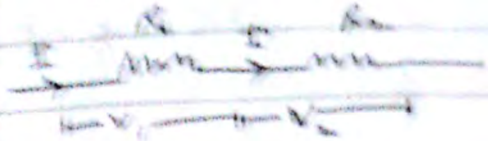
Resistances in Series

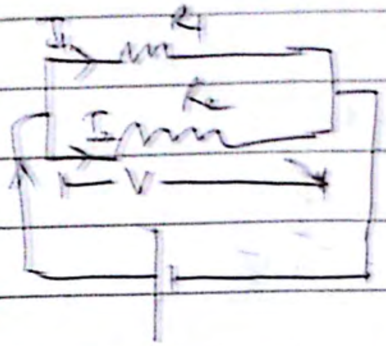


$$V = V_1 + V_2$$

$$V = I(R_1 + R_2) = IR$$

$$\boxed{R = R_1 + R_2}$$

Resistances in parallel



$$I = I_1 + I_2$$

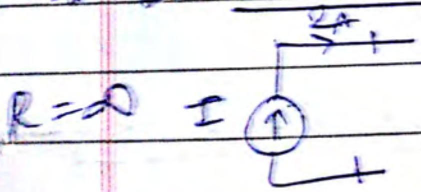
$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

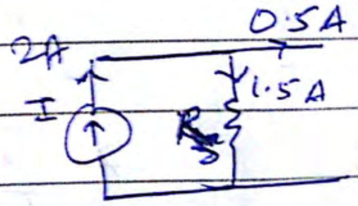
$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

Sources → Current source
Sources → voltage source

* In current source ideal ~~curren~~ internal impedance is infinite



Practical (connected in parallel) → There are losses but not so in ideal



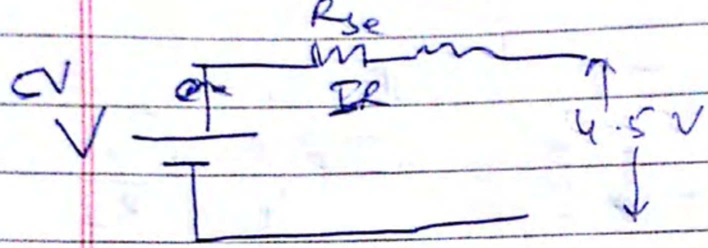
* Voltage source

Resistance of ideal voltage source

$R = 0$

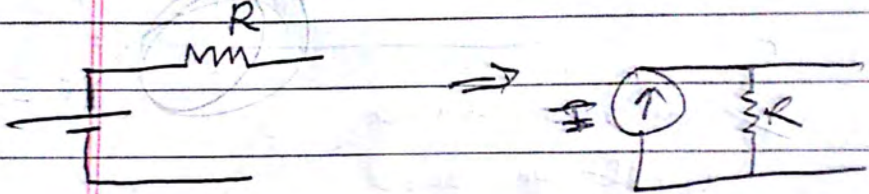
Internal ~~resistance~~ resistance is connected in series

Prac.



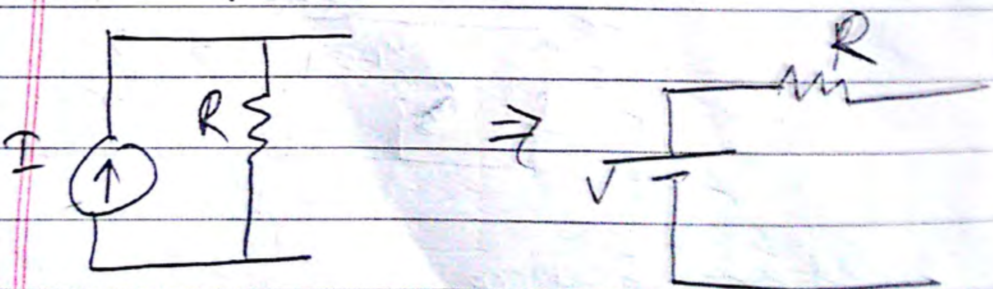
$V - IR_{se} = 4.5$

Conversion from voltage source to current source.



$I = \frac{V}{R}$

Conversion from current source to voltage source:

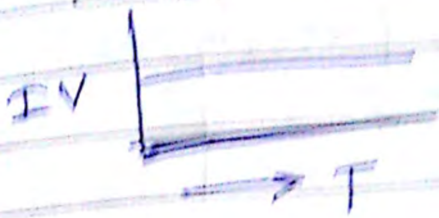


$V = IR$

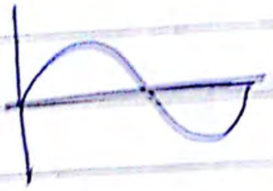
D.C → constant value

$\nu = 0$

$T = \infty$



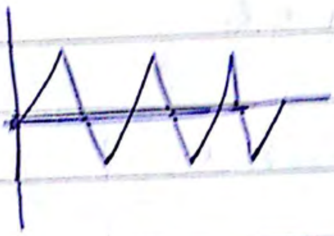
A.C → varies with time

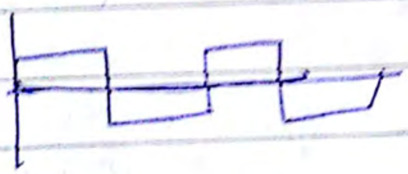


general (sinusoidal)

peak sharp → losses are more

as losses are less

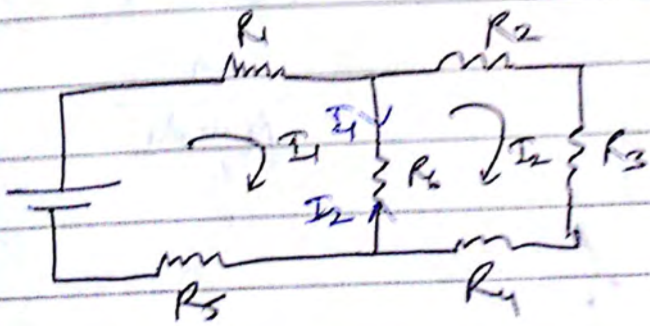




∝ No. of loops

= No. of eqⁿ

θ



∴

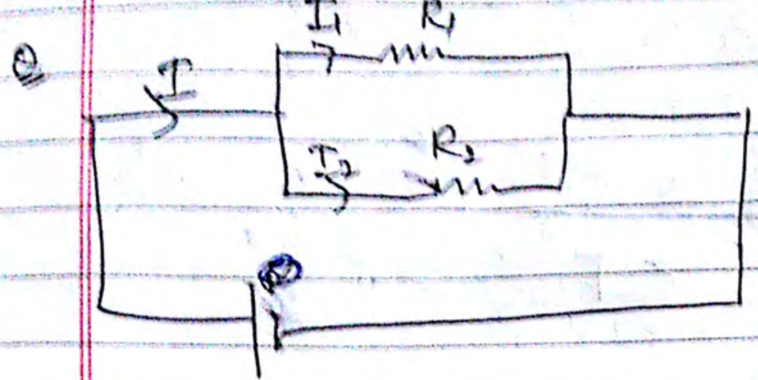
$R_1 \rightarrow I_1$ $\quad$ $R_2 \rightarrow I_2$

$R_5 \rightarrow I_1$ $\quad$ $R_3 \rightarrow I_2$

$R_4 \rightarrow I_2$

$R_6 \rightarrow I_1 - I_2$

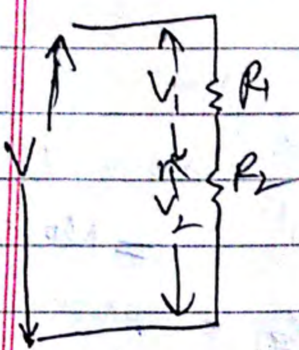
Current divider rule: (Resistances conn. in parallel)



$$I_1 = \frac{I R_2}{R_1 + R_2}$$

$$I_2 = \frac{I R_1}{R_1 + R_2}$$

Voltage divider rule: {when resistances are connected in series}

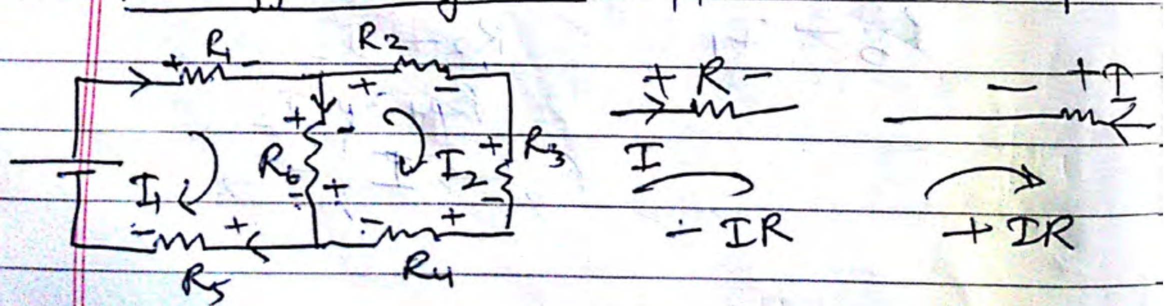


$$V_1 = \frac{V R_1}{R_1 + R_2}$$

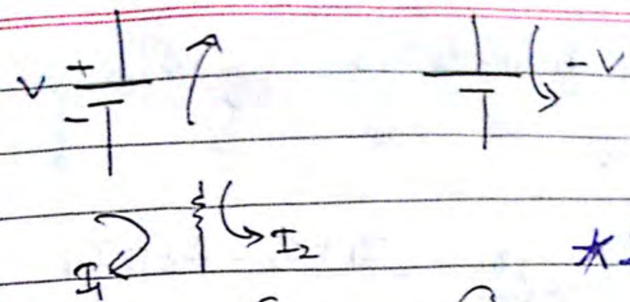
$$V_2 = \frac{V R_2}{R_1 + R_2}$$

4/1/17

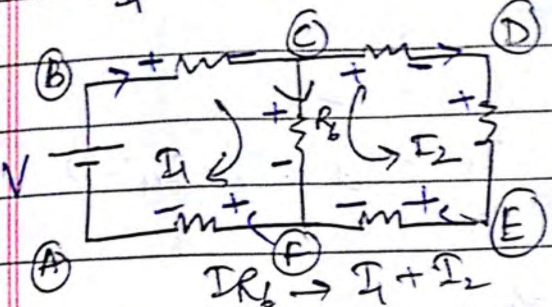
Kirchoff's Voltage law: {Applied across the loop}



* loop considered will be taken greater and subtracted from other current



* Same current direction → add



* Diff current direction → sub^n.

* Sign convention will be considered.

$IR_6 \to I_1 + I_2$

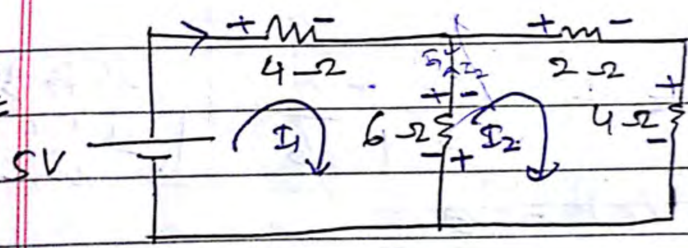
loop (I)

$$V - I_1 R_1 - (I_1 - I_2) R_6 = 0 \to (1)$$

loop (II)

$$-(I_2 - I_1) R_6 - I_2 R_2 - I_2 R_3 - I_2 R_4 = 0 \to (2)$$

Q



Sol^n: loop I.

$$5 - \cancel{4} I_1 (4) - (I_1 - I_2)6 = 0$$

$$5 - 10 I_1 + 6 I_2 = 0 \to (1)$$

loop II

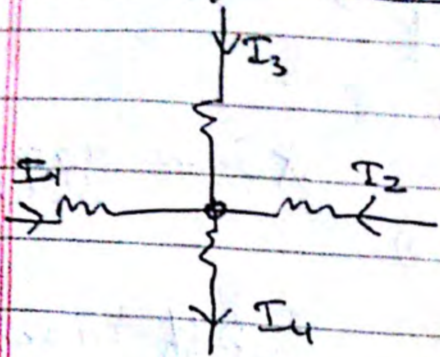
$$-(I_2 - I_1)6 - (2 I_2) - 4(I_2) = 0$$

$$\cancel{6 I_1 = 0}$$

$$-12 I_2 + 6 I_1 = 0 \to (2)$$

$$I_1 = \frac{5}{7} \qquad I_2 = \frac{5}{14}$$

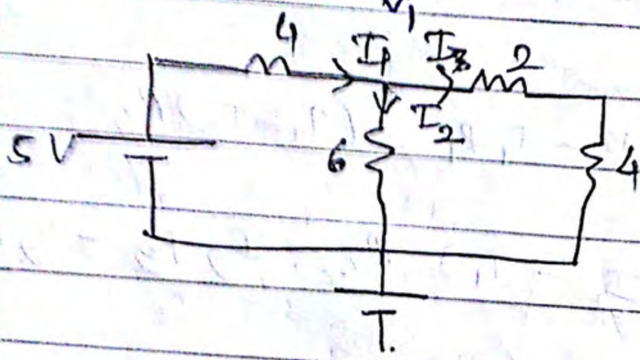
Kirchoff's Current law:



$$I_1 + I_2 = I_3 + I_4$$

$$I_1 + I_2 - I_3 - I_4 = 0$$

Q



$$I_1 + I_2 + I_3$$

$$\frac{5 - V_1}{4} = \frac{V_1 - 0}{6} + \frac{V_1}{6}$$ [as 2Ω & 4Ω are in series]

$$\cancel{\frac{(5 - V_1)}{4} = \frac{2V_1}{3}}$$

$$15 - 3V_1 = 4V_1$$

$$\frac{15}{7} = V_1$$

$\cancel{I_1 = }$

$$\cancel{\frac{5 - \frac{15}{7}}{4}} = \cancel{\frac{15}{7 \cdot 42}} \; \cancel{\frac{30}{42}}$$

$$I_2 = 5/14$$

$$I_1 = 5/7.$$

$$I_3 = 5/14$$

Date ______ Page ______

* lower pt is ground pt.
* Give direction of current
* Identify node (same as no. of eq^n) V → potential diff. = $V_1 - V_2$

Q

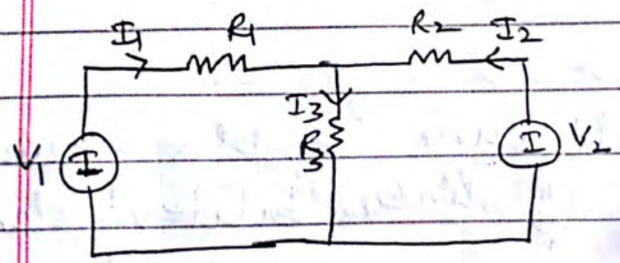
at Node (I) $I_1 = I_2 + I_3$

$$\frac{V - V_1}{R_1} = \frac{V_1 - 0}{R_2} + \frac{V_1 - V_2}{R_3}.$$

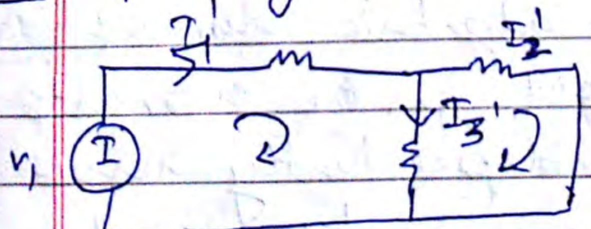
at Node (II) $I_3 + I_5 = I_4$

$$\frac{V_2 - 0}{R_4} = \frac{V_3 - V_2}{R_5} + \frac{V_1 - V_2}{R_3}$$

9/1/17 superposition theorem:



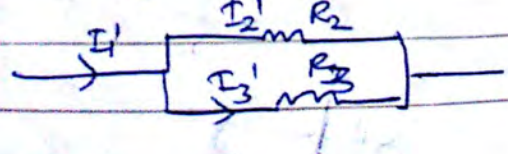
Case I: V_1 acting alone:



$V_2 = 0$

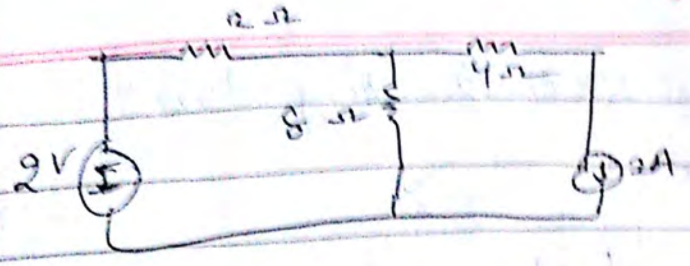
$$I_1 = \frac{V_1}{R_{eq}}.$$

$$R_{eq} = R_1 + (R_2 \parallel R_3)$$



* No of cases : No of sources

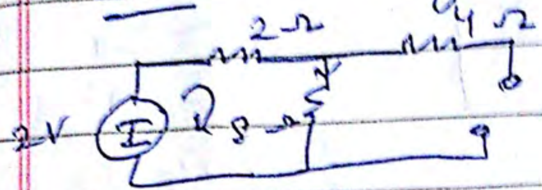
Q



Find current through 8 Ω

$I_{8\Omega} = ?$

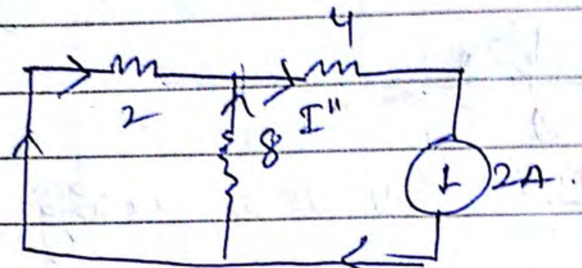
Sol^n: Case I : 2V acting alone

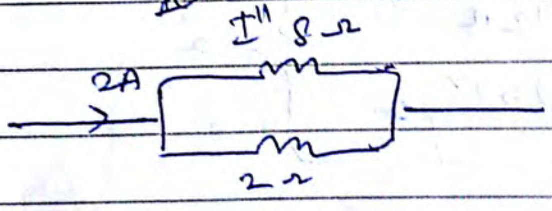


(Internal impedance = 0)

$$I_8 = \frac{2}{8+2} = \frac{1}{5} A$$

Case 2: 2A source is acting



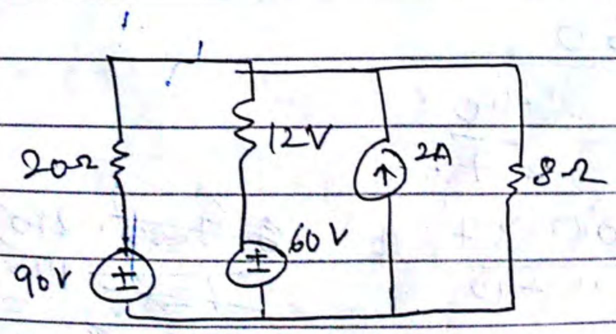


$$I'' = 2 \times \frac{2}{10}$$

$$= 2/5$$

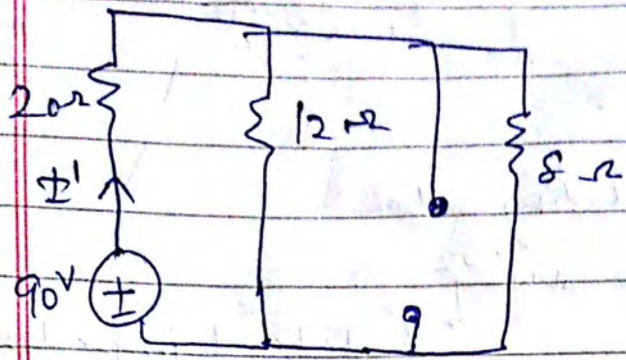
$$I_{8\Omega} = \frac{1}{5} - \frac{2}{5} = \frac{-1}{5} A \cdot \left[\frac{1}{5} \text{ dir.} \uparrow\right]$$

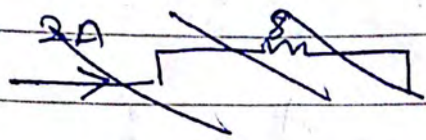
Q



$I_{20\Omega} = ?$

Case I: 90 V is acting alone





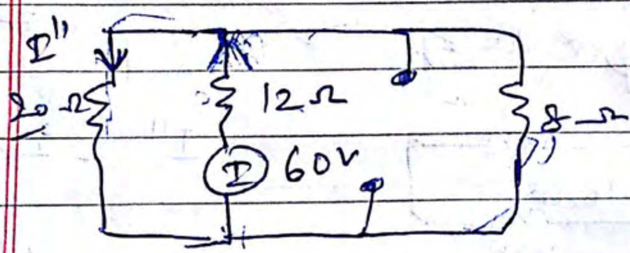
$$I' = \frac{90}{20+(12\parallel 8)} = 3.6\,A \uparrow$$

$\cancel{\frac{1}{12}} + \frac{1}{8}$

$\cancel{20}$

$= \cancel{\frac{90}{8}}$

Case II: 60 V is acting alone



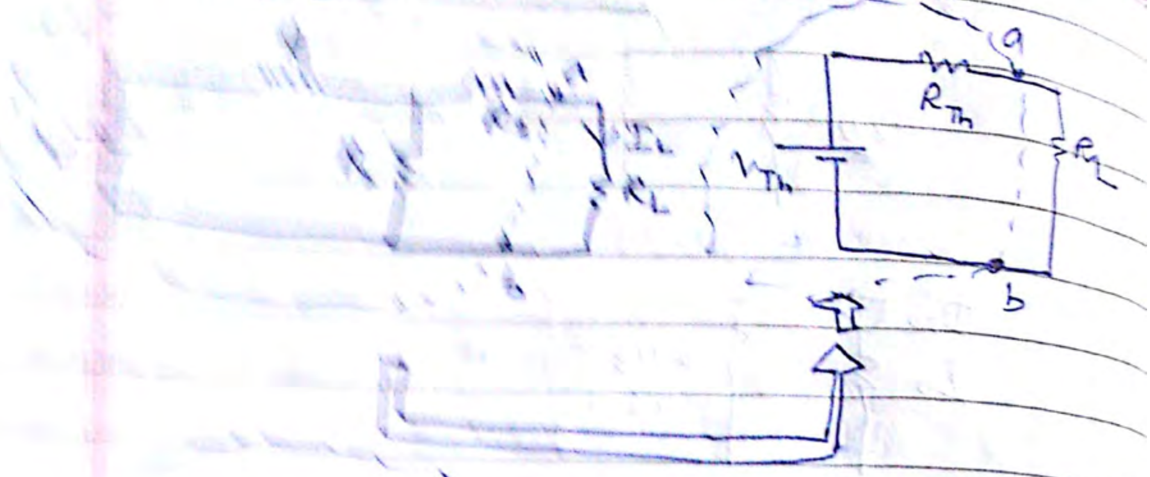
$$I'' = \frac{60}{12+(20\parallel 8)}$$

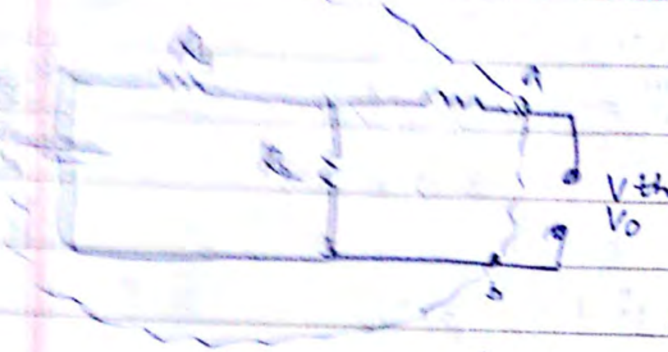
8×5

$\frac{2+5}{40} \Rightarrow \frac{7}{40}$

$$= \frac{60}{12+\frac{40}{7}}$$

$$= \frac{60\times 7}{84+40} = \frac{\cancel{420}\ \cancel{210}\ 105}{\cancel{124}\ \cancel{62}\ 31} = 3.3\,A \downarrow$$



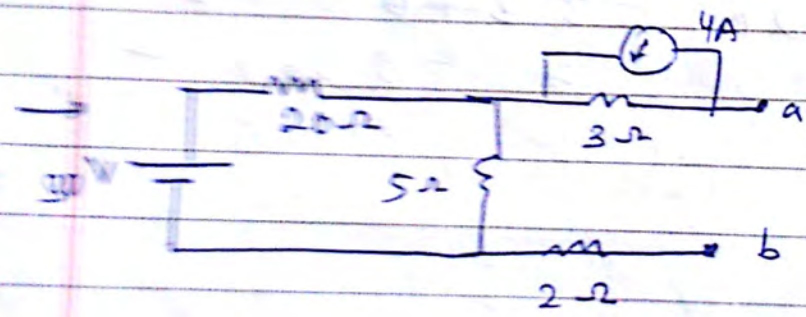


$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

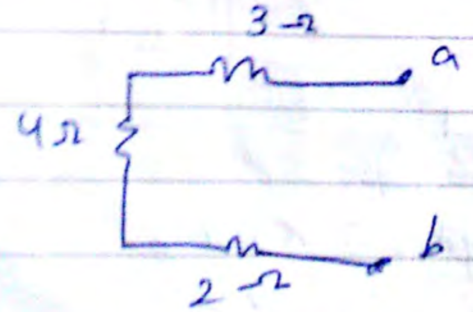
$$R_{Th} = R_{eq} = (R_1 \parallel R_2) + R_3$$

Step I

$$V_0 = V_{Th} = I R_2.$$

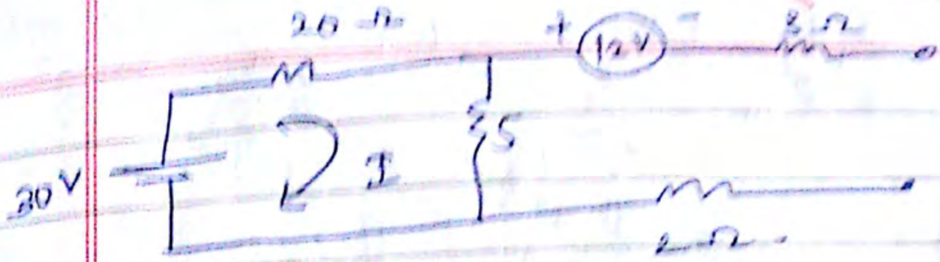


Q. find thevenin's eq. across point a, b
voltage source short circuited
current source is open



4 + 3 + 5

$$\boxed{R_{th} = 9\,\Omega}$$

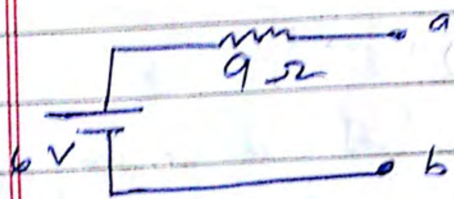


$$I = \frac{30}{25} = 1.2\,A.$$

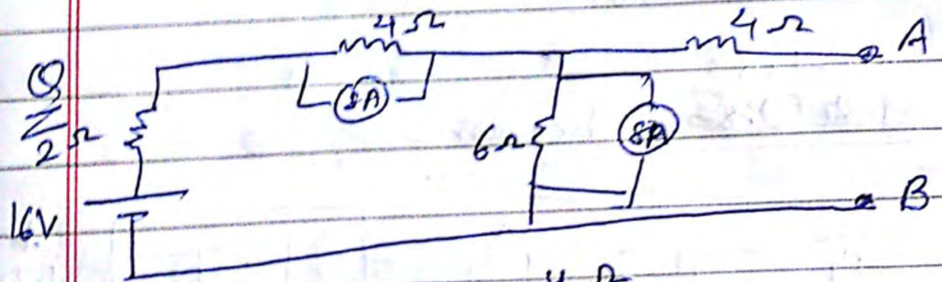
applying KVL,

$$12 - (1.2 \times 5) + V_{Th} = 0$$

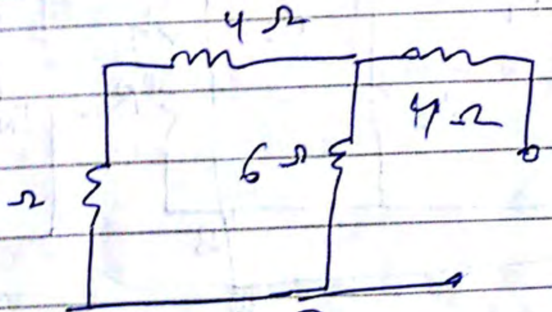
$$V_{Th} = -6.$$



$$I_1 = \frac{6}{9}.$$

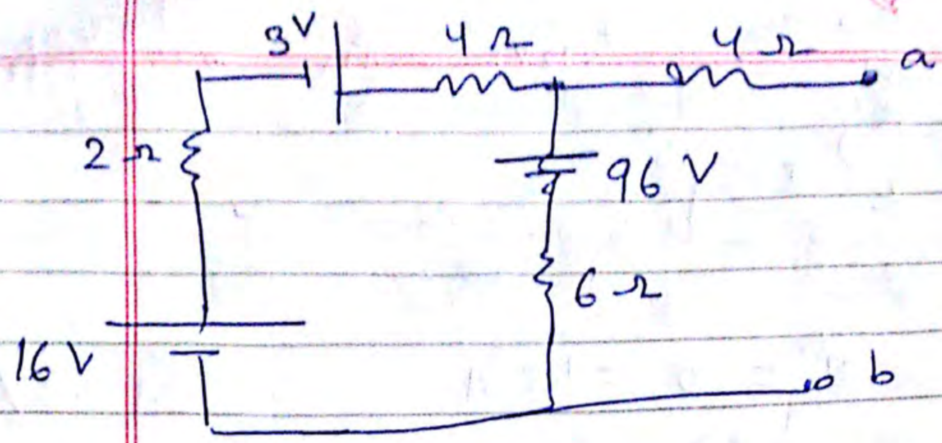


$R_{Th} =$



$$R_{Th} = 7$$

max power transf. → A

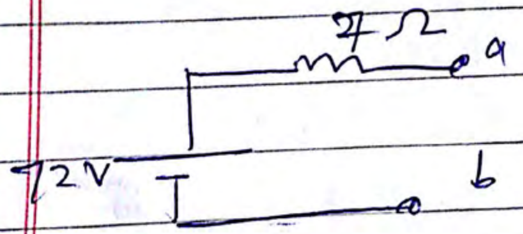


$$16 - 2I + 32 - 4I - 96 - 6I = 0$$

$$I = -4A$$

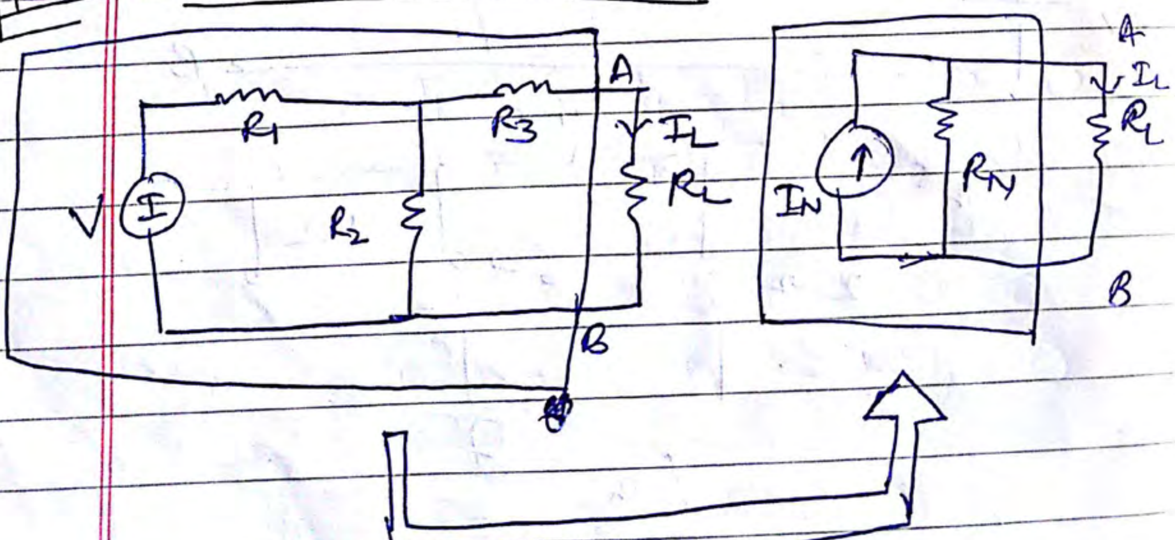
$$-4 \times 6 + 96 = V_{ab}$$

$$V_{ab} = 72\ V$$



16/1/17

~~Nortons~~ Norton's Theorem:



$$I_L = I_N \frac{R_N}{R_N + R_L}$$

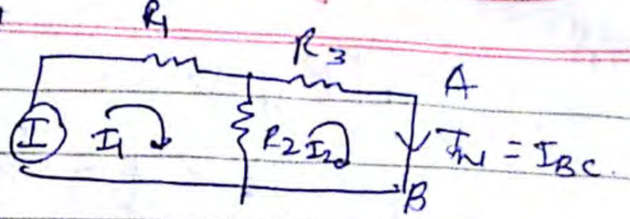
$R_N \Rightarrow R_{Th}$ (Eq. Resistance)

