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IE 306 Group 11 - HW1 Report

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1) Comments on Hand Simulation Results

We performed a hand simulation to capture the nature of the network queue using 15 arriving jobs at the system. We have generated random variates using random numbers coming from standard uniform distribution.
As expected, we have encountered different behaviors with each different job. Some of them had low renege times, leading them to leave the queue without being served. Here are the results of our hand simulation:

1) Average reneging rate (average number of people who reneged the system) = $\frac{2}{15} \approx 0.13$
2) Average sojourn time per job = 32.97
3) Proportion of time the server is blocked = $\frac{(93.75 - 79.03) + (101.56 - 96.48) + (110.25 - 110.17)}{156.99} \approx$ 0.126
4) Proportion of jobs completed = $1 - \frac{2}{15} \approx 0.86$

Then we tested the result of our hand simulation on Python using SimPy. To do so, we used the same random numbers we generated before. Result of the code simulation was in agreement with what we calculated before.

Note: You can see our work on hand simulation, In the section 4: “Hand Simulation”.

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2) Comments on Simulation Results

Statistics of simulations with different properties gave the following results;

Note: we used seed = 978. (in python code, it is the variable “RANDOM_SEED”)

Simulation with empty queuing network:

With 1000 items :

Average interarrival time: 7.119
Average renege time: 23.976
Total station #1 service time: 4648.453
Station #1 average service time: 6.412
Station #2 average service time: 33.647
Average waiting time in the queue of station #1: 5.926
Average sojourn time: 36.878
Proportion of time the server was blocked: 0.261
Average reneging rate: 0.275
Proportion of jobs that completed the service: 0.725
Average number of jobs in the system per unit time: 5.048
Little's Law - Calculated value of average number of jobs: 5.18

With 200 items:

Average interarrival time: 7.201
Average renege time: 22.936
Total station #1 service time: 865.063
Station #1 average service time: 6.314
Station #2 average service time: 35.954
Average waiting time in the queue of station #1: 6.17
Average sojourn time: 37.364
Proportion of time the server was blocked: 0.296
Average reneging rate: 0.315
Proportion of jobs that completed the service: 0.685
Average number of jobs in the system per unit time: 4.943
Little's Law - Theoretical value of average number of jobs: 5.189

With 20 items:

Average interarrival time: 7.527
Average renege time: 28.435
Total station #1 service time: 103.971
Station #1 average service time: 5.776
Station #2 average service time: 27.709
Average waiting time in the queue of station #1: 1.239
Average sojourn time: 31.376
Proportion of time the server was blocked: 0.0
Average reneging rate: 0.1

Proportion of jobs that completed the service: 0.9
Average number of jobs in the system per unit time: 2.697
Little's Law - Calculated value of average number of jobs: 4.168

Simulation with 5 jobs in the first system and half of the c servers (2) full:

With 1000 items :

Average interarrival time: 7.042
Average renege time: 24.486
Total station #1 service time: 4757.652
Station #1 average service time: 6.535
Station #2 average service time: 32.75
Average waiting time in the queue of station #1: 6.855
Average sojourn time: 37.291
Proportion of time the server was blocked: 0.249
Average reneging rate: 0.27
Proportion of jobs that completed the service: 0.73
Average number of jobs in the system per unit time: 5.194
Little's Law - Calculated value of average number of jobs: 5.296

With 200 items:

Average interarrival time: 6.88
Average renege time: 26.449
Total station #1 service time: 841.368
Station #1 average service time: 6.423
Station #2 average service time: 35.821
Average waiting time in the queue of station #1: 10.233
Average sojourn time: 40.34
Proportion of time the server was blocked: 0.309
Average reneging rate: 0.335
Proportion of jobs that completed the service: 0.665
Average number of jobs in the system per unit time: 5.712
Little's Law - Calculated value of average number of jobs: 5.863

With 20 items:

Average interarrival time: 7.354
Average renege time: 25.388
Total station #1 service time: 57.985

Station #1 average service time: 5.798
Station #2 average service time: 31.374
Average waiting time in the queue of station #1: 7.748
Average sojourn time: 30.674
Proportion of time the server was blocked: 0.325
Average reneging rate: 0.4
Proportion of jobs that completed the service: 0.6
Average number of jobs in the system per unit time: 5.039
Little's Law - Calculated value of average number of jobs: 4.171

Simulation with 10 jobs in the rst system and all of the c servers full in the second system:

With 1000 items:

