

# Neutrino Theory

Karan Kumar

# Class Policy

- Classes from 10:00 AM to 12:30 PM (5 min break at ~ 11:00 AM, 5 min break at noon).
- Up to four absences
- Lateness or leaving early counts as half-absence.
- Send email notifications of all absences to  
[shpattendance@columbia.edu](mailto:shpattendance@columbia.edu)

# Class Policy

- No cell phone uses during the class
- Feel free to step outside to the hallway in case of emergencies, bathroom, and starvations.
- Feel free to stop me and ask questions / ask for clarifications.

# Curriculum

Lecture	Topic
1	Introduction
2	History of Particle Physics
3	Special Relativity
4	Quantum Mechanics
5	Detectors By Sarah Vicker
6	Standard Model
7	Beyond the Standard Model
8	Neutrino Theory
9	Neutrino Experiment
10	Large Hadron Collider (LHC)
11	Higgs Boson and Beyond
12	Cosmology

**I am presenting these lesson with as little bias as possible. I highly encourage you to take this information, continue researching as you progress in your studies, and form your own informed opinions.**

# Lecture Material

- <https://github.com/Karan-Kumar22/Particle-Physics>

# Outline

- Introduction
- The Solar Neutrino Problem
- Neutrino Oscillations
- Neutrino Mass

# **Quiz [From Previous Lecture]**

# Questions 1

How much of the universe do you think is made of dark matter?

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**What is the Higgs fine-tuning problem?**

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# Question 9

**Which three forces are included in Grand Unified Theories (GUTs)?**

- A) Gravity, magnetism, and electricity
- B) Electromagnetic, weak, and strong nuclear forces
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# Question 11

**What is the basic idea of string theory?**

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- B) Atoms are made of strings of protons and neutrons
- C) Light travels through invisible strings in space
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Source: Wikipedia

9/27/25

Karan Kumar

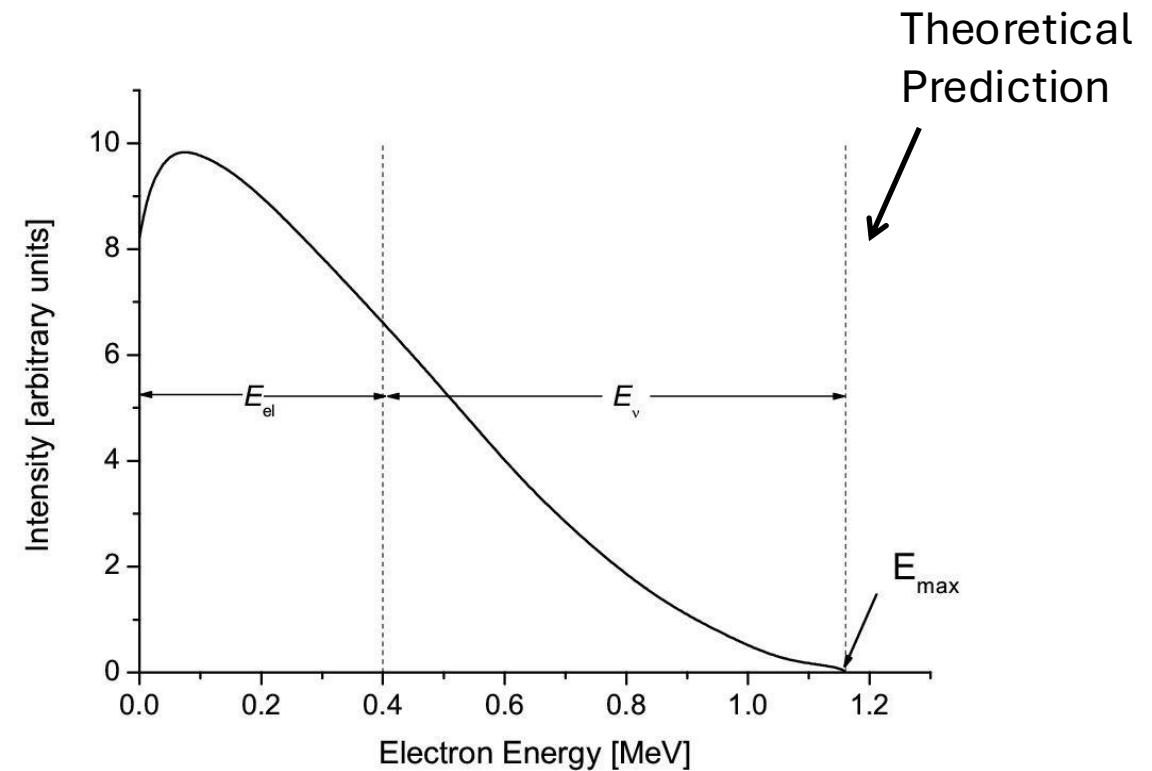
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- In theory, when a nucleus decays, it should release an electron with **one fixed energy**.



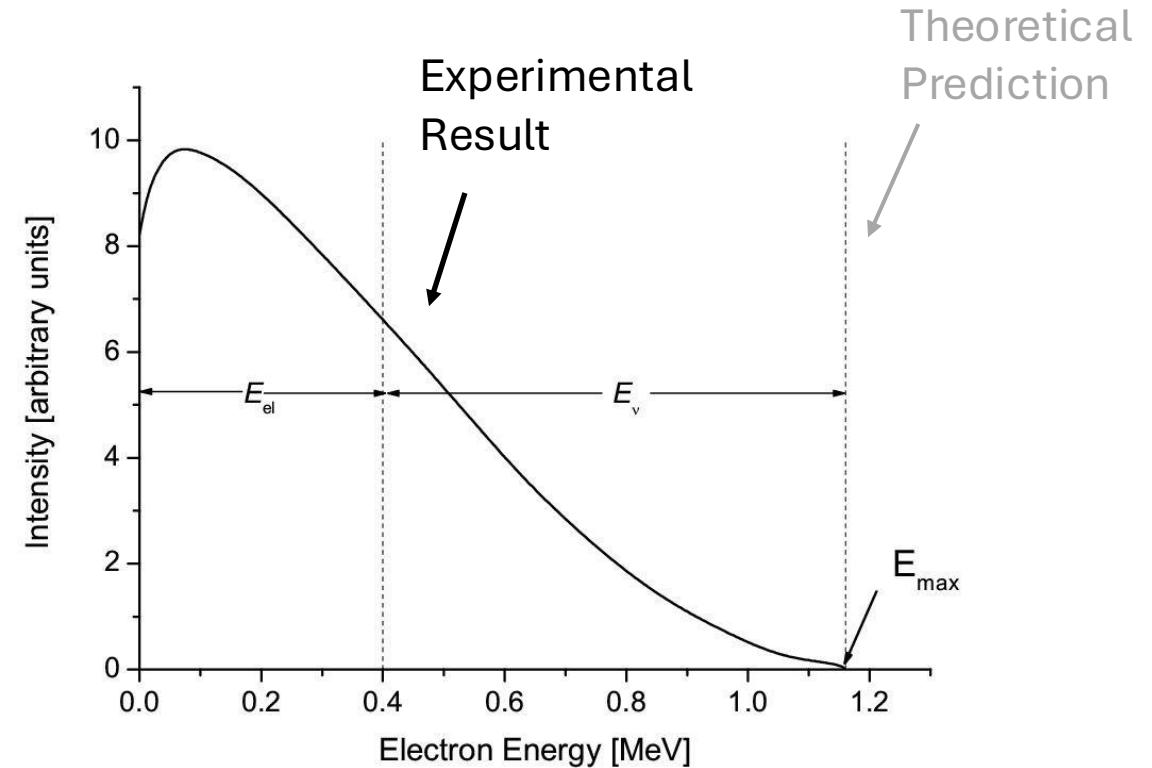
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# Neutrinos

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- In theory, when a nucleus decays, it should release an electron with **one fixed energy**.
- But experiments showed the electrons had a **range of energies**.



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- But Wolfgang Pauli suggested a better idea: a new, invisible particle must be carrying away the missing energy. He called it the “neutron.”
- In 1932, James Chadwick discovered the actual neutron (the heavy neutral particle in nuclei), so Enrico Fermi renamed Pauli’s lighter invisible particle the **neutrino** (meaning “little neutral one”).

# Neutrinos

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$$n \rightarrow p + e^- + \nu_e$$

- The energy is now shared by the electron and the neutrino, and that explains why some electrons have more, and some less, up to a maximum of  $E$ .

# Neutrinos

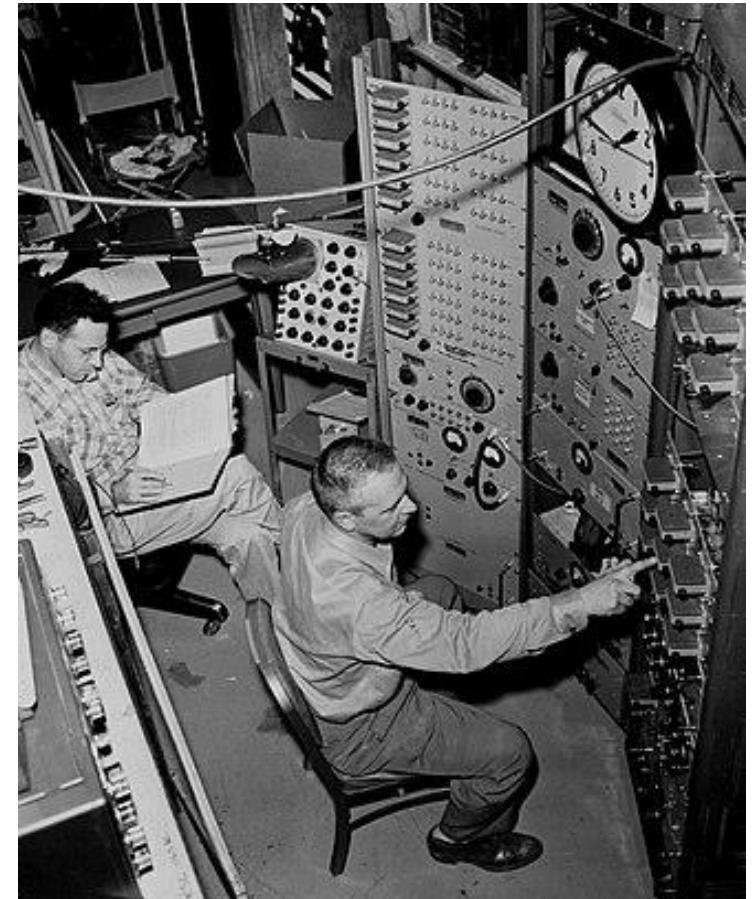
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- Neutrinos are **extremely hard to detect** because they almost never interact with matter. In fact, a single neutrino could travel through **light-years of solid lead** without being stopped.
- Every second, **hundreds of billions of neutrinos from the Sun** pass straight through your thumb.