↓ Same method
SC voltage source
Open circuit load current

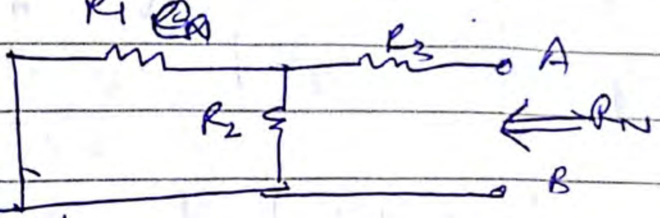
$I_2 = I_{sc} =$

Step I:

I_N or I_{SC}



Step II: R_N



look from load resp side

$$(R_1 \parallel R_2) + R_3 = R_N$$

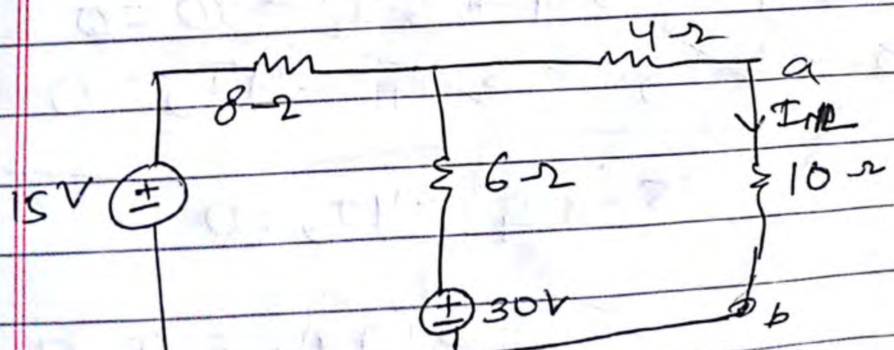
~~I_N~~ →

~~Step~~ Northon's theorem:

Step I: To find I_N / Norton's Current. I_N is short circuit current flowing through the terminal AB.

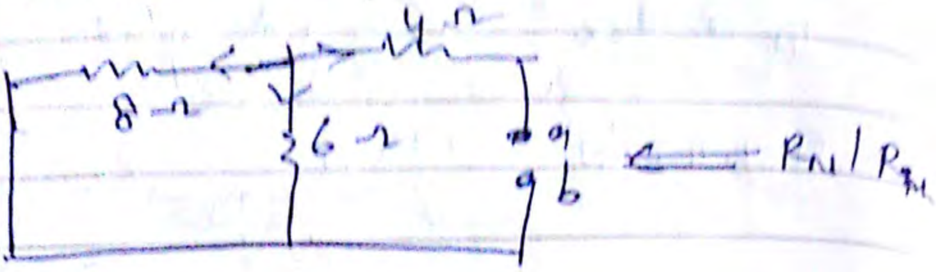
Step II: R_N (Norton's Resistance) → called as equivalent resistance. It is found out by short circuiting voltage sources and open circuiting current sources (Sources replaced by their internal ~~inde~~ impedances) and load resistance is removed. R_N is viewed from open circuited terminals AB.

Q



Find ~~curr~~ I_{10} through Norton's theorem.

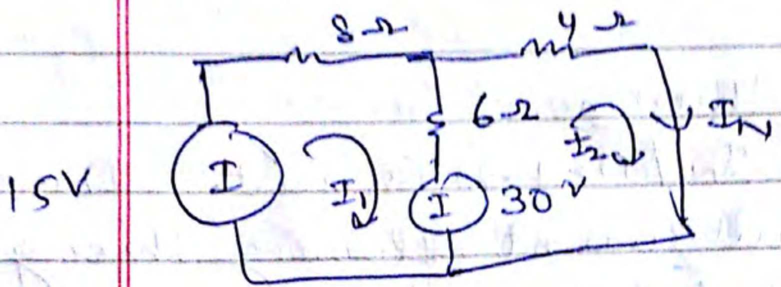
Soln: $R_N / R_{Th} \Rightarrow$



$$R_{Th} = (8 \| 6) + 4$$
$$= 7.4\Omega$$

$V = IR$
$I = \frac{V}{R}$

$I_N \Rightarrow$



(using KVL)

$I_2 = I_N (I_{SC}) =$ ________

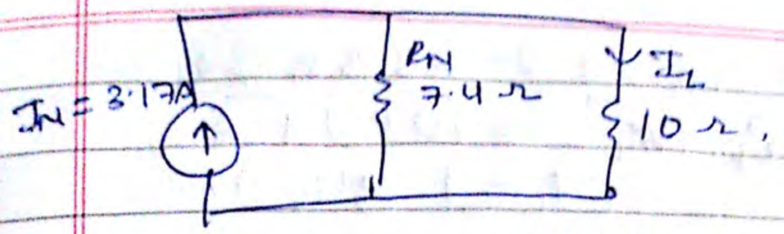
$$\cancel{15 - 8(I_1) - 6(I_1) + 30V = 0}$$
$$\cancel{+ 6 I_2 - 30V = 0}$$

$$15 - 8 I_1 - 6(I_1 - I_2) - 30 = 0$$
$$30 - 6(I_2 - I_1) - 4 I_2 = 0$$

$$15 - 8 I_1 - \cancel{6 I_1} + \cancel{6 I_2} - \cancel{30} = 0$$
$$\cancel{30} - \cancel{6 I_2} + \cancel{6 I_1} - 4 I_2 = 0$$

$$15 - 8 I_1 - 4 I_2 = 0$$

$$I_2 = 3.17 A. = I_N (I_{SC})$$
$$I_1 = 0.288 A.$$

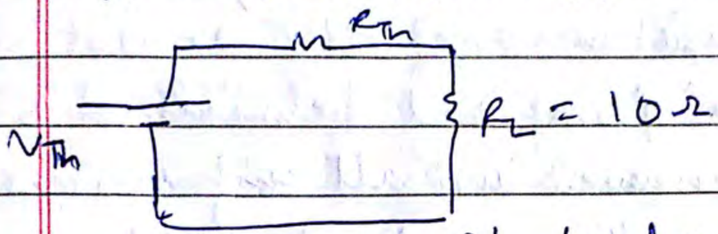


$$I_L = \frac{3.17 \times 7.4}{17.4}$$

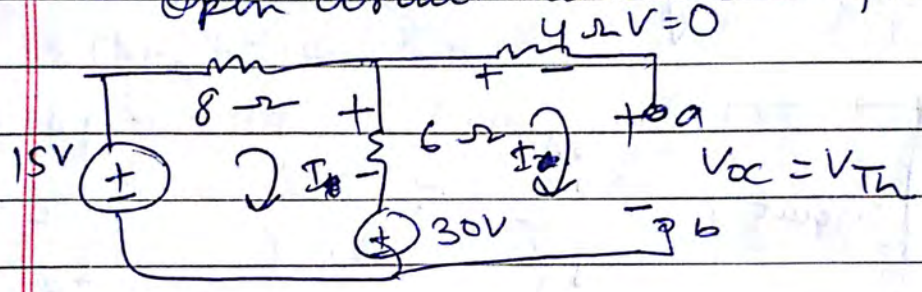
$$\boxed{I_L = 1.35A}$$

Thevenin's Theorem:

V_{Th} in series with R_{Th}



Open circuit load res to find V_{Th}.



$$\cancel{-8I_1 - 6(I_1 - I_2) + 15 = 0}$$

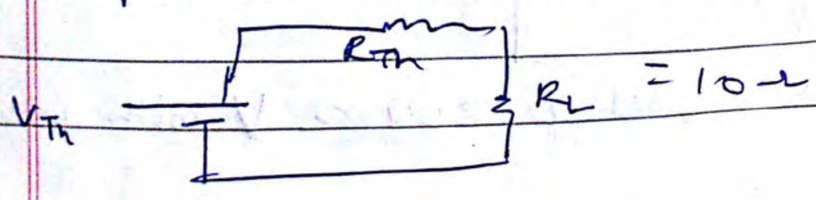
$$I = \frac{15-30}{14} = -1.07\,A.$$

$$30 + 6I - V_{ab} = 0.$$

$$V_{ab} = V_{oc} = V_{th} = \cancel{36.43}\ 23.58$$

$$\cancel{I_L = \frac{23.58}{10}} =$$

Put value in Thevenin's circuit.



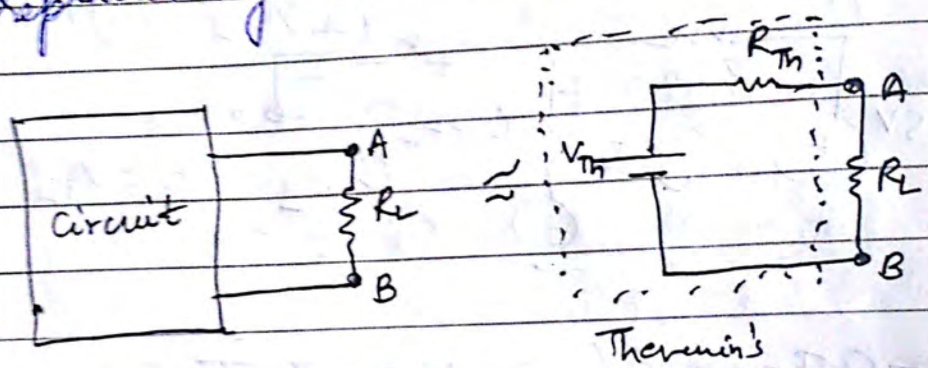
$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{23.58}{10+74}$$

$$I_L = 1.348\text{ A}$$

~~Imp~~ Maxi Power Transfer theorem:

11/1/22

In DC circuits, maximum power is transferred from source to load when load resistance is equal to internal resistance of the circuit as viewed from load terminals with the load removed and all emf sources replaced by their internal resistances.

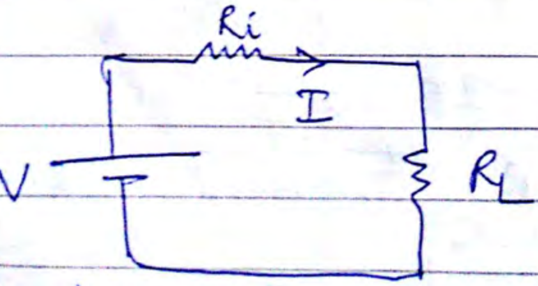


Thevenin's equivalent

$$\boxed{R_L = R_{Th} \text{ (or internal resistance of the circuit)}}$$

~~Imp~~

Proof:



Consider voltage source V with internal

resistance R_i delivering power to load R_L

Then, $I = \frac{V}{R_i + R_L} \longrightarrow (1)$

Power delivered, $P = I^2 R_L \longrightarrow (2)$

Putting (1) in (2)

$$P = \frac{(V)^2}{(R_i + R_L)^2} R_L$$

As V and R_i are constant P depends on R_L $\therefore$ Differentiating P w.r.t R_L, we can get condition for P_{max}.

$$\frac{dP}{dR_L} = 0$$

$$V^2 \left[\frac{[R_L + R_i]^2 - 2R_L(R_L + R_i)}{(R_i + R_L)^2} \right] = 0$$

So num. $= 0$.

$$(R_L + R_i)^2 - 2R_L(R_i + R_L) = 0$$

$$\Rightarrow (R_L + R_i)^2 = 2R_L R_i + 2R_L^2.$$

$$R_L^2 + R_i^2 + \cancel{2R_L R_i} = \cancel{2R_L R_i} + 2R_L^2$$

$$R_i^2 - R_L^2 = 0$$

$$\underbrace{(R_i + R_L)}_{\text{cannot be } 0}(R_i - R_L) = 0$$

$\cancel{R_i + R_L \neq 0}$ $R_i + R_L \neq 0$

$\therefore R_i - R_L = 0$

$\Rightarrow R_i = R_L$

or $\boxed{R_L = R_i}$

Imp

Points to be noted:

① Efficiency: The circuit efficiency at maximum power transfer is only 50% as one half of the total power is dissipated in internal resist. R_i of the source.

$$\eta = \frac{O/P \text{ power}}{I/P \text{ power}} = \frac{I^2 R_L}{I^2(R_i + R_L)}$$

For max. power

$\because$ i.e. $R_i = R_L$

$\therefore \eta = \frac{1}{2} = 50\%$

② Load voltage: (For max power)

$$V_L = IR_L = \frac{V}{(R_i + R_L)} \cdot R_L = \frac{V}{2}$$

$V \rightarrow$ supply voltage

Imp
Numerical + Theory

③ Max. power: (For max.)

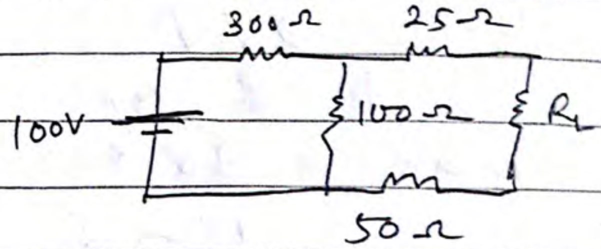
$$P_{max} = I^2 R_L$$

$$= \left(\frac{V}{R_i + R_L}\right)^2 \times R_L$$

$$\boxed{P_{max} = \frac{V^2}{4R_L}}$$

$V \rightarrow$ Thevenin's voltage

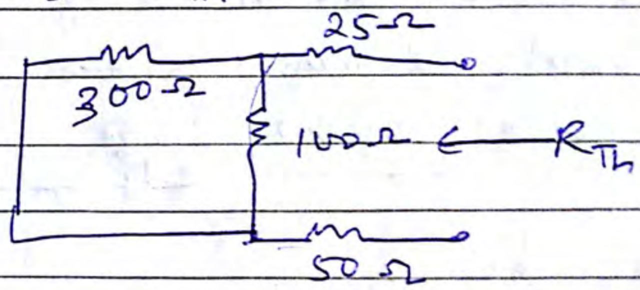
Q Find the ~~max.~~ value of R_L necessary to ~~obtain~~ obtain max. power in R_L. Also find the max. power in R_L



$R_L = ?$ $P_{max} = ?$

Sol^n: Acc. to max. Power transfer theorem (MPT theorem)

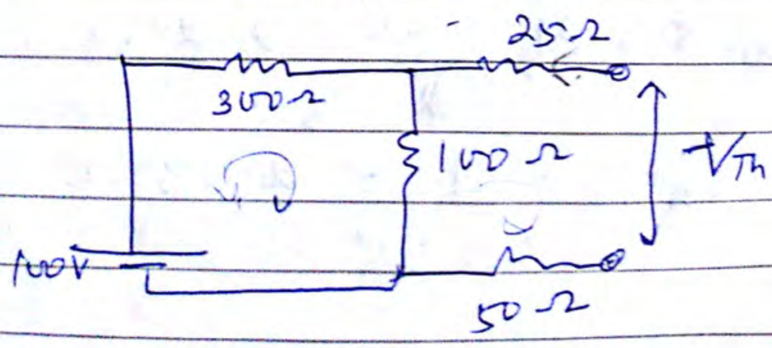
$$R_L = R_{Th}$$



$$R_{Th} = (300 \parallel 100) + 25 + 50$$
$$= 75 + 25 + 50$$
$$= 150\ \Omega$$

∴ Value of $R_L = 150\,\Omega$ for M.P.T.

