) to account for possible asymmetries. This asymmetric DCC model is estimated using quasi maximum likelihood methods. For a more technical understanding we refer readers to Idier et al. (2014) and Cappiello et al. (2006). Lastly, Equation 1 reveals that MES also depends on the tail expectations of the disturbances $\varepsilon_{m,t}$ and $\xi_{i,t}$. A non-parametric kernel estimation approach is used in order to estimate these tail expectations, so that the estimators are not unstable when $\frac{C}{\sigma_{m,t}}$ is large.

Estimation procedure of the SRISK

SRISK, the expected capital shortfall of an institution conditional on a systemic event, is a function of the size of the firm, its degree of leverage and the Long Run Marginal Expected Shortfall (LRMES). The capital shortfall of firm i at time t is formally defined as:

$$\mathbf{CS}_{i,t} = k\mathbf{A}_{i,t} - \mathbf{E}_{i,t} \ = k\left(\mathbf{L}_{i,t} + \mathbf{E}_{i,t}\right) - \mathbf{E}_{i,t}$$

where $\mathbf{E}_{i,t}$ is the value of equity, $\mathbf{L}_{i,t}$ is the book value of debt, $\mathbf{A}_{i,t}$ is the implied value of assets and k is the prudential capital fraction. When a firm is in distress this quantity will be positive, indicating insufficient working capital, whereas a negative value will be indicative of a capital surplus.

We are concerned with predicting the capital shortfall as a result of a systemic event which we will define as a market decline below a threshold C over a time horizon h, drawing from the work of Acharya et al. (2012), where the capital shortfall of a firm generates negative externalities if it occurs when the system is already in distress. We formally define SRISK as the expected capital shortfall conditional on a systemic event

$$
\begin{aligned}
SRISK_{i,t} &= E_t\left(\mathbf{CS}_{i,t+h} | R_{mt+1:t+h} < C\right) \\
&= kE_t\left(\mathbf{L}_{i,t+h} | R_{mt+1:t+h} < C\right) - (1-k)E_t\left(\mathbf{E}_{i,t+h} | R_{mt+1:t+h} < C\right).
\end{aligned}
$$

It is further assumed that in the event of the systemic event defined by C, debt can not be renegotiated, implying that $E_t\left(\mathbf{L}_{i,t+h} | R_{mt+1:t+h} < C\right) = \mathbf{L}_{i,t}$. From this it follows that

$$
SRISK_{i,t} = k\mathbf{L}_{i,t} - (1-k)\mathbf{E}_{i,t}\left(1 - LRMES_{i,t}\right),
$$

where $LRMES_{i,t}$ is the expectation of the firm equity multi-period return conditional on the systemic event. Fundamentally, this is the average of the fractional returns of the firm's equity in the crisis scenarios. Estimation of the Long Run MES is typically done using Monte-Carlo methods to simulate the system over the time horizon h and computing the expected loss of equity value of firm i. Alternatively, predictions can be made using a GARCH-DCC model as shown by Brownlees and Engle (2015). Without simulation, as we chose to do in our model, $LRMES$ is approximated as $1 - exp(-18 \times MES)$ (Acharya et al., 2012) where MES is the one day loss expected if the market makes a loss in excess of the threshold C, which we set at the daily 95% VaR. By aggregating the positive values of firm SRISK at any time t, we are able to obtain a system wide measure of financial distress. Formally, the total amount of systemic risk in the financial system at time t is given by

$$
SRISK_t = \sum_{i=1}^{N} \left(SRISK_{i,t}\right)_+,
$$

where $(x)_+$ denotes $max(x, 0)$. We have chosen to explicitly ignore capital surpluses for the very reason that in a crisis it is unlikely that the surplus capital of a firm will easily be mobilized to cover any shortfalls in the system and support failing firms through acquisitions or loans. This capital shortage can therefore be used to measure the extent to which the firm contributes to systemic risk. Formally, the contribution to systemic risk by any firm is given by:

$$
SRISK\%_{i,t} = \frac{\left(SRISK_{i,t}\right)_+}{\sum_{i=1}^{N}\left(SRISK_{i,t}\right)_+}
$$

Bibliography

Acharya, V., Engle, R., Richardson, M., 2012. Capital shortfall: A new approach to ranking and regulating systemic risks. The American Economic Review 102 (3), 59–64.

Acharya, V. V., Pedersen, L. H., Philippon, T., Richardson, M. P., 2010. Measuring systemic risk.

Alexander, C., 2008. Market risk analysis: Practical financial econometrics (vol. ii).

Brownlees, C. T., Engle, R. F., 2015. Srisk: A conditional capital shortfall measure of systemic risk. Available at SSRN 1611229.

Cappiello, L., Engle, R. F., Sheppard, K., 2006. Asymmetric dynamics in the correlations of global equity and bond returns. Journal of Financial econometrics 4 (4), 537–572.

Idier, J., Lamé, G., Mésonnier, J.-S., 2014. How useful is the marginal expected shortfall for the measurement of systemic exposure? a practical assessment. Journal of Banking & Finance 47, 134–146.

Thomson, J. B., 2009. On systemically important financial institutions and progressive systemic mitigation. FRB of Cleveland Policy Discussion Paper (7).