# Neutrinos

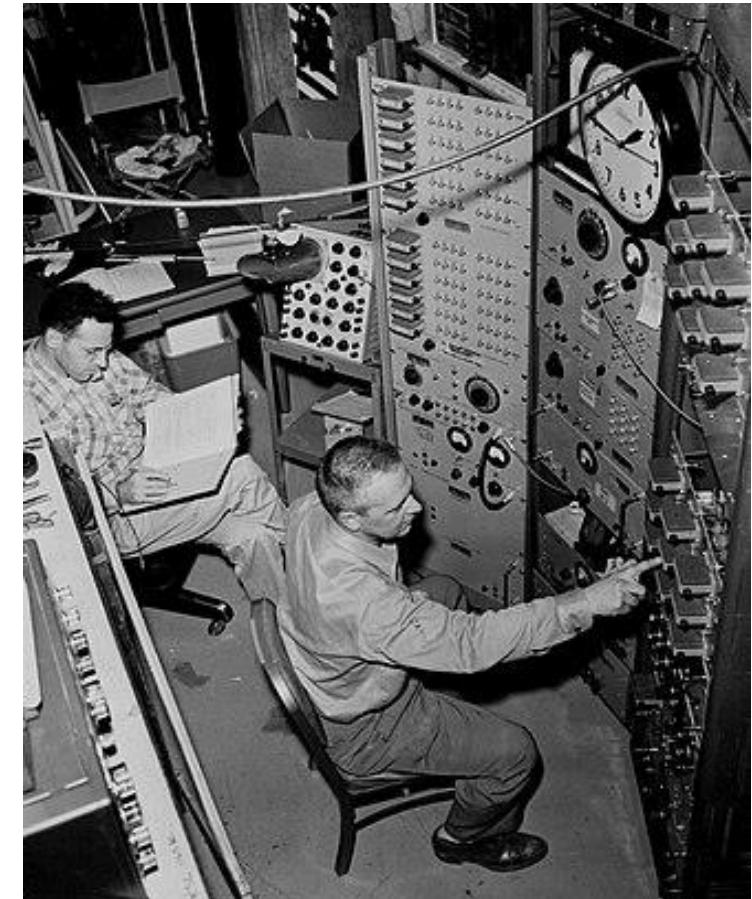
- Because they interact so weakly, neutrinos weren't detected until **1956**, when Frederick Reines and Clyde Cowan set up detectors near the **Savannah River nuclear reactor**, which produces huge numbers of neutrinos.



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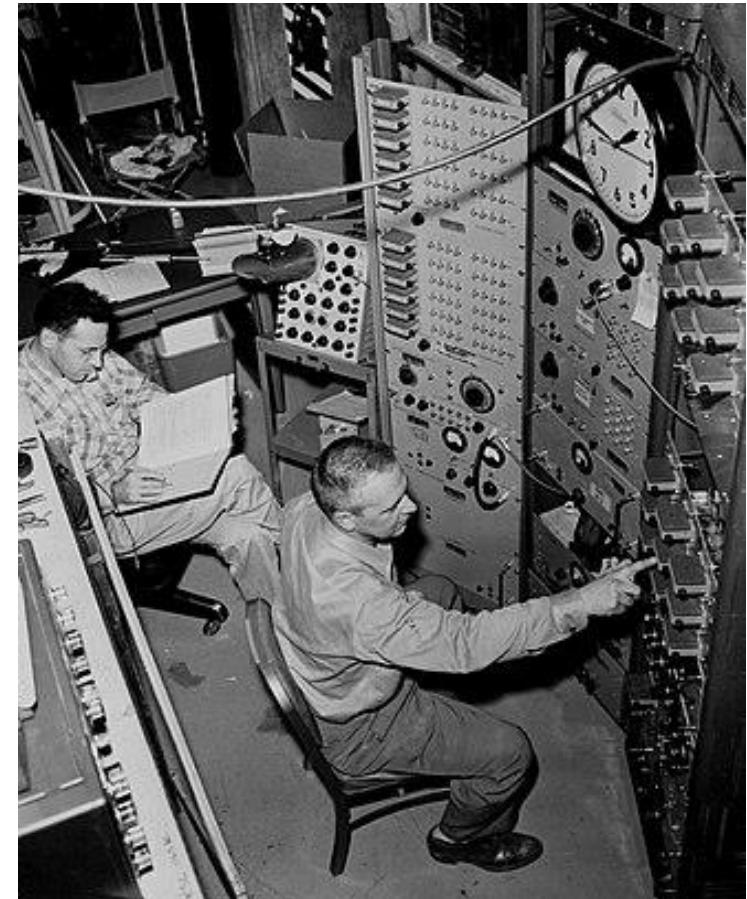
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- They looked for inverse beta decay, where an antineutrino interacts with a proton to produce a positron and a neutron. The positron created a quick flash of light, followed by a delayed flash when the neutron was captured, and seeing both flashes together confirmed the event.



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- This experiment provided the first direct detection of neutrinos and proved that they were real particles, not just a theoretical idea.



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- But in 2001, experiments on **neutrino oscillations** showed that neutrinos actually do have a **small but nonzero mass**. To this day, the exact value of that mass is still unknown.

# Neutrinos

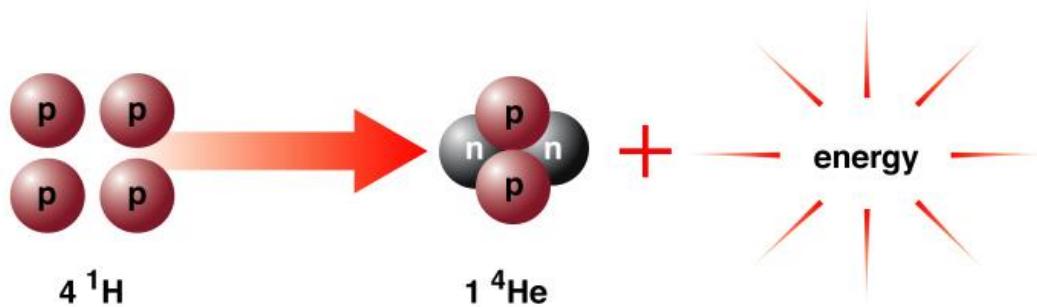
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- **It was wrong to assume neutrino mass to be zero at the first place.**  
**More on this in neutrino theory lecture.**

# The Solar Neutrino Problem

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Discovery of Nuclear Fusion as the Sun's Energy Source

- By 1920, Francis Aston measured atomic weights and found that four hydrogen atoms weigh slightly more than one helium atom.

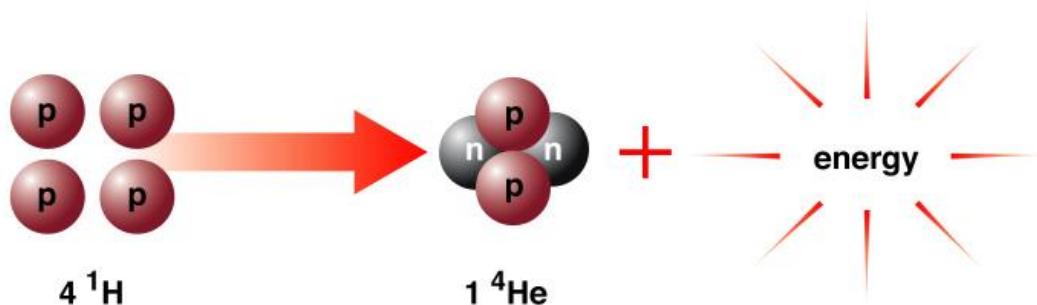


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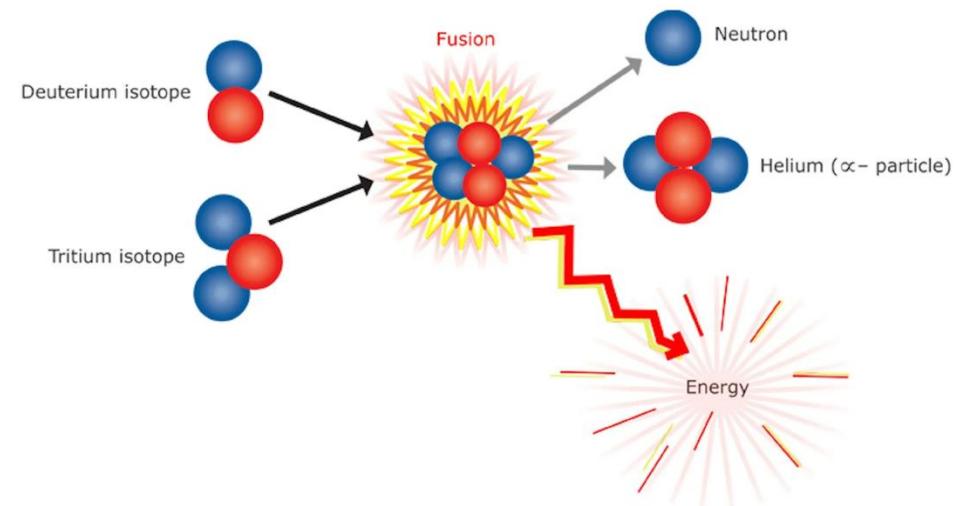


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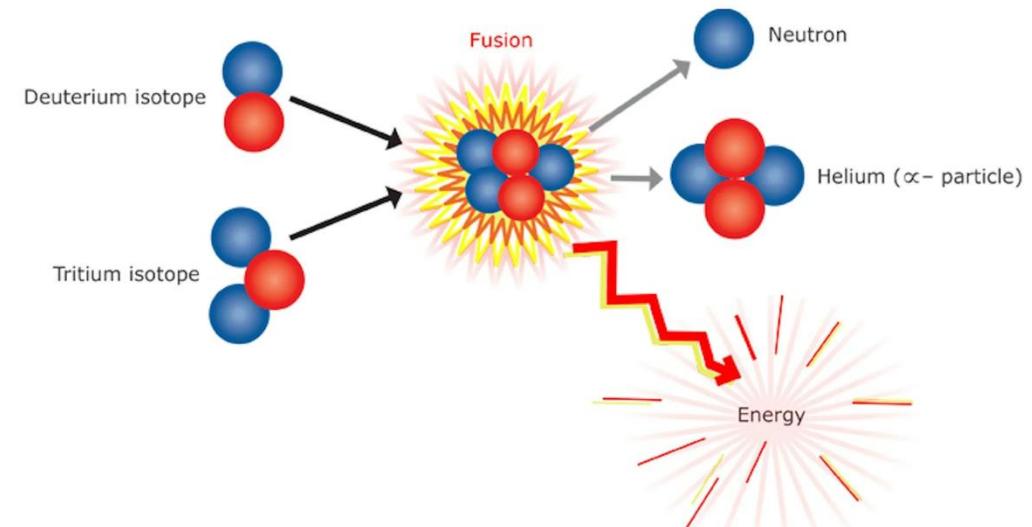
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www.sciencelearn.org.nz

<https://www.sciencelearn.org.nz/resources/242-plasmas-and-nuclear-fusion>

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## Discovery of Nuclear Fusion as the Sun's Energy Source

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- Eddington's idea was later supported by discoveries in the 1930s — Chadwick's neutron and Pauli's neutrino — which completed the picture of nuclear reactions in stars.



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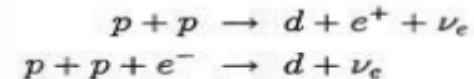
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# The Solar Neutrino Problem

- In 1938, Hans Bethe explained how the Sun produces energy through nuclear fusion.

## The pp Chain

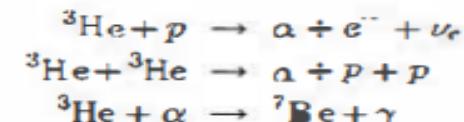
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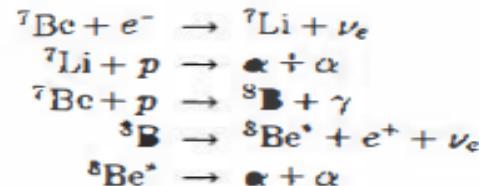
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Step 3: Helium-3 makes alpha particle or  ${}^7\text{Be}$ .



Step 4: Berillium makes alpha particles.

