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Index-tracking optimal portfolio selection

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This paper considers a portfolio selection problem with multiple risky assets where the portfolio is managed to track a benchmark market barometer, such as the S&P 500 index. A tracking optimization model is formulated and the tracking-efficient (TE) portfolios are shown to inherit interesting properties compared with Markowitz mean-variance (MV) optimal portfolios. In comparison to an MV-portfolio, both the beta and the variance of a TE-portfolio are higher by fixed amounts that are independent of the expected portfolio return. These differences increase with index variance, are convex quadratic in the asset betas, and depend on the asset means and covariance matrix. Furthermore, a TE portfolio is obtained by simply extending an MV portfolio by constant adjustments to portfolio weights, independent of the specified mean return of the portfolio, but dependent on the index variance and asset return parameters. Consequently, at lower thresholds of risk, TE portfolios are better-diversified than MV portfolios.

Keywords: Index-tracking; Mean-variance optimization; Portfolio selection

1. Introduction

According to the Investment Company Institute (2012) Fact Book, the mutual funds in the USA had total net assets of \$11.6 trillion in 2011, and of which \$1.1 trillion was under the management of 383 index mutual funds, highlighting their growing popularity. A hefty 78% of these assets were invested in stock index funds, and in particular, funds indexed to the S&P 500 were 34% of all assets invested in index mutual funds.

The problem of fund management with benchmark index tracking has been well studied in the literature, see, e.g. Dembo and King (1992), El-Hassan and Kofman (2003), Frino and Gallagher (2001), and Roll (2008). Given an underlying stock index, the basic idea is to design an index-tracking portfolio focusing on holding a relatively few stocks, rather than all of the stocks in the index with their associated market-capitalization weights, see Jansen and van Dijk (2002). This will prevent portfolios from holding very small and illiquid positions, not to mention the transaction costs associated with forming and rebalancing such portfolios. The classical index-tracking approach represents the problem in a least-squares framework with (tracking) errors computed using a sample of historical data. Such a problem becomes a quadratic optimization model where tracking errors between index returns and asset returns are minimized to construct equity index funds, see, e.g. Meade and Salkin (1990) or Montfort *et al.* (2008). For a survey of heuristic methods in tracking index portfolios, see di Tollo and Maringer (2009).

This paper follows a related, but different, approach in which the classical risk measure of portfolio variance is replaced with the variance of the tracking error between the exogenous (stochastic) index return and the return on

the portfolio of n risky assets. However, any discussion on transaction and other costs in portfolio management is avoided, under the condition that n is small relative to the number of constituent assets in the market index. The index-tracking optimization model is formulated in Section 2 for a specified mean return on the portfolio, and it is shown to be an extension of the (Markowitz) mean-variance model. The optimal target tracking-efficient (TE) portfolio is characterized in closed-form using Karush–Kuhn–Tucker (KKT) optimality conditions. In Section 3, a partitioned matrix inversion formula is utilized, which uses orthogonal complements of off-diagonal blocks to discover the correspondence between the TE-optimal and the classical MV optimal portfolios. Further properties are developed that allow an investor to conveniently augment a MV optimal portfolio to an index-tracking portfolio, or vice-versa, with relatively little effort in computation. Section 4 presents an illustrative example, where five risky assets are used to form an index-tracking portfolio. Section 5 concludes the paper.

2. Index-tracking optimization model

Consider a benchmark (stochastic) market index M , say the S&P500 index. Suppose a universe of n risky assets is identified, denoted by the set $\{1, 2, \dots, n\}$, and a portfolio P of the n assets is formed. The portfolio P is said to be ‘tracking’ the benchmark index if the return on P , denoted by r_P , in every unit time period follows the return on the index, denoted by r_M , very closely. The premise is that the pair of univariate random variables r_P and r_M is in-synch with respect to direction and magnitude if the variance of the difference, $\text{Var}(r_P - r_M)$, is the lowest possible for a desired level of expected return on the portfolio.

