The Max-Tree Data Structure

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Introduction

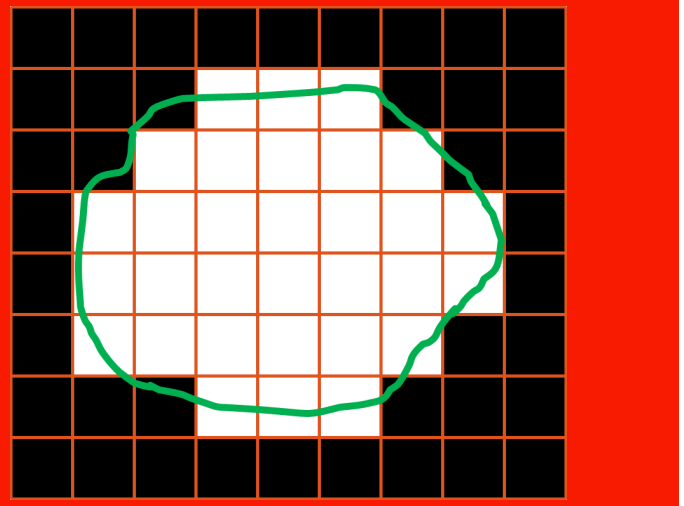


Image Connectivity

Connected component *C*: It is a connected set of maximal extent. i.e. there is no other connected set that is a superset of *C* that adhers to the same connectivity rules.

Maximality condition! Connected sets of maximal extent are easy to retrieve.

A connected component in the discrete image plane is often defined as a region consisting of foreground pixels that are all pair-wise connected with respect to some adjacency connectivity rule.

Examples are: 4- or 8- way connectivity on a grid.

Introduction

Connected Operators

A binary connected operator accesses all images points (in theory) and

- if a point belongs to the foreground the operator returns the connected component it belongs to;
- or the empty set (background) otherwise.

An attribute filter relies on a connected operator. The latter extracts the connected components of the image and the filter subjects each one separately to a binary attribute criterion

Example:

Component A : sizeof(A) = 100 pixels
Criterion. : sizeof(X)>= threshold

If threshold < 100 the component is rejected in its entirety.
If threshold >=100 the component is preserved intact.

Edge preserving operator; i.e. does not distort the shape/size of retained components.

Introduction

Image Partitions

A **partition of the image space** is the division of its definition domain into a set of non-overlapping cells (regions) the union of which covers the entire image space.

A connected attribute filter is an operator that produces a coarser partition of the image space with respect to the original.

A gray-scale connected attribute filter is the equivalent operator over a gray-scale image and can be computed by:

1. thresholding the input for each gray level;
2. applying the binary filter on each threshold set;
3. superimpose the stack of binary results to obtain the gray-scale output.

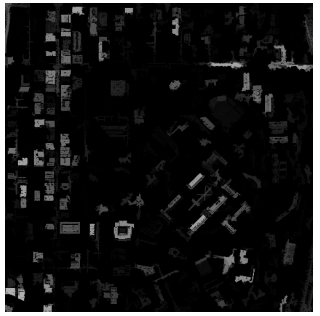
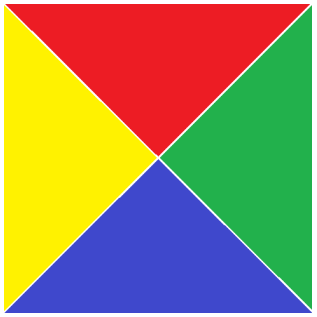
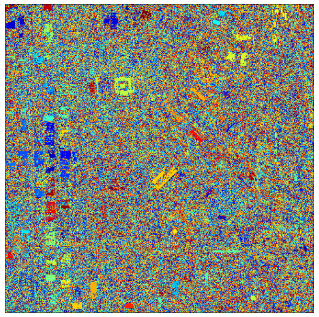
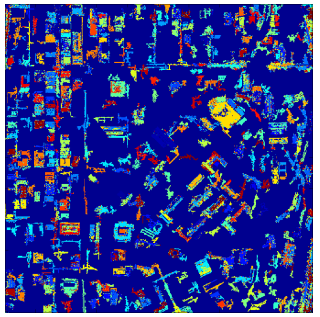
…OR RATHER NOT !

- very computationally expensive;
- very slow;
- not elegant !

Introduction

Image Partitions



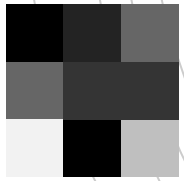
| Original image | silly partition | partition of flat zones | partition of conn. filters |

Input image

has 5 +1 levels (background is @ 0)

Threshold set is the set of all pixels of intensity greater or equal to a given threshold value (binary).

