IE 310 - Homework III

January 21, 2022

1 Answers

1)

	x_1	x_2	x_3	x_4	x_5	x_6	
x_2	g	e	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
x_6	2	0	0	-2	1	f	20
	a	b	0	c	d	0	$-z-1350$

(a) In order for x_2 to be in basis, e should be equal to 1. In order for x_6 to be in basis, f should be equal to 1. Coefficients of basic variables should be zero in the objective function, therefore $b = 0$.

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

$$A^* = \begin{bmatrix} g & 1 & 0 & \frac{1}{2} & -\frac{1}{4} & 0 \\ \frac{3}{2} & 0 & 1 & 0 & \frac{1}{2} & 0 \\ 2 & 0 & 0 & -2 & 1 & 1 \end{bmatrix}$$

$$A^* = B^{-1} \cdot A$$

$$\text{Therefore;} \quad g = \frac{1}{2} - \frac{3}{4} = -\frac{1}{4}$$

$$
\begin{aligned}
c &= \begin{bmatrix} -3 & -2 & -5 & 0 & 0 & 0 \end{bmatrix} \\
c_B &= \begin{bmatrix} -2 & -5 & 0 \end{bmatrix} \\
c^* &= c - c_B \cdot A^* \\
c_B \cdot A^* &= \begin{bmatrix} -7 & -2 & -5 & -1 & -2 & 0 \end{bmatrix} \\
c - c_B \cdot A^* &= \begin{bmatrix} 4 & 0 & 0 & 1 & 2 & 0 \end{bmatrix} \\
&\text{Therefore;} \quad a = 4,\ b = 0,\ c = 1,\ d = 2
\end{aligned}
$$

To summarize, values of the unknowns are:

$$
\begin{aligned}
a &= -4 \\
b &= 0 \\
c &= -1 \\
d &= -2 \\
e &= 1 \\
f &= 1 \\
g &= -\frac{1}{4}
\end{aligned}
$$

(b)

$$
\begin{aligned}
b^* &= \begin{bmatrix} 100 \\ 230 \\ 20 \end{bmatrix} \\
B^{-1} &= \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \\
b^* &= B^{-1} * b \\
\begin{bmatrix} 100 \\ 230 \\ 20 \end{bmatrix} &= \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \\
400 &= 2 \cdot b_1 - b_2 \\
460 &= b_2 \\
20 &= -2 \cdot b_1 + b_2 + b_3 \\
b_1 &= 430 \\
b_2 &= 460 \\
b_3 &= 420
\end{aligned}
$$

(c) Optimal dual solution can be found from values under slack variables.

$$
x_4 = 1\ (y_1),\quad x_5 = 2\ (y_2)\quad x_6 = 0\ (y_3)
$$

To convert the problem into dual, first multiply the constraints with (-1)

They have to be "greater than or equal to" in the min problem.

$$\max v = -430y_1 - 460y_2 - 420y_3$$

$$v = -430 \cdot (1) - 460 \cdot (2) - 420 \cdot (0) = -1350$$

Optimal value of the objective function: $v = -1350$

Optimal dual solution $= (1, 2, 0)$

(d) b^* should stay non-negative, otherwise basis changes.

$$\begin{aligned} b^* &= B^{-1} * b \\ &= \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 430 - \lambda \\ 460 \\ 420 \end{bmatrix} \\ &= \begin{bmatrix} 100 - \frac{\lambda}{2} \\ 230 \\ 20 + 2 \cdot \lambda \end{bmatrix} \end{aligned}$$

$$100 - \frac{\lambda}{2} \geq 0 \longrightarrow \lambda \leq 200$$

We can decrease the b_1 by at most 200. Otherwise, optimal basis changes.