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Denote the return on asset j by r_j and its mean by μ_j , for $j = 1, \dots, n$. The covariance between returns of assets i and j is denoted by σ_{ij} , and for $i = j$, $\sigma_j^2 \equiv \sigma_{jj}$ is the variance of r_j . The variance-covariance matrix of asset returns is denoted by the $n \times n$ matrix V . Let the portfolio weight in asset j be x_j , the percent of the budget invested in asset j , i.e. $\sum_j x_j = 1$. Thus, $r_P = \sum_j x_j r_j$. The covariance between asset returns and market index return is denoted by $\sigma_{jM} = \text{Cov}(r_j, r_M)$, $j = 1, \dots, n$, and the variance of r_M by σ_M^2 . Then, the ‘beta’ of asset j , relative to the market index M , is given by

$$\beta_j = \frac{\sigma_{jM}}{\sigma_M^2}. \quad (1)$$

The portfolio beta is $\beta_P = \sum_j \beta_j x_j = \beta'x$, where ‘prime’ denotes the transposition of the n -vector β of asset betas. The preceding ‘measure of goodness’ of portfolio P tracking the index M , $\text{Var}(r_P - r_M)$, is then

$$\begin{aligned} \text{Var}(r_P - r_M) &= \sigma_P^2 + \sigma_M^2 - 2 \text{Cov}(r_P, r_M) \\ &= \sigma_P^2 + \sigma_M^2 - 2 \sum_j x_j \sigma_{jM} \\ &\quad (\text{since } r_P = \sum_j x_j r_j) \\ &= \sigma_P^2 + \sigma_M^2 - 2\sigma_M^2 \beta_P. \end{aligned} \quad (2)$$

Since the index variance is independent of portfolio positions, the index-tracking objective is to minimize $\frac{1}{2}\sigma_P^2 - \sigma_M^2 \beta_P$, subject to the desired threshold on portfolio mean return, m , i.e.

$$\begin{aligned} \min_x \quad & \frac{1}{2}x'Vx - \sigma_M^2 \beta'x \\ \text{s.t.} \quad & \mu'x = m \\ & \mathbf{1}'x = 1, \end{aligned} \quad (3)$$

where $\mathbf{1}$ is a n -vector of 1’s. Observe that equation (3) is different from the classical MV-model only in the objective term that expresses the portfolio beta.

2.1. Optimality conditions

Denoting the Lagrange multipliers for portfolio mean return and budget constraints by λ and ρ , respectively, consider the Lagrangian:

$$L(x, \lambda, \rho) := \frac{1}{2}x'Vx - \sigma_M^2 \beta'x - \lambda(\mu'x - m) - \rho(\mathbf{1}'x - 1). \quad (4)$$

The necessary and sufficient KKT optimality conditions are

$$\nabla_x L = Vx - \sigma_M^2 \beta - \lambda \mu - \rho \mathbf{1} = \mathbf{0}, \quad (5)$$

$$\mu'x = m, \quad (6)$$

$$\mathbf{1}'x = 1. \quad (7)$$

Define the $(N+2) \times (N+2)$ matrix A by

$$A = \begin{bmatrix} V & -B \\ B' & \mathbf{0} \end{bmatrix}, \quad (8)$$

where $B = [\mu : \mathbf{1}] \in \mathbb{R}^{n \times 2}$ and $\mathbf{0}$ is a 2×2 matrix of zeros. Then, the optimal portfolio and Lagrange multipliers are

determined by the system of equations

$$A \begin{bmatrix} x \\ \lambda \\ \rho \end{bmatrix} = \begin{bmatrix} \sigma_M^2 \beta \\ m \\ 1 \end{bmatrix}. \quad (9)$$

Noting that B has full column rank, denote its *orthogonal complement* by B_\perp . Then, the nonsingularity of A and its inverse are provided by the following result.