Level set is a set of pixels with their original intensities masked by the foreground elements of a threshold set.



0	1	3
3	2	2
5	0	4

Threshold sets

Level sets

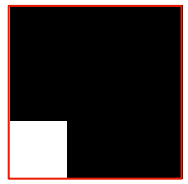
th=5

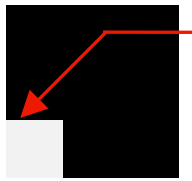
th=4

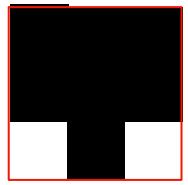
th=3

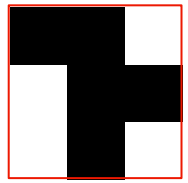
th=2

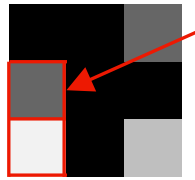
th=1

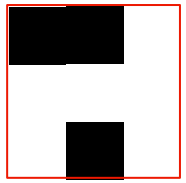


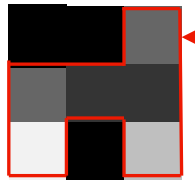


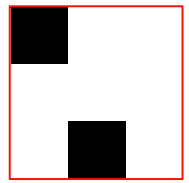


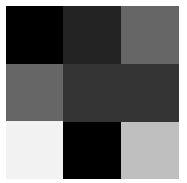












Regional Maximum: peak component defining a single flat zone of the same extent.

Flat zone: connected component of the level set. It can be seen as the 'visible part' of the component at level 3.

Peak Component: connected component of the threshold set.

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Max-Tree

A non-redundant hierarchical image representation structure

- The Max-Tree is a rooted uni-directed tree encoding the sequence of nested peak components along the grey scale.
- A Max-Tree node corresponds to a set of flat zones for which there exists a unique mapping to peak component.
- Every node is addressed by its level h and an index k ϵ Kh and points to its parent.
- The root node corresponds to the set of pixels belonging to the background and it points to itself.
- The leaves of the tree are regional maxima.
- The Max-Tree encodes a bright (leaves) to dark (root) nesting order.
- The inverse order is encoded in a **Min-Tree** structure