Average interarrival time: 7.119
Average renege time: 24.171
Total station #1 service time: 4543.026
Station #1 average service time: 6.372
Station #2 average service time: 33.728
Average waiting time in the queue of station #1: 6.231
Average sojourn time: 36.845
Proportion of time the server was blocked: 0.276
Average reneging rate: 0.283
Proportion of jobs that completed the service: 0.717
Average number of jobs in the system per unit time: 5.22
Little's Law - Calculated value of average number of jobs: 5.176

With 200 items:

Average interarrival time: 7.176
Average renege time: 23.225
Total station #1 service time: 787.987
Station #1 average service time: 6.254
Station #2 average service time: 37.782
Average waiting time in the queue of station #1: 7.553
Average sojourn time: 37.947
Proportion of time the server was blocked: 0.313
Average reneging rate: 0.348
Proportion of jobs that completed the service: 0.652
Average number of jobs in the system per unit time: 5.333
Little's Law - Theoretical value of average number of jobs: 5.288

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With 20 items:

Average interarrival time: 6.962
Average renege time: 32.23
Total station #1 service time: 39.151
Station #1 average service time: 7.83
Station #2 average service time: 42.037
Average waiting time in the queue of station #1: 19.342
Average sojourn time: 36.615
Proportion of time the server was blocked: 0.301
Average reneging rate: 0.524
Proportion of jobs that completed the service: 0.476
Average number of jobs in the system per unit time: 5.517
Little's Law - Calculated value of average number of jobs: 5.259

As we increased the number of jobs in the simulation, we have noticed that practical values started to tend to theoretical values. According to that, the behavior of the jobs started to stabilize, therefore leading to more predictable and stable utilization of the system.

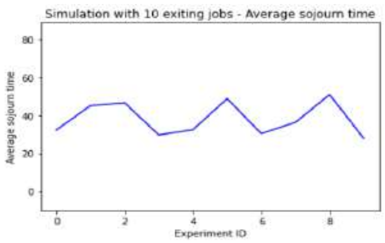
Simulation with empty queuing network and 100.000 items (and explanation of some statistics):

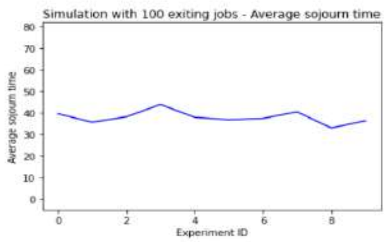
Average interarrival time: 6.998
Average renege time: 24.858
Station #1 average service time: 6.423
Station #1 average service time: 33.526

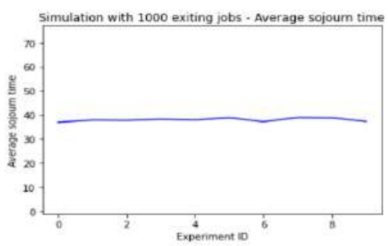
Average reneging rate: 0.291 → 29100 reneged jobs
Proportion of jobs that completed the service: 0.709 → 70900 completed jobs.
Total of reneging + completed jobs = 100,000

Total station #1 service time: 455490.081 → Average service time: 455490.081/70900 = 6.423
Total waiting time in the queue of station #1: 720687.527 → average waiting time: 720687.527 / 100000 = 7.207
Average sojourn time: 37.518
Proportion of time the server was blocked: 0.283
Average number of jobs in the system per unit time: 5.361
Little's Law - calculated value of average number of jobs: 5.361

At the end of our code, we provided a snippet to run the **empty queuing network** simulation with a different number of exiting jobs and varying random seeds. You can visualize the output by running the very last cell (keep in mind that you have to have **matplotlib** installed). Here, you can observe some of the outcomes.







Several experiments are carried out with different number of exiting jobs. As it can be seen from the results, as we increase the sample size (number of jobs), fluctuations in the values of average sojourn time decrease. Increasing the sample decreases the variance of the mean estimator. It is as expected since variance of the estimator is proportional to $\frac{1}{n}$ as shown below:

Mean estimator → $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

Variance of the estimator → $\sigma(\overline{X}) = \sigma(\frac{1}{n}\sum_{i=1}^{n} X_i)$

$$\sigma(\overline{X}) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma(X_i)$$

$$\sigma(\overline{X}) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma(X_i)$$

$$\sigma(\overline{X}) = \frac{1}{n^2}\, n \cdot \sigma^2(X_i)$$

$$\sigma(\overline{X}) = \frac{\sigma^2(X_i)}{n}$$

- ## 2.1) Theoretical Analysis

According to the parameters given for our group:

Expected interarrival times - Uniform Distribution:

$$E[X] = \frac{10+4}{2} = 7$$

Expected renege time - Exponential Distribution with $\lambda = 0.04$

$$E[X] = \frac{1}{\lambda} = \frac{1}{0.04} = 25$$

Expected station #1 service time - $Erlang(k, \mu_1)$ with $k = 3\ and\ \mu_1 = 0.468$:

$$E[X] = \frac{k}{\mu_1} = \frac{3}{0.468} = 6.41$$

Expected station #2 service time - Exponential Distribution with $\lambda = 0.03$

$$E[X] = \frac{1}{\lambda} = \frac{1}{0.03} = 33.3$$

When we increase the number of jobs in the simulation, we see that the experiment values get closer to the values that we calculated theoretically above. That gave us confidence that we calculated random numbers correctly.

The Conservation Equation:
L = λW (Little's Law) (λ = arrival rate) (W = average sojourn time)
Conservation equation holds for almost all queueing systems or subsystems regardless of the number of servers, the queue discipline, or any other special circumstances. As you see from the statistical results, when we increase the sample size, the statistical value of L (average number of jobs in the system per unit time) gets close to the calculated value. Calculated value is equal to the multiplication of $\frac{1}{mean\ interarrival\ time} \cdot mean\ sojourn\ time$

Indeed, in our last experiment with 100,000 jobs, they are exactly the same when rounded to 3 decimal places. For small samples, it is normal that there are differences between calculated value and statistical result since Little's Law is an approximation for long-term measurement. Our result is just another example of the fact that the conservation equation is "not influenced by the arrival process distribution, the service distribution, the service order, or practically anything else".

3) Pseudo Code and the Flowcharts

In the source code, we have used Python's yield keyword for getting availability of the resources and/or determining whether a job reneges before getting into the server instead of the while loops shown below in the pseudo-codes and the flowcharts.

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Arrival 1:

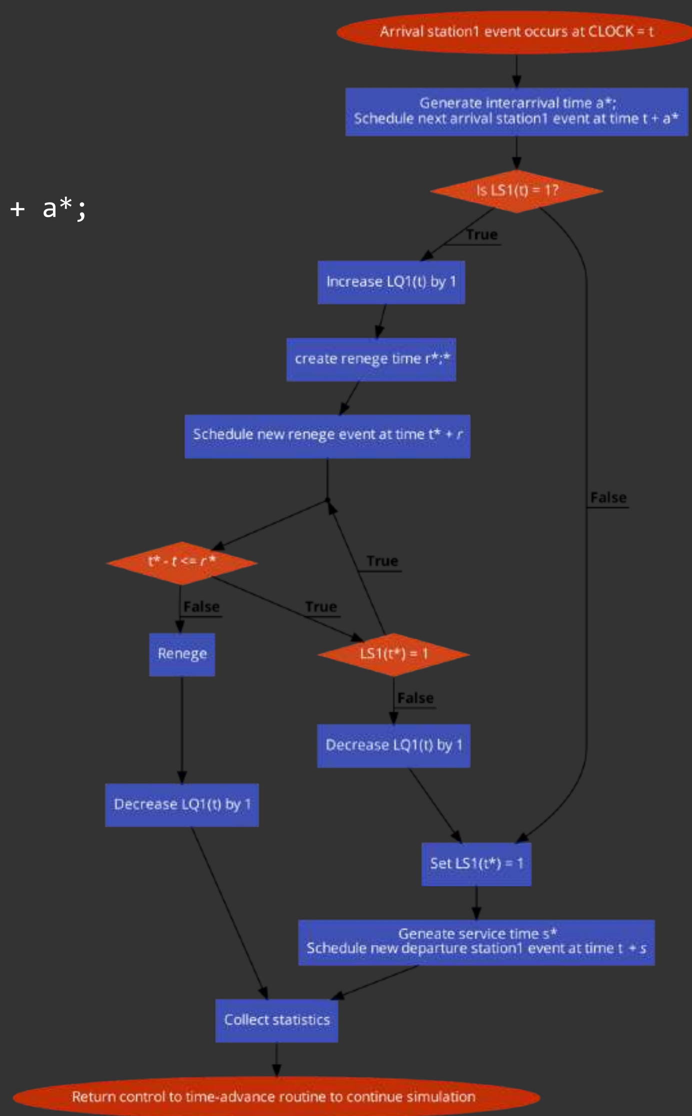
```
Arrival station1 event occurs at CLOCK = t;
Generate interarrival time a*;
Schedule next arrival station1 event at time t + a*;

if(Is LS1(t) = 1?) {
  Increase LQ1(t) by 1;
  create renege time r*;

  Schedule new renege event at time t*+r*;

  while(t* - t <= r*){

    if(LS1(t*) = 1){}
    else{
      Decrease LQ1(t) by 1;
      loop x
      return;
    }
  }
    Renege;
    Decrease LQ1(t) by 1;
}
else{
  x:
  Set LS1(t*) = 1;
  Generate service time s*
  Schedule new departure station1 event at time t* + s*;
}
Collect statistics;
Return control to time-advance routine to continue simulation;
```



