

## Satellite Navigation

### Calculation of GPS Satellite Position

#### **TASK 1**

Considering that the “main.m” file is loading the RINEX files “11.n” and “11.o”, the numerical values of the RINEX files are the same with those obtained after executing the “main.m” program.

#### **TASK 2**

For the second task, in order to compute the position of GPS satellite PRN 17, we created a function which calls all the needed values from the RINEX format files. Further, we applied the algorithm provided in the seminar’s sheet.

Algorithm:

WGS 84 value of the Earth’s gravitational constant for GPS user

$$\mu = 3.986005 \cdot 10^{14} \text{ m}^3/\text{sec}^2$$

WGS 84 value of the Earth’s rotation rate

$$\dot{\Omega}_E = 7.2921151467 \cdot 10^{-5} \text{ rad/sec}$$

Semi major axis

$$A = (\sqrt{A})^2$$

Computed mean motion (rad/sec):

$$n_0 = \sqrt{\frac{\mu}{A^3}}$$

Time from ephemeris reference epoch:

$$t_k = t - t_{oe}$$

Corrected mean motion:

$$n = n_0 + \Delta n$$

Mean anomaly:

$$M_K = M_0 + nt_k$$

Kepler’s equation for Eccentric Anomaly:

$$M_K = E_k - e \cdot \sin E_k$$

True anomaly:

$$v_k = \tan^{-1} \left\{ \frac{\sqrt{1-e^2} \cdot \sin E_k / (1 - e \cdot \cos E_k)}{(\cos E_k - e) / (1 - e \cdot \cos E_k)} \right\}$$

Eccentric Anomaly:

$$E_k = \cos^{-1} \left\{ \frac{e + \cos v_k}{1 + e \cdot \cos v_k} \right\}$$

Argument of latitude:

$$\phi_k = v_k + \omega$$

Second Harmonic Perturbations:

Argument of Latitude Correction

$$\delta u_k = C_{us} \sin 2\phi_k + C_{uc} \cos 2\phi_k$$

Radius Correction

$$\delta r_k = C_{rs} \sin 2\phi_k + C_{rc} \cos 2\phi_k$$

Inclination Correction

$$\delta i_k = C_{is} \sin 2\phi_k + C_{ic} \cos 2\phi_k$$

Corrected argument of Latitude:

$$u_k = \phi_k + \delta u_k$$

Corrected radius:

$$r_k = A(1 - e \cdot \cos E_k) + \delta r_k$$

Corrected Inclination:

$$i_k = i_0 + \delta i_k + (IDOT)t_k$$

Corrected longitude of ascending node

$$\Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e)t_k - \dot{\Omega}_e \cdot t_{oe}$$

Earth fixed coordinates:

$$x_k = x'_k \cos \Omega_k - y'_k \cdot \cos i_k \cdot \sin \Omega_k$$

$$y_k = x'_k \sin \Omega_k + y'_k \cdot \cos i_k \cdot \cos \Omega_k$$

$$z_k = y'_k \cdot \sin i_k$$

After the computation, the obtained position of the GPS satellite PRN 17 in ECEF (Earth centered, Earth fixed) coordinates for GPS week second 511200 in GPS week 1658 will be:

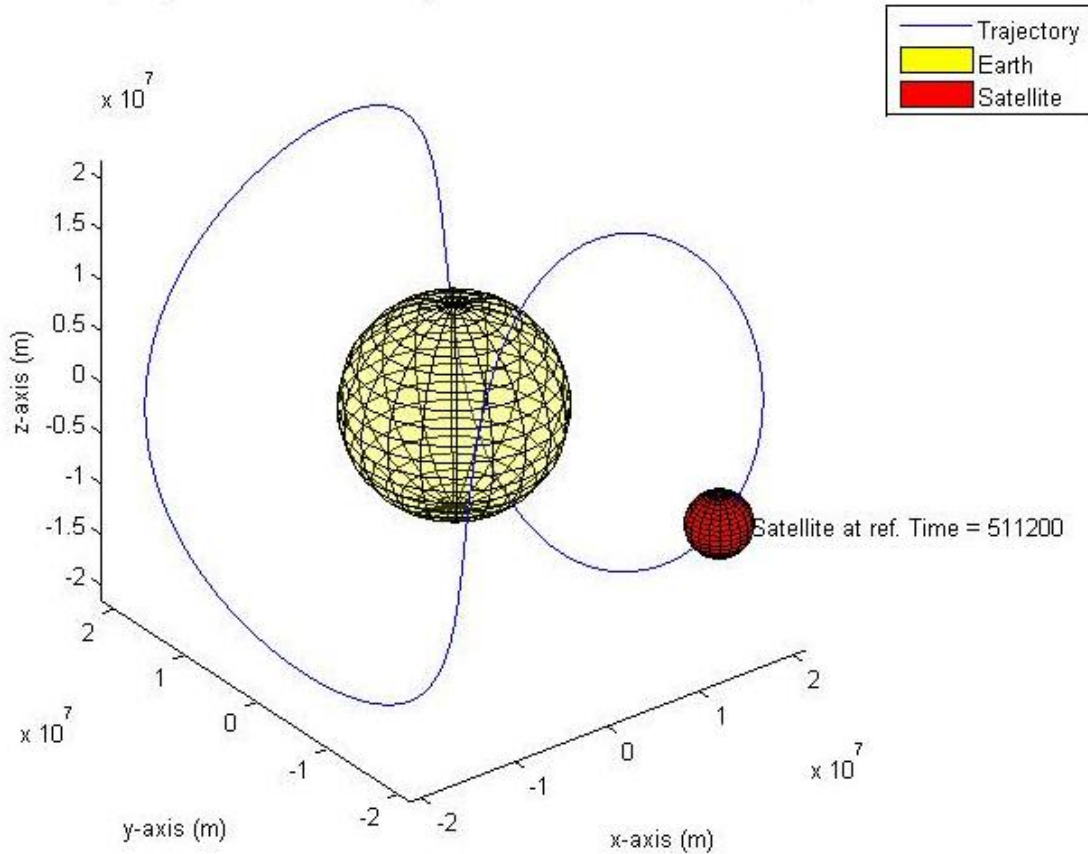
$$x_k = 19597195.64210617 \text{ m}$$

$$y_k = -11716658.28687484 \text{ m}$$

$$z_k = 13218383.92357504 \text{ m}$$

Taking all the positions during a sidereal day and also the Earth rotation rate, we plotted the results for satellite PRN 17.

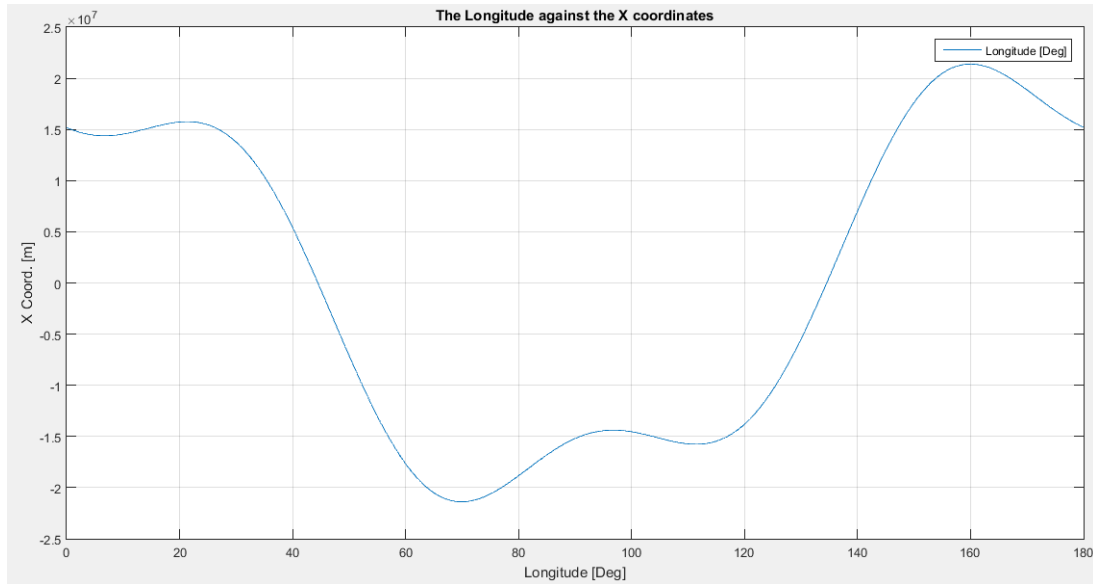
The trajectory of the Satellite with respect to the EC-inertial coordinates system