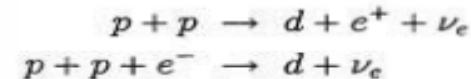


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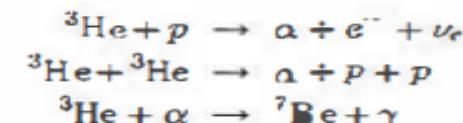
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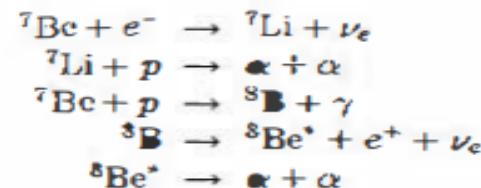
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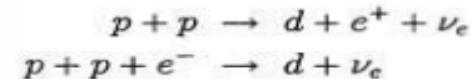


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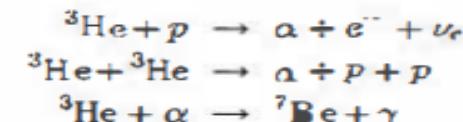
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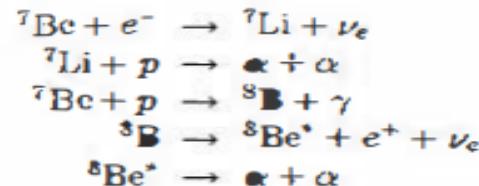
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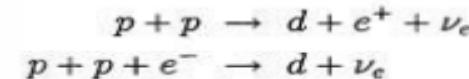


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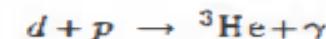
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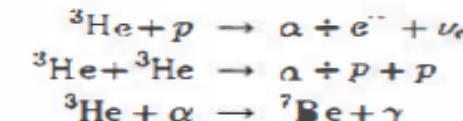
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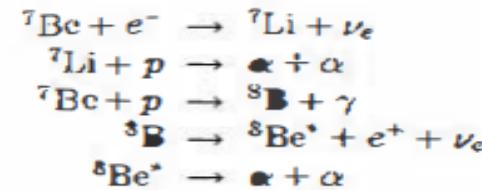
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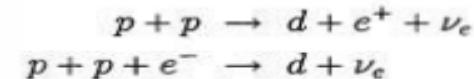


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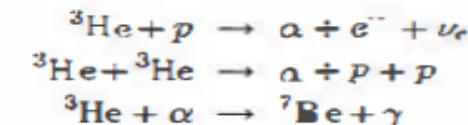
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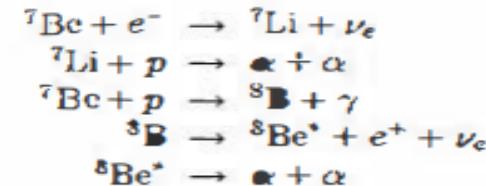
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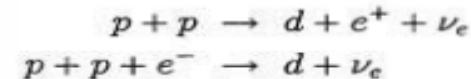


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- Each cycle converts hydrogen into helium and produces energy and neutrinos that escape from the Sun.

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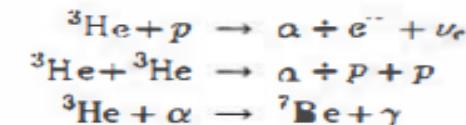
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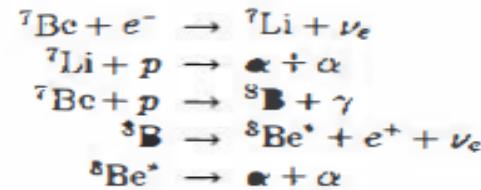
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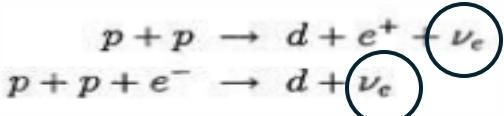
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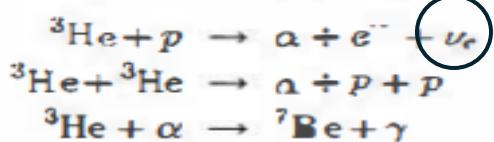
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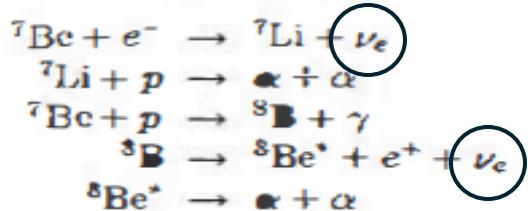
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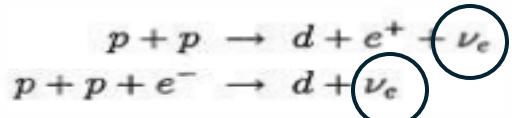
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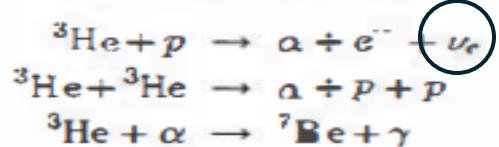
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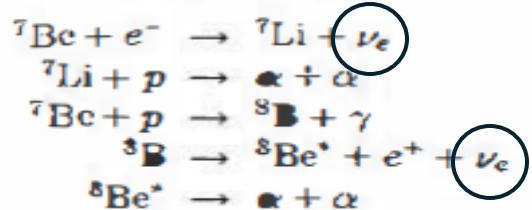
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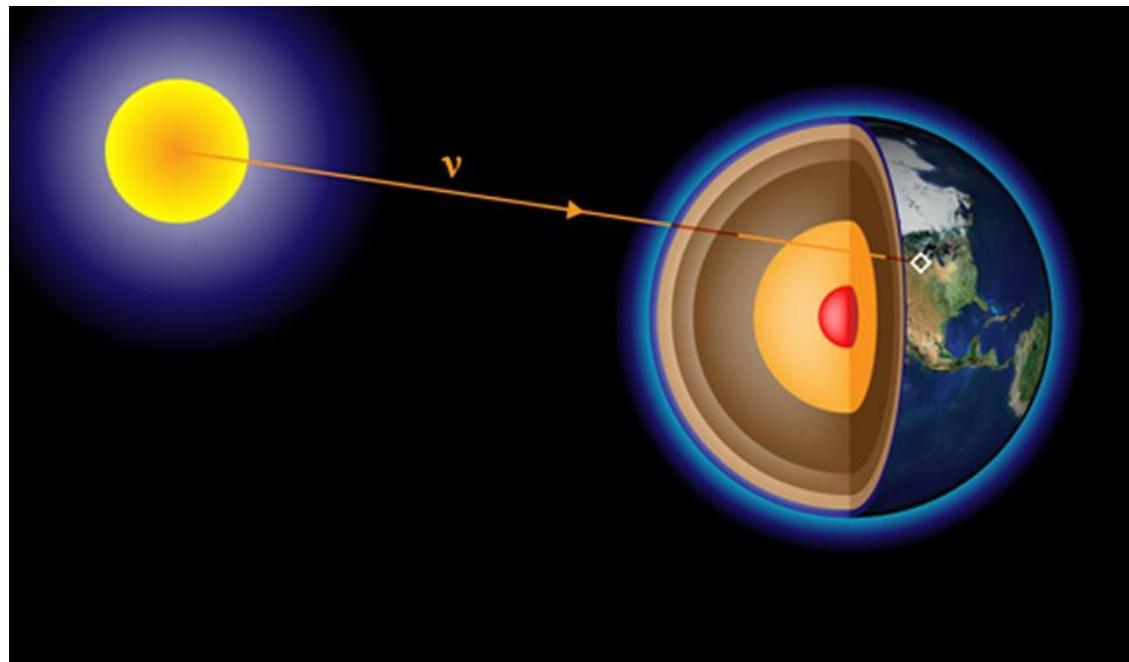
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It only producing electron neutrino

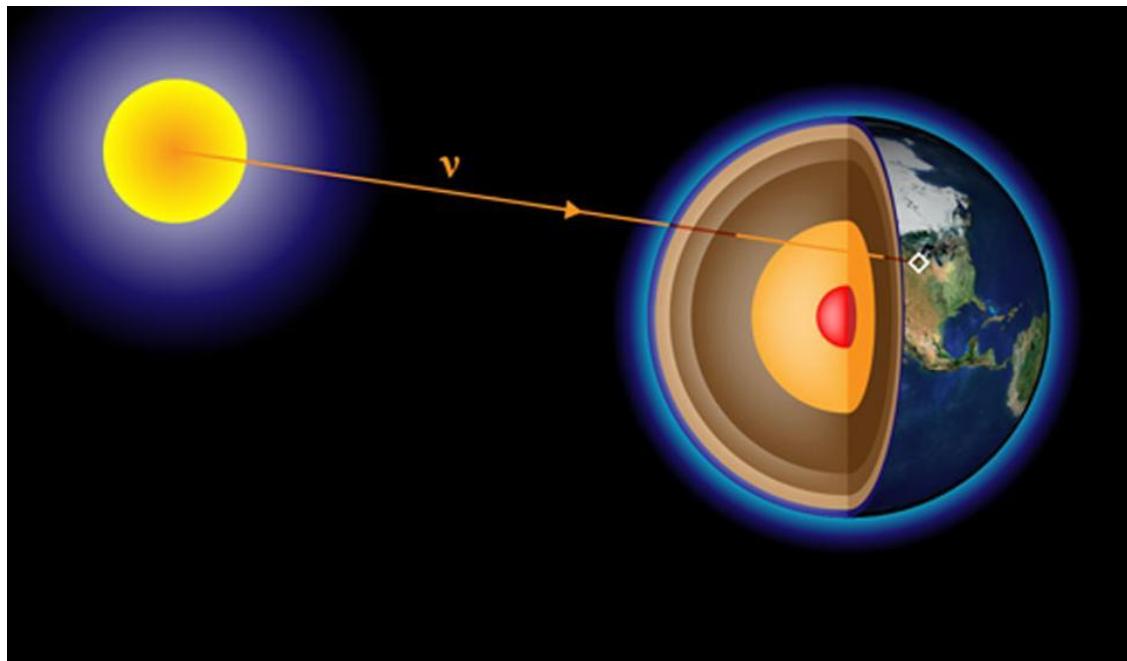
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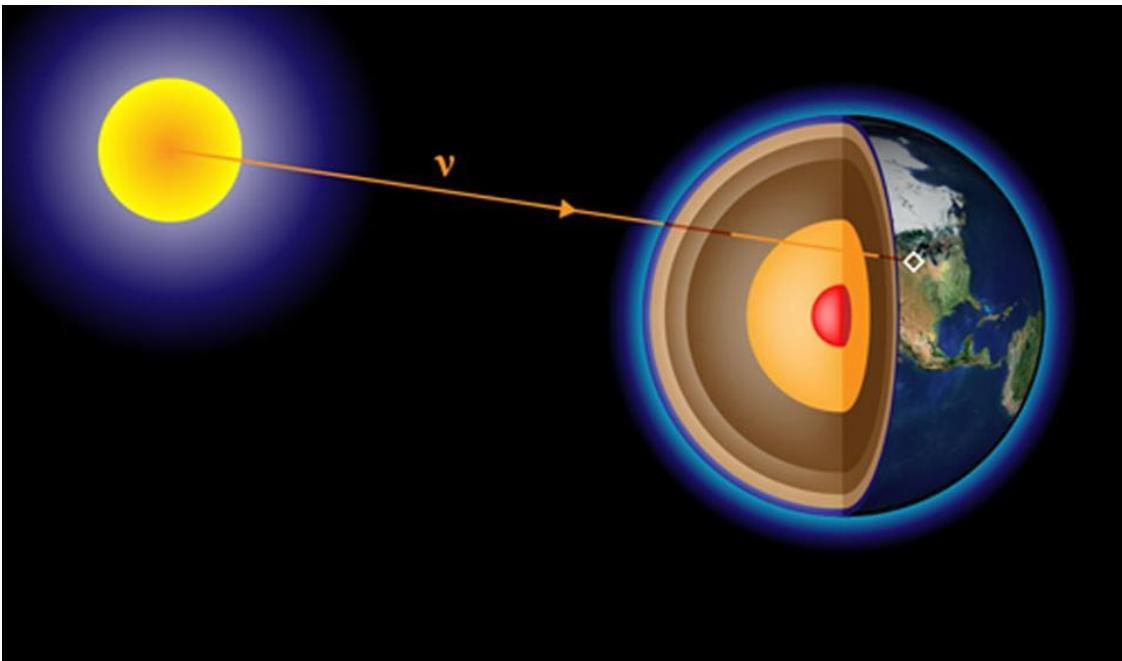
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- John Bahcall calculated the expected flux (billions pass through your thumb each second!).
- Neutrinos rarely interact, so detecting them is extremely difficult.