(e) For multiple optima, coefficient of a nonbasic variable should be zero and RHS in the final tableau should be feasible.

$$\begin{aligned} c^{*(1)} &= c^{(1)} - c_B \cdot A^{*(1)} \\ c^{*(1)} &= c^{(1)} - c_B \cdot B^{-1} \cdot A^{(1)} \\ 0 &= c_1 - \begin{bmatrix} -2 & -5 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ -2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \\ 0 &= c_1 - \begin{bmatrix} -2 & -5 & 0 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{4} \\ \frac{3}{2} \\ 2 \end{bmatrix} \\ 0 &= c_1 + 7 \\ c_1 &= -7 \end{aligned}$$

For the coefficient of x_1, $c_1 = -7$, there are multiple optima. Since other coefficients are not modified, $c^* \geq 0$, therefore basis doesn't change. Final tableau is still valid. We can switch from one end point to another by introducing x_1 to basic variables.

2) To solve the problem, objective function is converted to minimization form:

$$\min \quad -z = -3x_1 - 7x_2 - 5x_3$$

(a) Profit of the type 1 dessert represents the coefficient of x_1 in the objective function. In the final tableau, x_1 is not a basic variable. As long as $c^*(1) \geq 0$, optimal basis doesn't change.

$$
\begin{aligned}
c^{*(1)} &= c^{(1)} - c_B \cdot A^{*(1)} \\
c^{*(1)} &= c_1 - \begin{bmatrix} -5 & -7 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \\
&= c_1 + 6
\end{aligned}
$$

For $c_1 \geq -6$ ($0 \leq$ profit for type $1 \leq 6$), optimal basis doesn't change. If the profit for Type 1 dessert was 7 TL, then c_1 would be -7. Since it is less than -6, new pivot operations are needed. Continue the simplex iteration to find the new optimal solution. Basic variables are listed on the left side of the tableau in each iteration. Pivot elements are marked with asterisk (*).

	x_1	x_2	x_3	s_1	s_2	
x_3	$\frac{1}{2}$	0	1	$\frac{3}{2}$	$-\frac{1}{2}$	25
x_2	$\frac{1}{2}$*	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	25
	-1	0	0	4	1	$300 - z$

	x_1	x_2	x_3	s_1	s_2	
x_3	0	1	1	2	-1	0
x_1	1	2	0	-1	1	50
	0	2	0	3	2	$350 - z$

New optimal value of the objective function is 350, and optimal solution is:

$$x_1 = 50, \quad x_2 = 0, \quad x_3 = 0, \quad s_1 = 0, \quad s_2 = 0$$

(b) Profit of the type 2 dessert represents the coefficient of x_2 in the objective function. x_2 is in the basis and require detailed analysis. As long as $c^* \geq 0$, optimal basis doesn't change.

$$
\begin{aligned}
c^* &= c - c_B \cdot A^* \\
c^* &= \begin{bmatrix} -3 & c_2 & -5 & 0 & 0 \end{bmatrix} - \begin{bmatrix} -5 & c_2 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 0 & 1 & \frac{3}{2} & -\frac{1}{2} \\ \frac{1}{2} & 1 & 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \\
&= \begin{bmatrix} -3 & c_2 & -5 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \frac{c_2-5}{2} & c_2 & -5 & \frac{-15-c_2}{2} & \frac{5+c_2}{2} \end{bmatrix} \\
&= \begin{bmatrix} \frac{-c_2-1}{2} & 0 & 0 & \frac{15+c_2}{2} & \frac{-5-c_2}{2} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
-c_2 - 1 \geq 0 &\longrightarrow c_2 \leq 1 \\
15 + c_2 \geq 0 &\longrightarrow c_2 \geq -15 \\
-5 - c_2 \geq 0 &\longrightarrow c_2 \leq -5
\end{aligned}
$$

Therefore; $-15 \leq c_2 \leq -5$

For $-15 \leq c_2 \leq -$ ($5 \leq$ profit for type 2 ≤ 15), optimal basis doesn't change. However, since x_2 is in the basis, optimal solution changes. If the profit for the type 2 dessert was 13 TL, then c_2 would be -13. Therefore basis doesn't change. However, optimal value of the objective function should be calculated.

$$\begin{aligned}
z_0^* &= z_0 - c_B \cdot b^* \\
z_0^* &= 0 - \begin{bmatrix} -5 & -13 \end{bmatrix} \cdot \begin{bmatrix} 25 \\ 25 \end{bmatrix} \\
z_0^* &= 0 - (-450) \\
z_0^* &= 450
\end{aligned}$$