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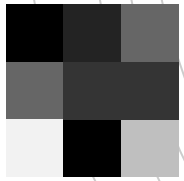
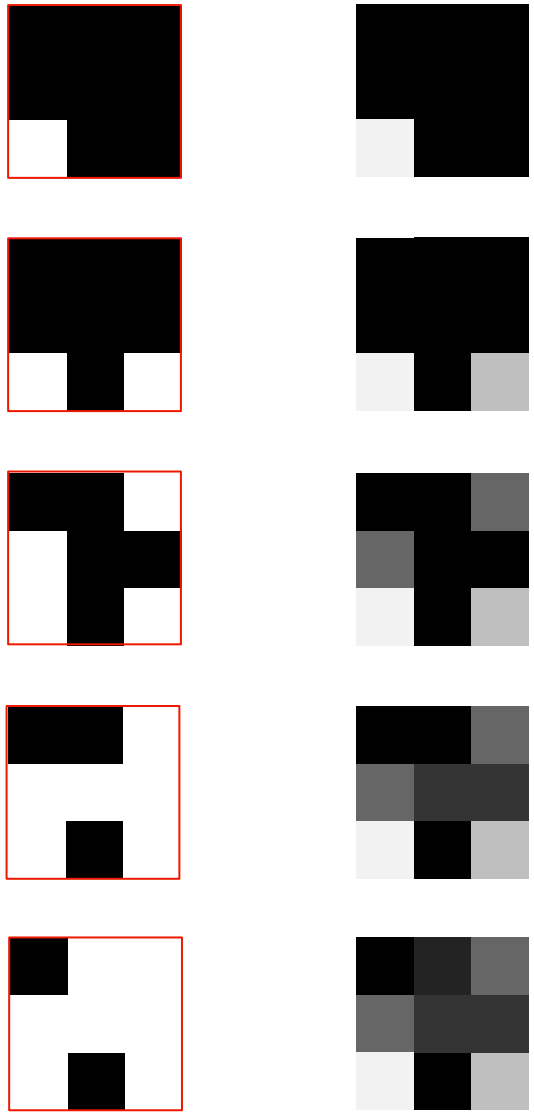
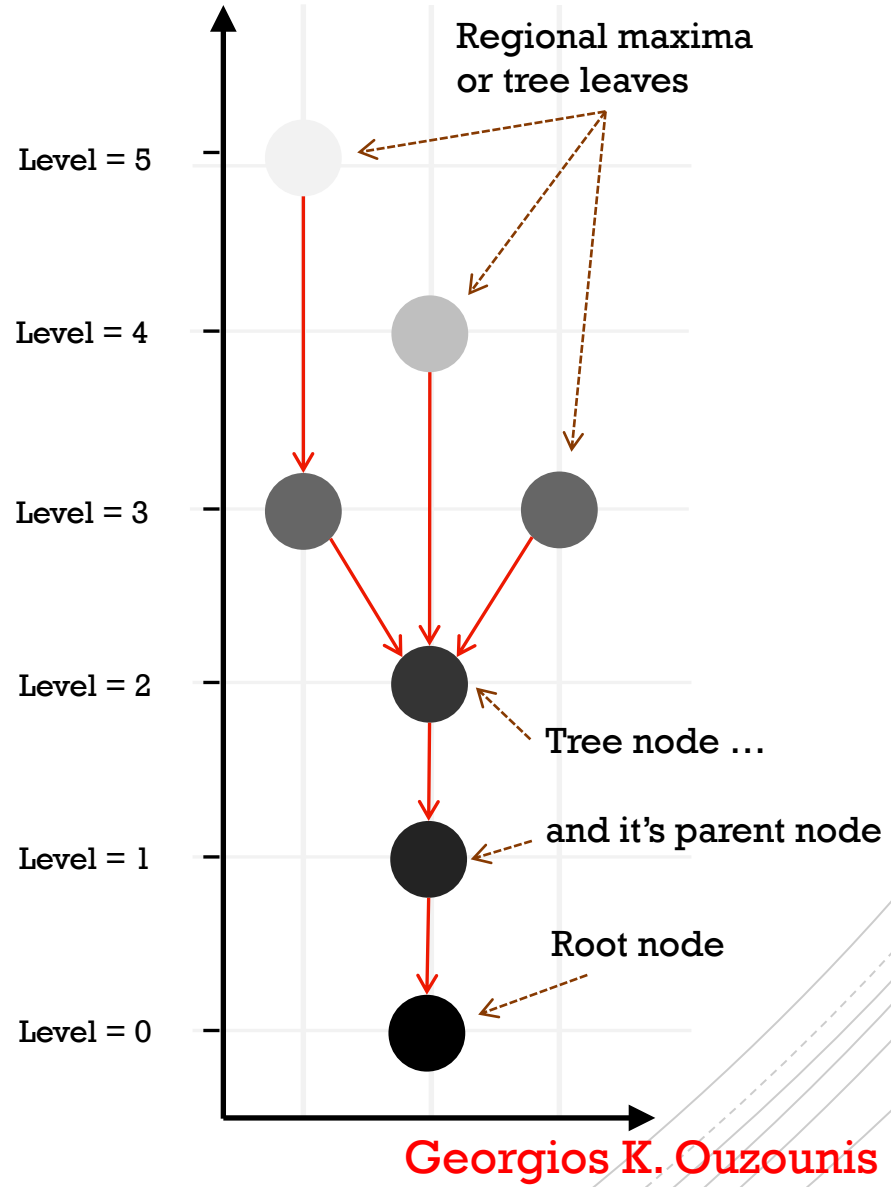
Max-Tree

A tree node is assigned to every component that associates to one or more flat-zones.



0	1	3
3	2	2
5	0	4





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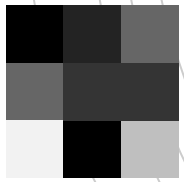
Node Attributes

Flat zone of the peak component

Higher level component

Background

\# Area (attribute)



0	1	3
3	2	2
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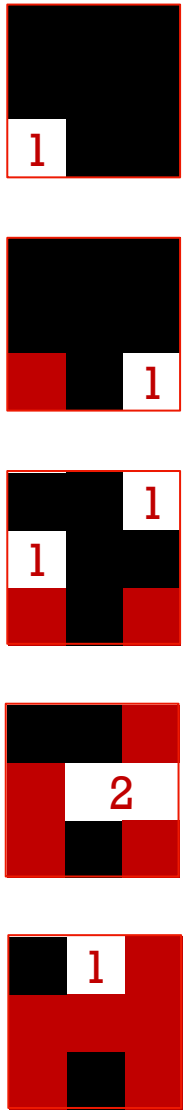