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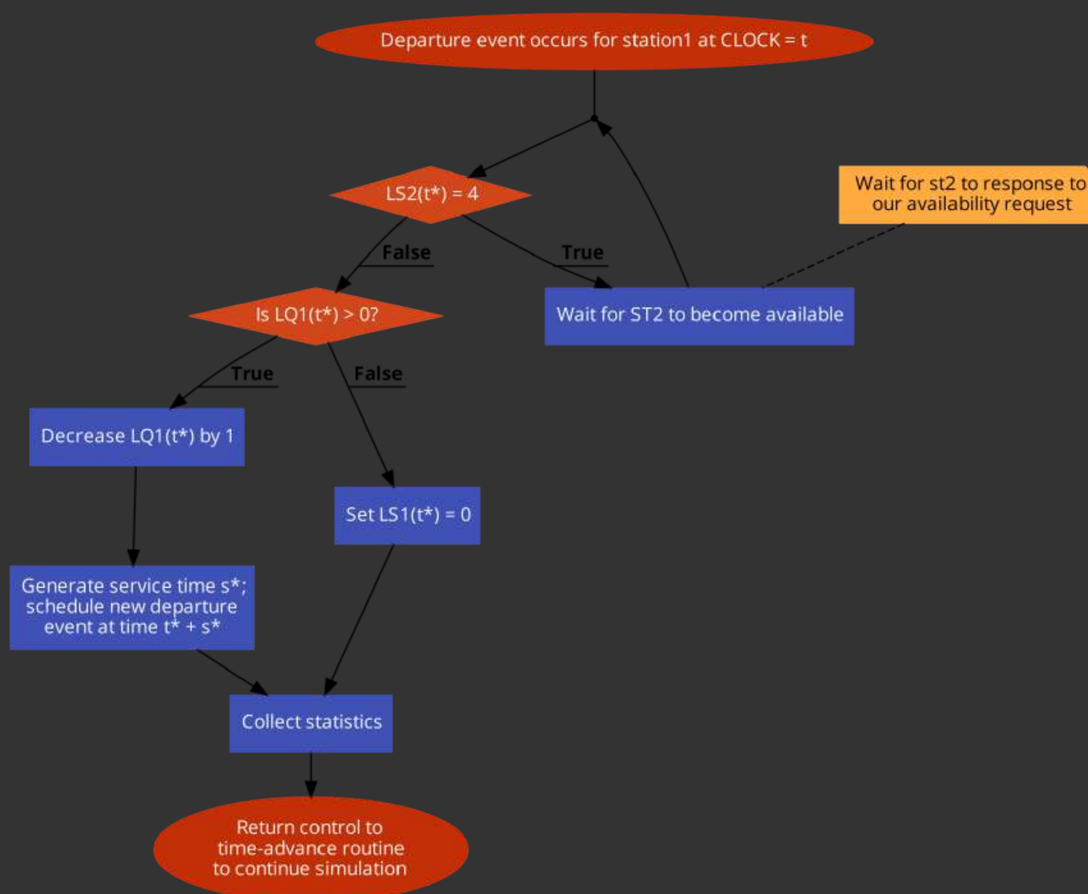
Departure 1:

```
Departure event occurs for station1 at CLOCK = t;
while(LS2(t*) = 4) {
  //Wait for st2 to response to our availability request
  Wait for ST2 to become available
}

if(Is LQ1(t*) > 0?) {
  Decrease LQ1(t*) by 1;
  Generate service time s*;
schedule new departure
event at time t* + s*;
}
else {
  Set LS1(t*) = 0
}
Collect statistics;
Return control to
time-advance routine
to continue simulation
```