$$P_{max} = \frac{V_{Th}^2}{4R_L}$$
$$= \frac{V_{Th}^2}{4R_L}$$



V_{Th} = voltage across $100\,\Omega$ resistor

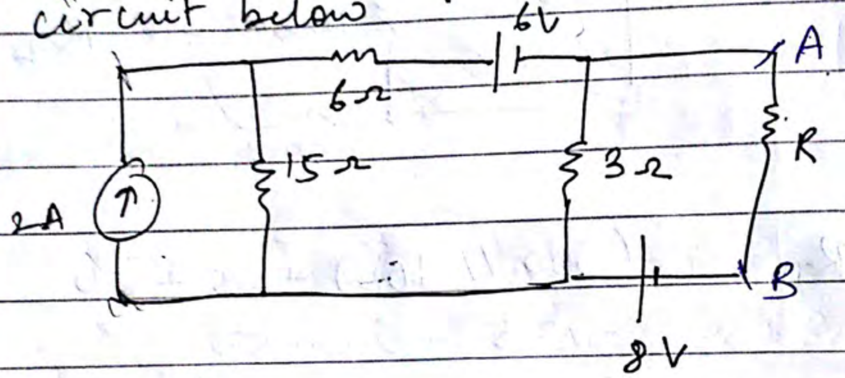
$= \frac{100}{400} \times 100$

$V_{Th} = 25\,V.$

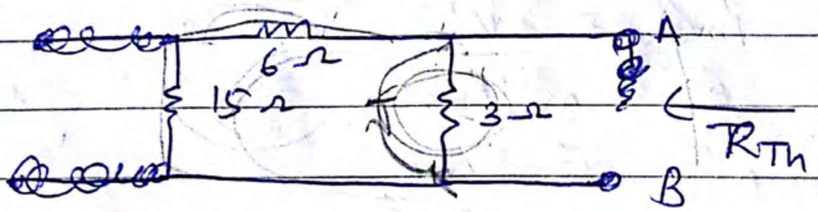
$P_{max} = \frac{\cancel{25} \times 25}{4 \times \cancel{150}} = \frac{25}{6 \times 4}$

$= 1.04W$

Q Calculate the value of R which will absorb maximum power from the circuit below



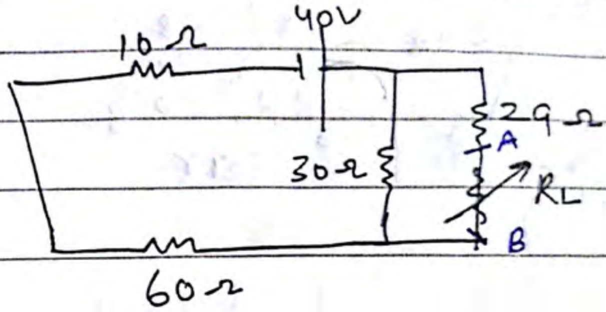
Soln: $R = ?$



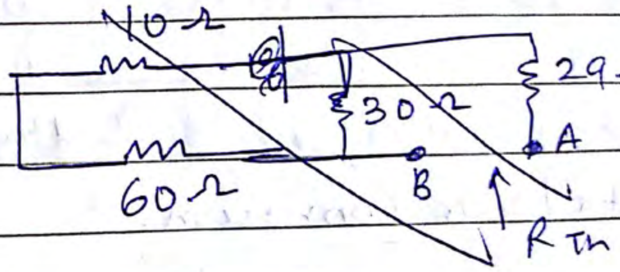
$\frac{1+7}{21} \rightarrow \frac{21}{8}$

$R_{Th} = R = \frac{21}{8} = 2.63\,\Omega$

Q. Find the value of R_L for which power transfer is maximum. Also calculate this power.



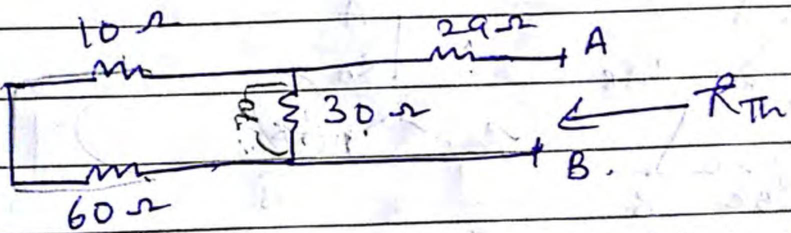
Soln:



$$\frac{1}{70} + \frac{1}{30}$$

$$\frac{30+70}{210}$$

$$\Rightarrow \frac{100}{210}$$



$$R_L = R_{Th} = 21 + 29$$

$$= 31.1\,\Omega$$

$$P_{max} = \frac{V^2}{4R_L}$$

$$= \frac{(40)(40)}{4(31.1)}$$

$$= \frac{400}{31.1}$$

$$= \quad W$$

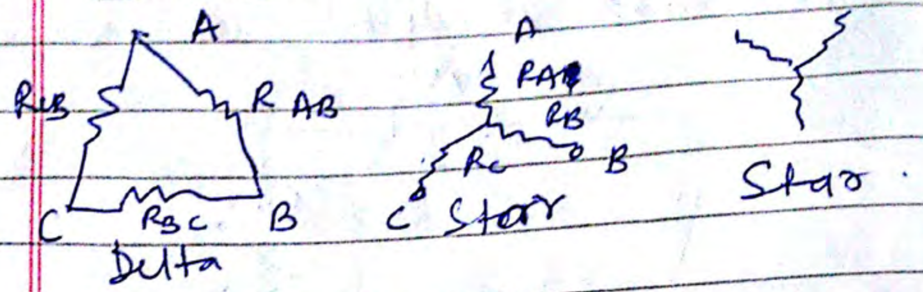
~~100~~ $V = IR$

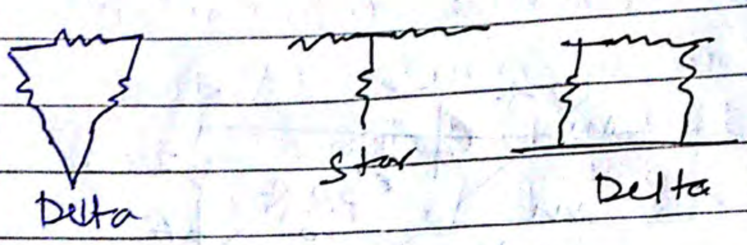
$10I +$

* eq. R_{eq} in Y = eq. R_{eq}

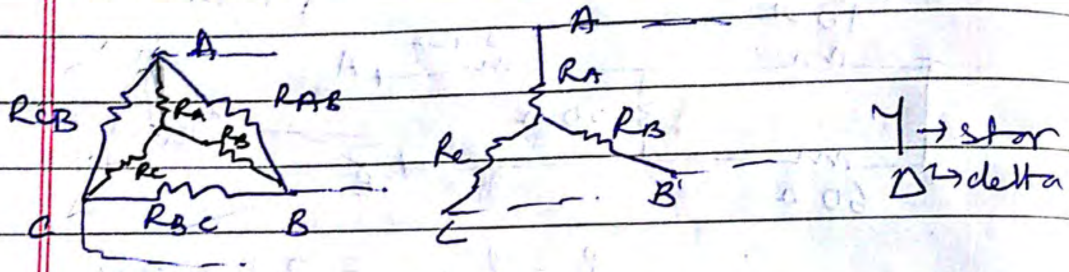
23/1/17 (5M) Imp.

Q) Delta/Star & Star/Delta Transformation





Delta/Star Conversion:



A & B in Y = $R_A + R_B = R_{AB} \parallel (R_{BC} + R_{CA})$

$$R_A + R_B \; \cancel{A \& B \text{ in } \Delta} = \frac{R_{AB}(R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (1)$$

B & C in Y = B & C in Δ

$$R_B + R_C = \frac{(R_{AB} + R_{CA})\, R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (2)$$

$$R_{CA} + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (3)$$

Subtract (2) from (1) and add to (3)

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (4)$$

$$R_B = \frac{R_{BC} R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (5)$$

$$R_C = \frac{R_{CA} R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \rightarrow (6)$$

Any arm of star connection = $\frac{\text{product of 2 adjacent arms of } \Delta}{\text{sum of arms of } \Delta}$

Star/Delta conversion:

divide (4) by (5)

$$R_A / R_B = R_{CA} / R_{BC}$$

$$R_{CA} = \frac{R_A R_{BC}}{R_B}$$

divide (4) by (6).

$$R_A / R_C = R_{AB} / R_{BC}$$

$$R_{AB} = \frac{R_A R_{BC}}{R_C}$$

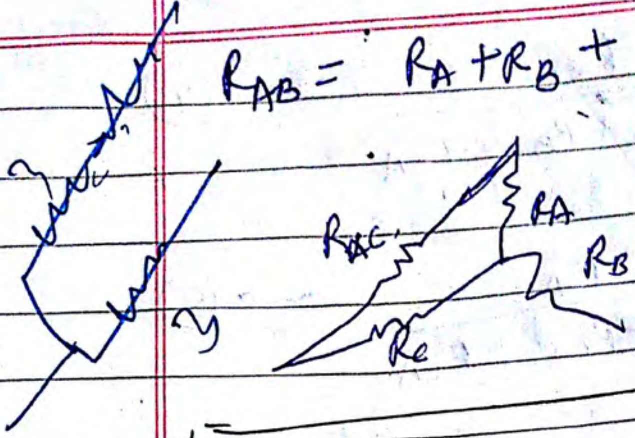
Put in (4)

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}.$$

Similarly

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$



Resistance b/w 2 terminals of Δ = sum of Y resistances connected to those terminals + (Product of same 2 resistances / IIIrd star resistance)

Q Find the value of current supplied by the battery by using star/delta transformation.

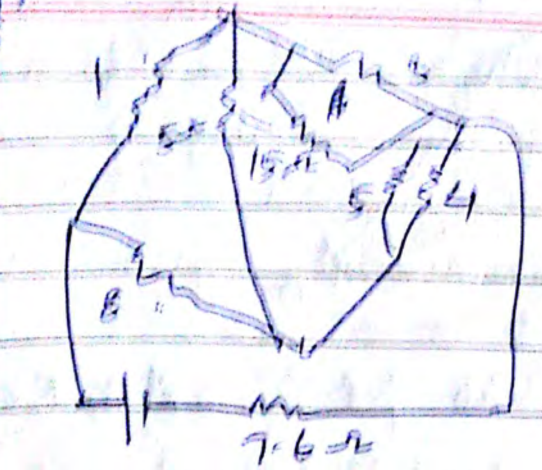
Sol^n:

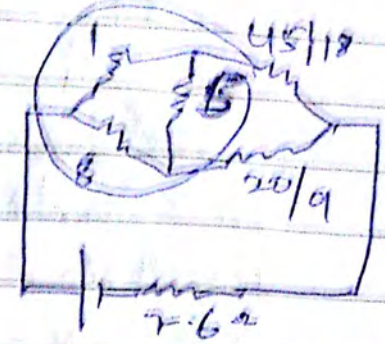
$$R_{ab} = 3 + 3 + \frac{(3\times3)}{1} = 15\ \Omega$$

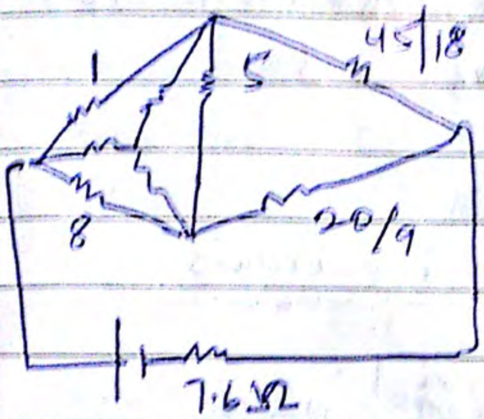
$$R_{bc} = 3 + 1 + \frac{3\times1}{3} = 5\ \Omega$$

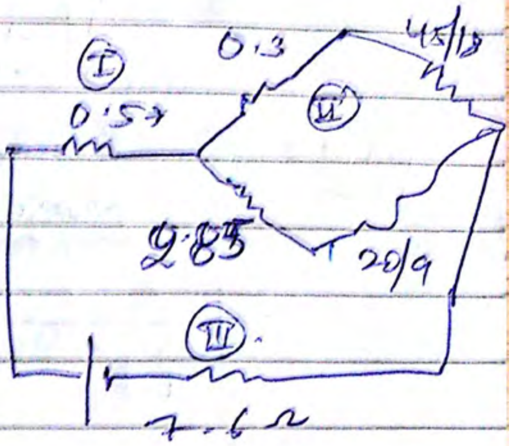
$$R_{ca} = 3 + 1 + \frac{3\times1}{3} = 5\ \Omega$$

9/10









~~Req~~ $R_{(II)} = 0.3 + \frac{45}{18} \parallel 2.85 + \frac{20}{9}$

$= \frac{1}{2.8} + \frac{1}{5.02}$

$\cancel{= 0.55}\ 4.8$

$R_{eq} = 0.57 + \cancel{0.55}\ 7.6 + 4.8$

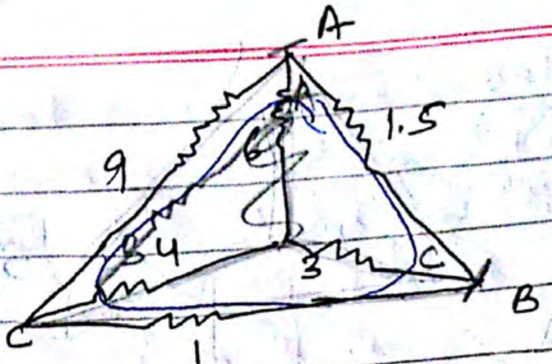
$= 9.9 \approx 10\ \Omega$

$V = IR$

$I = \frac{10}{10} = \underline{1\ A}$ (approx)

$3 = 121.5$

Q



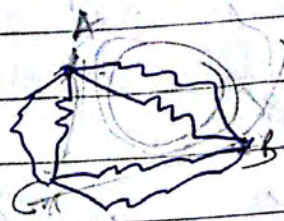
$R_{AB} =$

$R_{BC} =$

$R_{CA} =$

Find Req b/w pt.s AB, BC & CA

Sol^n:



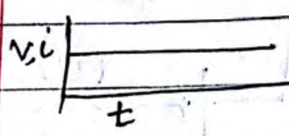
$R_a = 6$

$R_b = 4$

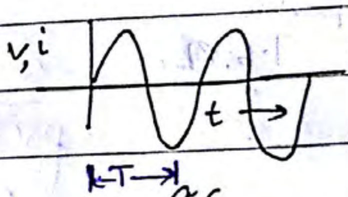
$R_c\ 3$

25/1/17

Module 2: A.C. Circuits:

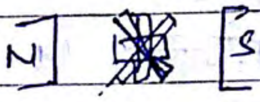


dc



ac

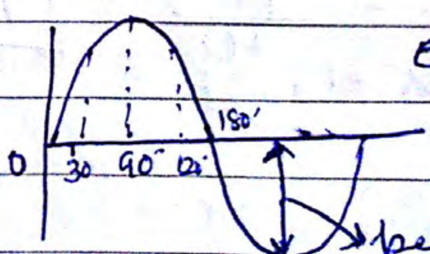
$$e = \frac{-d\phi}{dt}$$ (-ive due to Lenz's law & emf generated will oppose cause of motion.)



$\theta = \omega t$, $\omega = 2\pi f$ (rad/sec)

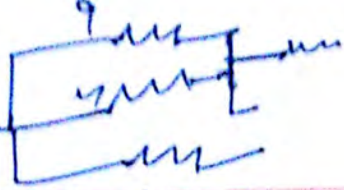
$= 2\pi f t = 2\pi \left(\frac{1}{T}\right) t$

$f = \frac{1}{T} \Rightarrow sec^{-1}$ or Hz



$e = Blv \sin\theta$

peak value / max. value



NG
CC
3 Cl.

$v_1 = V_m \sin \omega t_1$

$v_2 = V_m \sin \omega t_2$

or $V_m \sin \theta$

or $V_m \sin 2\pi f t_2$

or $V_m \sin \frac{2\pi}{T} t_2$

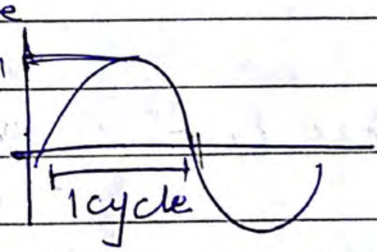
~~$2\pi f t + \theta = \omega t$~~

$\theta = 2\pi f t = \omega t$

Instantaneous value: value that changes from time to time

- ~~rms~~ rms value
- Peak value / Max value
- Average value: $\frac{\text{Area under curve}}{\text{Interval of curve}}$ [but 0 in case of sinusoidal wave]

Only calculate avg. value of one cycle in case of sinusoidal wave



Integrate V_m to get area under the curve

$A = \int_0^{\pi} V_m \sin \omega t$ [for full cycle]

★ Max value is also called amplitude of the wave form. (highest value)

★ Avg. value = $\frac{\text{~~Late~~ Net area under curve for time 't'}}{\text{time 't'}}$

Root mean square (rms): Effective or rms value of an alternating current is that steady state (D.C.) which when flowing through a given resistance for a given time produces same amount of heat as produced by alternating current when flowing through the same resistance for the same time.