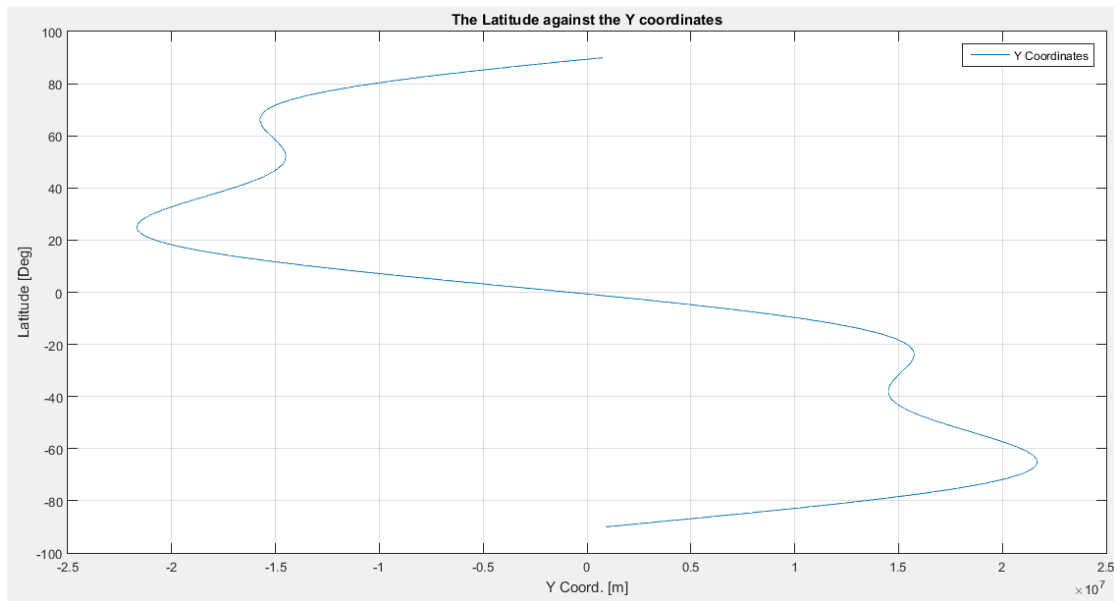
*Fig. 1 – Satellite position considering the Earth's rotation rate, for a total orbital period*

Because of the rotation of the earth, the Earth centered inertial system and the non-symmetrical fields on the Satellite are considered. We observe the strange behavior (non-elliptical) of the satellite's orbit. The total period is needed to draw a total orbit is one sidereal day

In addition, we plotted the longitudinal and latitudinal movement of the satellite, with respect to X and Y coordinates.



*Fig.2 – The Longitude against the X coordinate*



*Fig.3 – The Latitude against the Y coordinate*

### **TASK 3**

For the third task, we applied the same procedures like in task 2. The only difference is that the Earth rotation rate is fixed to be 0. As we can see, the difference between those 2 satellite's trajectories are pretty big. When the Earth's rotation rate is considered, the trajectory is a random closed curve. When the Earth's rotation rate is 0, the trajectory is circular. The results for task 3 are presented in fig. 4. *Note:* Here there is no need to consider a whole sidereal day. The half is sufficient because the rotational of the Earth is fixed. However, it will make no difference.