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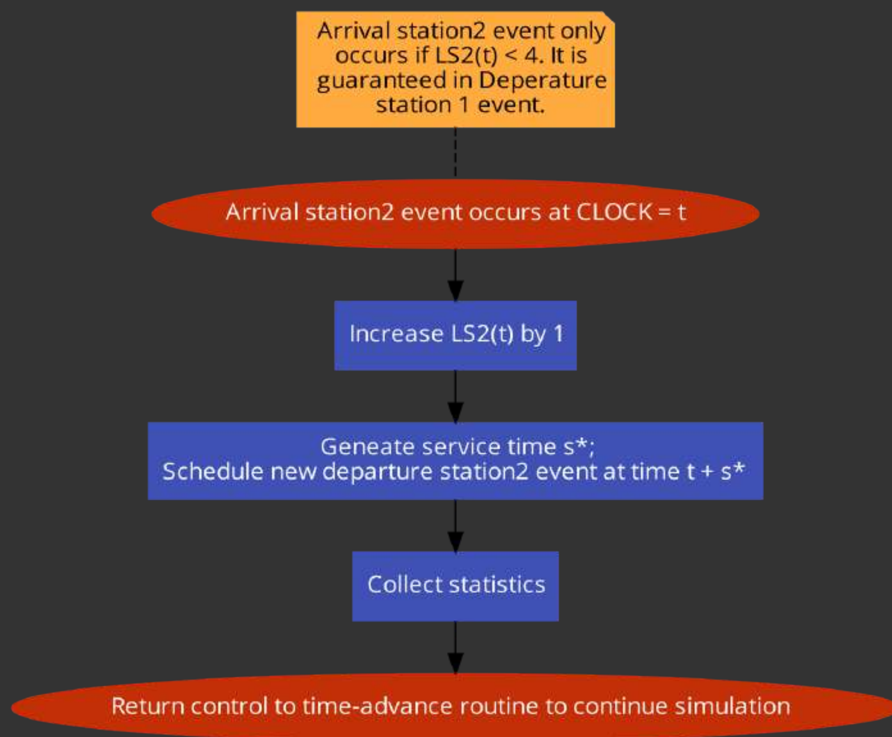
```
Arrival 2:

// Arrival station2 event only occurs if LS2(t) < 4. It is guaranteed in the Departure station 1 event.
Arrival station2 event occurs at CLOCK = t;
  Increase LS2(t) by 1;
  Generate service time s*;
  Schedule new departure station2 event at time t + s*;

Collect statistics;
Return control to time-advance routine to continue simulation;
```



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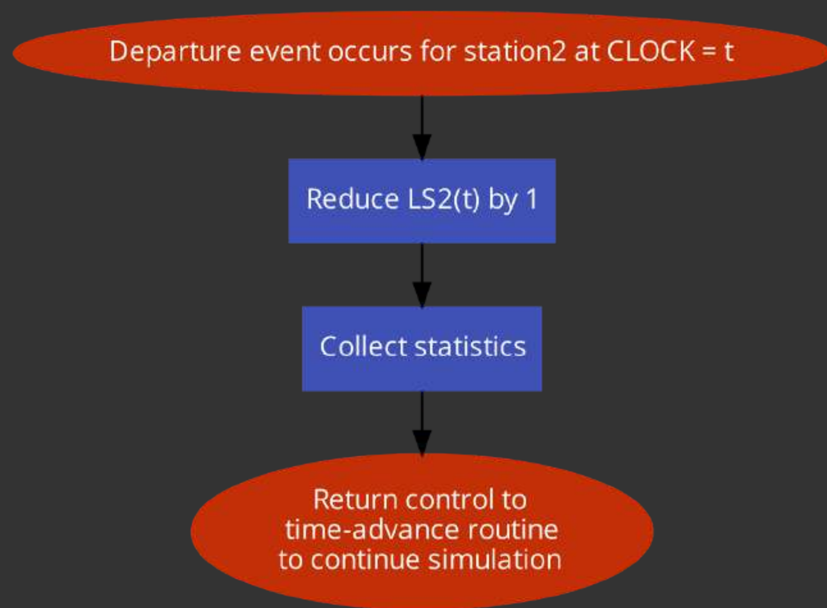
```
Departure 2:

Departure event occurs for station2 at CLOCK = t;

Reduce LS2(t) by 1

Collect statistics;
Return control to
time-advance routine
to continue simulation
```



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4) Hand Simulation:

See the next page.