Factors:

① **Form factor:** $\dfrac{\text{Rms value}}{\text{Avg. value}}$

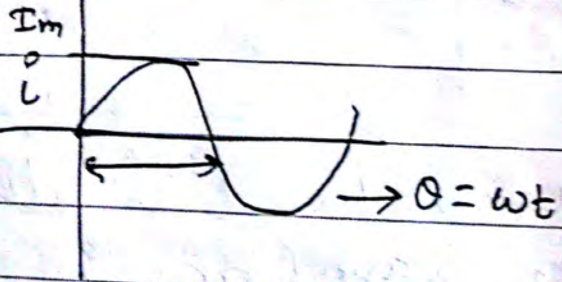
② **Peak factor:** $\dfrac{\text{max. value}}{\text{Rms. value}}$

To find rms value for sinusoidal value:

① ~~Find avg. value / mean value~~

② ~~Take square~~

③ ~~Take root~~



rms/Eff value $\int_0^{\pi} i\,d\theta$ $\qquad i = I_m \sin\theta$

$$= \int_0^{\pi} \frac{I_m \sin\theta \, d\theta}{\pi}$$

~~rms value~~ $\cancel{= \sqrt{\int_0^{\pi} \frac{I_m^2 \sin^2\theta \, d\theta}{\pi}}}$

$$\cancel{= \sqrt{\frac{I_m^2 \int_0^{\pi} \sin^2\theta \, d\theta}{\pi}}}$$

$$\text{rms value} = \sqrt{\int_0^{\pi} \frac{I_m^2 \sin^2\theta \, d\theta}{\pi}}$$

$$= \sqrt{I_m^2 \frac{\int_0^{\pi} \sin^2\theta \, d\theta}{\pi}}$$

$$= \frac{I_m}{\sqrt{2}}$$

$$\boxed{\cancel{\text{rms value}} = 0.707 I_m = I_{rms}}$$

$$\text{avg. value} = \frac{2 I_m}{\pi}$$

$$\boxed{I_{avg} = 0.637 I_m}$$

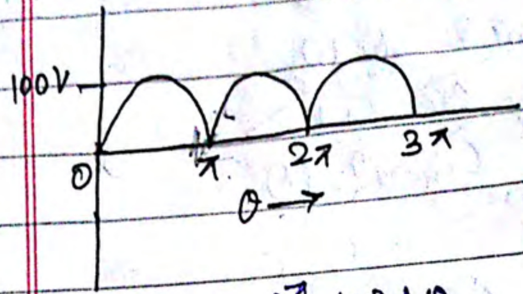
$$\boxed{\text{Form factor} = \frac{0.707 I_m}{0.637 I_m} = 1.11}$$

$$\boxed{\text{Peak factor} = \frac{I_m}{0.707 I_m} = 1.41}$$

also called as crest factor

1/11/17

Q Find rms value, avg. value & form factor of wave form given:



soln:

$$V_{av} = \frac{\int_0^{\pi} v(\theta) d\theta}{\pi}$$

$$v(\theta) = V_m \sin\theta$$

$$V_{av} = \int_0^{\pi} \frac{V_m \sin\theta \, d\theta}{\pi}$$

$$V_{av} = \frac{1}{\pi} \int_0^{\pi} V_m \sin\theta \, d\theta$$

$$= \frac{-1}{\pi} V_m [\cos\pi - \cos 0]$$

$$= \frac{-100}{\pi} [-2]$$

$$V_{av} = \frac{200}{\pi}$$

$$V_{rms} = \sqrt{\frac{\int_0^{\pi} v^2(\theta) d\theta}{\pi}}$$

$$v(\theta) = V_m \sin\theta$$

$$V_{rms} = \sqrt{\frac{\int_0^{\pi} V_m^2 \sin^2\theta \, d\theta}{\pi}}$$

$$= \sqrt{\frac{\int_0^{\pi} (100)^2 \sin^2\theta \, d\theta}{\pi}}$$

$$= (100)\sqrt{\int_0^{\pi} \frac{1-\cos 2\theta \, d\theta}{2\pi}}$$

$$\cancel{= 100 \quad \pi - \cos(2\pi) - 0 + \cos 0}$$

$$\cancel{= \frac{100}{\sqrt{2\pi}} \sqrt{\left[\theta - \frac{\sin \theta}{2}\right]_0^{\pi}}}$$

$$\cancel{= \frac{100}{\sqrt{4\pi}} \sqrt{\pi - \sin\pi - 0 + \sin 0}}$$

$$\cancel{= \frac{100}{2\sqrt{\pi}} \sqrt{\pi}}$$

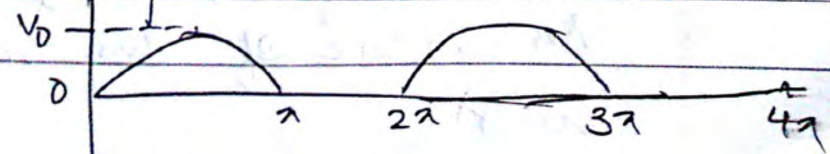
$$\cancel{= \frac{100}{2} = 50}$$

$$V_{rms} = \frac{100}{\sqrt{2}}$$

$$F.F = \frac{Rms}{Avg.} = \frac{100/\sqrt{2}}{200/\pi} = 1.11$$

$$P.F = \frac{max.}{rms} = \frac{100}{100/\sqrt{2}} = \sqrt{2} = 1.414$$

Q. Find avg. value, rms value & f.f of given wave form:



→ Here $0 \to 2\pi$ is complete cycle

Sol^n: $V_{avg} = \int_0^{2\pi} \frac{v(\theta)\,d\theta}{2\pi}$

$v(\theta) = V_0 \sin\theta$ $(0 \text{ to } \pi)$
$= 0$ $(\pi \text{ to } 2\pi)$

$V_{avg} = \int_0^{\pi} \frac{V_0 \sin\theta + 0}{2\pi}$

$= \frac{V_0}{\pi}$

$V_{rms} = \sqrt{\int_0^{2\pi} \frac{v^2(\theta)\,d\theta}{2\pi}}$

$= \sqrt{\frac{\int_0^{\pi} V_0^2 \sin^2\theta\,d\theta}{2\pi}}$

$= \frac{V_0}{2}$

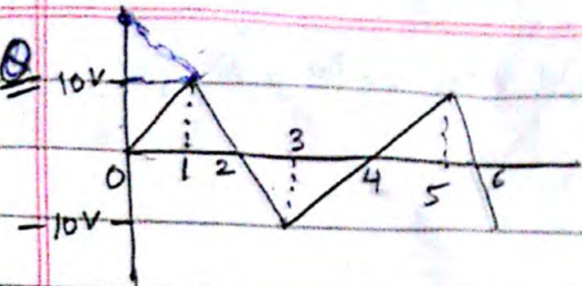
$F.F = \frac{Rms}{Avg} = \frac{V_0/2}{V_0/\pi} = \pi/2$

P.F (Crest factor) $= \frac{V_0}{V_0/2} = 2$

* In rectangular wave form P.F = F.F = 1 (as all values are const.) and let max. length = A, so avg. value & rms value of this wave form is A

$100t + (20)\,t = 200t$

Q



$(2,0)$

$0 = -10\times 2 + c$

$\Rightarrow c = 20$

$$V_{avg} = \frac{\frac{1}{2}\times 2\times 10}{2} = 5$$

$$V_{rms} = \sqrt{\frac{\int_0^2 v^2(t)\,dt}{2}} = \sqrt{\frac{\int_0^1 (10t)^2 dt + \int_1^2 (-10t+20)^2 dt}{2}}$$

$$= \left[\frac{100t^3}{3}\right]_0^1 + \cancel{\left[\frac{-10t^2}{2} + 20t\right]}$$

$$= \frac{100}{3} + \left[\frac{100t^3}{3} + 400t - \frac{200t^2}{2}\right]_1^2$$

$$= \frac{100}{3} + \left[\frac{800}{3} - \frac{100}{3} + 400 - 100 - \frac{800}{2} + \frac{200}{2}\right]$$

$$= \frac{100}{3}\left[\cancel{300} + \frac{700}{3} - \cancel{300}\right]$$

$$= \sqrt{\frac{1000}{3\times 2}}$$

$$= \sqrt{\frac{1000}{6}} = 5.773V$$

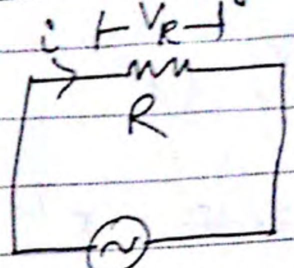
Check

$V_{rms} = 5.773V$

$$F.F = \frac{Rms}{Avg} = \frac{5.773}{5} = 1.154$$

2/17

A.C. through resistance

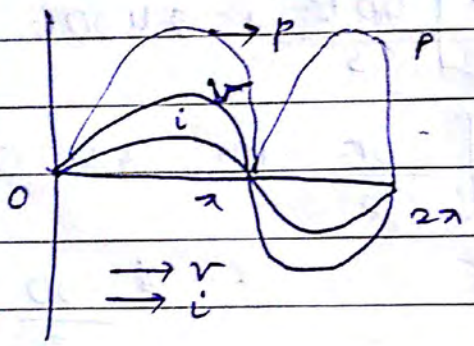


$v = V_m \sin \omega t \longrightarrow (1)$

$i = \frac{v}{R}$

$= \frac{V_m \sin \omega t}{R}$

$i = I_m \sin \omega t \longrightarrow (2)$



$P = vi$

$= V_m \sin \omega t \, I_m \sin \omega t$

$= V_m I_m \sin^2 \omega t$

$= I_m V_m \left(\frac{1 - \cos 2\omega t}{2} \right)$

$= \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\omega t}{2}$

avg. power = true power = active power

$= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} - \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m \cos 2\omega t}{2}$

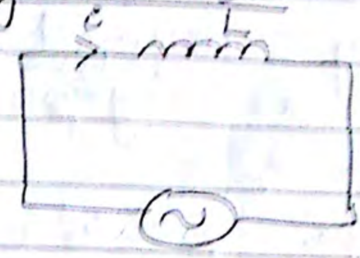
$= \frac{V_m I_m}{2} = V_{rms} I_{rms} = VI$

Power factor = 1 = $\cos \phi$

angle b/w V & I

Power factor: Amount of power absorbed by the load.

A.C through Inductor:



$V = V_m \sin \omega t \rightarrow (1)$

$$V = V_m \sin \omega t \rightarrow (1)$$

$$V_m \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin \omega t$$

$$\int di = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$i = \frac{V_m}{\omega L}(-\cos \omega t)$$

$$i = I_m (-\cos \omega t)$$

$$i = I_m \sin(\omega t - \pi/2) \rightarrow (2)$$

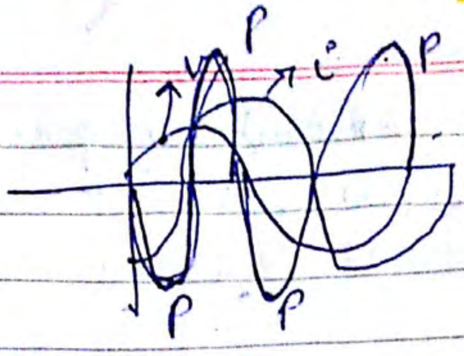
$$I_m = \frac{V_m}{\omega L}$$

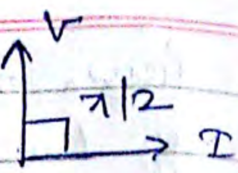
$$\Rightarrow \frac{V_m}{I_m} = \omega L$$

$$= 2\pi f L$$

$$= X_L \rightarrow \text{Inductive reactance}$$

(Ω)





~~P=0~~

Energy consumed by it = 0

= active ~~avg~~ power = 0 = avg. power

So, Reactive power will be considered

proof

$P = vi$

Instantaneous power

$= V_m \sin \omega t \; I_m \sin(\omega t + \pi/2)$

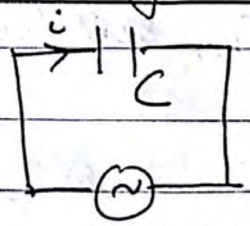
$= -V_m I_m \sin \omega t \cos \omega t.$

$= \frac{-V_m I_m}{2} \sin 2\omega t.$

avg power $= \frac{1}{2\pi}\int_0^{2\pi} \frac{-V_m I_m}{2} \sin 2\omega t$

$= 0$

A.C. through capacitance:



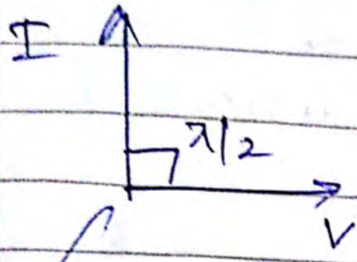
$v = V_m \sin \omega t$

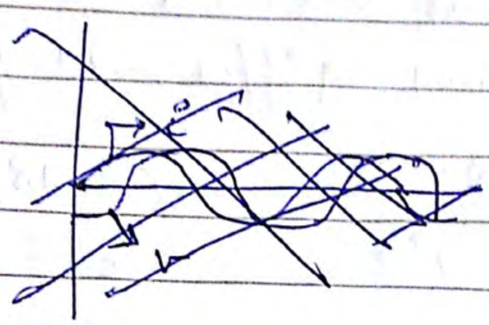
$V = V_m \sin \omega t \quad \rightarrow (1)$

$q = cv \quad = cV_m \sin \omega t$

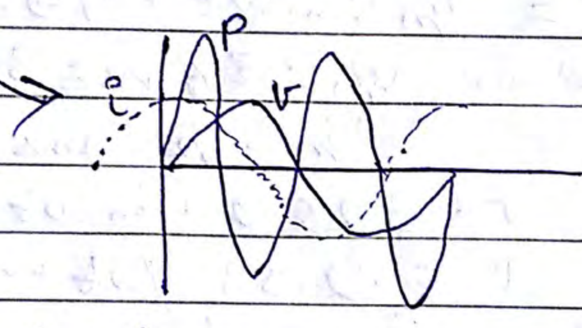
$i = \frac{dq}{dt}$

$$i = \omega C V_m \cos\omega t$$
$$= I_m \cos\omega t$$
$$= I_m \sin(\omega t + \pi/2) \rightarrow (2)$$





$$\frac{V_m}{I_m} = \frac{1}{\omega C}$$
$$= \frac{1}{2\pi f C}$$
$$= X_C \quad (\Omega)$$



$$p = vi$$
$$= V_m \sin\omega t \; I_m \sin(\omega t + \pi/2)$$
$$= V_m I_m \sin\omega t \cos\omega t$$
$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$\text{avg. } P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin\omega t = 0$$

Net power absorbed in an inductor and in capacitor is 0

Q ~~3/8 μF~~ 318 μF capacitor is connected across 230 V, 50 Hz system.