LEMMA 2.1 *If $B'_\perp VB_\perp$ is of full rank, then the matrix A is nonsingular. Moreover, its (partitioned) inverse is*

$$A^{-1} = \begin{bmatrix} Q & (I_n - QV) \cdot (B')^r \\ -B^l \cdot (I_2 - VQ) & -B^l \cdot (VQV - V) \cdot (B')^r \end{bmatrix}, \quad (10)$$

where

$$Q = B_\perp \cdot (B'_\perp VB_\perp)^{-1} \cdot B'_\perp, \quad (11)$$

and B^l and $(B')^r$ are left and right inverses of B and B' , respectively, given by $B^l = (B'B)^{-1}B'$ and $(B')^r = B(B'B)^{-1}$.

Proof Follows directly from a more general result derived in Faliva and Zoia (2002). ■

It is straightforward to show that $B'_\perp VB_\perp$ is of full rank. Then, an optimal solution of the index-tracking problem in equation (3) is given by:

$$\begin{bmatrix} x^* \\ \lambda^* \\ \rho^* \end{bmatrix} = A^{-1} \begin{bmatrix} \sigma_M^2 \beta \\ m \\ 1 \end{bmatrix}. \quad (12)$$

Thus, defining the $n \times 2$ matrix $R = (I_n - QV) \cdot (B')^r$, the TE portfolio is

$$x^* = R \begin{bmatrix} m \\ 1 \end{bmatrix} + \sigma_M^2 Q\beta. \quad (13)$$

3. Comparison with MV optimal portfolios

The classical MV (Markowitz) portfolio optimization model is

$$\begin{aligned} \min_x \quad & \frac{1}{2}x'Vx \\ \text{s.t.} \quad & \mu'x = m \\ & \mathbf{1}'x = 1, \end{aligned} \quad (14)$$

specified with a mean return m , its optimal portfolio composition can be determined by referring to the corresponding KKT optimality conditions; denoting the MV-optimal portfolio weights by \tilde{x} and the Lagrange multipliers by $\tilde{\lambda}$ and $\tilde{\rho}$

$$\begin{bmatrix} \tilde{x} \\ \tilde{\lambda} \\ \tilde{\rho} \end{bmatrix} = A^{-1} \begin{bmatrix} 0 \\ m \\ 1 \end{bmatrix}. \quad (15)$$

Using the partitioned matrix inverse, therefore, MV-optimal portfolio is determined by the weighting vector

$$\tilde{x} = R \begin{bmatrix} m \\ 1 \end{bmatrix}. \quad (16)$$

By comparison to the TE-optimal portfolio in equation (13), the following result holds.

PROPOSITION 3.1 Given n risky assets with mean vector μ and variance–covariance matrix V , the MV optimal portfolio composition \tilde{x} for specified portfolio mean return m can be extended to a benchmark index-TE portfolio x^* by

$$x^* = \tilde{x} + \sigma_M^2(Q\beta), \quad (17)$$

where β is the vector of betas of the n risky assets determined with respect to the benchmark index whose variance is σ_M^2 and the $n \times n$ matrix Q is given by equation (11) which depends only on asset parameters μ and V .

The portfolio extension from \tilde{x} to x^* does not depend on the desired mean return m for the portfolio, but is directly proportional to the variance of the index return, with the constant of proportionality dependent on the asset return parameters. That is, denoting $s_j := Q^{(j)}\beta$ where $Q^{(j)}$ is the j th row of Q , and thus $s_j = s_j(\mu, \beta, V)$, the adjustment of portfolio positions is

$$x_j^* - \tilde{x}_j = \sigma_M^2 s_j, \quad \forall j = 1, \dots, n. \quad (18)$$

Moreover, denoting the MV-optimal portfolio beta and variance by $\tilde{\beta}_P$ and $\tilde{\sigma}_P^2$, respectively, and the TE-optimal portfolio beta and variance by β_P^* and σ_P^{2*} , it follows that $\beta_P^* > \tilde{\beta}_P$ and $\sigma_P^{2*} > \tilde{\sigma}_P^2$ as shown below.