# The Solar Neutrino Problem

- This plot shows predicted neutrino energy spectra — most from pp reactions.

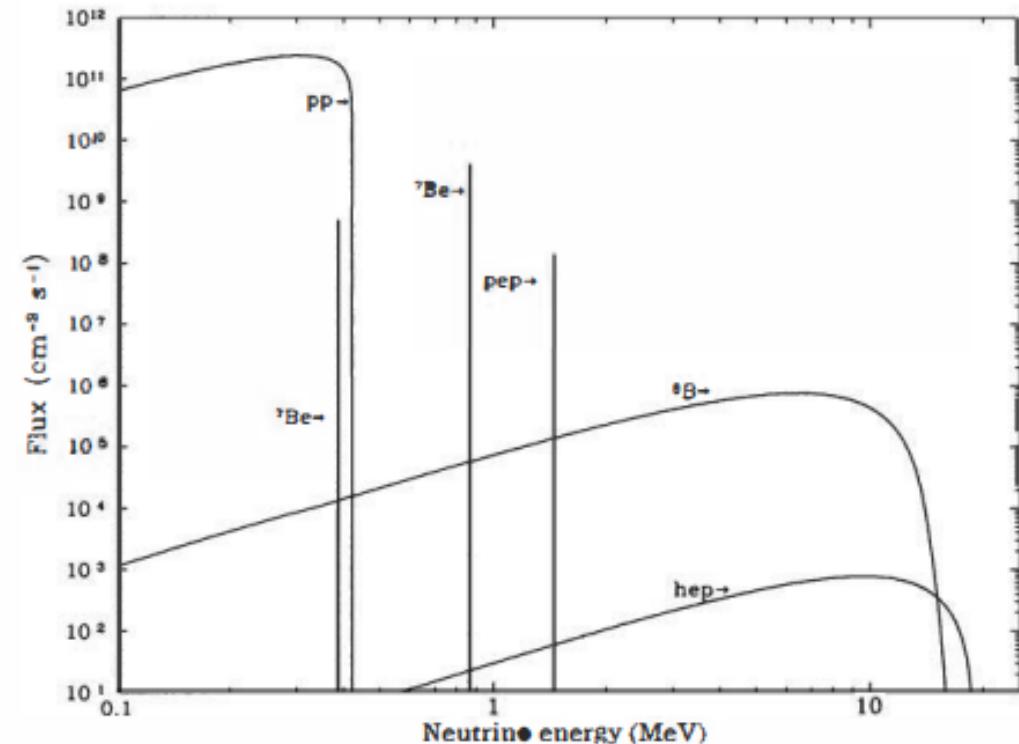
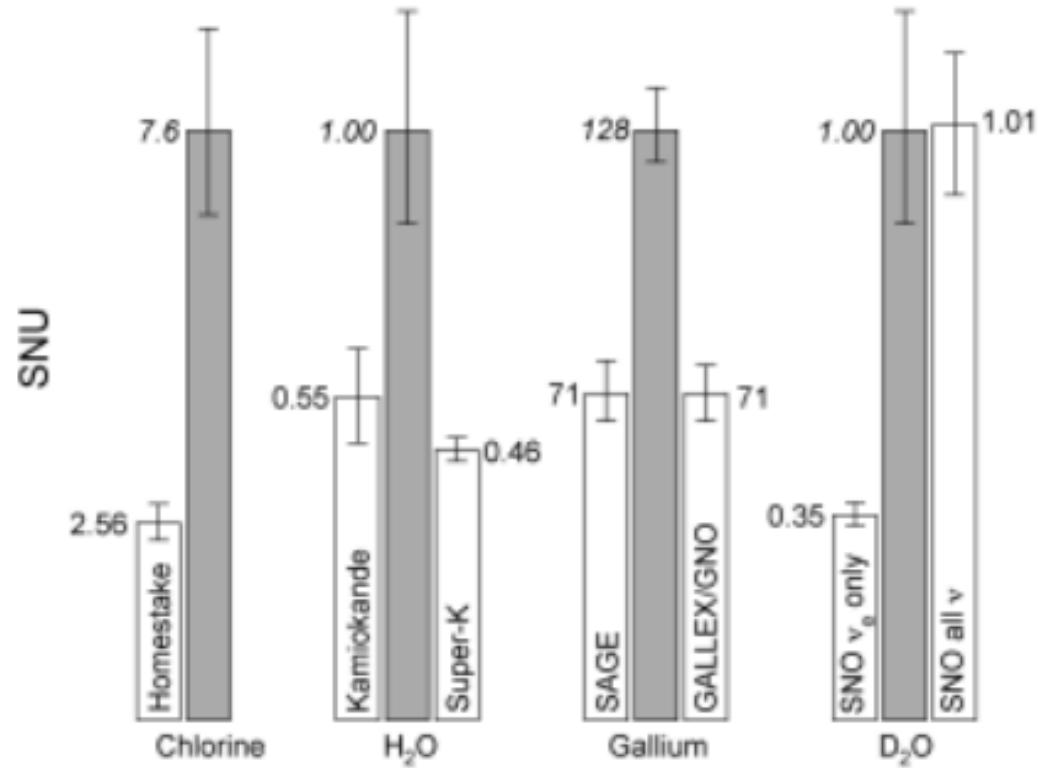


Fig. 11.2 The calculated energy spectra for solar neutrinos.  
(Source: J. N. Bahcall, A.M. Serenelli, and S. Basu, *Astrophysical Journal* 621, L85 (2005).)

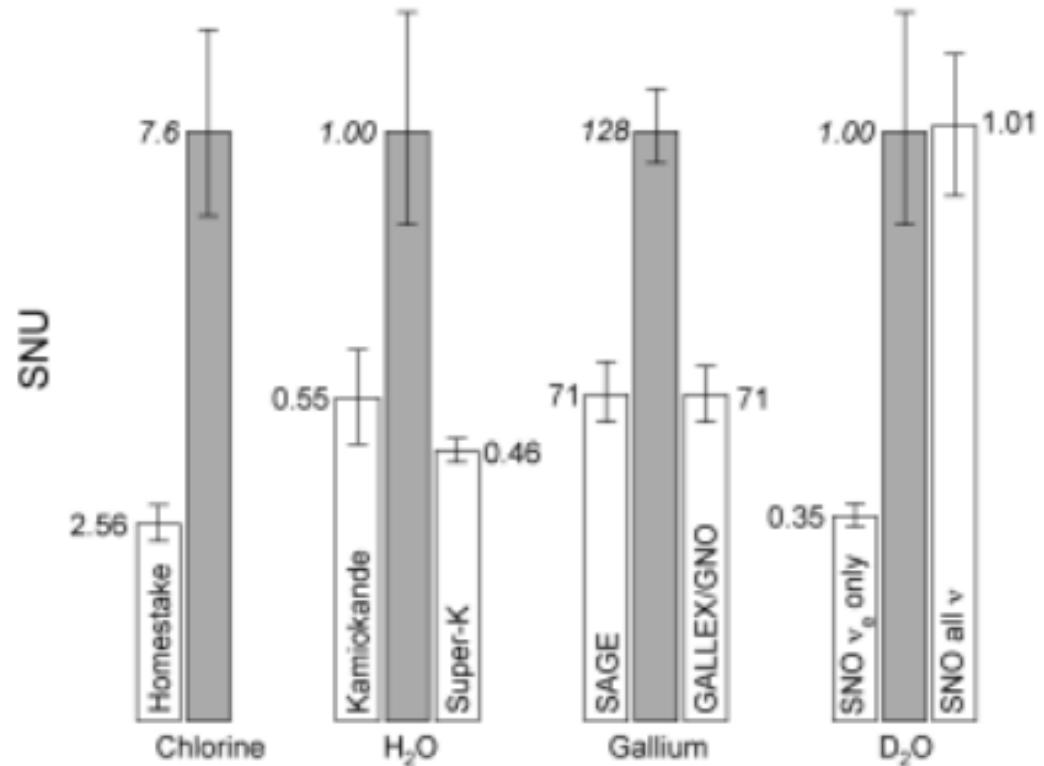
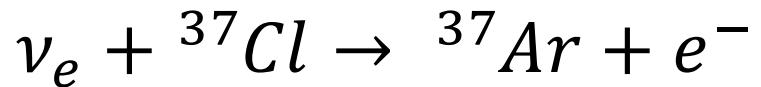
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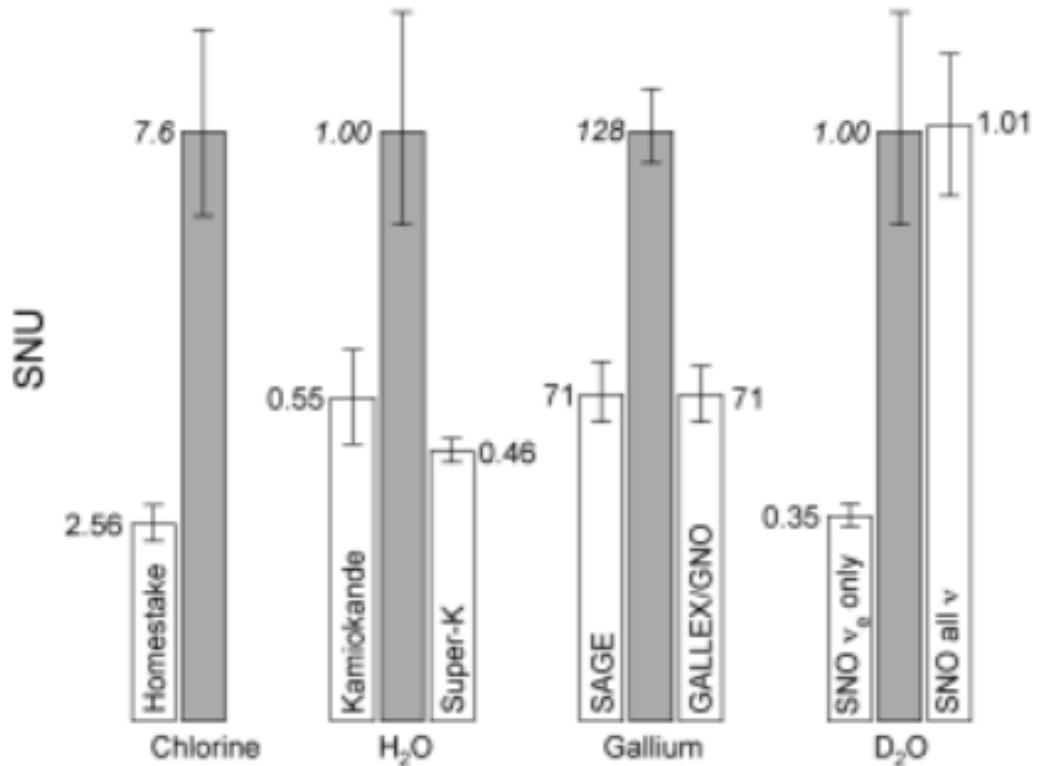
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- Used a tank of chlorine ( $\text{C}_2\text{Cl}_4$ ) deep underground to detect neutrinos via:



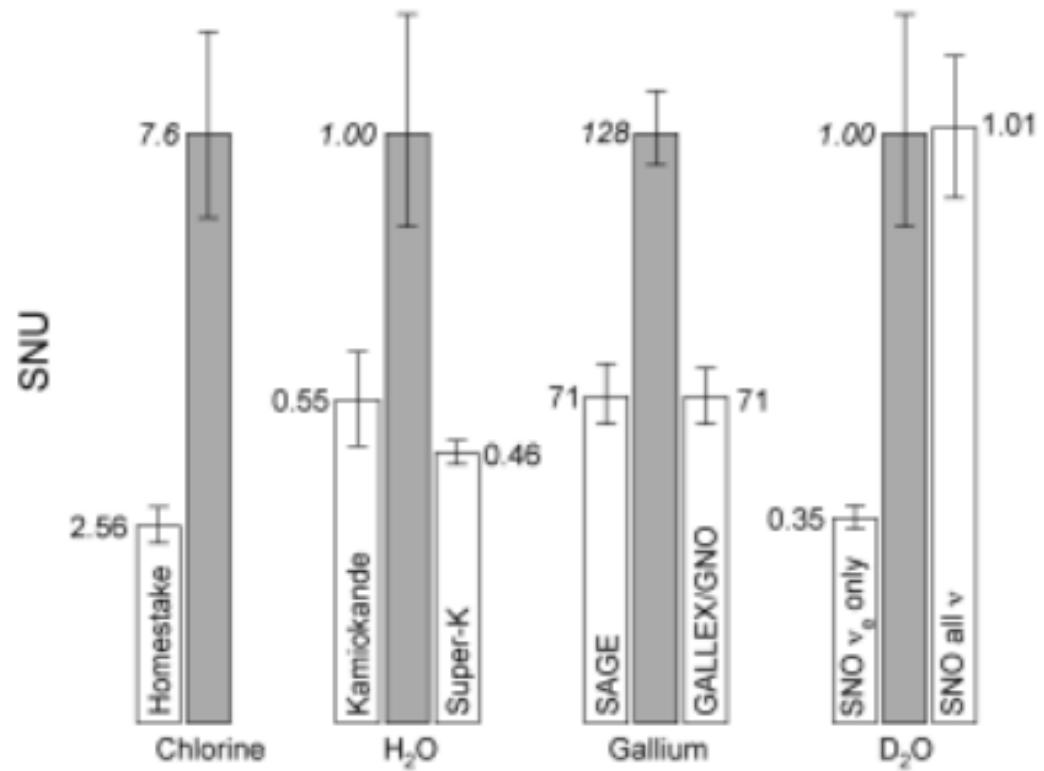
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# The Solar Neutrino Problem

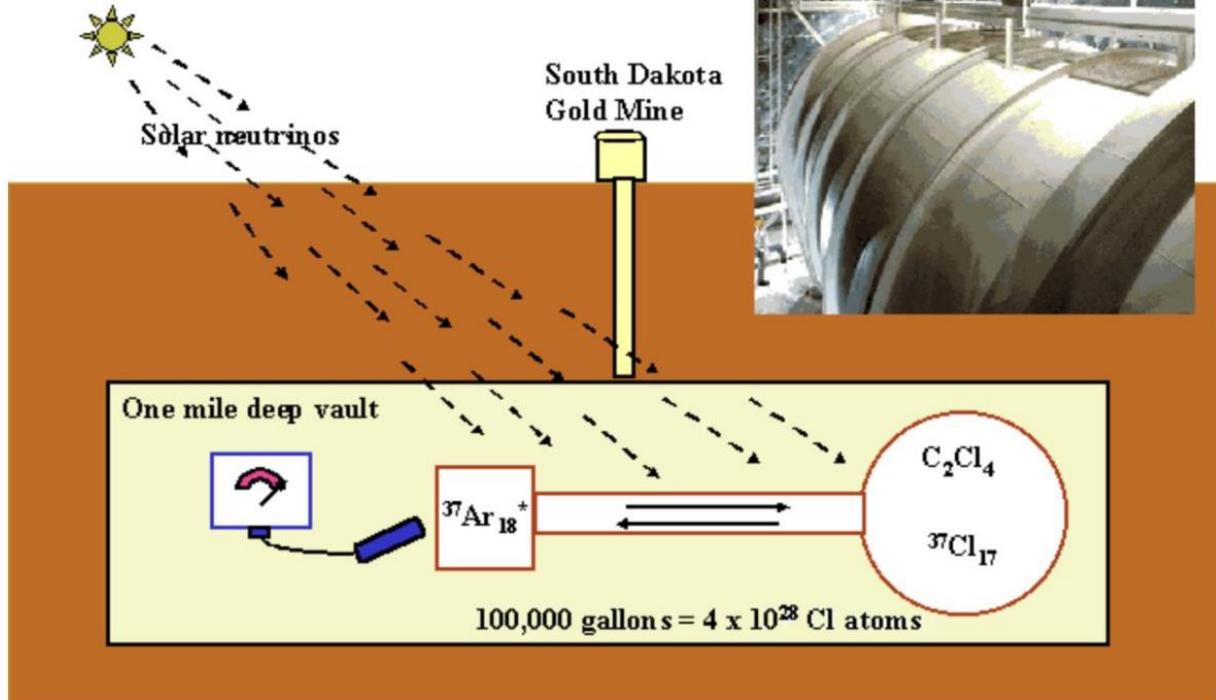
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$$\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$$
- Predicted rate:  $\sim 1$  atom every 2 days  
→ Observed only 1/3 of that!
- Result: fewer neutrinos detected than theory predicted — the Solar Neutrino Problem.



# Solar Neutrino Problem

- In the Sun's core, nuclear fusion reactions convert hydrogen into helium. These reactions release an enormous number of neutrinos.
- The sun only produces **electron** neutrinos.
- Ray Davis, a chemist at Brookhaven National Laboratory, led the first experiment to detect electron neutrinos produced by the sun.
- However, the experiment found only one-third the number of solar neutrinos that was predicted, leading to the “**solar neutrino problem**.”
- The detector was only sensitive to electron neutrinos.

## Solar Neutrino Problem

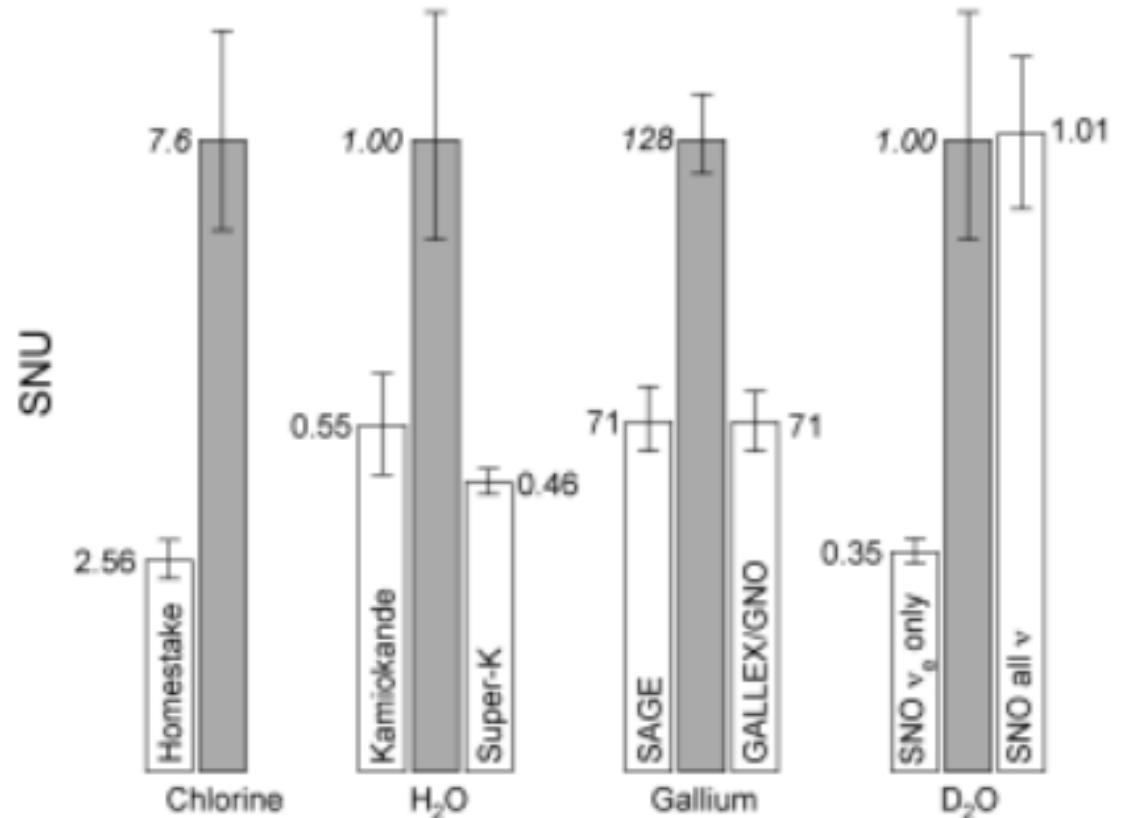


Source: <http://physics.gmu.edu/~hgeller/tsa724/sld029.htm>

# Neutrino Oscillations

# Neutrino Oscillations

- Early experiments (like Davis's) detected fewer electron neutrinos from the Sun than predicted. Initially, scientists thought this was due to experimental error, but later confirmed the deficit through independent experiments.



# Neutrino Oscillations

- In 1968, Bruno Pontecorvo proposed that electron neutrinos could transform into other types (like muon or tau neutrinos) as they travel — a process known as neutrino oscillation. This would explain why detectors sensitive only to electron neutrinos saw fewer events.

# Neutrino Oscillations

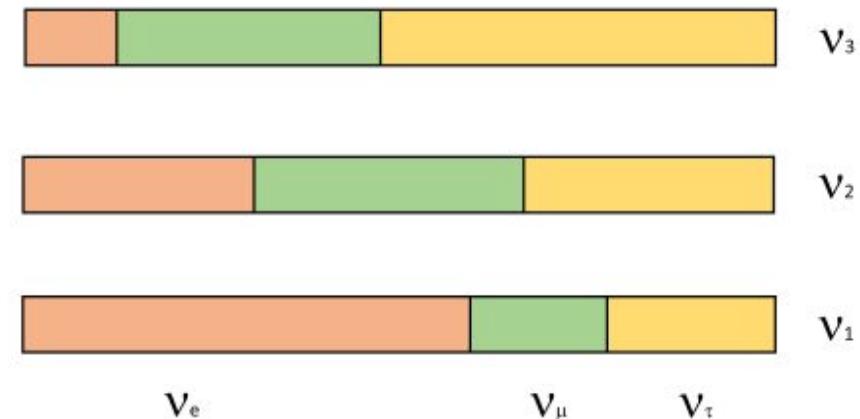
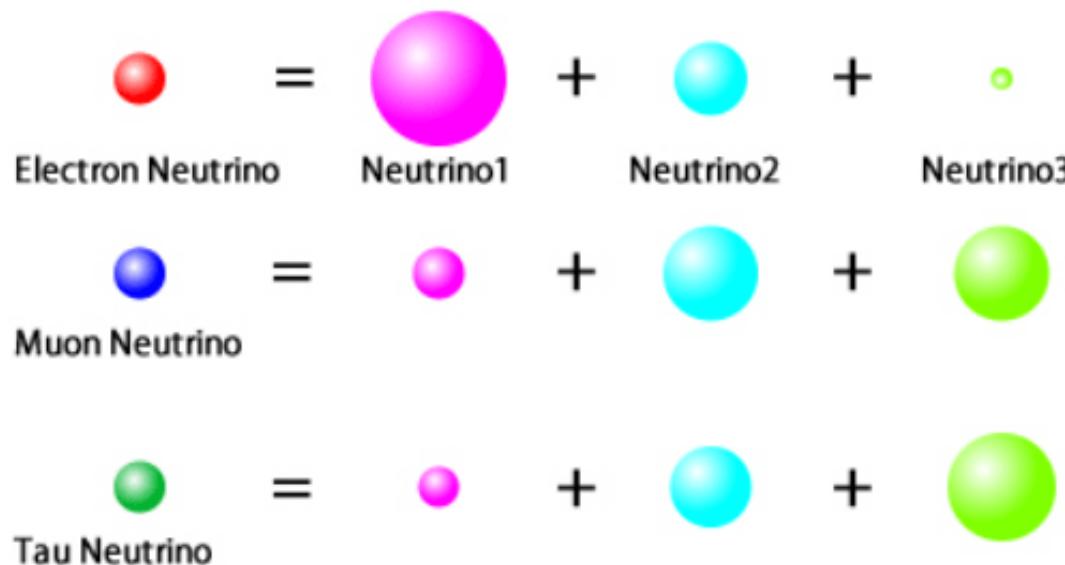
**Quantum SHAPE-SHIFTING: Neutrino Oscillations by minutephysics**

- <https://www.youtube.com/watch?v=7fgKBJDMO54>

# Neutrino Oscillations

## How It Works (Conceptually)

- Neutrinos come in three “flavors”: electron, muon, and tau.  
But these flavors are not the same as their mass states — the versions of the neutrino that have definite mass. Each flavor is actually a mix of two or more mass states.