New optimal value of the objective function is 450, and optimal solution stays the same:

$$x_1 = 0, \quad x_2 = 25, \quad x_3 = 25, \quad s_1 = 0, \quad s_2 = 0$$

(c) In order for current basis to remain optimal when there is a change in the right hand side b_i, b^* must stay non-negative. Otherwise dual simplex algorithm is used to solve the problem and basis changes.

$$\begin{aligned}
b^* &= B^{-1} * b \\
b^* &= \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} * \begin{bmatrix} b_1 \\ 100 \end{bmatrix} \\
b^* &= \begin{bmatrix} \frac{3b_1-100}{2} & \frac{-b_1+100}{2} \end{bmatrix} \\
&3b_1 - 100 \geq 0 \longrightarrow b_1 \geq \frac{100}{3} \\
&-b_1 + 100 \geq 0 \longrightarrow b_1 \leq 100 \\
&\text{Therefore; } \frac{100}{3} \leq b_1 \leq 100
\end{aligned}$$

For values $\frac{100}{3} \leq b_1 \leq 100$, current basis remains optimal. Assuming amount of packs of sugar is integer, amount of packs should be greater than 33 and less than or equal to 100.

(d) 60 stays in the allowed range. Basis doesn't change, however optimal solution changes.

$$\begin{aligned}
z_0^* &= z_0 - c_B \cdot b^* \\
z_0^* &= z_0 - c_B \cdot B^{-1} \cdot b \\
z_0^* &= z_0 - \begin{bmatrix} -5 & -7 \end{bmatrix} \cdot \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 60 \\ 100 \end{bmatrix} \\
z_0^* &= 0 - \begin{bmatrix} -5 & -7 \end{bmatrix} \cdot \begin{bmatrix} 40 \\ 20 \end{bmatrix} \\
z_0^* &= 0 - (-340)
\end{aligned}$$

New optimal value of the objective function is 340, and optimal solution stays the same:

$$x_1 = 0, \quad x_2 = 20, \quad x_3 = 40, \quad s_1 = 0, \quad s_2 = 0$$

If there were 30 packs of sugar, optimal basis would change. Then we have to continue with the dual simplex iterations. First calculate the b^* and z_0^*.

$$b^* = B^{-1} \cdot b$$

$$b^* = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 30 \\ 100 \end{bmatrix}$$

$$b^* = \begin{bmatrix} -5 \\ 35 \end{bmatrix}$$

$$z_0^* = z_0 - c_B \cdot b^*$$

$$z_0^* = z_0 - \begin{bmatrix} -5 & -7 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ 35 \end{bmatrix}$$

$$z_0^* = 0 - (-220)$$

$$z_0^* = 220$$

Basic variables are listed on the left side of the tableau in each iteration. Pivot elements are marked with asterisk (*).

	x_1	x_2	x_3	s_1	s_2	
x_3	$\frac{1}{2}$	0	1	$\frac{3}{2}$	$-\frac{1}{2}*$	-5
x_2	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	35
	3	0	0	4	1	$220 - z$

	x_1	x_2	x_3	s_1	s_2	
s_2	-1	0	-2	-3	1	10
x_2	1	1	1	1	0	30
	4	0	2	7	0	$210 - z$

New optimal value of the objective function is 210, and optimal solution is given below:

$$x_1 = 0, \quad x_2 = 30, \quad x_3 = 0, \quad s_1 = 0, \quad s_2 = 10$$

(e) Upon modifying the inital matrix A, for basis to remain optimal, $c^{*(n)} \geq 0$ must be ensured.

$$c^{*(1)} = c_1 - c_B \cdot B^{-1} \cdot A^1$$

$$c^{*(1)} = -3 - \begin{bmatrix} -5 & -7 \end{bmatrix} \cdot \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$c^{*(1)} = -3 - \begin{bmatrix} -5 & -7 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$$

$$c^{*(1)} = -3 - (-\frac{5}{2})$$

$$c^{*(1)} = -\frac{1}{2}$$

Basis changes. $c^{*(1)}$ and $A^{*(1)}$ are already calculated. Basic variables are listed on the left side of the tableau in each iteration. Pivot elements are marked with asterisk (*).