PROPOSITION 3.2 When extending a MV optimal portfolio to a benchmark index-tracking portfolio with portfolio mean return unchanged,

- (i) the portfolio beta increases by $\sigma_M^2(\beta'Q\beta) > 0$,
- (ii) the portfolio variance increases by $(\sigma_M^2)^2\beta'Q\beta$ and
- (iii) the portfolio index-tracking error variance decreases by $(\sigma_M^2)^2\beta'Q\beta$.

Proof Noting equation (17), $\beta'x^* = \beta'\tilde{x} + \sigma_M^2(\beta'Q\beta)$, and thus, $\beta_P^* - \tilde{\beta}_P = \sigma_M^2\beta'Q\beta$. Then for the first part, it suffices to show that Q is a positive-definite matrix. For some $y \in \mathbb{R}^n$

$$\begin{aligned} y'Qy &= y'B_\perp(B'_\perp VB_\perp)^{-1}B'_\perp y = (B'_\perp y)'(B'_\perp VB_\perp)^{-1}(B'_\perp y) \\ &= z'(B'_\perp VB_\perp)^{-1}z \quad (\text{for } z = B'_\perp y) \\ &> 0 \quad (\text{due to positive definiteness of } B'_\perp VB_\perp). \end{aligned}$$

To show the second part, pre-multiplying by $x^{*'}V$ on both sides of equation (17)

$$\begin{aligned} x^{*'}Vx^* &= (\tilde{x} + \sigma_M^2 Q\beta)'(V\tilde{x} + \sigma_M^2 VQ\beta) \\ &= \tilde{x}'V\tilde{x} + 2\sigma_M^2 \tilde{x}'VQ\beta + (\sigma_M^2)^2 \beta'(QVQ)\beta, \end{aligned} \quad (19)$$

since Q and V are symmetric. Notice that

$$QVQ = [B_\perp(B'_\perp VB_\perp)^{-1}B'_\perp]V[B_\perp(B'_\perp VB_\perp)^{-1}B'_\perp] = Q$$

and

$$\begin{aligned} \tilde{x}'VQ\beta &= (m, 1)R'VQ\beta \quad (\text{due to equation (16)}) \\ &= (m, 1)[(I_n - QV)(B')^r]'VQ\beta \\ &= (m, 1)[(B')^r]'[(I_n - VQ)]VQ\beta \\ &= (m, 1)[(B')^r]'[VQ - VQVQ] \\ &= (m, 1)[(B')^r]'[VQ - VQ] \quad (\text{since } QVQ = Q) \\ &= 0. \end{aligned}$$

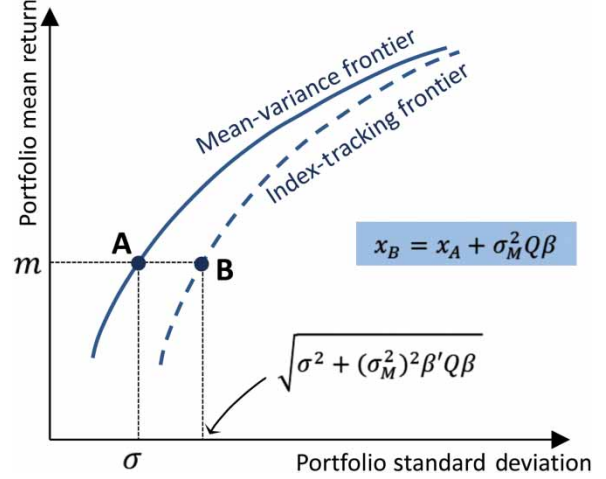


Figure 1. Shift in efficient frontiers of Markowitz and index-tracking portfolios.