Random Numbers for Hand Simulation

Interarrival times ~ U[4,10]

Random numbers = 0.39, 0.78, 0.06, 0.79, 0.96, 0.45, 0.09, 0.18, 0.44, 0.31, 0.61, 0.47, 0.65, 0.65, 0.22

Interarrival times = 6.34, 8.68, 4.36, 8.74, 9.76, 6.70, 4.54, 5.08, 6.64, 5.86, 7.66, 6.82, 7.90, 7.90, 5.32

$(R_i \cdot (b-a) + a)$ $a=4$, $b=10$

Random numbers = 0.13, 0.65, 0.55, 0.11, 0.61, 0.26, 0.06, 0.48, 0.58, 0.81

Reneging times = 51.01, 10.77, 14.95, 55.18, 12.36, 33.68, 70.34, 18.35, 13.62, 5.27

(3) (4) (6) (7) (8) (9) (10) (11) (12) (13)

$X_i = -\frac{1}{\lambda} \ln(R_i)$

$\lambda = 0.04$

0.04 → 80.47 (14)

0.78 → 6.21 (15)

Reneging times are generated when a job enters the queue of the first station.

Random numbers = 0.98, 0.92, 0.75, 0.58, 0.65, 0.08, 0.20, 0.31, 0.78, 0.60, 0.15, 0.70, 0.46

(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12) (14)

Station 2 service times = 0.67, 2.78, 9.59, 18.16, 14.36, 84.19, 53.65, 39.04, 8.28, 17.03, 63.24, 11.89, 25.88 (14)

$X_i = -\frac{1}{\lambda} \ln(R_i)$

$\lambda = 0.03$

(There is no service time for job 13 and 15 since they renege)

Station 2 service times are generated when a job enters the second station.

Erlang(3, 0.468)

$k = 3$

$\beta = 0.468$

$X_i = -\frac{1}{\beta} \sum_{j=1}^{k} \ln(R_j)$

Station 1 service times are generated whenever a job enters the server of the first station.

Random numbers - Station 1 service time mapping → Station 1 service times

(1) [0.23, 0.83, 0.26] → 6.42

(2) [0.56, 0.43, 0.22] → 6.28

(3) [0.09, 0.12, 0.98] → 9.72

(4) [0.90, 0.59, 0.17] → 5.14

(5) [0.22, 0.79, 0.06] → 9.75

(6) [0.47, 0.65, 0.64] → 3.49

(7) [0.41, 0.94, 0.20] → 5.48

(8) [0.95, 0.12, 0.55] → 5.92

(9) [0.19, 0.88, 0.55] → 5.10

(10) [0.04, 0.59, 0.60] → 9.1

(11) [0.88, 0.77, 0.50] → 2.31

(12) [0.70, 0.95, 0.42] → 2.73

(14) [0.09, 0.25, 0.79] → 8.61

(No service time for job 13 and 15 since they renege from the queue)

A → arrival to the system
A2 → arrival to the station 2
D1 → departure from station 1
D2 → departure from station 2
R → renege from queue of station 1

Simulation time	FEL	number in queue one	state of server 1	state of server 2	L	R	wait in queue
0	A(1, 6.34)	–	idle	i,i,i,i	0	–	–
6.34 (1 has arrived)	D1(1, 12.76) A(2, 15.02)	–	1	i,i,i,i	1	–	–
12.76 (1 → st2)	A(2, 15.02) D2(1, 13.43)	–	idle	1,i,i,i	1	–	–
13.43 (1 leaves the system)	A(2, 15.02)	–	idle	i,i,i,i	0	–	–
15.02 (2 has arrived)	D1(2, 21.3) A(3, 19.38)	–	2	i,i,i,i	1	–	–
19.38 (3 has arrived)	D1(2, 21.3) A(4, 28.12)	1(3)	2	i,i,i,i	2	–	–
21.3 (2 → st2 3 → st1)	A(4, 28.12) D1(3, 31.02) D2(2, 24.08)	–	3	2,i,i,i	2	–	1.92
24.08 (2 has left the system)	A(4, 28.12) D1(3, 31.02)	–	3	i,i,i,i	1	–	1.92
28.12 (4 has arrived)	D1(3, 31.02) A(5, 37.88)	1(4)	3	i,i,i,i	2	–	1.92
31.02 (4 → st1 3 → st2)	A(5, 37.88) D1(4, 36.16) D2(3, 40.61)	–	4	3,i,i,i	2	–	4.82

