CH #2

Date

- Set Representation :-
  - → Tabular ⇒ $A = \{1, 2, 3, 4, 5\}$
  - → Descriptive ⇒ Set of first five natural numbers
  - → Set Builder Notation ⇒ $\{x \in N \mid x \le 5\}$

$A \subseteq B$ "A is a subset of B"

Subset ↵ ↳ Superset

$A \subset B$ "If A = B and at least one element of B is not in A"

Proper Subset

- Power Set :-

If A has "n" elements then the Power set of A will have "2^n" elements

$A = \{0, 1, 2\} = 2^3 = 8$

$P(A) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$

- Cartesian Product :-

$A = \{a, b\}$, $B = \{1, 2, 3\}$

$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

- Cardinality :-

Number of elements in a set $A = \{1, 2, 3\}$ $|A| = 3$

- Union :- $= (+)$ or $(\cup)$
- Intersection :- $= (\times)$ or $(\cap)$
- Difference :- $= (\sim p \to q)$

$A = \{a, b, c, d\}$ $B = \{c, d, f, g\}$

$A - B = \{a, b\}$

- Complement :- $= (\sim)$

$A' = U - A = \{\text{Everything}\} - \{a, b, c, d\}$

IMPORTANT LAWS :-

$A - B = A \cap \sim B \,/\, A \cap B'$

$C \in A$ & $C \in B$ then $C \subseteq A \cap B$

$A \in C$ & $B \in C$ then $A \cup B \subseteq C$

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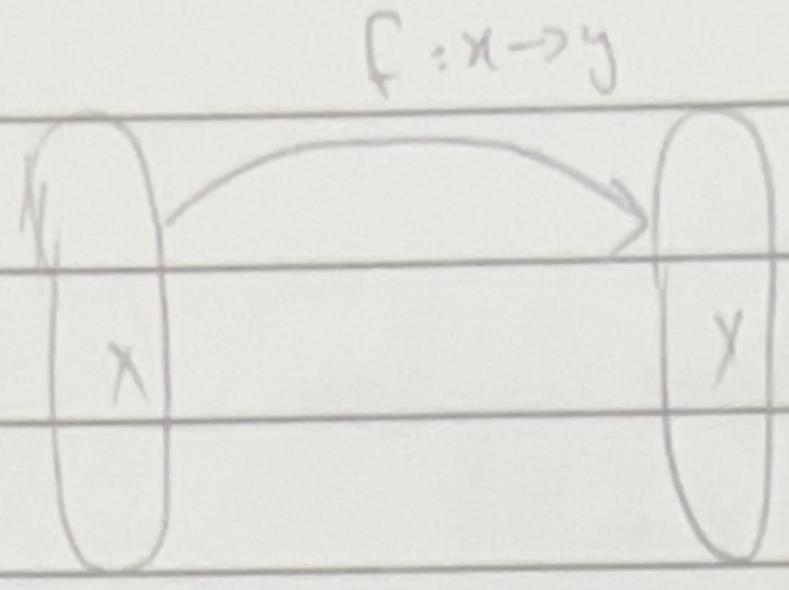
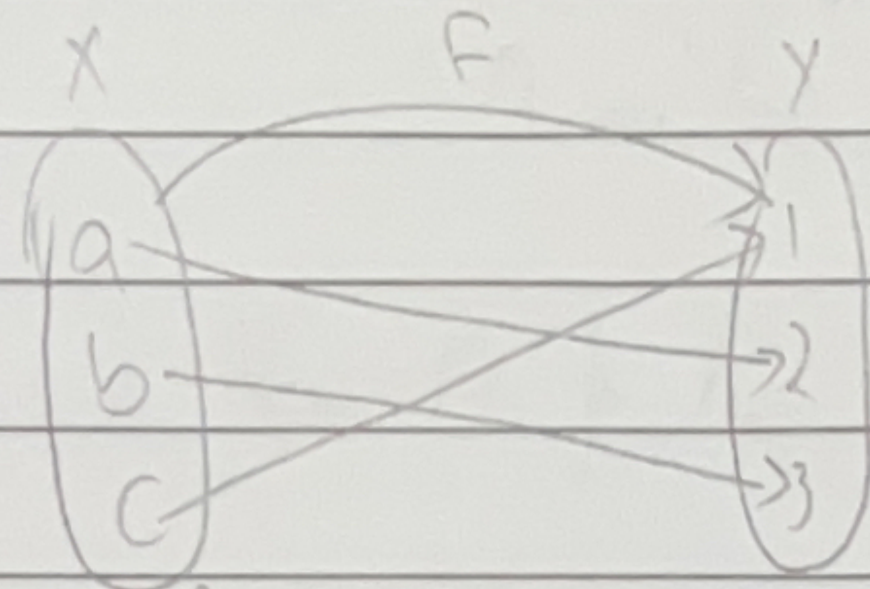
Functions :-



