02417: Time Series Analysis

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Assignment 2: ARMA Processes and Seasonal Processes

In this assignment you will study how the choice of coefficients (of the operator polynomials in ARMA processes) affects the structure of the process, through simulated data and empirical autocorrelation functions. The assignment is to some extent specified in terms of R commands. Users of other software packages should find similar functions to answer the questions. *Be aware that the model parametrizations in the assignment and in your favorite software package may not be identical.*

Question 2.1: Stability Let the process X_t be given by

$$X_t + \phi_1 X_{t-1} + \phi_2 X_{t-2} = \varepsilon_t$$

where ε_t is a white noise process.

Investigate analytically for which values of ϕ_2 the process is stationary when $\phi_1 = -1/3$. In addition it should be investigated for which values of ϕ_2 the autocorrelation function shows damping harmonic oscillations. Still for $\phi_1 = -1/3$.

Question 2.2: University activity A university counts the number of passed course modules every term. There are four terms per year. Based on historical data the following model has been identified:

$$(1 - 0.8B + 0.7B^2)(1 - 0.9B^4)(Y_t - \mu) = \varepsilon_t$$

where ε_t is a white-noise process with variance $^2_{\varepsilon}$. Based on 60 observations, it is found that $\sigma^2_{\varepsilon} = 50^2$. Furthermore, μ was estimated to 1000. The table below shows the last ten observations:

t	51	52	53	54	55	56	57	58	59	60
count	989	724	1013	1280	1092	811	952	1277	1111	848

Predict the values of Y_t corresponding to t = 61 and 62, together with 95% prediction intervals for the predictions.

Question 2.3: Random walk Let the process Y be given by

$$Y_t = \frac{1}{4} + \sum_{i=1}^{t} \varepsilon_i$$

where ε_t is a white noise process with mean zero and variance σ^2_{ε}.

1. Find the mean value, variance and covariance functions of the process.
2. Is the process Y stationary? If so, in what sense? If not, in what sense?
3. Simulate a white noise process of 1000 values with mean zero and $\sigma^2_{\varepsilon} = \sqrt{2}$. Then figure out a fast and compact way to calculate the process Y_t based on this.

4. Next, simulate 10 realizations of the process Y_t. Save the realizations and plot them in the same graph using `plot` and `lines` or `matplot`. Also plot their estimated autocorrelation functions (Preferably in one plot). Comment on the graphs, and confirm you answers in step 2.

Question 2.4: Simulating seasonal processes A process Y_t is said to follow a multiplicative $(p,d,q)\times(P,D,Q)_s$ seasonal model if

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D Y_t = \theta(B)\Theta(B^s)\varepsilon_t$$

where (ε_t) is a white noise process, and $\phi(B)$ and $\theta(B)$ are polynomials of order p and q, respectively. Furthermore, $\Phi(B^s)$ and $\Theta(B^s)$ are polynomials in B^s. All according to definition 5.22 in the textbook.

Simulate the following models (where monthly data are assumed). Plot the simulations and the associated autocorrelation functions (ACF and PACF).

1. A $(0,0,0)\times(1,0,0)_{12}$ model with the parameter $\Phi_1 = 0.6$.
2. A $(1,0,0)\times(0,0,0)_{12}$ model with the parameter $\phi_1 = 0.8$.
3. A $(1,0,0)\times(0,0,1)_{12}$ model with the parameters $\phi_1 = 0.8$ and $\Theta_1 = 0.5$.
4. A $(0,0,1)\times(0,0,1)_{12}$ model with the parameters $\theta_1 = 0.5$ and $\Theta_1 = 0.4$.
5. A $(1,0,0)\times(1,0,0)_{12}$ model with the parameters $\phi_1 = 0.8$ and $\Phi_1 = 0.6$.
6. A $(0,0,1)\times(1,0,0)_{12}$ model with the parameters $\theta_1 = 0.5$ and $\Phi_1 = 0.6$.

Are all models seasonal? Summarize your observations on the processes and the autocorrelation functions. Which conclusions can you draw on the general behavior of the autocorrelation function for seasonal processes? Note: `arima.sim` does not have a seasonal module, so model formulations as standard ARIMA processes have to be made.