Substituting in equation (19) and noting that TE-optimal portfolio variance $\sigma_P^{2*} = x^{*'}Vx^*$ and MV-optimal portfolio variance $\tilde{\sigma}_P^2 = \tilde{x}'V\tilde{x}$, it follows that $\sigma_P^{2*} - \tilde{\sigma}_P^2 = (\sigma_M^2)^2\beta'Q\beta > 0$.

To show the decrease in tracking error variance, noting equation (2), denote TE-optimal error variance by E^* and MV-optimal error variance by \tilde{E} . Then,

$$\begin{aligned} E^* &= \sigma_P^{2*} + \sigma_M^2(1 - 2\beta_P^*) \\ &= [\tilde{\sigma}_P^2 + (\sigma_M^2)^2\beta'Q\beta] + \sigma_M^2[1 - 2(\tilde{\beta}_P + \sigma_M^2\beta'Q\beta)] \\ &= \tilde{\sigma}_P^2 + \sigma_M^2(1 - 2\tilde{\beta}_P) - (\sigma_M^2)^2\beta'Q\beta \\ &= \tilde{E} - (\sigma_M^2)^2\beta'Q\beta. \end{aligned} \quad \blacksquare$$

Observe that Q depends only on asset parameters μ and V , but not on the specified mean return m for the portfolio. Consequently, TE-portfolio's variance risk increases by a constant amount given by $\theta := (\sigma_M^2)^2\beta'Q\beta$ for all levels of m . Therefore, the efficient frontier of tracking optimal portfolios is obtained from the classical mean-standard deviation efficient frontier by a lateral shift, at some MV-efficient point (m, σ) , by the amount $\sqrt{\sigma^2 + \theta}$ (see the illustration in figure 1). Thus, this shift is dependent on the desired mean m for the portfolio, and the shift is completely determined by the mean, beta, and variance–covariance information of the risky assets as well as the variance of the benchmark index. As the specified m increases (and thus the portfolio risk), it follows that $\sigma^2 \gg \theta$ and the index-tracking frontier begins to match that of the MV-frontier more closely.

4. Illustrative example

The purpose of this example is to illustrate the preceding methodology by using a sample of five stocks to track an index. Indeed, the choice of the n stocks would be an important dimension in a real-world implementation of an index fund. Consider monthly returns of the five technology stocks (tickers), AAPL, CSCO, IBM, MSFT and ORCL, indexed $j = 1, 2, 3, 4$, and 5. The S&P 500 index is used as the benchmark target to be tracked using a portfolio of

Table 1. Statistical parameters of monthly asset returns (2009–2012).

	Mean (μ) (%)	Std. dev. (σ) (%)	Correlation matrix					Beta (β)
			AAPL	CSCO	IBM	MSFT	ORCL	
AAPL	2.91	7.57	1	0.493	0.519	0.509	0.302	0.966
CSCO	0.73	8.91	0.493	1	0.631	0.599	0.615	1.464
IBM	1.74	4.29	0.519	0.631	1	0.462	0.563	0.682
MSFT	0.76	6.27	0.509	0.599	0.462	1	0.607	1.031
ORCL	1.46	7.98	0.302	0.615	0.563	0.607	1	1.277

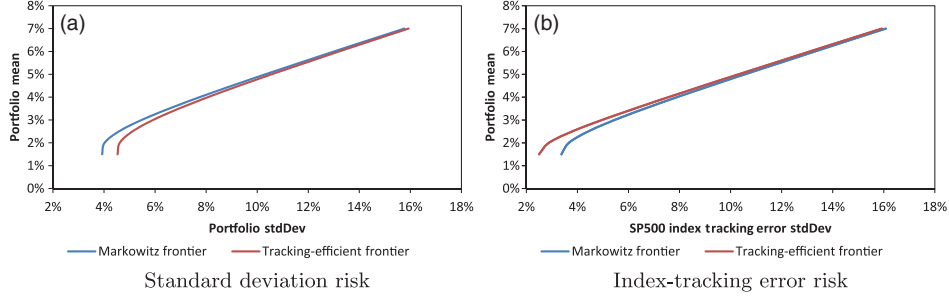


Figure 2. Efficient frontiers of five-stock portfolios. (a) Standard deviation risk. (b) Index-tracking error risk.