Domain Co-Domain

It becomes Range → only the used elements of Y

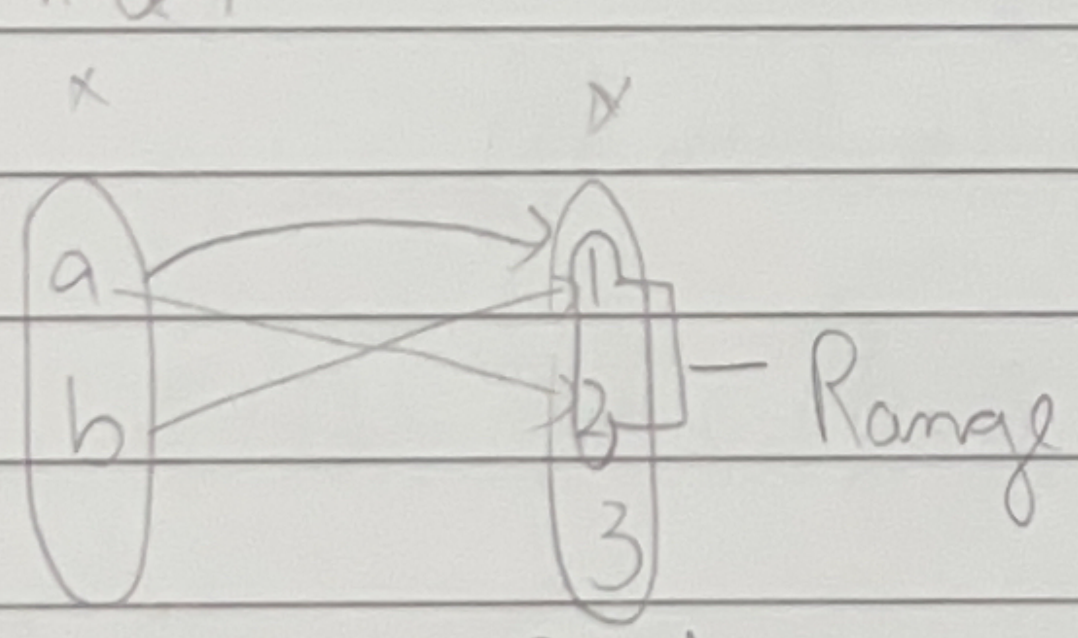
Function is also called as mappings, transformations



$f(a) = 2$
$f(b) = 3$
$f(c) = 1$

Two rules for function :-

→ All elements of X should have a image on y
→ Each element should have only one image on the Y

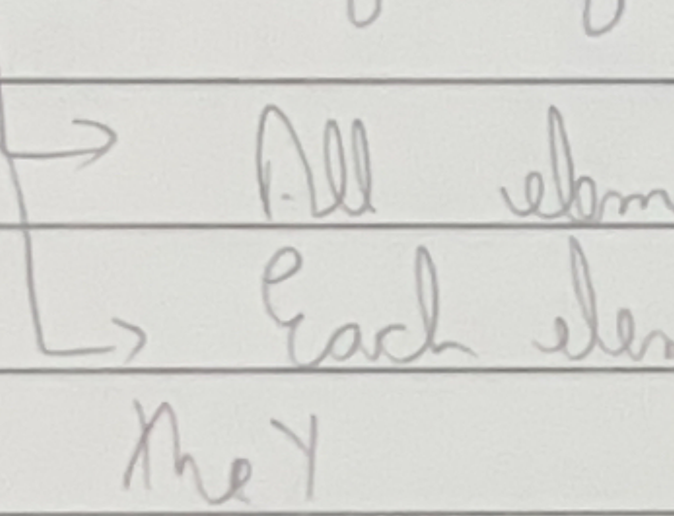


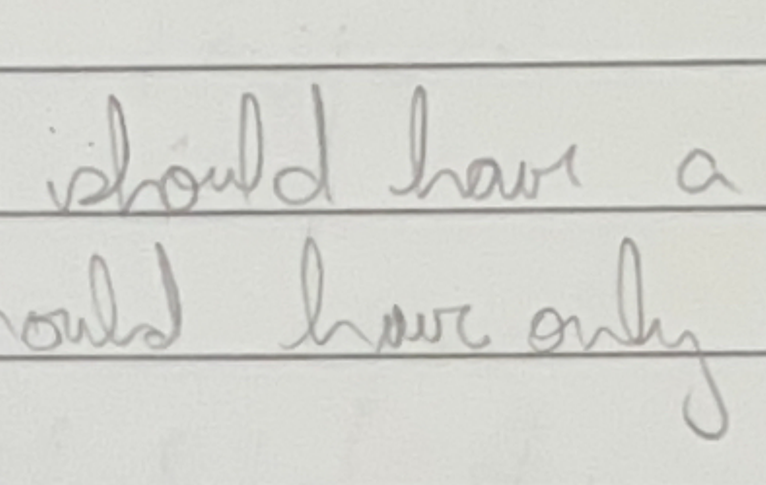
$[1, \infty)$

Domain Co-domain

One to One: (Injective)