	x_1	x_2	x_3	s_1	s_2	
x_3	$\frac{1}{2}$*	0	1	$\frac{3}{2}$	$-\frac{1}{2}$	25
x_2	0	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	25
	$-\frac{1}{2}$	0	0	4	1	$300 - z$

	x_1	x_2	x_3	s_1	s_2	
x_1	1	0	2	3	-1	50
x_2	0	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	25
	0	0	1	$\frac{11}{2}$	$\frac{1}{2}$	$325 - z$

New optimal value of the objective function is 325, and optimal solution is given below:

$$x_1 = 50, \quad x_2 = 25, \quad x_3 = 0, \quad s_1 = 0, \quad s_2 = 0$$

Indeed, now Şölen should make Type 1 dessert.

(f) Upon adding a new variable, in order for basis to remain optimal, $c^{*(n+1)} \geq 0$ must be ensured.

$$c^{*(6)} = c_6 - c_B \cdot B^{-1} \cdot A^{n+1}$$
$$c^{*(6)} = -17 - \begin{bmatrix} -5 & -7 \end{bmatrix} \cdot \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$
$$c^{*(6)} = -17 - \begin{bmatrix} -5 & -7 \end{bmatrix} \cdot \begin{bmatrix} \frac{5}{2} \\ \frac{1}{2} \end{bmatrix}$$
$$c^{*(6)} = -17 - (-16)$$
$$c^{*(6)} = -1$$

Basis changes. $c^{*(6)}$ and $A^{*(6)}$ are already calculated. Basic variables are listed on the left side of the tableau in each iteration. Pivot elements are marked with asterisk (*).

	x_1	x_2	x_3	s_1	s_2	x_4	
x_3	$\frac{1}{2}$	0	1	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$*	25
x_2	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	25
	3	0	0	4	1	-1	$300 - z$

	x_1	x_2	x_3	s_1	s_2	x_4	
x_4	$\frac{1}{5}$	0	$\frac{2}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	1	10
x_2	$\frac{2}{5}$	1	$-\frac{1}{5}$	$-\frac{8}{10}$	$\frac{6}{10}$	0	20
	$\frac{16}{5}$	0	$\frac{2}{5}$	$\frac{23}{5}$	$\frac{4}{5}$	0	$310 - z$

New optimal value of the objective function is 310, and optimal solution is given below:

$$x_1 = 0, \quad x_2 = 20, \quad x_3 = 0, \quad x_4 = 10, \quad s_1 = 0, \quad s_2 = 0$$

Indeed, Şölen should manufacture Type 4 dessert.

3) (a) First check the constraints:

$$\begin{aligned} 8\cdot(3)+3\cdot(1)-2\cdot(5)\le 18 &\longrightarrow 17\le 18\\ 2\cdot(3)-7\cdot(1)+(5)\le 4 &\longrightarrow 4\le 4\\ 1\cdot(3)+1\cdot(1)+2\cdot(5)\le 14 &\longrightarrow 14\le 14 \end{aligned}$$

Constraints are satisfied.

$$b - Ax_0 = \begin{bmatrix} 1\\ 0\\ 0 \end{bmatrix}$$

By Complementary slackness, we know that

$$A^T y_0 - c = \begin{bmatrix} 0\\ s_2\\ 0\\ s_4\\ 0 \end{bmatrix}$$

$$y_0 = (0, y_2, y_3)$$

LP model can be simplified by putting the y_0 values. If, $A^T y_0 - c$ turns out to be the same as we expect, then given point is indeed an optimal solution. Dual problem:

$$\begin{aligned} \min\ \ v = 18y_1 + 4y_2 + 14y_3&\\ 8y_1 + 2y_2 + y_3 &\ge 5\\ -2y_1 + 4y_2 + 3y_3 &\ge 16\\ 3y_1 - 7y_2 + y_3 &\ge -4\\ +3y_2 - y_3 &\ge -1\\ -2y_1 + y_2 + 2y_3 &\ge 7\\ y_1, y_2, y_3 &\ge 0 \end{aligned}$$

