Variance Reduction Method for Merton Process Monte-Carlo Process

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Codes Available at: Github Link

The variance of a Monte-Carlo estimator is an important component of the computional efficiency. A high MC variance will negatively affect the robustness and precision of the estimation, especially when the input paramters tend to change drastically. In this project, we implemented an importance sampling variance reduction method on pricing European put options. Under the project setting, we assue the underlying seurity price follows a Merton Jump-Diffusion model. We used change of measures to change the frequency distributions of jumps in order to obtain MC samples for security dynamics. We also derived the distribution paramter value that miminized the Mean-Squared-Error and minimized the computational efficiency.

Consider the price of the European put option under the risk-neutral measure, $E = \mathbb{E}[(S - X_t)_+]$, in the Merton Jump-Diffusion model with a log-normally distributed scock price at maturity:

$$X_T = X_0 \exp\left(-\frac{\sigma^2}{2}T + \sigma B_T\right)\prod_{n=1}^{N_T} e^{X^n}, \qquad X_0 = 2000,$$

Where the jumps $N_T \sim Pois_\lambda$ and $X^n \sim N(a, b^2), i.i.d.$ For simplicity we assume an interest rate of zero and

$$T = 1\ (Maturity);$$

$$\sigma = 0.17\ (Volatiltiy);$$

$$S = 1500\ (Stock\ Price);$$

$$\lambda = 2\ (Jump\ Intensity);$$

$$a = -0.05, b = 0.03\ (Distribution\ Parameters)$$

Consider a change of measure from $\mathbb{p}$ (normal measure) to $\mathbb{Q}$ (risk-neutral measure) given by the Radon-Nikodym density

$$\frac{d\mathbb{Q}}{d\mathbb{p}} = Z_T^N = e^{(\lambda - \ell)T}\prod_{n=1}^{N_t}\frac{\ell q(L^n)}{\lambda p(L^n)},$$

where $\ell > 0$, p is the dentisty of normal distribution $N(a, b^2)$, and q is the dentisty of a normal distribution $N(c, d^2)$. Because $\mathbb{Q}$ and $\mathbb{p}$ are equivalent measures, we have:

$$E = \mathbb{E}[(S - X_t)_+] = \mathbb{E}^{\mathbb{Q}}[\frac{(X_T - S)_+}{Z_T^N}]$$

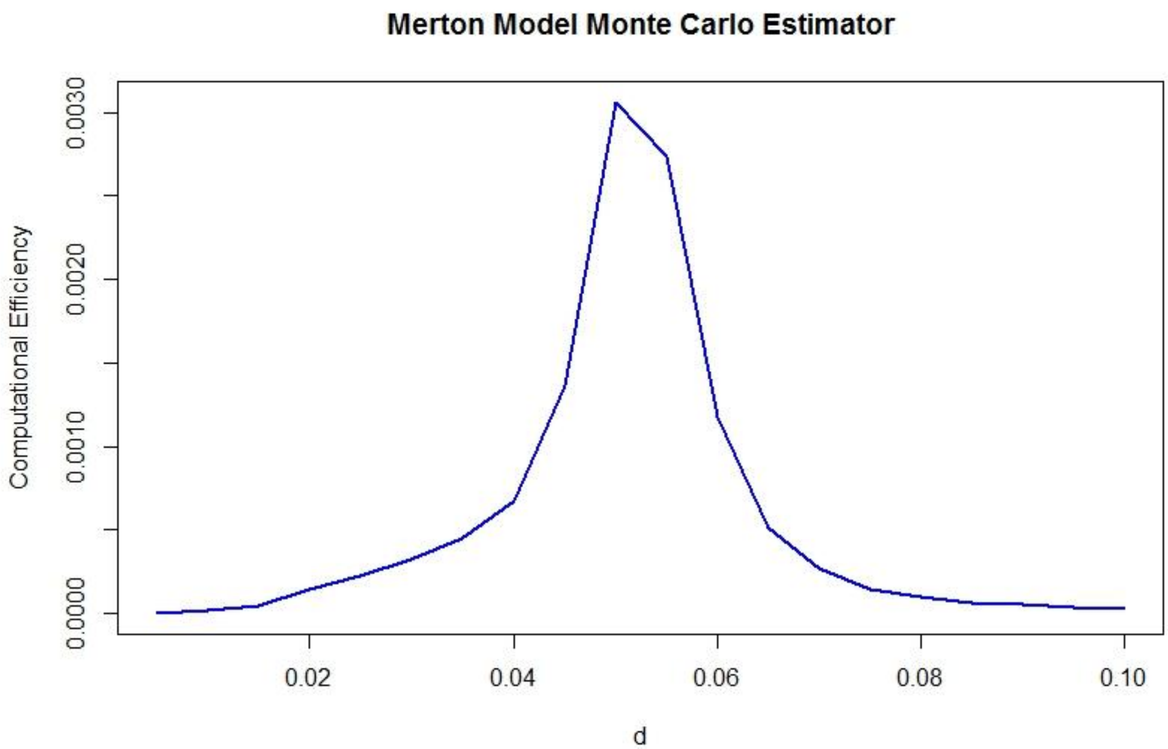
As a result, we can define an importance sampling Monte-Carlo Estimator Z_K^I using the above Radon-Nikodym density as follows:

$$Z_K^I = \frac{1}{K}\sum_{k=1}^{K}\frac{\left(S - X_T^{(k)}\right)_+}{Z_T^{n,(k)}}$$

where $(X_T^{(k)}, Z_T^{n,(k)})$ are i.i.d. samples of (X_T, Z_T^N).

Value of d maximizes the computational efficiency of the estimator:

We chose the number of Z_K^I 500, with each Z_K^I estimated by K = 10000 Monte Carlo samples. In addition We chose 20 values of d between (0, 0.1] with interval 0.005 between each value. Therefore, there are totally 10000 * 500 * 20 = 100 million runs.

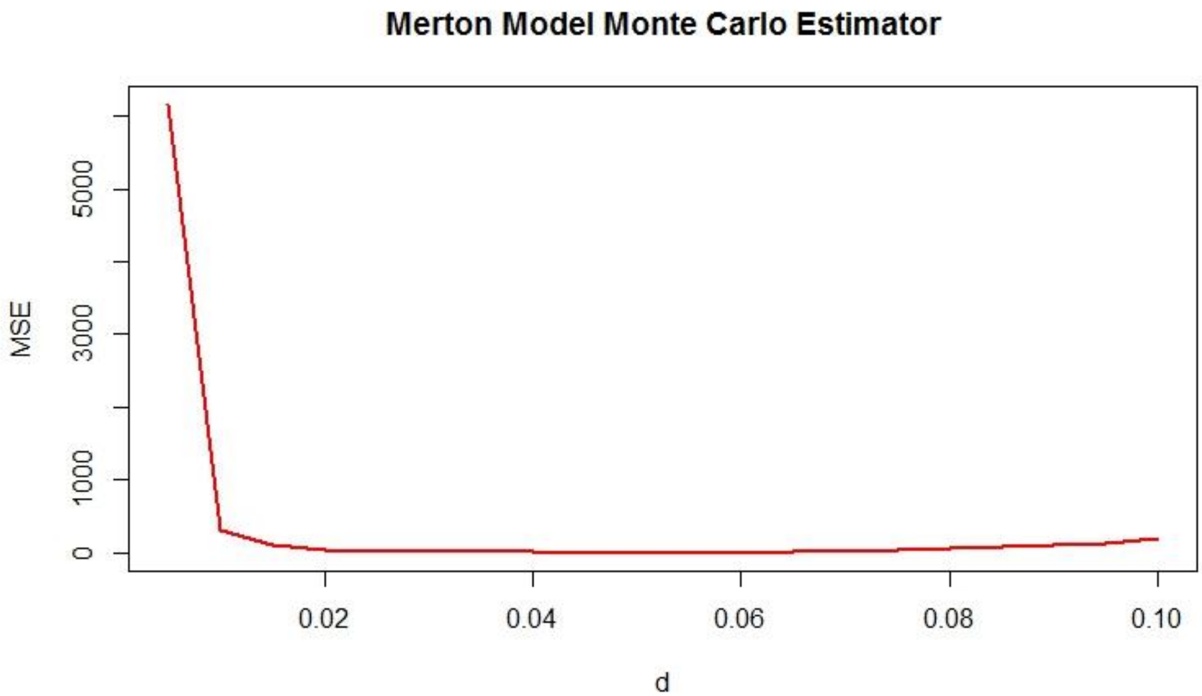


The true put option price is E = 29.97, as observed in market trading data.

From the above plot, the computational efficiency of the estimator is maximazed when d = 0.05. That is, when d is in the middle of the interval (0, 0.1]. In this setting, we also assume $\ell = \lambda$ and $c = a$.

Value of d maximizes the Mean Square Error (MSE) of the estimator:

Still, we choose the number of Z_K^I 500, with each Z_K^I estimated by K = 10000 Monte Carlo samples. In addition, we choose 20 values of d between (0, 0.1] with interval 0.005 between each value. Therefore, there are totally 10000 * 500 * 20 = 100 million runs.



From the above plot, the MSE of the Monte Carlo estimator is minimized when d = 0.05.

To conclude, when d is in the middle of the (0, 0.1], MSE is minimized.while computational efficiency is maximized.