(i) Determine capacitive reactance

(ii) rms value of current

(iii) Eqⁿ for voltage & current

Solⁿ: $C = 318\,\mu F$

(1) $X_c = \frac{1}{2\pi f C} = 10\,\Omega$

(2) rms value of current

$= \frac{V}{X_c} = \frac{230}{10} = 23\,A$

(3) $v = V_m \sin\omega t$

$i = I_m \sin(\omega t + \pi/2)$

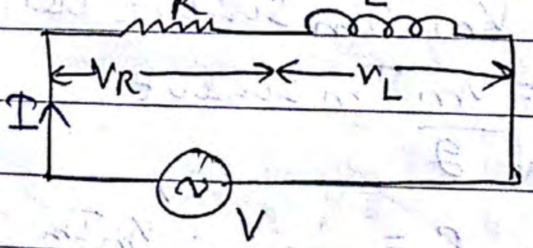
$V_m = \sqrt{2}\,V_{rms} = 325.27\,V$

$I_m = \sqrt{2}\,I_{rms} = 32.53\,A$

$v = 325.27 \sin\omega t$

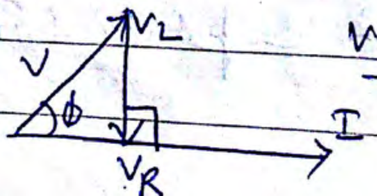
$i = 32.53\,(\omega t + \pi/2)$

17. R-L Series A.C Circuit:



$V_R = IR$

$V_L = IX_L$



Voltage triangle

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$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}.$$

$$V = IZ.$$

$$Z = \sqrt{R^2 + X_L^2} \text{ (Impedance)}$$

$\phi \rightarrow$ power factor angle

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

(Y) Admittance $\propto \frac{1}{\text{Impedance (Z)}}$

(X) Reactance $\propto \frac{1}{\text{susceptance (B)}}$

$$Y = \frac{1}{Z}, \mho \text{ Siemen.}$$

$$P = I_m V_m \sin\omega t \sin(\omega t - \phi)$$

$$= \frac{V_m I_m}{2}\left[2\sin\omega t - \sin(\omega t - \phi)\right]$$

$$= \frac{1}{2} V_m I_m \left[\cos\phi - \cos(2\omega t - \phi)\right]$$

$$P = \frac{1}{2} V_m I_m \cos\phi \; \frac{1}{2} V_m I_m \cos(\omega t - \phi)$$

Instantaneous power

~~Instantaneous power~~ o

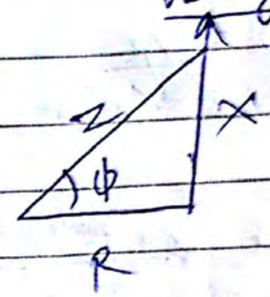
Integrate instantaneous power by Over a cycle to get avg. power

$$\text{avg. power} = \frac{1}{2} V_m I_m \cos\phi$$

$$= \frac{V_m I_m \cos\phi}{\sqrt{2} \times \sqrt{2}}$$

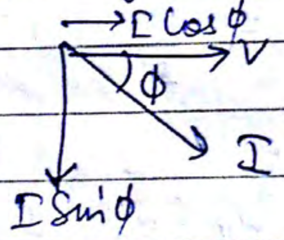
$$\boxed{P = VI\cos\phi}$$

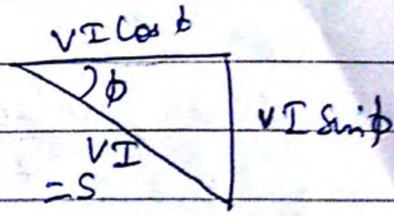
~~voltage triangle~~



Impedance triangle

voltage triangle:





VI → Real power / active power / true power or avg. power

Q ⇒ Reactive power

S → apparent power

$$S = \sqrt{P^2 + Q^2}$$

~~S → from source to the load~~
~~Q. [illegible] absorbed from load~~

* <u>Apparent Power</u>: Total power that appears to be transferred b/w source & load ($S = VI$). [V.A → volt amp.]

* <u>True Power</u>: It is that power which is actually consumed in the circuit. Power is consumed in resistance only since inductor and capacitor does not consume any active power. It is useful component on apparent power.

$$P = VI\cos\phi \qquad \text{[Watts]}$$

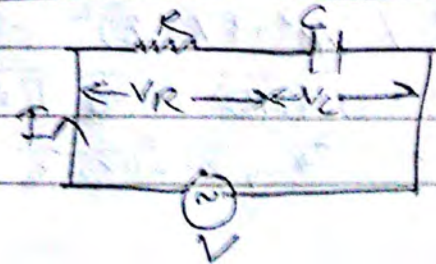
* <u>Reactive power</u>: component of apparent power which is neither consumed nor does any useful work in the circuit. Circulating power is called as reactive power.

$$Q = VI\sin\phi \qquad \text{[V.A.R]}$$

volt ampere Reactive

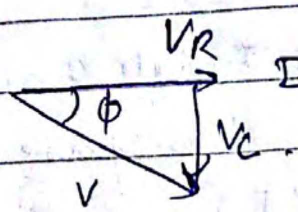
$$\boxed{\cos\phi = \frac{P}{S}}$$

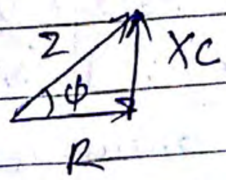
<u>RC- series A.C. circuit:</u>



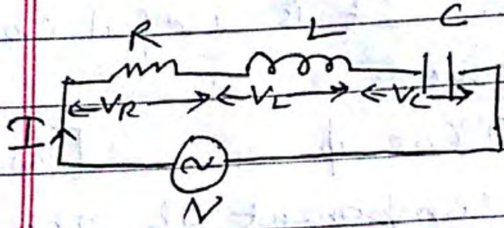
$$V_R = IR$$
$$V_C = IX_C$$

$$Z = \sqrt{R^2 + X_C^2}$$

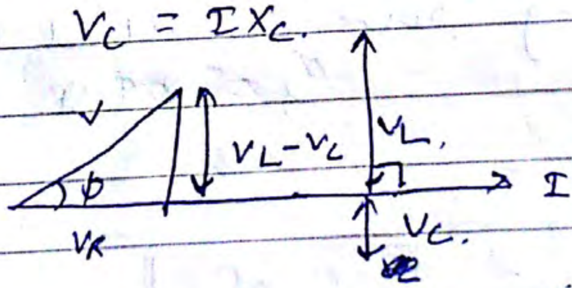


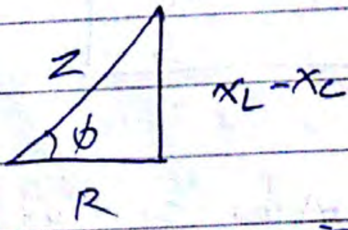


10|2|17 R-L-C Series A.C Circuit:



$$V_R = IR$$
$$V_L = IX_L$$
$$V_C = IX_C$$





$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

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$V = IZ$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = |R + j(X_L - X_C)|$ Check

$\cos\phi = R/Z$

$\tan\phi = \frac{V_L - V_C}{R} = \frac{X_L - X_C}{R}$

$X_L > X_C$

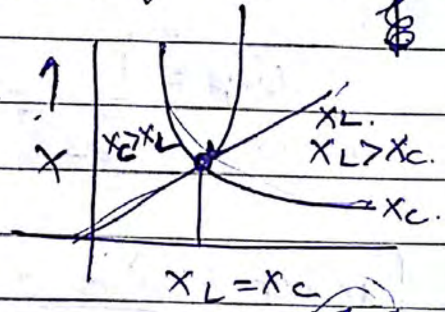
$X_L - X_C \rightarrow$ +ive I leads V by ϕ.

$X_L - X_C \rightarrow$ -ive I leads V by ϕ.

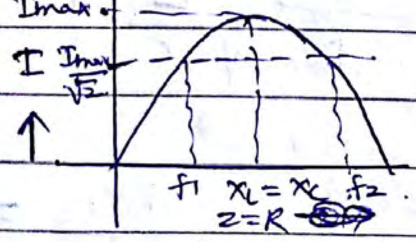
$X_L > X_L$

$X_L - X_C = 0$
$(X_L = X_C)$ I in phase with voltage
$\phi = 0$

$\boxed{Z = R}$ [Resonance condition)



→ frequency



→ frequency

Band width = $f_2 - f_1$ [~~lower & upper~~ & lower cut of frequency]

Resonant Frequency:

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz}$$

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}}} \text{ rad/sec}$$

$$Q \text{ factor} = \frac{X_C}{R} = \frac{\omega_r L}{R}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$Q \text{ factor} = \frac{X_C}{R} = \frac{1}{\omega_r C R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$Q \text{ factor} = \frac{V_L}{V} = \frac{I X_L}{I R} = \frac{X_L}{R}$$

$$= \frac{V_C}{V} = \frac{I X_C}{I R} = \frac{X_C}{R}$$

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Q - factor: Quality factor circuit.

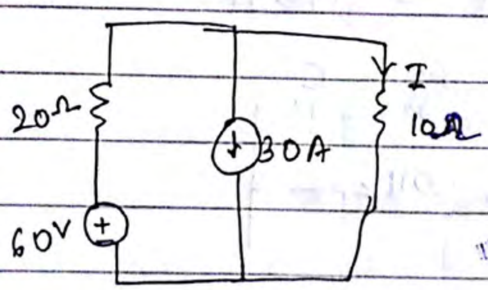
B.W → Band width (used in commⁿ network (radio frequen band).)

$$\boxed{\begin{aligned} BW \ominus \quad f_2 &= f_r + \frac{R}{4\pi L} \\ f_1 &= f_r - \frac{R}{4\pi L} \\ f_r &= \sqrt{f_1 + f_2} \end{aligned}}$$

$$\boxed{B.W = f_2 - f_1}$$

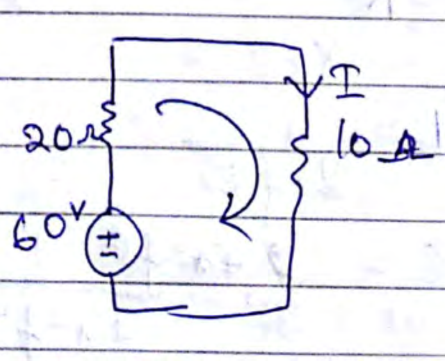
20/7/17 Superposition theorem:

Q



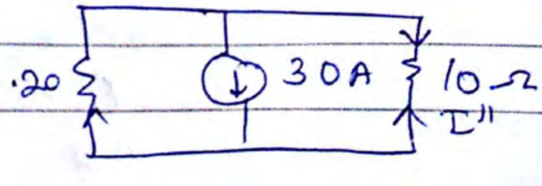
$I_{10\Omega} = ?$

Solⁿ: Case I:



$$I_{10} = \frac{60}{30} = 2\,A.$$

Case II:



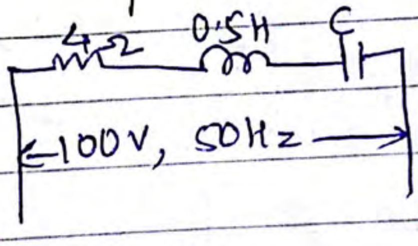
$$I'' = \frac{30 \times 20}{30} = 20A$$

$$I = 2 - 20$$
$$= -18A$$

7/2/17

Q A circuit consists of resistance of 4Ω, Inductance 0.5 Henry, variable capacitance in series across 100V, 50Hz supply.
(i) Calculate value of capacitance to produce resonance
(ii) Voltage across capacitance
(iii) Q factor of the circuit.

Sol^n:



$X_L = X_C$.

$Z = R$ $\qquad\qquad\qquad I = V/R$

$X_L = X_C$.

$$2\pi f L = \frac{1}{2\pi f C}$$

$$C = \frac{1}{4\pi^2 f^2 L}$$

$$C = \frac{1}{4(3.14)^2 (50)^2 (0.5)}$$

$$C = \frac{1}{39438.4}$$

$C = 20.26 \mu F$.

~~$V_C = I X_C$~~

$I = \frac{100}{4} = 25 A$.

$V_C = I X_C$

$= \frac{25}{2 \times \pi \times 50 \times 20.26}$

$= \frac{25}{6364.86}$

~~$= 3.92 \times 10^{3} V$~~

$= 3925 V$.

$Q = \frac{\omega L}{R} = 39.27$

Q Coil of resistance 40Ω, $L = 0.75 H$ are connected in series. Resonant freq: $= 55 Hz$. If supply voltage is $250V, 50 Hz$ find line current, power factor, power consumed (active power).

Soln: $f_r = \frac{1}{2\pi\sqrt{LC}}$

$55 = \frac{1}{2\pi\sqrt{0.75 C}}$

$\boxed{C = 11.16 \mu F}$

$X_L = 2\pi f L = 235.6 \Omega$

$X_C = \frac{1}{2\pi f C} = 285.10 \Omega$.

* Always write whether power factor is leading / lagging.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= 83.62\ \Omega$$

$$I = \frac{V}{Z} = 3.92A$$

$$\frac{R}{Z} = \cos\phi$$

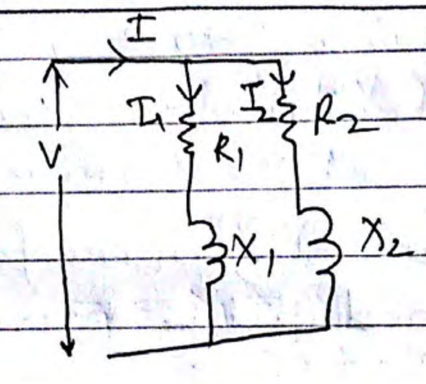
$$= 0.6287 \text{ leading}$$

$$P = VI\cos\phi$$

$$P = 617.63\ W$$

$$S = VI = \sqrt{P^2 + Q^2}$$

Parallel A.C Circuit:



$Z_1 = 4 + j0.5$ → rectangular form

$$Z = \sqrt{4^2 + (0.5)^2} \angle \underbrace{\tan^{-1}\left(\frac{X}{R}\right)}_{\text{Polar form}}$$

$$Z = M \angle \theta$$

$$M_1 \angle \theta_1 \qquad M_2 \angle \theta_2$$

$$M_1 M_2 \angle \theta_1 + \theta_2$$

$$\frac{M_1}{M_2} \angle \theta_1 - \theta_2$$

$$a + jb = m\angle\theta$$
$$= \sqrt{a^2+b^2}\angle\tan^{-1}(b/a)$$

$$M(\cos\theta + j\sin\theta) = M\angle\theta$$
(rectangular form)

$$I = I_1 + I_2$$
$$= \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$
$$= V\left(\frac{Z_1+Z_2}{Z_1 Z_2}\right)$$

$$I = VY_1 + VY_2 \qquad \frac{1}{Z_1} = Y_1$$
$$= V(Y_1 + Y_2) \qquad \frac{1}{Z_2} = Y_2$$

$Y \rightarrow$ ~~conductance~~ addmittance

$$I = VY$$

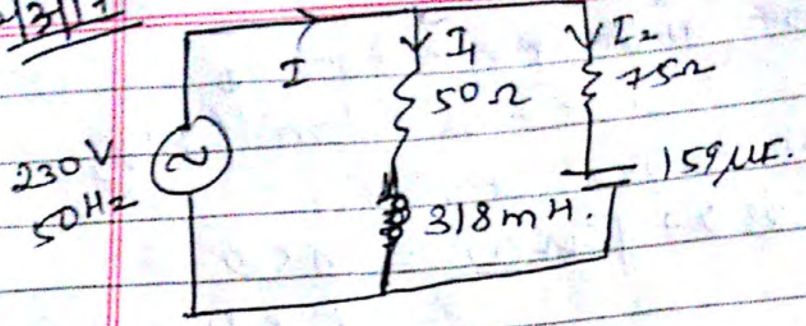
$$Z = \frac{1}{25\angle\theta} \qquad \angle\theta \rightarrow \text{at angle } \theta$$

$$Y = 25\angle(-\theta)$$

$$Z = \frac{1}{25 + j0.5}$$

$Y =$ Rationalise above & then reciprocate.

3/3/17



Determine supply current & circuit power factor.