Scanned with CamScanner

Simulation time	FEL	number in queue one	State of server 1	state of server 2	L	R	wait in queue
36.16 (4 completed its job in st 1. 4 → st 2)	A(5, 37.88) D2(3, 40.61) D2(4, 54.32)	–	idle	3,4,i,i	2	–	4.82
37.88 (5 has arrived)	D1(5, 47.63) D2(3, 40.61) D2(4, 54.32) A(6, 44.58)	–	5	3,4,i,i	3	–	4.82
40.61 (3 has left the system)	D1(5, 47.63) D2(4, 54.32) A(6, 44.58)	–	5	4,i,i,i	2	–	4.82
44.58 (6 has arrived)	D1(5, 47.63) D2(4, 54.32) A(7, 49.12)	1(6)	5	4,i,i,i	3	–	4.82
47.63 (5 completed its job in station 1. 5 → st 2 6 → st 1)	D2(4, 54.32) D2(5, 61.99) D1(6, 51.12) A(7, 49.12)	–	6	4,5,i,i	3	–	7.87
49.12 (7 has arrived)	D2(4, 54.32) D2(5, 61.99) D1(6, 51.12) A(8, 54.20)	1(7)	6	4,5,i,i	4	–	7.87
51.12 (6 completed its job in st 1 6 → st 2 7 → st 1)	D2(4, 54.32) D2(5, 61.99) D2(6, 135.31) D1(7, 56.6) A(8, 54.20)	–	7	4,5,6,i	4	–	9.87
54.20 (8 has arrived)	D2(4, 54.32) D2(5, 61.99) D2(6, 135.31) D1(7, 56.6) A(9, 60.84)	1(8)	7	4,5,6,i	5	–	9.87

Simulation time	FEL	number in queue one	State of server 1	State of server 2	L	R	wait in queue
54.32 (4 has left the system)	D2(5, 61.99) D2(6, 135.31) D1(7, 56.6) A(9, 60.84)	1 (8)	7	i, 5, 6, i	4	-	9.99
56.60 (7 → s+2 8 → s+1)	D2(5, 61.99) D2(6, 135.31) D2(7, 110.25) D1(8, 62.52) A(9, 60.84)	-	8	7, 5, 6, i	4	-	12.27
60.84 (9 has arrived)	D2(5, 61.99) D2(6, 135.31) D2(7, 110.25) D1(8, 62.52) A(10, 66.70)	1(9)	8	7, 5, 6, i	5	-	12.27
61.99 (5 has left the system)	D2(6, 135.31) D2(7, 110.25) D1(8, 62.52) A(10, 66.70)	1 (9)	8	7, i, 6, i	4	-	13.42
62.52 (8 completed its job in s+1. 8 → s+2 9 → s+1)	D2(6, 135.31) D2(7, 110.25) D2(8, 101.56) A(10, 66.70) D1(9, 67.62)	-	9	7, 8, 6, i	4	-	13.95
66.70 (10 has arrived)	D2(6, 135.31) D2(7, 110.25) D2(8, 101.56) A(11, 74.36) D1(9, 67.62)	1 (10)	9	7, 8, 6, i	5	-	13.95
67.62 (9 → s+2 10 → s+1)	D2(6, 135.31) D2(7, 110.25) D2(8, 101.56) A(11, 74.36) D2(9, 75.9) D1(10, 76.72)	-	10	7, 8, 6, 9	5	-	14.87

Simulation time	FEL	number in queue one	State of server 1	State of server 2	L	R	wait in queue
74.36 (11 has arrived)	D2(6,135.31) D2(7,110.25) D2(8,101.56) A(12,81.18) D2(9,75.9) D1(10,76.72)	1 (11)	10	7,8,6,9	6	-	14.87
75.9 (9 has left the system)	D2(6,135.31) D2(7,110.25) D2(8,101.56) A(12,81.18) D1(10,76.72)	1 (11)	10	7,8,6,i	5	-	16.41
76.72 (10→st2 11→st1)	D2(6,135.31) D2(7,110.25) D2(8,101.56) D2(10,93.75) D1(11,79.03) A(12,81.18)	-	11	7,8,6,10	5	-	17.23
79.03 (11 completed its work in st1, blocks it till 10 leaves)	D2(6,135.31) D2(7,110.25) D2(8,101.56) D2(10,93.75) A(12,81.18) A2(11,93.75)	-	idle (blocked by 11)	7,8,6,10	5	-	17.23
81.18 (12 has arrived)	D2(6,135.31) D2(7,110.25) D2(8,101.56) D2(10,93.75) A(13,89.08) A2(11,93.75)	1 (12)	idle (blocked by 11)	7,8,6,10	6	-	17.23
89.08 (13 has arrived)	D2(6,135.31) D2(7,110.25) D2(8,101.56) D2(10,93.75) A2(11,93.75) A(14,96.98) R(13,94.35)	2 (12,13)	idle (blocked by 11)	7,8,6,10	7	-	25.13