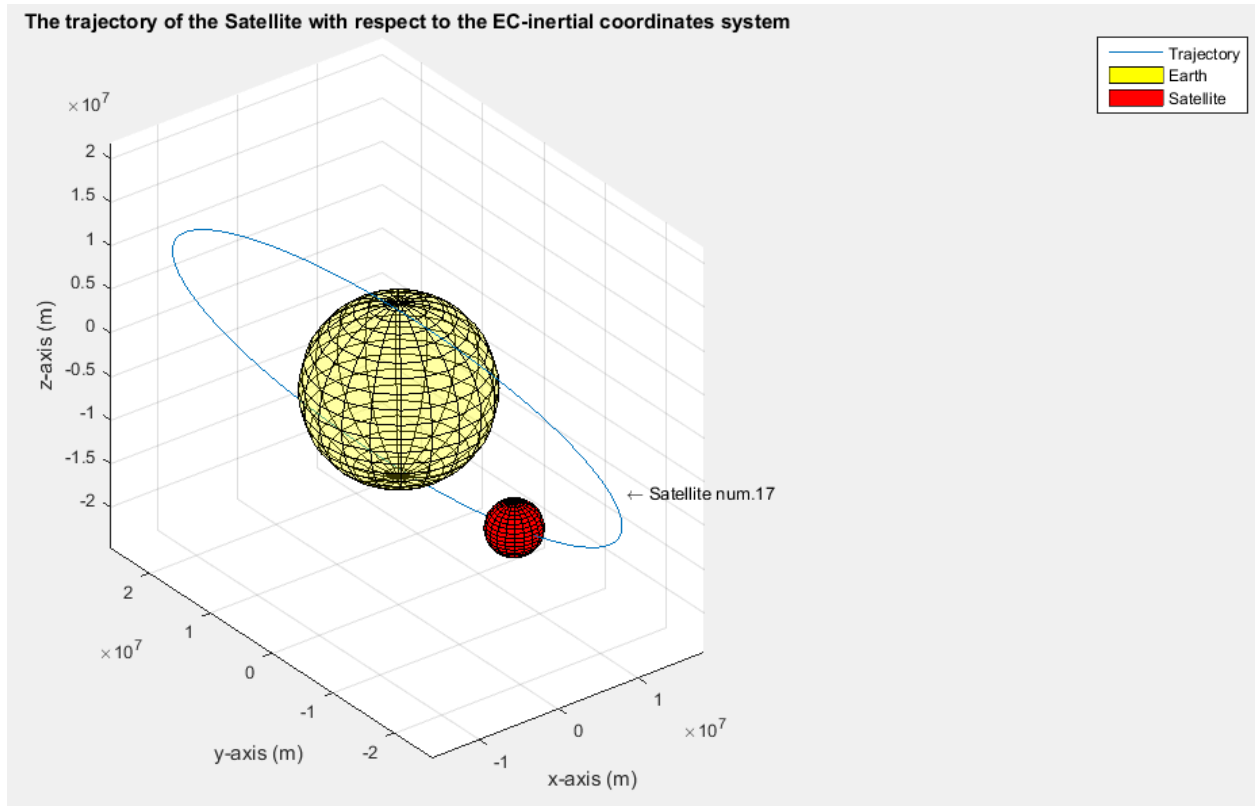
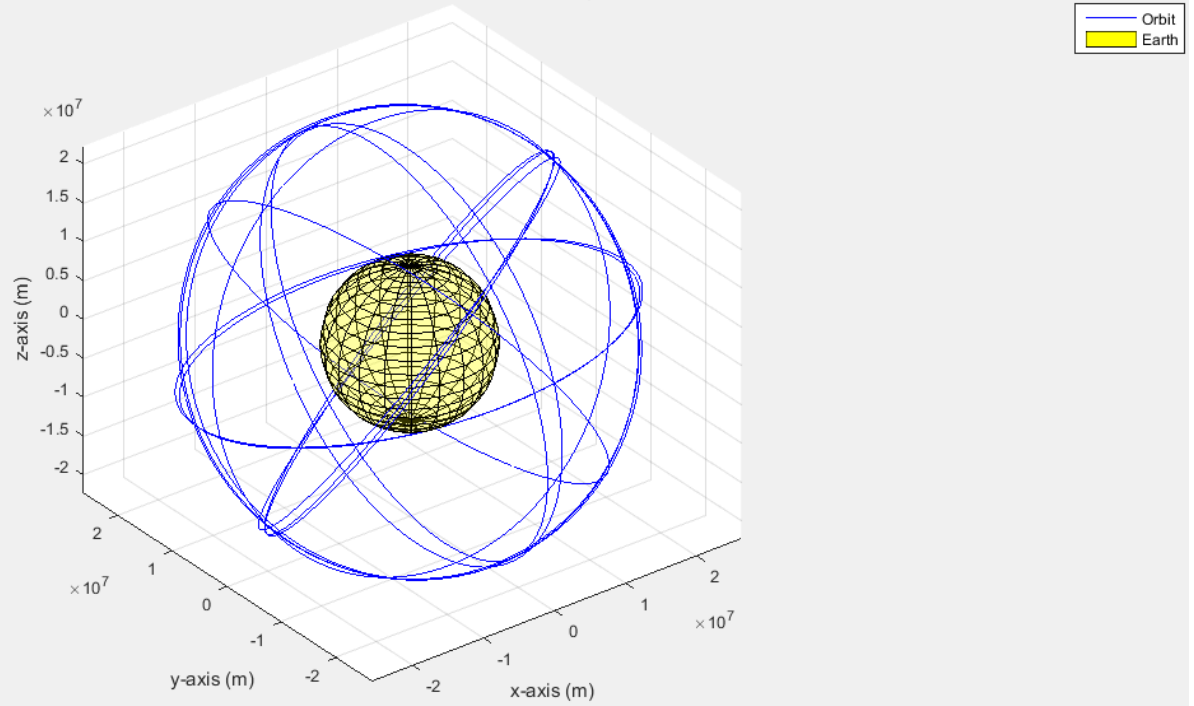


Fig. 4 – The trajectory of the Satellite with respect to the EC-inertial coordinates system

#### TASK 4

In order to complete task 4, we take into account all data provided by the RINEX files 11.n and 11.o. We set the Earth's rotation rate to 0 and then we applied the same algorithm, now, for more satellites. The final result is presented in fig. 5.

The Orbits of the Satellite with respect to the EC-inertial coordinates system and the rotational is zero



*Fig. 5 – The orbits of the satellites, when Earth's rotation is zero (in EC-inertial coordinates system)*