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Thapar Institute of Engineering and Technology, Patiala

School of Mathematics

Mid Semester Examination

B.E. (Sem: IV & VI)	Course Code: UMA035 Course Name: Optimization Techniques
Date: 06-04-2022	Day/Time: Wednesday, 16.30 – 18.30 hrs
Time: 2 Hours, M. Marks: 35	Name of Faculty: MKS, AK, MKR, SJK, NK, MG, JPR, BHU, SPNP, TV

Note: (1) **Attempt any five questions.**
(2) **Calculator without graphing mode and alphanumeric memory is permitted.**

1. (a) A chemical company produces two products X and Y. Each unit of product X requires 3 hours on operation I and 4 hours on operation II, while each unit of product Y requires 4 hours on operation I and 5 hours on operation II. Total available time for operation I and II is 20 hours and 26 hours, respectively. The production of each unit of product Y also results in 2 units of a by-product Z at no extra cost. Product X sells at profit of Rs 10/unit while Y sells at profit of Rs 20/unit. By-product Z brings a unit profit of Rs 6 if sold, in case it cannot be sold, the destruction cost is Rs 4/unit. Forecasts indicate that not more than 5 units of Z can be sold. Formulate the linear programming model to determine the quantities of X and Y to be produced, keeping Z in mind, so that profit earned is maximum. [4 marks]

   (b) State and prove fundamental theorem of Linear Programming Problem (LPP) for maximization case only. [3 marks]

2. (a) Show graphically that the set $S = \{(x_1, x_2) \in \mathbb{R}^2 : |x_1| + |x_2| \leq 1\}$ is convex set. [2 marks]

   (b) Solve the following LPP using algebraic and graphical methods, and show the correspondence between basic feasible solution (BFS) and vertices of the feasible region for given LPP.
   $Max\ Z = 6x_1 + 5x_2$
   Subject to $x_1 + x_2 \leq 5$; $3x_1 + 2x_2 \leq 12$; $x_1, x_2 \geq 0$. [5 marks]

3. (a) Write the following LPP in standard form.
   $Max\ Z = 2x_1 + 3x_2 - 7x_3$
   Subject to $x_1 + 2x_2 - x_3 \leq -16$; $2x_1 + x_2 + 4x_3 \geq 10$; $x_1 \geq 0, x_2 \leq 0$ and x_3 is unrestricted. [2 marks]

   (b) Solve the following LPP using Big-M method.
   $Max\ Z = 6x_1 + 4x_2$
   Subject to $2x_1 + 3x_2 \leq 30$; $3x_1 + 2x_2 \leq 24$; $x_1 + x_2 \geq 3$; $x_1, x_2 \geq 0$. [5 marks]

4. (a) Consider the following LPP as primal problem:
   $Max\ Z = 5x_1 - 4x_2 + 3x_3$
   Subject to $2x_1 + x_2 - 6x_3 = 20$; $6x_1 + 5x_2 + 10x_3 \leq 76$; $8x_1 - 3x_2 + 6x_3 \geq 50$; x_1 is unrestricted, $x_2 \geq 0, x_3 \leq 0$.
   Write the dual of above LPP. Using complementary slackness theorem, find the optimal solution of dual and it is given that s_3(surplus variable in third constraint), s_2 (slack variable in second constraint) and x_1 are the basic variables in the primal optimal table. [4 marks]

   (b) State and prove Weak-Duality theorem. [3 marks]

5. Consider the following LPP:
   $Min\ Z = x_1 - 2x_2 + x_3$
   Subject to $x_1 + 2x_2 - 2x_3 \leq 4$; $x_1 - x_3 \leq 3$; $2x_1 - x_2 + 2x_3 \leq 12; x_1, x_2, x_3 \geq 0$.
   Introducing s_1, s_2 and s_3 as starting slack variables in the first, second and third constraint respectively, the optimal table of given LPP is as follows:

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B.V.	x_1	x_2	x_3	s_1	s_2	s_3	Solution
$z_j - c_j$	$\frac{-9}{2}$	0	0	$\frac{-3}{2}$	0	-1	-8
x_2	3	1	0	1	0	1	6
s_2	$\frac{7}{2}$	0	0	$\frac{1}{2}$	1	1	7
x_3	$\frac{5}{2}$	0	1	$\frac{1}{2}$	0	1	4

Using sensitivity analysis, find the optimal solution and optimal value in each case separately (if exists).

(a) If R.H.S. is changed in the given LPP from $(4,3,2)^T$ to $(1,4,11)^T$. [1 mark]

(b) If a new constraint $x_1 + x_2 = 7$ is added in the given LPP. [2 marks]

(c) If a new variable with cost (-1) and column $(3,-2,-4)^T$ is added in the given LPP. [2 marks]

(d) In what range the cost coefficient of x_2 varies so that the optimality of the given LPP is unaffected?[2 marks]

6. (a) Use Gomory's cutting plane method, find the optimal solution of given integer programming problem (IPP).
$Max\ Z = 3x_1 + 2x_2$
Subject to $x_1 \leq 2$; $x_2 \leq 2$; $2x_1 + 2x_2 \leq 7$; $x_1, x_2 \geq 0$ and are integers.
The optimal Simplex table of above non-integer programming problem is given below.

B.V.	x_1	x_2	s_1	s_2	s_3	Solution
$z_j - c_j$	0	0	1	0	0	9
x_1	1	0	1	0	0	2
s_2	0	0	1	1	$\frac{-1}{2}$	$\frac{1}{2}$
x_2	0	1	-1	0	$\frac{1}{2}$	$\frac{3}{2}$

where, s_1, s_2 and s_3 are starting slack variables in the first, second and third constraint respectively[4 marks]

(b) Without performing Simplex iterations, find the missing of given LPP
$Max\ Z = -x_1 + 3x_2 - 2x_3$
Subject to $3x_1 - x_2 + 3x_3 \leq 7$; $-2x_1 + 4x_2 \leq 12$; $-4x_1 + 3x_2 + 8x_3 \leq 10$; $x_1, x_2, x_3 \geq 0$.

B.V.	x_1	x_2	x_3	s_1	s_2	s_3	Sol
$z_j - c_j$	$\frac{-1}{2}$	0	*	0	$\frac{3}{4}$	*	*
s_1	*	0	3	*	$\frac{1}{4}$	*	10
x_2	*	1	0	*	$\frac{1}{4}$	*	3
s_3	*	0	8	*	$\frac{-3}{4}$	*	1

where, s_1, s_2 and s_3 are slack variables in the first, second and third constraint respectively. [3 marks]

7. (a) Solve the following LPP by using Branch and Bound method:
$Max\ Z = 5.2x_1 + 6.3x_2$
Subject to $x_1 + x_2 \leq 5$; $4x_1 + 7x_2 \leq 28$; $x_1,\ x_2 \geq 0$ and are integers. [3 marks]

(b) Find the optimal solution of following LPP using Simplex method.
$Max\ Z = x_1 + 2x_2 + 3x_3$
Subject to $x_1 + 2x_2 \leq 20$; $3x_1 + 4x_3 \leq 30$; $x_1, x_2, x_3 \geq 0$. [4 marks]

End of Question Paper