Set y_0 to zero and put surplus variables.

$$\begin{aligned} 2y_2 + y_3 &= 5 \quad (1)\\ 4y_2 + 3y_3 - s_2 &= 16\\ -7y_2 + y_3 &= -4 \quad (2)\\ -y_3 - s_4 &= -1\\ y_2 + 2y_3 &= 7 \quad (3) \end{aligned}$$

Using (1), (2) and (3) we get $y_2 = 1, y_3 = 3$. Then $s_2 = -3$ and $s_4 = -2$. Surplus variables do not satisfy the non-negativity constraints. To find the optimal solution, Simplex iterations are performed. Basic variables are listed on the left side of the tableau in each iteration. Pivot elements are marked with asterisk (*).

$$\min \; -z = -5x_1 - 16x_2 + 4x_3 + x_4 - 7x_5$$

	x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	
s_1	8	-2	3	0	-2	1	0	0	18
s_2	2	$4*$	-7	3	1	0	1	0	4
s_3	1	3	1	-1	2	0	0	1	14
	-5	**-16**	4	1	-7	0	0	0	$0 - z$

	x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	
s_1	9	0	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	1	$\frac{1}{2}$	0	20
x_2	$\frac{1}{2}$	1	$-\frac{7}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	0	1
s_3	$-\frac{1}{2}$	0	$\frac{25}{4}*$	$-\frac{13}{4}$	$\frac{5}{4}$	0	$-\frac{3}{4}$	1	11
	3	0	**-24**	13	-3	0	4	0	$16 - z$

	x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	
s_1	$\frac{224}{25}$	0	0	$\frac{62}{50}$	$-\frac{7}{5}$	1	$\frac{22}{50}$	$\frac{2}{25}$	$\frac{522}{25}$
x_2	$\frac{9}{25}$	1	0	$-\frac{4}{25}$	$\frac{3}{5}$	0	$\frac{1}{25}$	$\frac{7}{25}$	$\frac{102}{25}$
x_3	$-\frac{2}{25}$	0	1	$-\frac{13}{25}$	$\frac{1}{5}$	0	$-\frac{3}{25}$	$\frac{4}{25}$	$\frac{44}{25}$
	$\frac{27}{25}$	0	0	$\frac{13}{25}$	$\frac{9}{5}$	0	$\frac{28}{25}$	$\frac{96}{25}$	$\frac{1456}{25} - z$

Given point is not an optimal solution. Optimal value of the objective function is 58.24, and optimal solution is as follows:

$$x_1 = 0, \quad x_2 = 4.08, \quad x_3 = 1.76, \quad x_4 = 0, \quad x_5 = 0, \quad s_1 = 20.88, \quad s_2 = 0$$

(b) First check the constraints. Multiply each constraint with (-1) to turn them into "less than or equal to" inequalities:

$$\begin{aligned} -x_1 - 2x_2 + x_3 - x_4 &\leq 0 \\ -2x_1 - 3x_2 - x_3 + x_4 &\leq -3 \end{aligned}$$

$$\begin{aligned} -1 \cdot (1) + 1 \cdot (1) \leq 0 &\longrightarrow 0 \leq 0 \\ -2 \cdot (1) - 1 \cdot (1) \leq -3 &\longrightarrow -3 \leq -3 \end{aligned}$$

Constraints are satisfied.

$$b - Ax_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

By Complementary slackness, we know that

$$A^T y_0 - c = \begin{bmatrix} 0 \\ s_2 \\ 0 \\ s_4 \end{bmatrix}$$

$$y_0 = (y_2, y_3)$$

LP model can be simplified by putting the y_0 values. If, $A^T y_0 - c$ turns out to be the same as we expect, then given point is indeed an optimal solution. Dual problem:

$$\begin{aligned} \min \ v = -3y_2 & \\ -y_1 - 2y_2 &\geq 5 \\ -2y_1 - 3y_2 &\geq 8 \\ y_1 - y_2 &\geq 4 \\ -y_1 + y_2 &\geq 2 \\ y_1, y_2 &\geq 0 \end{aligned}$$