five stocks. Using the monthly prices from 2009 to 2012, mean vector, correlation matrix and beta vector are determined (see table 1). The standard deviation of the index return is $\sigma_M = 4.28\%$ (and its mean return is $\mu_M = 1.23\%$). The orthogonal complement B_\perp and the matrix Q , see equation (11), can be obtained easily under MATLAB using the two lines of code

```

nullB=null([mu ones(n,1)]');
Q=nullB/(nullB'*V*nullB)*nullB';

```

where n is the number of assets, μ the asset return mean vector and V the variance–covariance matrix. The resulting matrices Q and R are

$$Q = \begin{bmatrix} 91.412 & 48.859 & -183.348 & 66.168 & -23.092 \\ 48.859 & 221.214 & -67.492 & -156.390 & -46.192 \\ -183.348 & -67.492 & 521.039 & -102.539 & -167.659 \\ 66.168 & -156.390 & -102.539 & 260.736 & -67.974 \\ -23.092 & -46.192 & -167.659 & -67.974 & 304.917 \end{bmatrix},$$

$$R = \begin{bmatrix} 39.609 & -0.672 \\ -20.182 & 0.152 \\ 0.991 & 0.963 \\ -39.525 & 0.921 \\ 19.107 & -0.364 \end{bmatrix}. \quad (20)$$

This yields $\beta'Q = [73.5398, 104.7338, -240.5721, -52.9182, 115.2166]$, $\beta'Q\beta = 152.9525$, $\sigma_M^2\beta'Q\beta = 0.2799$ and $(\sigma_M^2)^2\beta'Q\beta = 5.13 \times 10^{-4}$.

In particular, setting the desired portfolio mean equal to the S&P 500 index mean return, i.e. $m = 1.23\%$, the MV optimal portfolio in equation (16) is

$$\tilde{x} = R \begin{bmatrix} 1.23\% \\ 1 \end{bmatrix} = [-0.1843, -0.0963, 0.9751, 0.4345, -0.1290]'. \quad (21)$$

Since $\sigma_M^2 Q\beta = [0.1346, 0.1917, -0.4404, -0.0969, 0.2109]'$, referring to Proposition 3.1, the index-tracking

optimal portfolio is given by

$$x^* = \begin{bmatrix} -0.1843 \\ -0.0963 \\ 0.9751 \\ 0.4345 \\ -0.1290 \end{bmatrix} + \begin{bmatrix} 0.1346 \\ 0.1917 \\ -0.4404 \\ -0.0969 \\ 0.2109 \end{bmatrix} = \begin{bmatrix} -0.0497 \\ 0.0955 \\ 0.5347 \\ 0.3376 \\ 0.0819 \end{bmatrix}. \quad (22)$$

Therefore, the index-tracking portfolio (with market mean) has only one short position with $x(\text{AAPL}) = -4.97\%$, while the Markowitz portfolio has three short positions with $x(\text{AAPL}) = -18.43\%$, $x(\text{CSCO}) = -9.63\%$ and $x(\text{ORCL}) = -12.90\%$. Moreover, the extreme long position of 97.51% in IBM under the MV portfolio is changed in the index-tracking portfolio to 53.47%. Moreover, the variance of the MV portfolio is 1.67×10^{-3} , and by adding $(\sigma_M^2)^2\beta'Q\beta$ to it, we obtain the variance of the TE portfolio as $1.67 \times 10^{-3} + 5.13 \times 10^{-4} = 2.183 \times 10^{-3}$. The optimal beta of the MV portfolio is $\beta'\tilde{x} = 0.6289$, which is increased by $\sigma_M^2\beta'Q\beta = 0.2799$ to the TE-optimal portfolio beta of 0.9089.