Sol^n:

$$I = I_1 + I_2$$

$$= \frac{V}{Z_1} + \frac{V}{Z_2}$$

$$I = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right).$$

$$I = I_1 + I_2$$

$$= \frac{V}{Z_1} + \frac{V}{Z_2}$$

$$= V(Y_1 + Y_2)$$

$$I = VY$$

$$Y = Y_1 + Y_2$$

$$Z_1 = R_1 + jX_L$$

$$= \cancel{50 + j(318 \times 10^{-3})}$$

$$= 50 + j(2\pi f(318 \times 10^{-3}))$$

$$Z_2 = R_2 - jX_C$$

$$= \cancel{75 - j(159 \times 10^{-6})}$$

$$= 75 - j\left(\frac{1}{\omega C}\right)$$

$$= 75 - j\left(\frac{1}{2\pi f \times 159 \times 10^{-6}}\right)$$

$$Z_1 = 50 + j(99.99)\,\Omega$$

$$Z_2 = 75 - j(20.01)\,\Omega$$

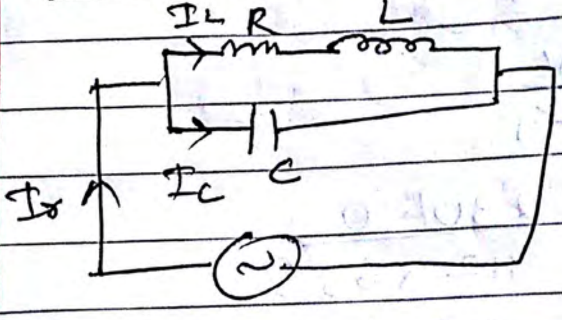
I (in polar form) $\rightarrow 3.92 \angle -15.7$

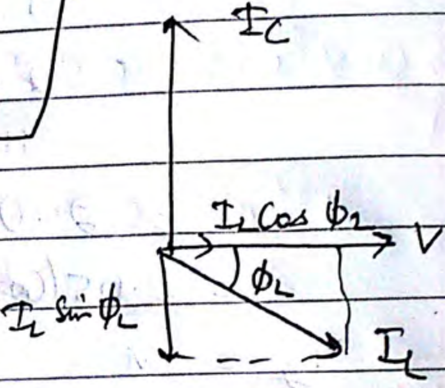
$-$ive sign means current is lagging

Power factor $= \cos(-15.7)$ ~~(lagging)~~

$= 0.962$ (lagging)

Resonance in parallel A.C. circuit:



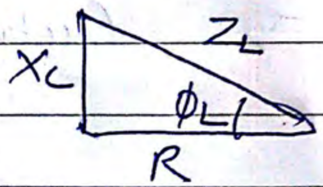


$Z_L = R + jX_L$

For resonance

, I should be equal to $I_L \cos\phi_L$

$\therefore\ I_C - I_L \sin\phi_L = 0$



$I_C = I_L \sin\phi_L$

$$\frac{V}{X_C} = \frac{V}{Z_L}\left(\frac{X_L}{Z_L}\right)$$

or $X_L X_C = Z_L^2$

$\omega L / \omega C = Z_L^2$

$$\frac{L}{C} = R^2 + (2\pi f_r L)^2$$

$$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$2\pi f_r L = \sqrt{\frac{L}{C} - R^2}$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

Impedance at resonance:

$$I_r = I_L \cos\phi_L$$

$$\frac{V}{Z_r} = \frac{V}{Z_L}\frac{R}{Z_L}$$

$$\frac{1}{Z_r} = \frac{R}{Z_L^2}$$

$$Z_L^2 = \frac{L}{C}$$

$$\frac{1}{Z_r} = \frac{R}{Z_L^2} = \frac{RC}{L}$$

$$\boxed{Z_r = \frac{L}{CR}}$$

Dynamic Impedance.

Q. Dynamic impedance of ∥ resonant circuit is 500 kΩ. Circuit consists of 250 pF capacitor in ∥ with coil of res = 10 Ω. Calculate

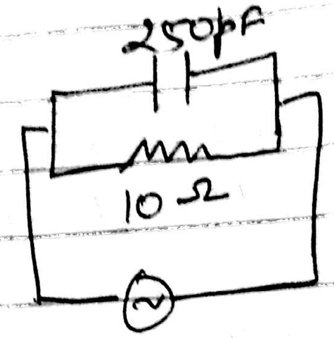
(i) coil inductance

(ii) resonant frequency.

(iii) Q-factor

Soln:

(i) $Z_x = 500\,k\Omega$



$$Z_x = \frac{L}{CR}$$

$$L = Z_x CR$$

$$= (500)\times 10^3 \times 10 \times 250 \times 10^{-12}$$

$$= 1.25\times 10^{-3}\,H$$

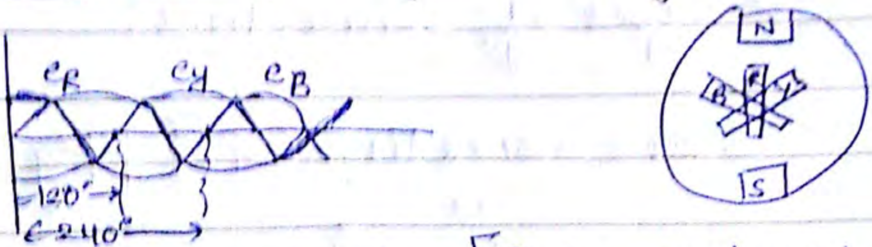
(ii) $$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{(1.25\times10^{-3})(250\times10^{-12})} - \frac{10^2}{(1.25\times10^{-3})}}$$

$$= \frac{1}{2\pi}\sqrt{}$$

9/3/17

3 phases Systems: Module IV

Generation of polyphase supply:



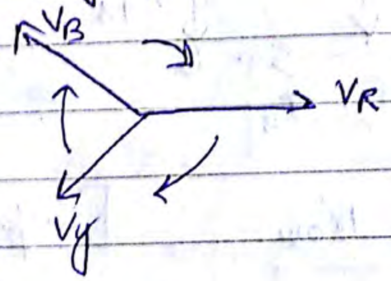
wave form.

$e_R = E_m \sin \omega t$. [E_m: max. value of Emf induced]

$e_Y = E_m \sin(\omega t - 120°)$

$E_B = E_m \sin(\omega t - 240°)$

Phasor diag:



* Power delivered by 3 phase is more (3 times $VI \cos\phi$) Power factor is better.

Advantages of 3 phase:

~~① Power delivered by 3 phase is more~~

① lesser conductor for transmission of same output of power at same voltage

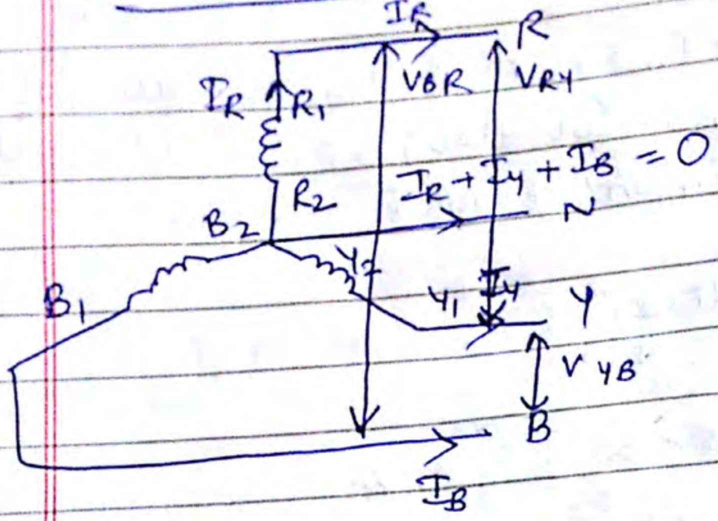
② Polyphase motors have uniform torque whereas it is ~~pure~~ pulsating in single phase

③ Power factor of single phase machine is ~~more~~ ~~low~~ lower than 3 phase machine

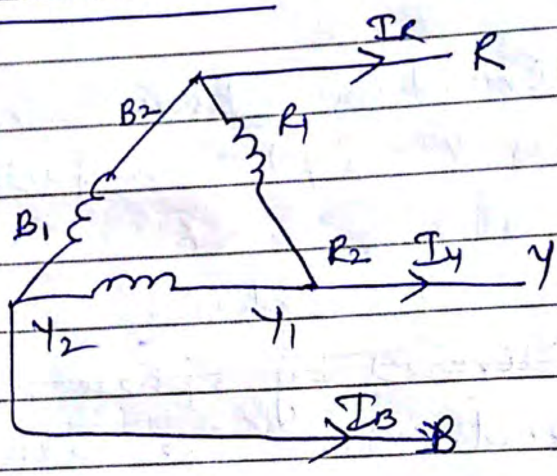
(4) Efficiency of 3 phase machine is higher than single phase machine.

(5) 2 types of connections:

Star Connection: [Supply]



Delta connection: [Supply].



* No neutral pt- in Delta connection
* Phase seq.: Sequence at which each phase reaches its max- value (General RYB)
  → due to this direction of torque can be changed

* 
If Voltage is measured b/w 2 lines which ~~these~~ ~~they~~ are line voltages (V_RY ~~&~~, V_YB, V_RY)

* Phase voltage : Voltage b/w each phase (voltage b/w 1 line & neutral)

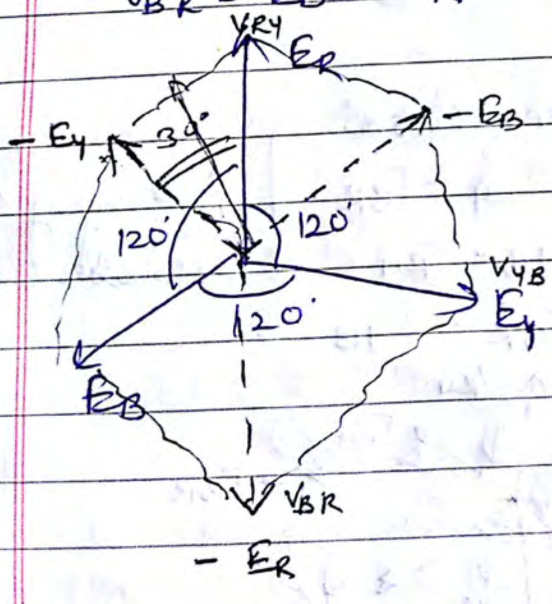
* In star connection : line current = phase current but line voltage ≠ phase voltage

* In delta connection : every phase is connected b/w 2 lines so line voltage = phase voltage ~~both~~ but line current ≠ phase current

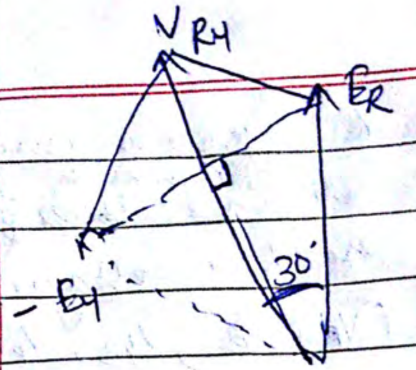
In star connection:

$$V_{RY} = E_R - E_Y$$
$$V_{YB} = E_Y - E_B$$
$$V_{BR} = E_B - E_R$$

} ~~phase voltage~~ in terms of line voltages



Apply ||gm law to obtain relation



$V_{RY} = 2 E_R \cos(60/2)$.

$= 2 E_R \frac{\sqrt{3}}{2}$.

$$\boxed{V_{RY} = \sqrt{3} E_R}$$

$$\boxed{V_L = \sqrt{3} V_{phase}}$$

$$\boxed{I_L = I_{phase}}$$

$P = 3 V_{phase} I_{phase} \cos\phi$.

$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$.

$$\boxed{P = \sqrt{3} V_L I_L \cos\phi}$$

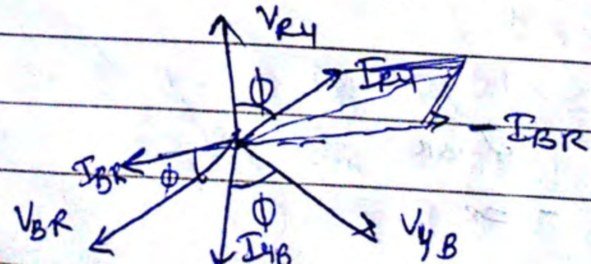
★ In Delta connection:

$I_R = I_{RY} - I_{BR}$

$I_Y = I_{YB} - I_{RY}$

$I_B = I_{BR} - I_{YB}$

[Applying KCL at nodes]



$I_{RY} \rightarrow$ phase current

$I_R \rightarrow$ line current

angle b/w $\angle V_{RY}$ & $I_R \rightarrow 30+\phi$

$$I_R = 2\, I_{RY} \cos 30^\circ$$
$$= 2\, I_{RY} \frac{\sqrt{3}}{2}$$
$$I_R = \sqrt{3}\, I_{RY}.$$

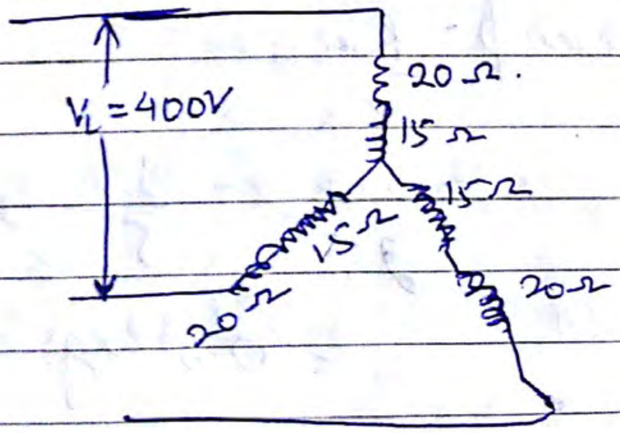
$$\boxed{I_L = \sqrt{3}\ I_{phase}}$$

$$\boxed{V_L = V_{phase}}$$

8|3|17

Q 3 coils each having res. of 20 Ω and inductive reactance 15 Ω are connected in star to 400V, 3-phase 50 Hz supply. Calculate line current, power factor & power supply.

~~Sol~~:



~~$X_L = 20\,\Omega$~~

~~$R = 15\,\Omega$~~

$R = 20\,\Omega$

$X_L = 15\,\Omega$

~~Phase Bal~~

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$ $V_L = 400V$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$= \frac{400}{\sqrt{3}}$$

$$V_{ph} = 231 V$$

$$Z_{ph} = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{(20)^2 + (15)^2}$$

$$= \sqrt{400 + 225}$$

$$= \sqrt{625}$$

$$= 25\,\Omega$$

$$I_{ph} = \frac{231\,\Omega}{25\,\Omega} = 9.24\,A = I_L$$

~~Power factor~~

$$\text{Power factor} = \cos\phi = \frac{R_{ph}}{Z_{ph}}$$

$$= \frac{20}{25} = \frac{4}{5}$$

$$= 0.8 \text{ lag}$$

$$\text{Power} = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= 3\, V_{ph}\, I_{ph} \cos\phi$$

$$= 3 \times 0.8 \times 9.24 \times 231 = 5121\,W$$

Q 3 similar coils, R = 5Ω, L = 0.02H are connected in delta to 440V, 3 phase 50 Hz, supply. Calculate line current & total power absorbed.

Soln:
$V_L = 440V$.
$R = 5\Omega$.
$X_L = 0.02H$.
$f_t = 50Hz$.

(i) Delta: $V_L = V_{phase}$.

$V = IR$.

$I_{ph} = \frac{V_{ph}}{Z_{ph}}$ &

$= \cancel{\frac{440}{\sqrt{(5)^2 + (0.02)^2}}}$

~~= 88 A~~

$\Rightarrow \cancel{\frac{440}{\sqrt{25 + 0.0004}}}$

$Z = \sqrt{(5)^2 + (2\pi(50)(0.02))}$

$= \sqrt{25 + (2\pi)}$.

$= \sqrt{25 + 2\pi}$

$= 8.05\Omega$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{440}{8.05} = 54.6A$$

$$I_L = \sqrt{3}\, I_{ph}$$

$$\boxed{I_L = 95.02A}$$

$$P = VI\cos\phi$$

$$\cos\phi = \frac{R_{ph}}{Z_{ph}}$$

$$\cos\phi = 0.62$$

$$P = 3V_{ph} I_{ph} \cos\phi = 45kW.$$

Star connection:

$$I_L = 54.6A = I_{ph}$$

$$V_L = \sqrt{3}\, V_{ph}$$

~~Vph =~~

$$\cancel{\cos\phi}\quad P = \sqrt{3}\, V_L I_L \cos\phi$$

$$= \sqrt{3}(440)(54.6)(0.8)$$

$$= 33\,288.630W$$

~~33.2886kW~~

15 kW

lagging → R_L
leading → R_C
Reactive power = $\sqrt{3} V_L I \sin\phi$
~~Active~~ $S = \sqrt{3} V_L I$

Q A Balanced (impedances are same) delta connected load takes a line current of 18A at power factor of 0.85 leading from 400 V, 3 phase 50Hz supply. Calculate res- & capacitance of each branch of load.