<https://antares.in2p3.fr/science/dark-matter/neutrino-oscillations-and-interactions/>

<https://www-sk.icrr.u-tokyo.ac.jp/en/sk/neutrino/kajita/vibration/>

# Neutrino Oscillations

## How It Works (Conceptually)

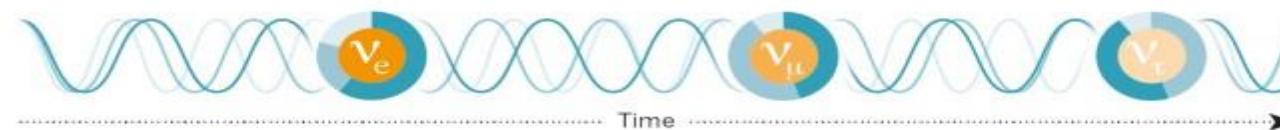
- Because the different mass states move at slightly different speeds, their waves slowly drift out of sync as the neutrino travels. When they overlap again, the mixture changes — so a neutrino that started as one flavor may appear as another.
- It's like two musical notes that interfere with each other — sometimes they add up, sometimes they cancel out. The result is a repeating pattern, or an oscillation.

### AN UNCONVENTIONAL PARTICLE

A neutrino ( $\nu$ ), or its antimatter counterpart the antineutrino, is always produced alongside an electron (e) or one of the electron's heavier cousins, the muon ( $\mu$ ) or tau ( $\tau$ ) particle — and the presence of this partner particle gives the neutrino a 'flavour'.



Unlike electrons, muons and tau particles, neutrinos do not have definite masses. Instead, every neutrino is a mixture — or quantum superposition — of three 'mass states', and those states mix in different proportions to make different flavours.



As a neutrino travels, each state contributes to its mass at a varying rate, causing the neutrino to change flavour over time.  
The frequency of the changes depends on the differences between the mass states, the neutrino's energy and parameters that govern how the states are allowed to mix.

© nature

Source: Nature

# Neutrino Oscillations

## The Big Picture

- Neutrinos switch back and forth between flavors as they move — that's why we call it “oscillation.”
- The rate of this switching depends on:
  - How strongly the flavors are mixed (the mixing angle).
  - The difference between the masses of the two neutrinos.
- If the masses were exactly equal, or if there were no mixing, no oscillation would happen.

# Neutrino Oscillations

*The solution to solar neutrino problem turned out to be that neutrino oscillate. Since the detector was sensitive only to electron neutrinos, muon and tau neutrinos were unobservable.*



$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

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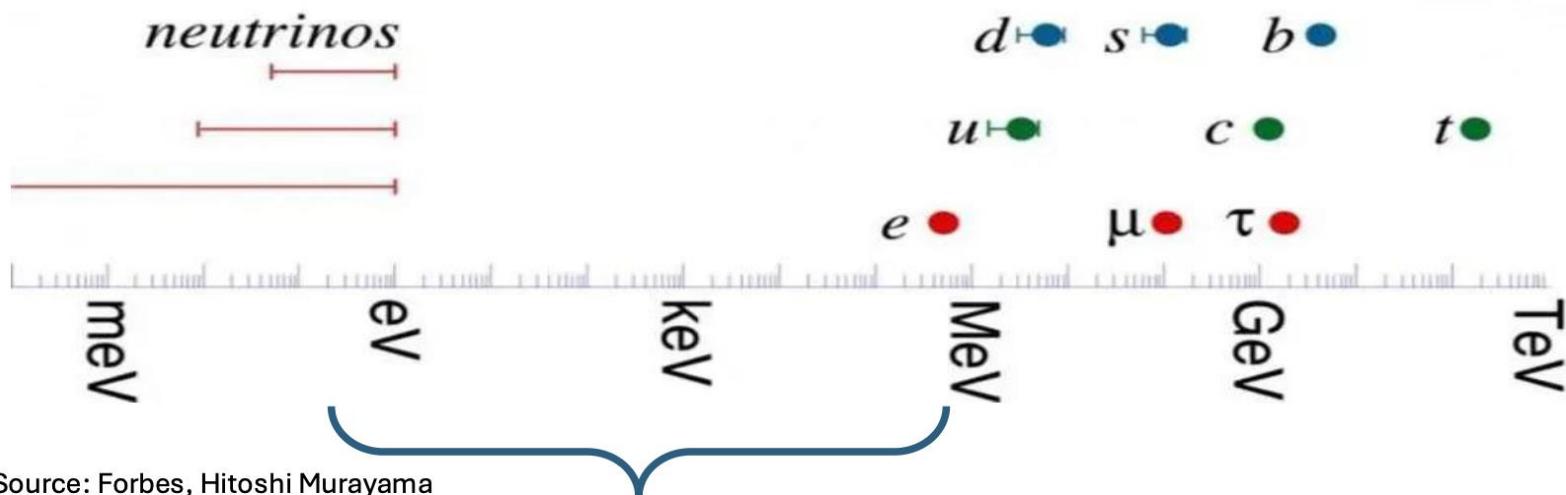
*Depends on the mass difference*

*The different types of neutrinos must have different masses in order to oscillate.*

# What do we currently know about neutrino mass?

We don't know the absolute mass of neutrino but we know at least two of the neutrinos must have mass.

$$m_3 \geq 0.05 \text{ eV}, \quad m_2 \geq 0.008 \text{ eV} \quad [\text{Particle Data Group}]$$



***Why is the neutrino mass so much smaller than other elementary particles?  
There must be an explanation for this.***

# Neutrino Masses

There are three neutrino flavors (electron, muon, tau), so there must be three mass eigenstates:

$$\nu_1, \nu_2, \nu_3$$

The differences in their squared masses are what matter in oscillations:

$$\Delta m_{21}^2 = m_2^2 - m_1^2, \quad \Delta m_{32}^2 = m_3^2 - m_2^2, \quad \Delta m_{31}^2 = m_3^2 - m_1^2$$

Only two of these are independent, because:

$$\Delta m_{31}^2 = \Delta m_{32}^2 + \Delta m_{21}^2$$

# Neutrino Masses

## Measured Splittings

From solar and atmospheric neutrino experiments:

- **Solar neutrinos** give  
 $\Delta m_{21}^2 \approx 7.4 \times 10^{-5} \text{ eV}^2$
- **Atmospheric neutrinos** give  
 $|\Delta m_{32}^2| \approx 2.5 \times 10^{-3} \text{ eV}^2$

So the splittings are tiny — meaning all three neutrinos are **very light**, much lighter than electrons.

# Neutrino Masses

## Normal vs. Inverted Hierarchy

The experiments tell us the **differences**, not which mass is the largest.

There are two possibilities:

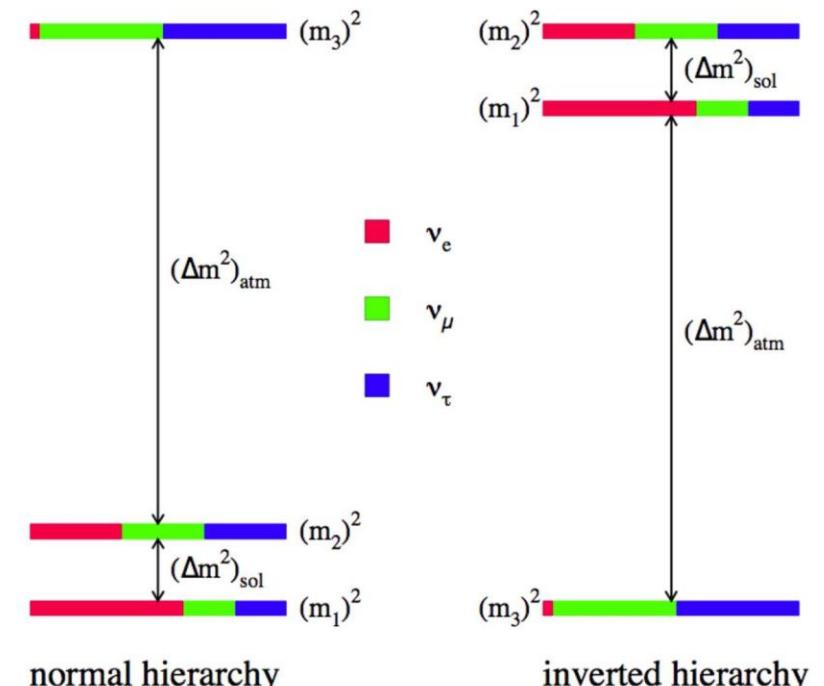
- **Normal hierarchy:**

$$m_1 < m_2 < m_3$$

(like other leptons: electron < muon < tau)

- **Inverted hierarchy:**

$m_3$  is the lightest, and  $m_1, m_2$  are heavier.



*Source: Wikipedia*

Both patterns fit the oscillation data — only **future experiments** can determine which one is true.

# The Mixing Matrix

## 1. From Two Flavors to Three

Earlier, we described how two flavors ( $\nu_e, \nu_\mu$ ) can mix through one angle  $\theta$ .

But in reality, there are **three neutrino flavors**:

$$\nu_e, \nu_\mu, \nu_\tau$$

and each is a **mixture** of three mass eigenstates:

$$\nu_1, \nu_2, \nu_3$$

# The Mixing Matrix

## 2. The PMNS Mixing Matrix

The flavor and mass states are related by a **unitary mixing matrix**  $U$ :

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

This matrix tells us how much of each **mass eigenstate** contributes to each **flavor eigenstate**.

# The Mixing Matrix

### 3. Parameterization (Three Angles + One Phase)

Like the quark-mixing (CKM) matrix, the neutrino matrix can be expressed using **three mixing angles** ( $\theta_{12}, \theta_{23}, \theta_{13}$ ) and one **CP-violating phase**  $\delta$ :

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$ .

# The Mixing Matrix

## 4. Meaning of the Angles

Each angle controls mixing between two neutrinos:

- $\theta_{12}$ : solar neutrino mixing ( $\nu_e \leftrightarrow \nu_\mu$ )
- $\theta_{23}$ : atmospheric neutrino mixing ( $\nu_\mu \leftrightarrow \nu_\tau$ )
- $\theta_{13}$ : small but crucial mixing connecting  $\nu_e$  and  $\nu_3$

Typical measured values:

$$\theta_{12} \approx 34^\circ, \quad \theta_{23} \approx 45^\circ, \quad \theta_{13} \approx 8.5^\circ$$

# The Mixing Matrix

## 5. CP Violation in Neutrinos

The complex phase  $\delta$  can cause **CP violation** — meaning oscillation probabilities for neutrinos and antineutrinos could differ.