The efficient frontiers of MV and TE portfolios are shown in figure 2; note at higher levels of risk, these frontiers begin to coincide. Furthermore, optimal portfolio compositions too are quite different at lower levels of risk, but they begin to follow each other at higher levels of risk, see figure 3(a) and 3(b). The simulation of the two portfolios for $m = 1.23\%$, i.e. the index mean return, was performed for the period April 2009–December 2012. It is evident from the cumulative monthly performance, see figure 4, that the TE-portfolio tracks the S&P 500 market index more closely with a tracking error root mean square (RMS) of 2.71%, compared with the MV-portfolio RMS of 3.53%.

S&P 500 index covers 10 major sectors of the economy, and the five stocks used here is a small sample of information technology, a sector with a weight of 18.6% in the index. As such, the above example is not an attempt to track the index, but it is presented to illustrate the method. In

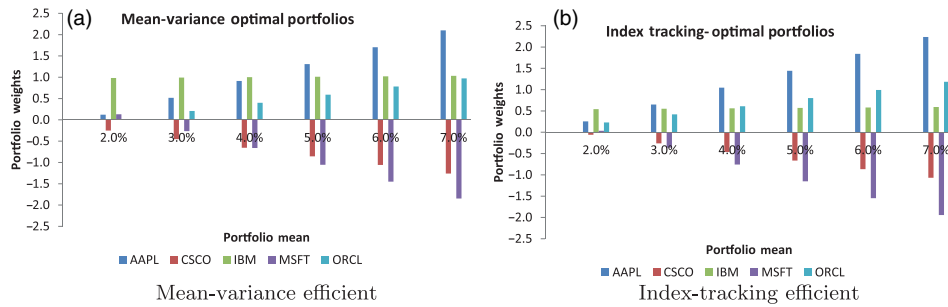


Figure 3. Composition of optimal portfolios. (a) MV efficient. (b) Index-TE.

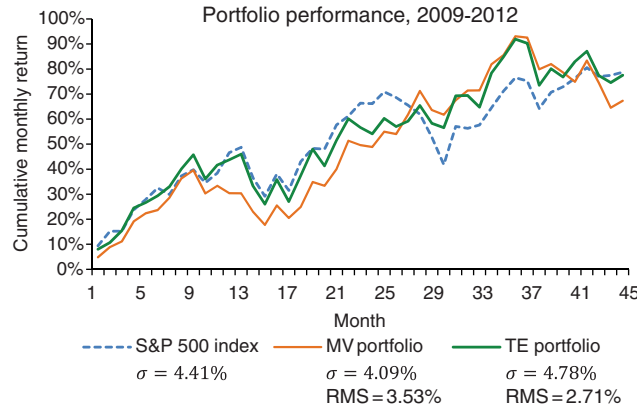


Figure 4. Cumulative monthly portfolio performance, 2009–2012.

practice, according to Morningstar data in 2003, the average S&P 500 index fund had tracking errors of 0.38%; however, a well-run S&P 500 index fund is expected to yield a tracking error of no more than 0.05%.

5. Concluding remarks

This paper developed a closed-form solution to the index-tracking portfolio selection problem. The portfolio positions are shown to be a constant vector adjustment to the MV portfolio positions regardless of the expected return of the portfolio, but they depend on the index and asset covariance matrix. TE portfolios have the property that the optimal positions begin to converge to those of MV optimal portfolios as portfolio risk threshold is increased. At lower risk thresholds, TE-portfolios are more-diversified than the MV counterparts. As one would expect, TE-optimal portfolio beta is higher than that of an MV-optimal portfolio for a specified mean, and this increase in beta is independent of the desired mean return, but it depends on the index and asset covariance matrix. The availability of closed-form expressions does imply that an investment strategy may be devised to quickly rebalance a portfolio of the risky assets to move between an index-tracking mode and a MV efficient mode as market and asset parameters change under evolving market regimes.

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