Soln:

~~$V_L = 400V$~~

~~$\cos\phi = 0.85$~~

~~$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{\sqrt{\ }}$~~

~~$I_L = \sqrt{3}\, V_{ph}\, I_{ph}$~~

Delta connected load,

$I_L = \sqrt{3}\, I_p$, $V_L = V_{ph}$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$= \frac{18}{\sqrt{3}}$$

$$= 10.37\,A.$$

$$\cos\phi = 0.85 \text{ Leading} = \frac{R_{ph}}{Z_{ph}}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{10.37} = 38.5\,\Omega$$

~~$R_{ph} = Z_{ph}\,(38.5)(0.85)$~~

~~$= 32.72\,\Omega$~~

$$Z_{ph} = \sqrt{\quad}$$

~~Rph~~

$$R_{ph} = Z_{ph} \times \cos\phi$$
$$= 32.72\,\Omega.$$

$$Z_{ph} = \sqrt{R^2 + X_C^2}.$$

$$38.5 = \sqrt{(32.72)^2 + X_C^2}.$$

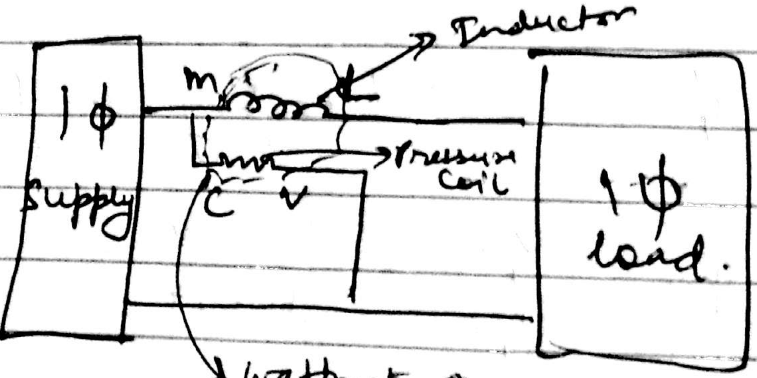
$$X_C = 20.99\,\Omega.$$

$$X_C = \frac{1}{2\pi f C}$$

$$C = 156.9\,\mu F$$

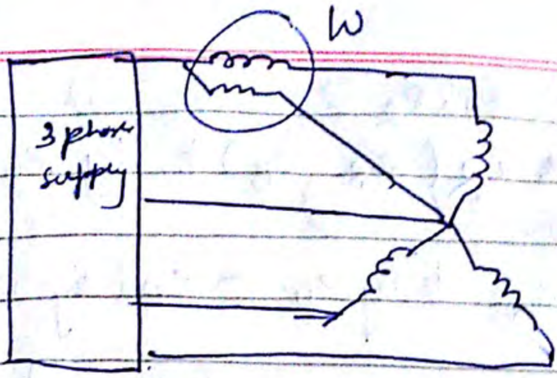
$X_L - X_C \rightarrow$ net reactance

Measurement of power in 3 phase circuit:

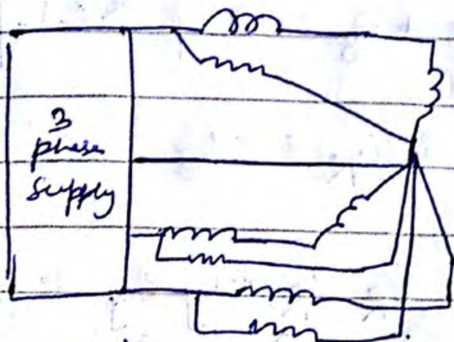


→ wattmeter : measures power

φ → phase



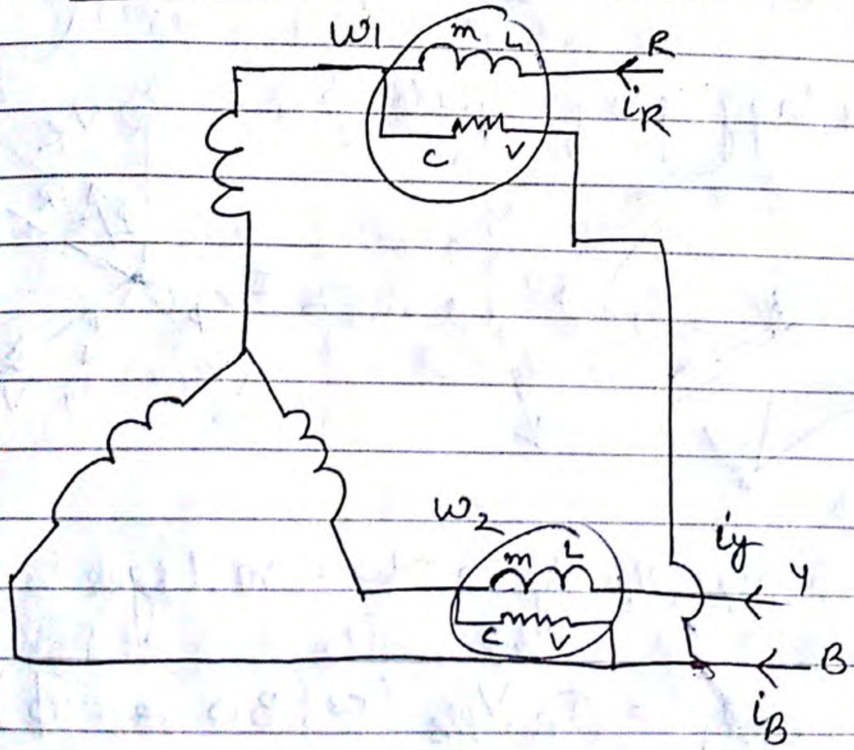
$P = 3W$



$P = 3W$

Derivation Surely Always come.

15/3/17 **Two Wattmeter Method:**



$$W_1 = \underbrace{i_R}_{\text{current coil}}\underbrace{(e_R - e_B)}_{\text{Find potential (Higher - lower potential)}}$$

$$W_2 = i_y(e_y - e_B)$$

~~P + W + V~~ $P = W_1 + W_2$

$$P = W_1 + W_2$$
$$= i_R(e_R - e_B) + i_y(e_y - e_B)$$

$$= i_R e_R + i_y e_y - e_B(i_y + i_R)$$

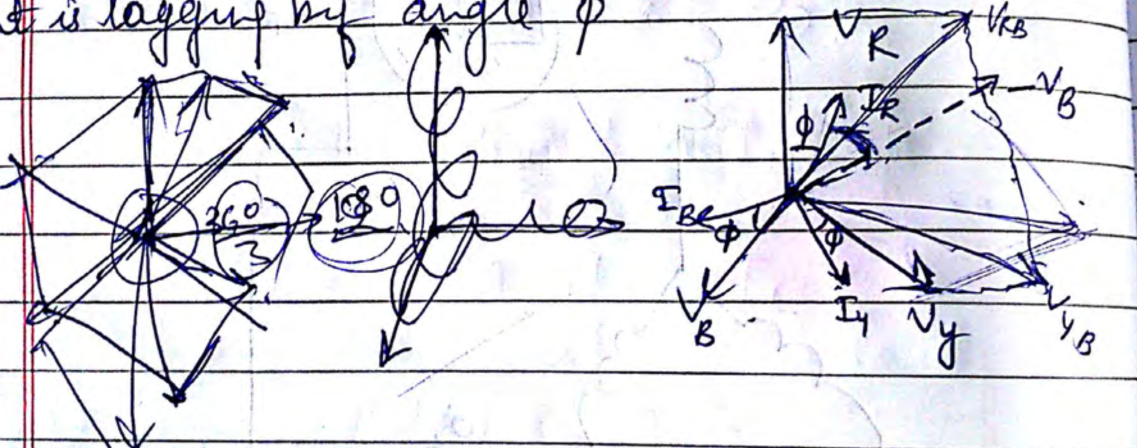
$$i_R + i_y + i_B = 0 \quad [\text{KCL}]$$

$$\therefore i_y + i_R = -i_B$$

$$W_1 + W_2 = \underbrace{i_R e_R}_{P_R} + \underbrace{i_y e_y}_{P_y} + \underbrace{i_B e_B}_{P_B}$$

$$W_1 + W_2 = P_y + P_B + P_R \quad \left(\text{Total power of load} \right)$$

current is lagging by angle ϕ



$V_R, V_B, V_y \rightarrow$ phase voltages

$$W_1 = I_R V_{RB} \cos(30 - \phi)$$

$$W_2 = I_y V_{yB} \cos(30 + \phi).$$

load is balanced,

$$|I_R| = |I_y| = |I_B| = I_C$$

$$|V_{RY}| = |V_{YB}| = |V_{BR}| = V_L$$

$$W_1 = V_L I_L \cos(30-\phi)$$
$$W_2 = V_L I_L \cos(30+\phi).$$
$$P = W_1 + W_2$$
$$= V_L I_L (\cos(30-\phi) + \cos(30+\phi))$$
$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi].$$

$$= V_L I_L (2 \cos 30 \cos\phi)$$
$$= V_L I_L \times 2 \times \frac{\sqrt{3}}{2} \cos\phi.$$

$$\boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi} \rightarrow (1)$$

Similarly

$$\boxed{W_1 - W_2 = V_L I_L \sin\phi} \rightarrow (2)$$

$$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right),$$

$$\therefore \phi = \tan^{-1}\left(\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right)$$

$$P.F = \cos\phi$$
$$= \cos\left(\tan^{-1}\left(\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right)\right)$$

~~If $W_1 - W_2 = 0$.~~

(1) If $\phi = 0$ $(W_1 = W_2)$.

P.F = 1 (unity)

$$W_1 = W_2 = \frac{W}{2}$$

(2) If P.F = 0.5, $\phi = 60°$

$$W_1 = \frac{\sqrt{3}}{2} V_L I_L$$

$$W_2 = 0.$$

(3) $\phi = 90°$, P.F = 0 → eff. = 0

$$W_1 = -W_2.$$

$$W_1 = \frac{1}{2} V_L I_L.$$

$$W_2 = \frac{1}{2} V_L I_L$$

20/3/17

Q. 2 wattmeters are connected to measure input of 15 HP, 50 Hz, 3 phase induction motor at full load. Full load efficiency and power factor are 0.9 and 0.8 lagging respectively. Find readings of 2 wattmeters.

Solⁿ: Power = 15 H.P

= 15 × 746

= 11190 W

$$\cos\phi = \cos\left[\tan^{-1}\left(\frac{\sqrt{3}(w_1 - w_2)}{w_1+w_2}\right)\right]$$

$$\eta = \frac{O/P}{I/P} = \frac{15 \times 746}{I/P}$$
$$= 0.9.$$

$P_{in} = 12433.3W.$
$P_{in} = \sqrt{3}\, V_L I_L \cos\phi.$
$w_1 = V_L I_L \cos(30 - \phi) = 8783.17W$
$w_2 = V_L I_L \cos(30+\phi) = 3476.569W$

$V_L I_L = 8972$

Q For a certain load 1 of the wattmeter read 20 kW and other 5 kW. After the voltage circuit transformer ~~wall~~ has been reversed. Calculate power and power factor of the load.

$W_1 = 20 \text{ kW}$
$W_2 = -5 \text{ kW}$

$P = w_1 + (w_2)$
$= 20 - 5$
$P = 15\text{kW}$ 46.09

~~$P = V_L I_L \cos\phi$~~

Power factor: $\cos\phi = \cos\left[\tan^{-1}\frac{\sqrt{3}(15)}{(25)}\right].$

$$= \cos\left[\tan^{-1}(1.039)\right].$$

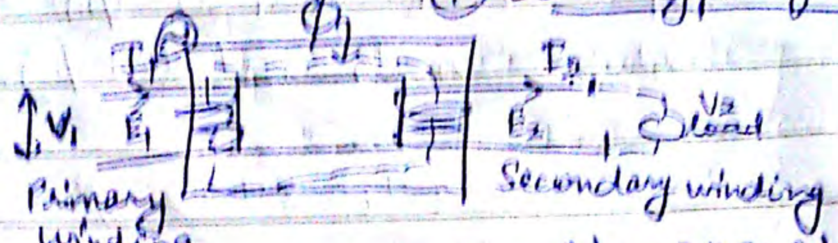
~~$= 0.693°$~~ (0.3273)

Module V

1. Transformer
2. D.C. Machine < Motor / Generator

Transformer

1. Core type of transformer



$N_1 \Rightarrow$ no. of turns in primary $N_2 \rightarrow$ no. of turns in secondary

Flux is generated on passing current in primary & secondary winding

$$e = -\frac{d\phi}{dt} \quad \Bigg| \quad e = -N\frac{d\phi}{dt}$$

If secondary winding is open
$I_2 = 0$

If secondary winding is connected to load,

$$V_2 = E_2 - I_2 Z_2$$

$I_2 = 0$ [no load]

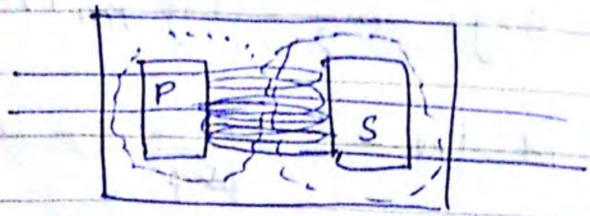
$V_2 = E_2$

E_2 : due to mutual induction
E_1 : due to self induction

If $N_2 > N_1 \Rightarrow$ step up transformer
$N_2 < N_1 \Rightarrow$ stepdown transformer
$N_2 = N_1 \Rightarrow$ ~~no power gen.~~

Transformer used for isolating circuit

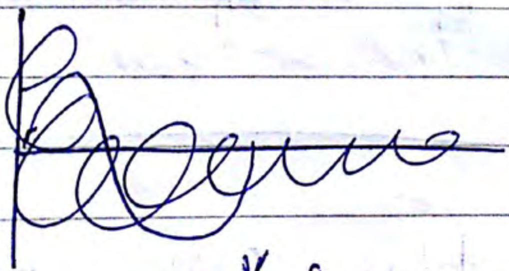
(2) Shell type of transformer:

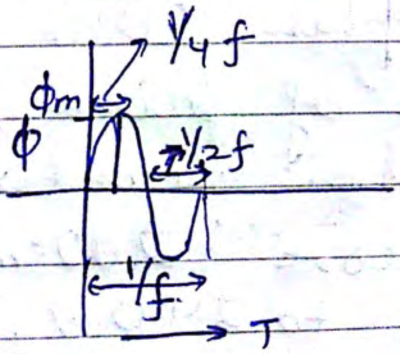


It is used to have 0 leakage flux. ~~and~~

Core type of transformer	Shell type transf.
1. Winding encircles the core	1. Core encircles most part of winding
2. Construction is preferred for low voltage	2. ~~Construction~~ is High voltage application.
3. Cooling is better	3. Cooling is poor.
4. Maintainance is easy	4. Maintainance is difficult.

Emf equation of Transformer:





$$e = \frac{\text{change in flux}}{\text{Time}}$$

$$= \frac{\phi_m - 0}{1/4f} = \cancel{4f\phi_m}$$

Avg emf / turn = $4 f \phi_m$

Avg. emf = $4 f \phi_m N$.

Form factor = $\frac{rms}{avg} = 1.11$

$$\boxed{Rms \text{ emf} = 4.44 f \phi_m N}$$

$N_1 \rightarrow$ primary winding (no. of turns)

$N_2 \rightarrow$ no. of turns of secondary winding

$E_1 = 4.44 f \phi N_1$

$E_2 = 4.44 f \phi N_2$

$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K =$$ ~~Freq~~ Turn's ratio of transformer

Q. Single phase 50 Hz transformer has 80 turns of primary winding and 400 turns on secondary windings. Cross section Area of core : 200 cm². Primary is connected to 240 V, 50 Hz supply

(i) Determine emf induced in secondary winding

(ii) Max. value of flux density in the core. $\hookrightarrow \phi / A$.

$f = 50Hz$ if not g

Sol:

$N_1 = 80$

$N_2 = 400$

$\cancel{e = \frac{N d\phi}{dt} = (400-80)}$

$E_1 = 240V.$

$a = 200 cm^2.$

$K = \frac{N_2}{N_1} = \frac{E_2}{E_1}$

$E_2 = 240 \times 5 = 1200V.$

$E_2 = 4.44 \times 50 \times \phi_m \times N_2.$

$\phi_m = 0.0135$ wb.

$B_m = \frac{\phi}{a} = 6.75$ wb/m^2

→ Transformer rating is given as KVA, VA.

Ideal transformer: I/P KVA = O/P KVA

$V_1 I_1 = V_2 I_2.$

$$\boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2} = K}$$

$$\boxed{K = \frac{N_2}{N_1} = \frac{E_2}{E_1}}$$

Turn's Ratio / Transformation ratio