Onto: (Surjective)





also onto
bijective

One to One & Onto
bijective

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By Defination (One to One)

Let $(x_1 = x_2) \in R$ Lets say $f(x) = x^2+1$

$x_1^2 = x_2^2$

$x_1^2 + 1 = x_2^2 + 1$

$f(x_1) = f(x_1)$ is ~~not~~ one to one else failed

By defination (Onto)

Let $f(x) = y$ Lets say $f(x) = 1/x$

$f(x) = y$

$y = 1/x$

$x = 1/y$

and

$f(x) = 1/(1/y)$

$f(x) = y$ its onto

If both true then bijective.

Negation of conditional statements removes the if $p \wedge \sim q$

$p \to q$

Inverse $\sim p \to \sim q$

Converse $q \to p$

Contrapositive $\sim q \to \sim p$

Implication law (if p then q)

Negation of Implication $(p \wedge \sim q)$

To find dual just chang $\wedge$ into $\vee$ and vice versa

Use $(\to)$ for $\forall$; $(\wedge)$ same for both

and $(\wedge)$ for $\exists$; $(\wedge)$

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- Modus Ponens

$p \to q$

p

$\therefore q$

- Modus Tollens

$p \to q$

$\sim q$

$\therefore \sim p$

- Hypothetical Syllogism

$p \to q$

$q \to r$

$\therefore p \to r$

- Disjunction Syllogism

$p \cup q$

$\sim p$

$\therefore q$

- Addition

p

$\therefore p \cup q$

- Simplification

$p \cap q$

$\therefore p$

- Conjunction

p

q

$\therefore p \cap q$

- Resolution

$p \cup q$

$\sim p \cup r$

$\therefore q \cup r$

$d \mid n$ d divides n if and only if $n = d \cdot k$

$n = 2k+1$ odd

$n = 2k$ even

$n = k^2$ perfect square

Sequence & Summations

* to have a alternating sequence use $(\times(-1)^n)$

it will be like this $-1/2, 2/3, -3/4, 4/5, -5/6$ etc

Arithmetic sequence:-

$5, 9, 13, 17$

$5, 5+4, 5+8, 5+12$ $\qquad 5+(n\times 4)\ n \geq 0$

$a_n = a + (n-1)d \qquad n \geq 1$

↳ common difference

Geometric sequence:-

$1, 2, 4, 8, 16 \ldots$

$\times 2 \quad \times 1/2 \quad \times 1/2 \quad \times 1/2$

$a_n = ar^{n-1} \qquad n \geq 1$

* If one & two terms are given and you have to find sequence then

sub. equations for $\Rightarrow$ A.P

divide equations for $\Rightarrow$ G.P

Summation:-

$$\sum_{k=1} a_k = a_1 + a_2 + a_3 \ldots\ldots$$

$$\sum_{k=b-i}^{a-i} (k+i) = \sum_{k=b}^{a} (k)$$

$$\sum_{k=b+i}^{a+i} (k-i) = \sum_{k=b}^{a} (k)$$

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Arithematic series sum:-

- $S_n = \frac{n(a+L)}{2}$ (When first (a) & last (l) term is given)

- $S_n = \frac{n}{2}(2a+(n-1)d)$ (When first term (a) & difference (d) is given)

Geometric Series Sum:-

- $S_n = \frac{a(1-r^n)}{1-r}$ $(r \neq 1)$

Mathematical Induction:-

Basic Step : $P(1) = R.H.S$

Induction Step : $1+2+3 \cdots\cdots (2k-1) = k^2$

$1+2+3 \cdots\cdots (2k+2-1) = (k+1)^2$

$k^2 + 2k + 1 = (k+1)^2$

$(k+1)^2 = (k+1)^2$ proved

For $\sum_{i=1}^{n+1} i2^i = n \cdot 2^{n+2} + 2$

$(n+1)2^{(n+1)} = n \cdot 2^{n+2} + 2$

$(n+1+1)2^{(n+2)} = (n+1) \cdot 2^{n+3} + 2$

$(n+2) \cdot 2^{n+2} = (n+1) \cdot 2^{n+3} + 2$

$n \cdot 2^{n+2} + 2 + (n+2) \cdot 2^{n+2} = (n+1) \cdot 2^{n+3} + 2$

$2^{n+2}(n+n+2) + 2 = //$

$2^{n+2} \cdot 2(n+1) + 2 = //$

$2^{n+3}(n+1) + 2 = //$ proved

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Check divisibility by Mathematical Induction :-