This is an active area of research because it might help explain the **matter–antimatter asymmetry** in the universe.

# The Mixing Matrix

## 6. Physical Picture

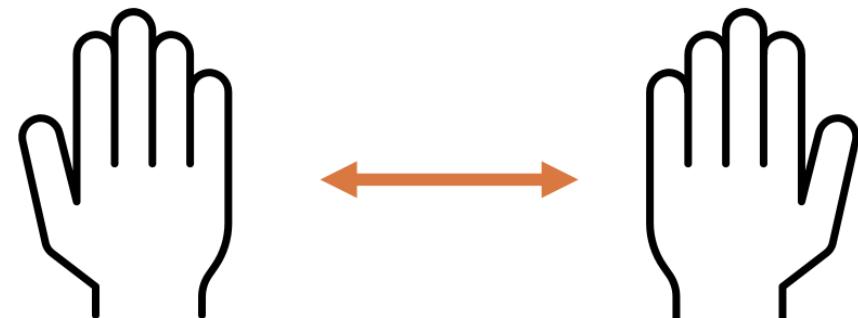
So, a flavor neutrino (say  $\nu_\mu$ ) is not a particle with a definite mass.

Instead, it's a **quantum mixture** of three mass states that each evolve differently over time — producing the oscillation effects observed in experiments.

# **Neutrino Mass Models**

# Why was the neutrino mass was set to zero in Standard Model?

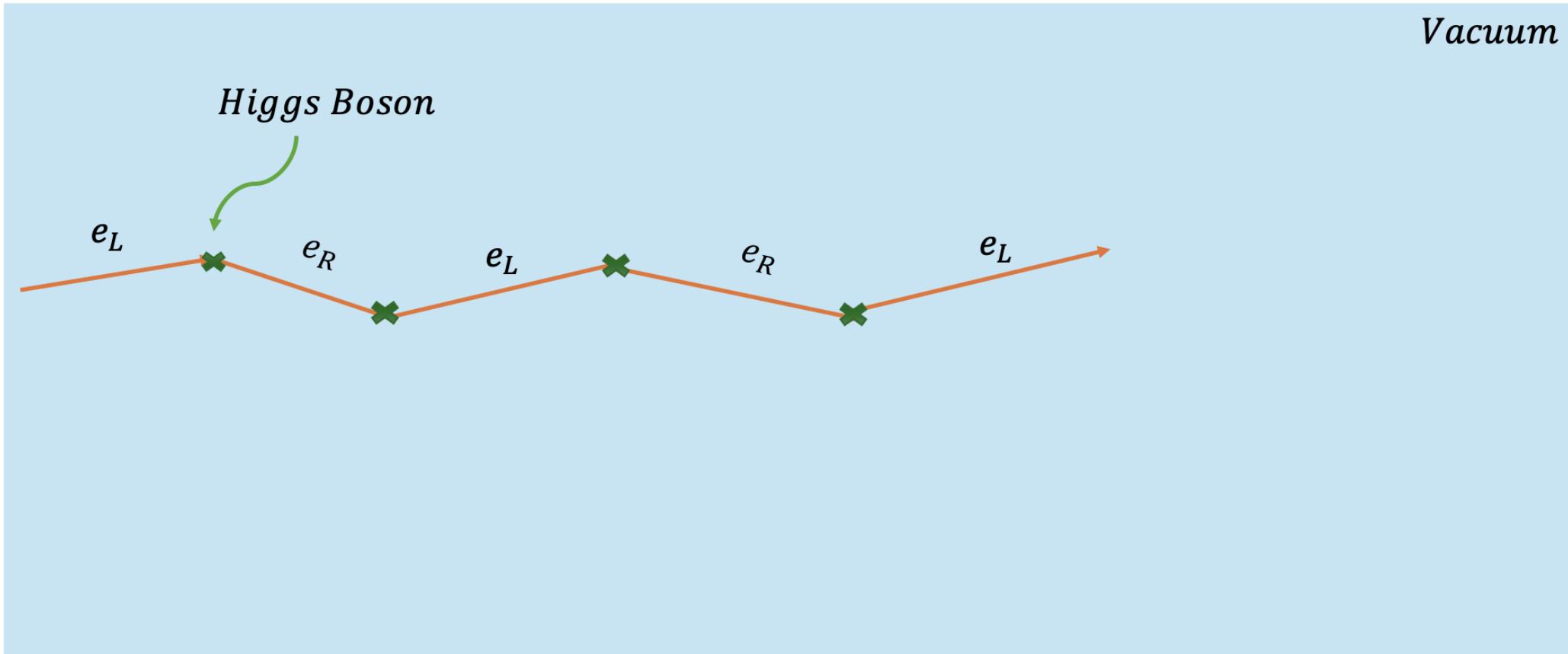
Standard model doesn't predict mass of these elementary particles but it does tell you how they get mass.



Left-Handed Particle

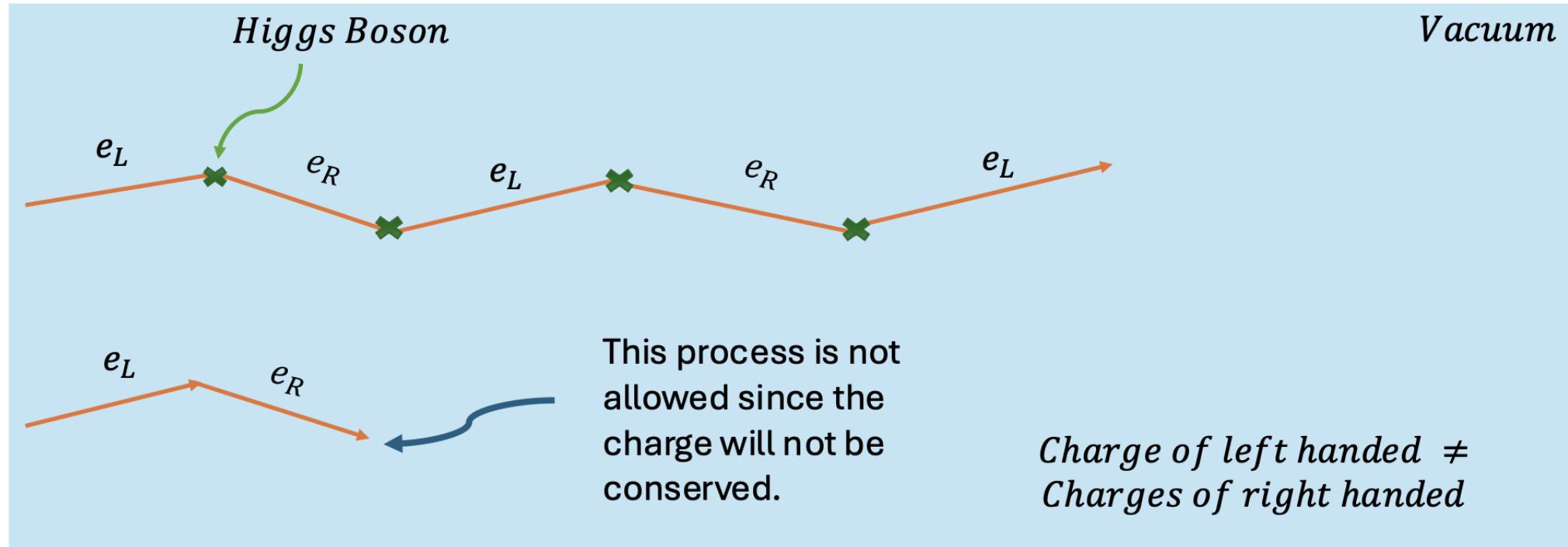
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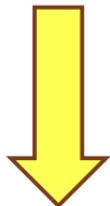
*If you have done some basic physics, you must have heard about electric charges and charge conservation. The Standard Model have other charges called color charge and hypercharge. All these charges must be conserved.*



# Why was the neutrino mass was set to zero in Standard Model?

All the known neutrinos are left-handed and all the known anti-neutrinos are right-handed.

*Neutrino travelled very close to the speed of light.  
Scientist didn't find any right handed neutrinos.*

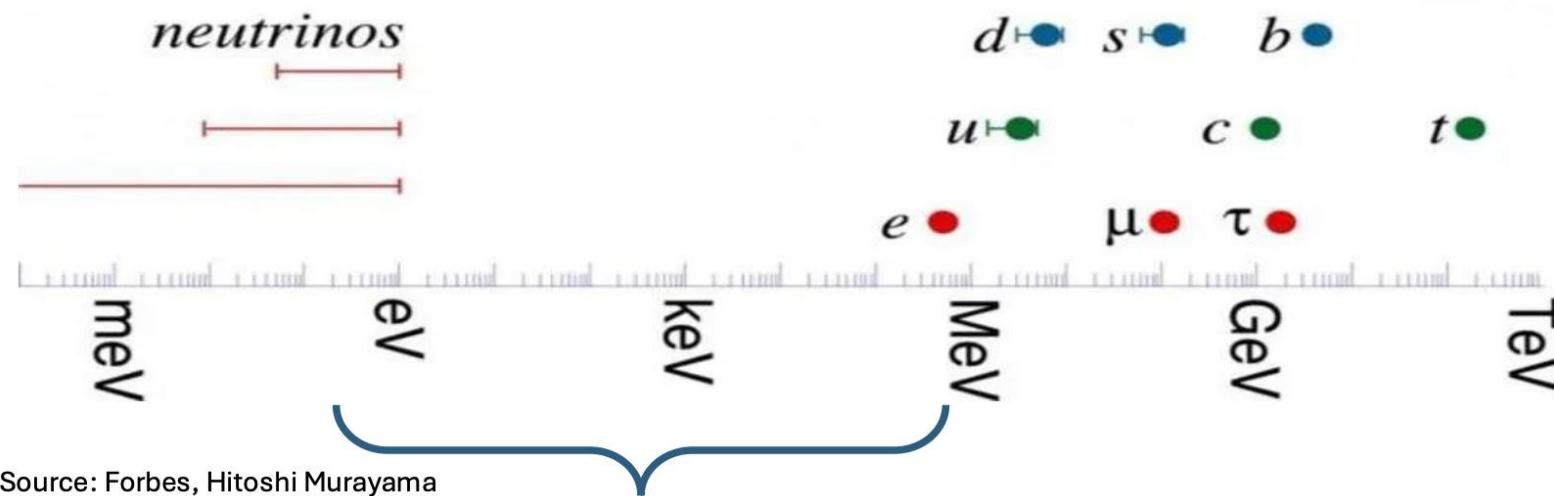


*The Standard model was written so that the neutrino mass was zero.*

# What do we currently know about neutrino mass?

We don't know the absolute mass of neutrino but we know at least two of the neutrinos must have mass.