Simulation time	FEL	number in queue one	State of server 1	State of server 2	L	R	wait in queue
93.75 (10 has left the system 11 → st2 12 → st1)	D2(6, 135.31) D2(7, 110.25) D2(8, 101.56) D2(11, 156.99) D1(12, 96.48) A(14, 96.98) R(13, 94.35)	1(13)	12	7,8,6,11	6	-	29.80 + 4.67
94.35 (13 reneges)	D2(6, 135.31) D2(7, 110.25) D2(8, 101.56) D2(11, 156.99) D1(12, 96.48) A(14, 96.98)	-	12	7,8,6,11	5	1	30.4 + 4.67
96.48 (12 completes its job in st1, blocks st1)	D2(6, 135.31) D2(7, 110.25) D2(8, 101.56) D2(11, 156.99) A(14, 96.98) A2(12, 101.56)	-	idle (blocked by 12)	7,8,6,11	5	1	30.4 + 4.67
96.98 (14 has arrived)	D2(6, 135.31) D2(7, 110.25) D2(8, 101.56) D2(11, 156.99) A2(12, 101.56) A(15, 102.30)	1(14)	idle (blocked by 12)	7,8,6,11	6	1	30.4 + 4.67
101.56 (8 has left the system 12 → st2 14 → st1)	D2(6, 135.31) D2(7, 110.25) D2(11, 156.99) D2(12, 113.45) A(15, 102.30) D1(14, 110.17)	-	14	7,12,6,11	5	1	34.98 + 4.67
102.30 (15 has arrived)	D2(6, 135.31) D2(7, 110.25) D2(11, 156.99) D2(12, 113.45) D1(14, 110.17) R(15, 108.51)	1(15)	14	7,12,6,11	6	1	34.98 + 4.67

Simulation time	FEL	number in queue one	state of server 1	State of server 2	L	R	wait in queue
108.51 (15 reneges)	D2 (6, 135.31) D2 (7, 110.25) D2 (11, 156.99) D2 (12, 113.45) D1 (14, 110.17)	–	14	7, 12, 6, 11	5	2	41.19 +4.67
110.17 (14 has completed its job in st 1)	D2 (6, 135.31) D2 (7, 110.25) D2 (11, 156.99) D2 (12, 113.45) A2 (14, 110.25)	–	idle (blocked by 14)	7, 12, 6, 11	5	2	41.19 +4.67
110.25 (7 has left the system 14 → st 2)	D2 (6, 135.31) D2 (11, 156.99) D2 (12, 113.45) D2 (14, 136.13)	–	idle	14, 12, 6, 11	4	2	41.19 +4.67
113.45 (12 has left the system)	D2 (6, 135.31) D2 (11, 156.99) D2 (14, 136.13)	–	idle	14, i, 6, 11	3	2	41.19 +4.67
135.31 (6 has left the system)	D2 (11, 156.99) D2 (14, 136.13)	–	idle	14, i, i, 11	2	2	41.19 +4.67
136.13 (14 has left the system)	D2 (11, 156.99)	–	idle	i, i, i, 11	1	2	41.19 +4.67
156.99 (11 has left the system)	–	–	idle	i, i, i, i	0	2	41.19 +4.67

total waiting time = 41.19 + 4.67 = 45.86

	Arrival time	Departure from the system	Total time	Waiting time	
1	6.34	13.43	7.09	0	
2	15.02	24.08	9.06	0	
3	19.38	40.61	21.23	1.92	
4	28.12	54.32	26.20	2.90	
5	37.88	61.99	24.11	0	
6	44.58	135.31	90.73	3.05	
7	49.12	110.25	61.13	2.00	
8	54.20	101.56	47.36	2.40	
9	60.84	75.90	15.06	1.68	
10	66.70	93.75	27.05	0.92	
11	74.36	156.99	82.63	2.36	
12	81.18	113.45	32.27	12.57	
13	89.08	94.35*	5.27	5.27*	
14	96.98	136.05	39.15	4.58	
15	102.30	108.51*	6.21	6.21	

Sum = 494.55 mean sojourn = $\frac{494.55}{15} = 32.97$ //

average reneging rate = $\frac{2}{15} = 0.13$ //

proportion of jobs completed = $1 - \frac{2}{15} = 0.86$ //

proportion of time the server is blocked:

$$\frac{(93.75 - 79.03) + (101.56 - 96.48) + (110.25 - 110.17)}{156.99}$$

$= 0.126$ //