$9^n + 3$ by 4

$9^n + 3 = 4m$ $\quad\therefore 9^n = 4m - 3$

$9^{n+1} + 3 = 9 . 9^n + 3$

$= 9(4m - 3) + 3$

$= 36m - 27 + 3$

$= 36m - 24$

$= 4(9m - 6)$ proved divisible by 4

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Relations:-

null relation = ϕ

universal relation = $A \times A$

$aRB \Rightarrow (a, B)$

Reflexive :-

If $A = \{(1,1), (2,2), (3,3), (2,3)\}$ it is reflexive

every point in relation has a loop

Symmetric :-

If $A = \{(1,2), (2,1), (3,4), (4,3)\}$ it is symmetric

all order pairs's opposite should be there like $(1,2) \Leftrightarrow (2,1)$

$A = \{(1,1), (2,2), (3,3), (4,3), (4,4)\}$ not symmetric

Transitive :-

If $A = \{(1,2), (2,3), (1,3), (2,4)\}$ is transitive

$(a,b), (b,c), (a,c) \Rightarrow$ transitivity

Equivalence :-

Where all reflexive, symmetric & transitivity are true

To find linearly congruent :-

$2 \equiv 0 \pmod 2$

$2 \bmod 2 = 0 \bmod 2$ ✓

$as + mt = 1 \rightarrow$ prime

↳ inverse of equation Let say

$a \equiv m \pmod n$

(a, n) s would be inverse of

a modulo b

⇓

(a, b)

$t \equiv s \pmod n$

any t that meets the condition will be inverse of a m(mod n)

$a^{p-1} = 1 \pmod P \Rightarrow$ Fermat's theorem

↳ not divisible by p ↳ prime

Recursive Relations:-

For recursive relations just add or multiply the the difference in previous value if a_0 = Base then if Result = 3 × Base

$a_n = 3a_{n-1}$

in each iteration.

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Fibonacci :-

$f_k = f_{k-1} + f_{k-2} \quad k \geq 3 \quad f_0, f_1$ Base

Tower of Hanoi :-

Moves $= 2^n - 1$ (One move at a time)

Counting Bits :-

$B_n = n B_{n-1}$

- Inclusion-exclusion Principle

either n_1 ways or n_2 ways $\Rightarrow |n_1| + |n_2| - |n_1 \cap n_2|$

- Binomial Theorem :-

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j = {}^nC_j \cdot x^{n-j} \cdot y^j$$

n = Power

j = term -1

- Pascal's triangle :-

				${}^1C_1$					Power 0
			${}^2C_1$ 1		${}^2C_1$				" 1
		${}^3C_1$ 1		${}^1C_2$ 2		${}^3C_3$ 1			" 2
	${}^4C_1$ 1		${}^4C_2$ 3		${}^4C_3$ 3		${}^4C_4$ 1		" 3
1		4		6		4		1	" 4

Repetition	=	Not allowed	,	Allowed
permutation	=	${}^nP_r$	,	n^r
combination	=	${}^nC_r$	,	${}^{(n+r-1)}C_{r}$

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$

INVERSE $\sim p \to \sim q$

Contradict $= p \wedge \sim q$ (False) then $p \to q$ is true

CONVERSE $q \to p$

NEGATION $\sim(p \to q)$

CONTRAPOSITION $\sim q \to \sim p$ true ~~then~~ then $p \to q$ is true

1, 13, 20, 22, 23, 27

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$A - B = A \cap B^c$

$g : R \to R^+$ $\quad g(x) = x^2 + 1$

Let say x_1 & x_2 are two numbers from $\mathbb{R}$

where $x_1 = x_2$

$x_1^2 = x_2^2$

$x_1^2 + 1 = x_2^2 + 1$

$g(x_1) = g(x_2)$

Thus if $x_1 = x_2$ then $g(x_1) = g(x_2)$

$g : R_1 \to R+$ is well defined

$A \subseteq A \cup B$

$x \in A$ and $x \notin B$

$x \in A$

x is arbitrary so

$A \subseteq A \cup B$

Injection:

Let $f(x_1) = f(x_2)$

$x_1 = x_2$ proved

~~Bijection~~ Surjection:-

$f(x) = y \Rightarrow x = y + 2$

$\cancel{f(y) = y}$

$f(y+2) = y$