Put surplus variables.

$$\begin{aligned} -y_1 - 2y_2 &= 5 \quad (1) \\ -2y_1 - 3y_2 + s_2 &= 8 \\ y_1 - y_2 &= 4 \quad (2) \\ -y_1 + y_2 + s_4 &= 2 \end{aligned}$$

Using (1) and (2), $y_1 = 1, y_2 = -3$ and then $s_2 = 1$ and $s_4 = 6$. Variables don't satisfy the non-negativity constraints. Given point is **not optimal**. Before proceeding to solve the problem, observe the constraints. It can be easily seen that problem is unbounded due to the non-binding constraints. For example, for any given x_2, x_3, x_4 values, x_1 can be incremented as much as wanted to maximize the objective function and still it satisfies the constraints. (max) problem is **unbounded** and therefore dual (min) problem has no feasible solutions.

(c) First check the constraints.

$$\begin{aligned} 7 \cdot (60) + 2 \cdot (4) - 16 \cdot (25) \leq 28 &\longrightarrow 28 \leq 28 \\ 3 \cdot (60) + (4) - 7 \cdot (25) \leq 9 &\longrightarrow 9 \leq 9 \end{aligned}$$

$$b - Ax_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

By Complementary slackness, we know that

$$A^T y_0 - c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y_0 = (y_1, y_2)$$

Dual problem is constructed to proceed.

$$\begin{aligned} \min\ \ v = 28y_1 + 9y_2 \\ 7y_1 + 3y_2 \geq 10 \\ 2y_1 + y_2 \geq 3 \\ -16y_1 - 7y_2 \geq -23 \\ y_1, y_2 \geq 0 \end{aligned}$$

Put surplus variables.

$$\begin{aligned} 7y_1 + 3y_2 &= 10 \\ 2y_1 + y_2 &= 3 \\ -16y_1 - 7y_2 &= -23 \end{aligned}$$

Therefore $y_1 = 1$ and $y_2 = 1$. It satisfies the non-negativity constraints. Therefore given point is an **optimal solution**.
Although not necessary, it can be shown that this LP problem has multiple optima.

4) Linear Programming Model is constructed below. Variables are given in the order of foods (oats, chicken, eggs, milk, kuchen, beans).

$$\begin{aligned} \text{Variables: } & x_1, x_2, x_3, x_4, x_5, x_6 \text{ (number of portions)} \\ \text{Objective function: } & \text{minimize } \ z = 30x_1 + 240x_2 + 130x_3 + 90x_4 + 200x_5 + 60x_6 \\ \text{Constraints: } & \\ \text{Energy (Kcals): } & 110x_1 + 205x_2 + 160x_3 + 160x_4 + 420x_5 + 260x_6 \geq 2000 \\ \text{Protein (g): } & 4x_1 + 32x_2 + 13x_3 + 8x_4 + 4x_5 + 14x_6 \geq 55 \\ \text{Calcium (mg): } & 2x_1 + 12x_2 + 54x_3 + 285x_4 + 22x_5 + 80x_6 \geq 800 \\ \text{Limit (portions/day): } & x_1 \leq 4 \\ & x_2 \leq 3 \\ & x_3 \leq 2 \\ & x_4 \leq 8 \\ & x_5 \leq 2 \\ & x_6 \leq 2 \\ \text{Further : } & x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{aligned}$$

Optimal solution is obtained at:

$$x_1 = 4.0,\ x_2 = 0,\ x_3 = 0,\ x_4 = 2.0877948,\ x_5 = 1.6808401,\ x_6 = 2.0$$

$$z = \$\ 764.0695$$

Notice that numbers represent the portions. Same answer can be stated as shown below:

$$\text{Oats} = 28 * 4.0 = 112 \text{ gram}$$

$$\text{Chicken} = 0 \text{ gram}$$

$$\text{Eggs} = 0 \text{ egg}$$

$$\text{Milk} = 237 * 2.0877948 = 494.8073676 \text{ cc}$$

$$\text{Kuchen} = 170 * 1.6808401 = 285.742817 \text{ gram}$$

$$\text{Beans} = 520 \text{ gram}$$