Discrete Mathematics

Logic

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Introduction

Logic has many important applications to mathematics, computer science, and numerous other disciplines.

- In the specification of software and hardware.

- To design computer circuits, to construct computer programs, to verify the correctness of programs, and to build expert systems.

- To analyze and solve many familiar puzzles.

- For constructing some, but not all, types of proofs automatically.

2 Applications of Propositional Logic

Translating English Sentences

Translating sentences into compound statements removes the ambiguity.

Once we have translated sentences from English into logical expressions, we can use rules of inference to reason about them.

Example

How can this English sentence be translated into a logical expression?

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

2 Applications of Propositional Logic

System Specifications

System and software engineers take requirements in natural language and produce precise and unambiguous specifications that can be used as the basis for system development.

Example

Express the specification "The automated reply cannot be sent when the file system is full" using logical connectives.

Consistent

System specifications should be **consistent**, that is, they should not contain conflicting requirements that could be used to derive a contradiction.

Determine whether these system specifications are consistent:

"The diagnostic message is stored in the buffer or it is retransmitted."

"The diagnostic message is not stored in the buffer."

"If the diagnostic message is stored in the buffer, then it is retransmitted."

Boolean Searches

Logical connectives are used extensively in searches of large collections of information, such as indexes of Web pages.

In Boolean searches, the connective *AND* is used to match records that contain both of two search terms,

the connective *OR* is used to match one or both of two search terms,

and the connective *NOT* (sometimes written as *AND NOT*) is used to exclude a particular search term.

2 Applications of Propositional Logic

Logic Puzzles

Puzzles that can be solved using logical reasoning are known as logic puzzles.

Solving logic puzzles is an excellent way to practice working with the rules of logic.

Example

An island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie.
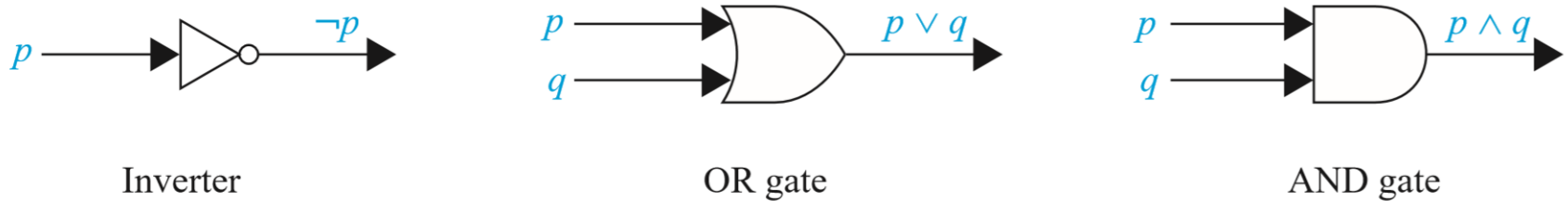
You encounter two people A and B.

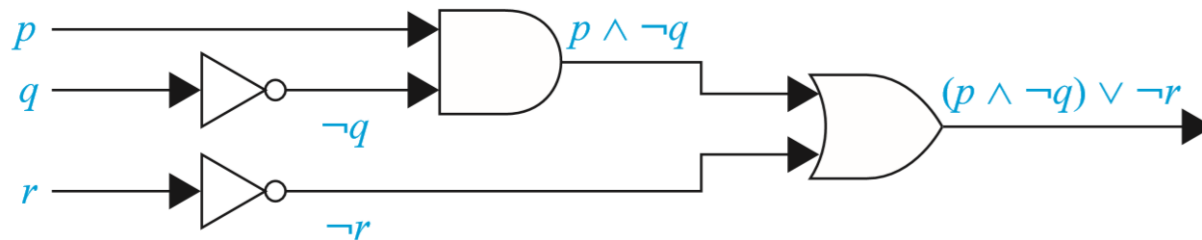What are A and B if A says "B is a knight" and B says "The two of us are opposite types?"

Logic Circuits

A **logic circuit** (or **digital circuit**) receives input signals p_1, p_2, ..., p_n, each a bit [either 0 (off) or 1 (on)], and produces output signals s_1, s_2, ..., s_n, each a bit.

Inverter   OR gate   AND gate

FIGURE 1   Basic logic gates.

FIGURE 2   A combinatorial circuit.