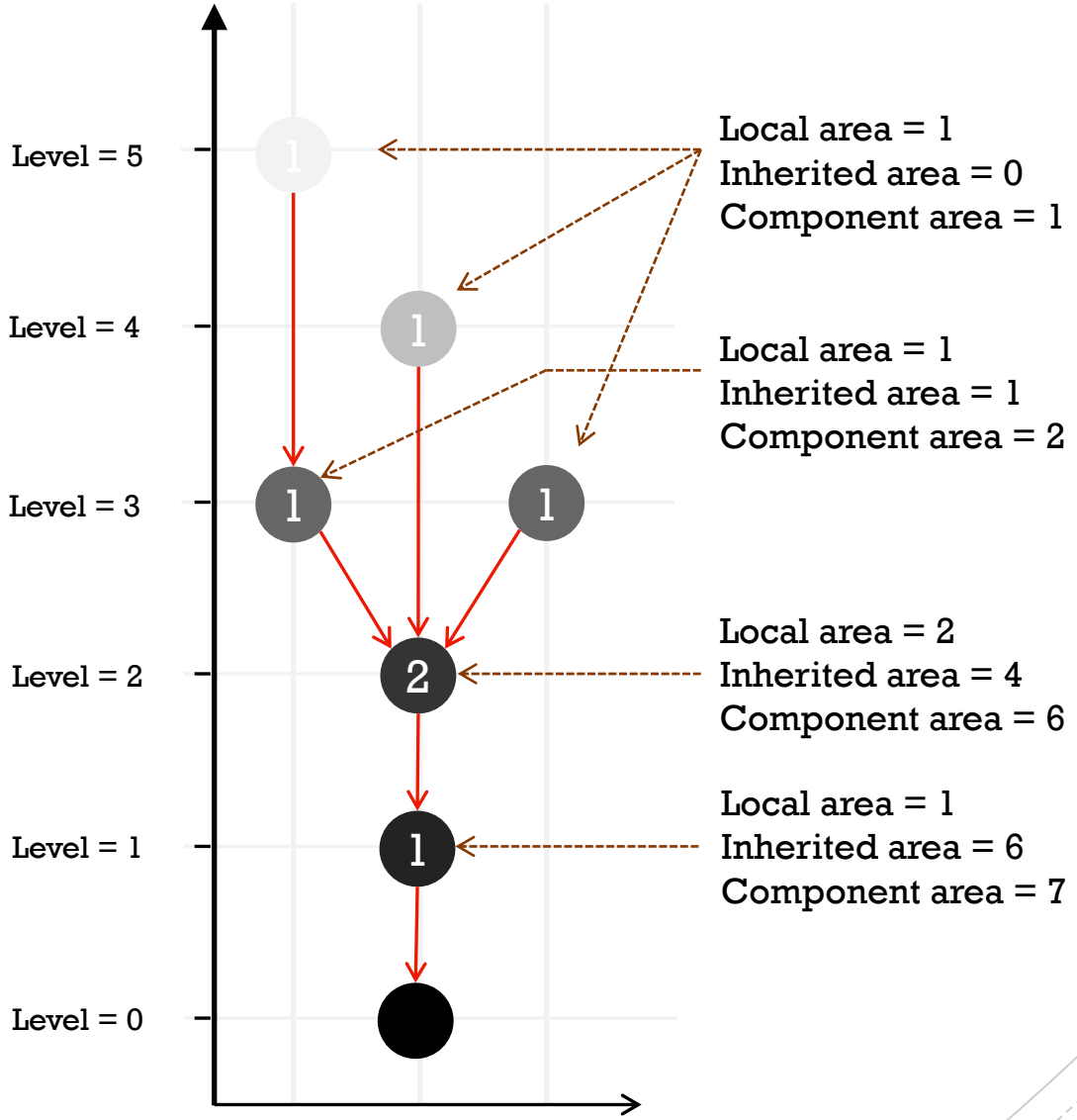

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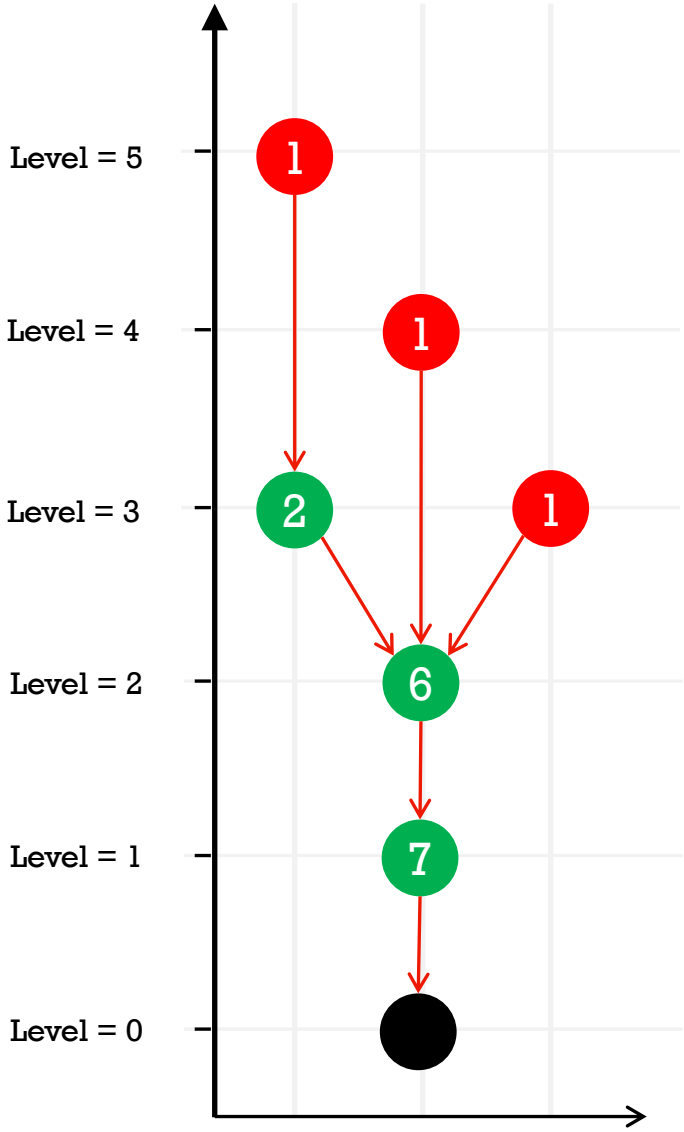
Attribute Filters

Evaluate a logical predicate on the attribute value(s) of each node and accept or reject it accordingly

0	1	3
3	2	2
5	0	4



Consider an area criterion:

if Area(peak component) >= 2,
retain it,
else reject it.

If a component is rejected, the new level of its respective node gets the intensity of its highest surviving ancestor.

This is known as the **'direct filtering rule'**.

The root node, i.e. the background, is not processed.

Numbers on each node indicate the area of the component it associates with.

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Attribute Filters

Filtering rules determine the way the output image is reconstructed, i.e. the new intensities of the nodes following the application of a filter

Supported Operators (filters):

- Connected Attribute Openings/Closings;
  Increasing, idempotent and anti-extensive/extensive
- Connected Attribute Thinnings/Thickenings;
  Non-increasing, idempotent and anti-extensive/extensive
- Hyper-connected equivalents of the above;

Filtering rules:

- Subtractive (regular connectivity, all attributes);
- Direct (regular connectivity, increasing attributes);
- k-subtractive (hyper-connectivity, all attributes);
- k-absorption (hyper-connectivity, all attributes);

Attribute Filters

Filtering rules determine the way the output image is reconstructed, i.e. the new intensities of the nodes following the application of a filter

Filter Application Modes
(th : attribute threshold)

- Reject if less than th;
- Reject if greater than th;
- Accept if greater than th1 **AND** less than th2 (th1<th2);
- Reject if less than th1 **OR** greater than th2 (th1<th2);

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Attributes

Attributes are computed from node auxiliary data. These are the 'essential ingredients' needed to compute a component descriptor/feature.

ATHOS (Attribute Horizon Setting) is a smart auxiliary data management system that decides what is to be computed and where is it when needed.

Size/Shape attributes
Area; Perimeter;
Perimeter Simplicity/Complexity;
Inertia;
Compactness/non-compactness;
Contour roughness/smoothness;
Linearity;
Sparseness;
PCA major/minor axis;
Orientation;
Centroid
X-, y-, z- position (raster, UTM, LatLong)
Bounding box
Bounding box area, perimeter & diagonal
Hue Moments
Rectangularity

Intensity attributes
Intensity;
Mean;
Standard deviation;
Variance;
Homogeneity;
Persistence;

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Examples

Min-Tree structure, 4-connectivity

Attributes used:
size, non-compactness, level.

Filtering rule: subtractive

Examples

Min-Tree structure, 4-connectivity

Attributes used:
size, non-compactness, level.

Filtering rule: subtractive

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Examples

Min-Tree structure, 4-connectivity

Attributes used:
size, non-compactness.

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Thank you

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