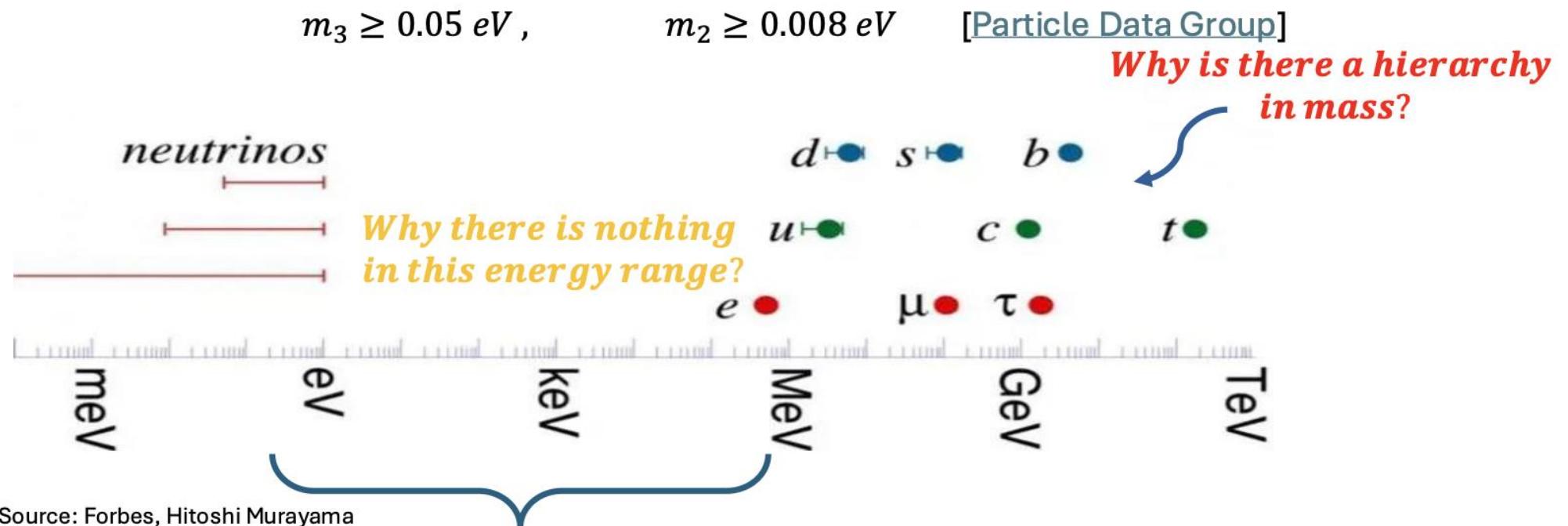
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# How do Neutrinos Get Mass?

There are many models that add neutrino mass to the Standard Model, and we don't know yet which one is correct.

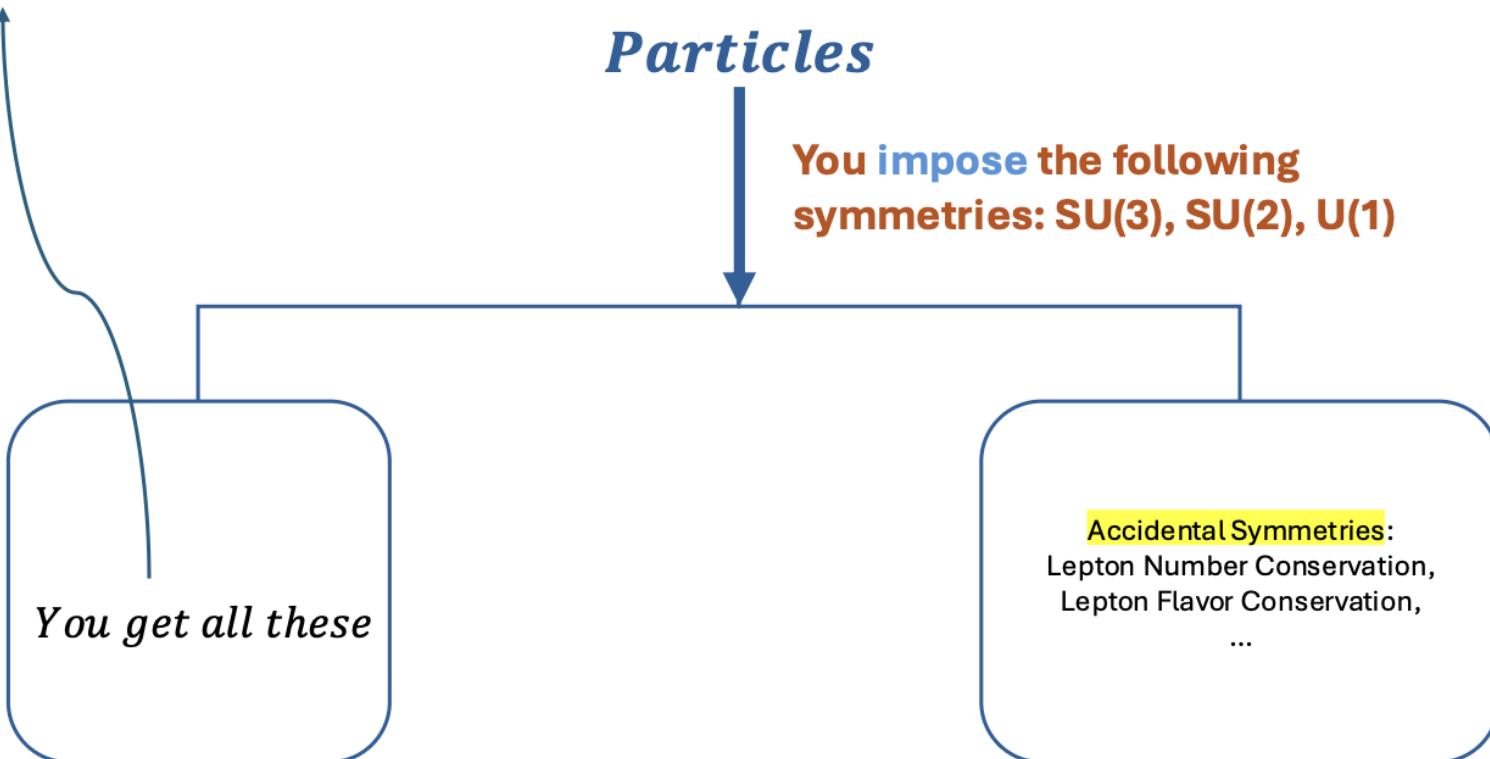
In this discussion, I will focus on two relatively simple and widely popular models.

***Before we get to that we need to understand  
a little bit about symmetries.***

# Symmetries in the Standard Model

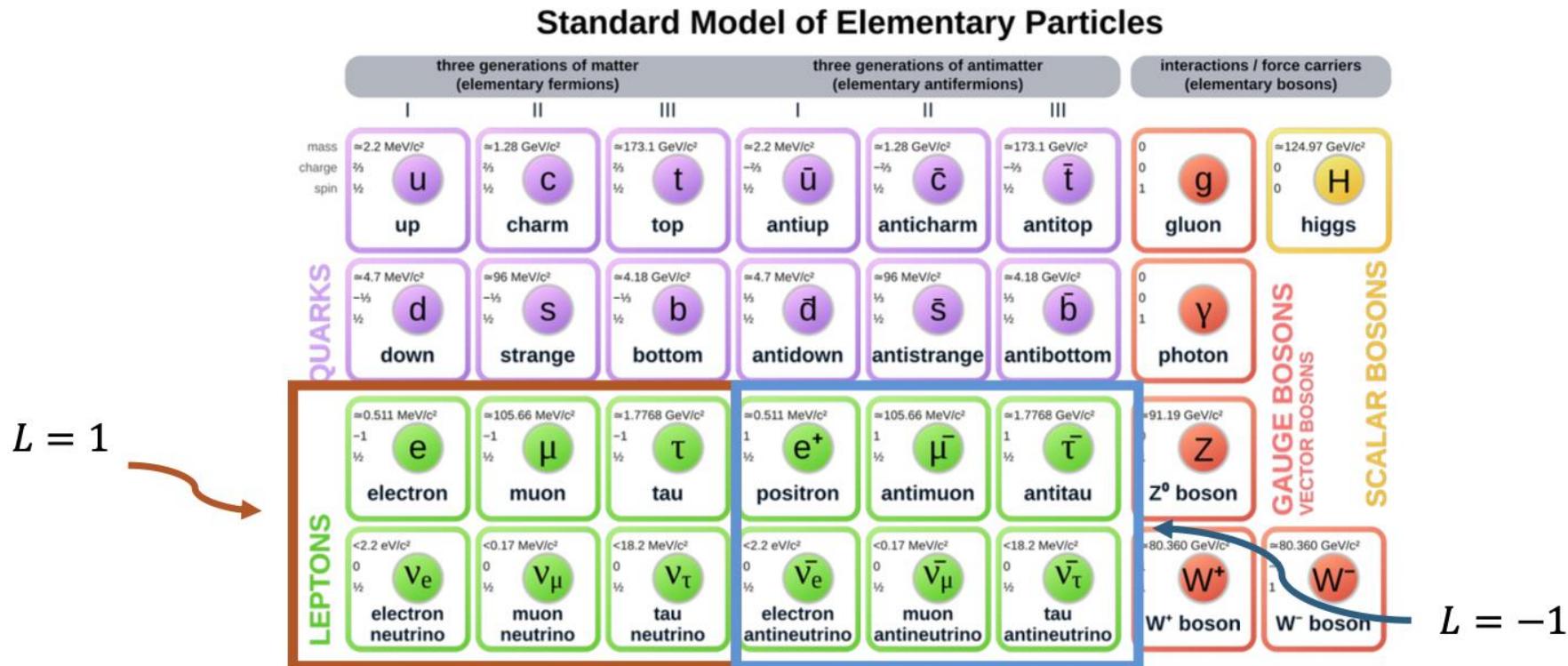
Symmetries tells us

- what kinds of interactions can happen
- it predicts what force carriers should be included
- It tells us which charges must be conserved in these interactions.



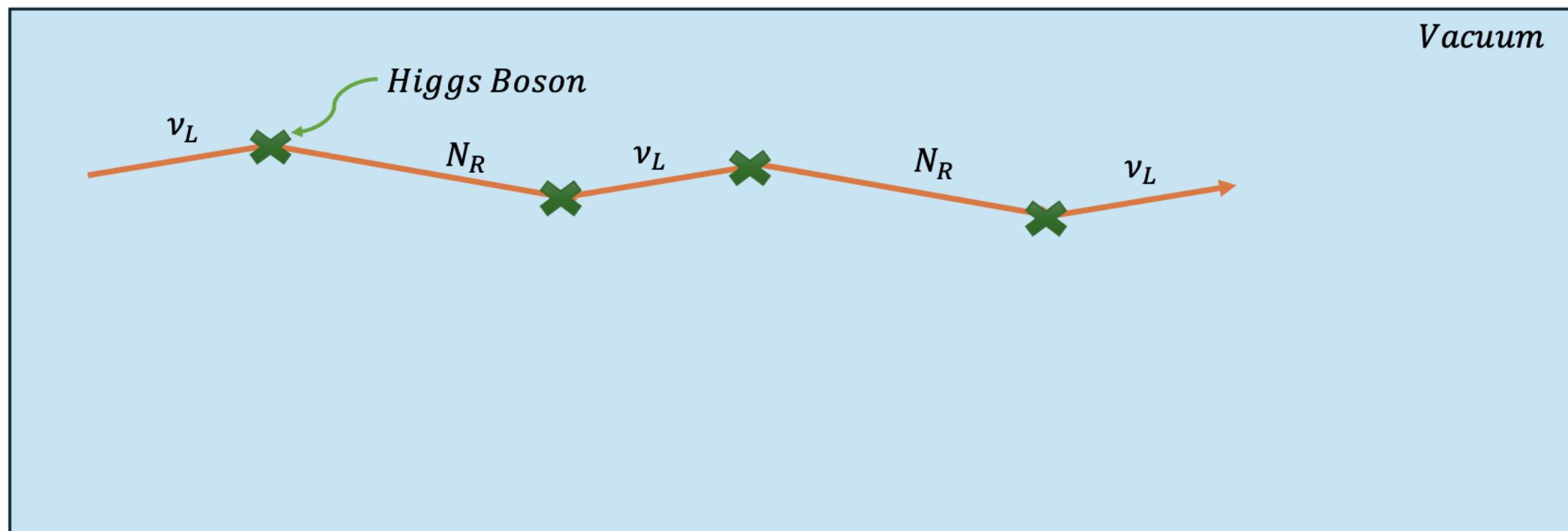
# Lepton Number Conservation

We assign a quantum number called the Lepton Number to each particle in the standard model.



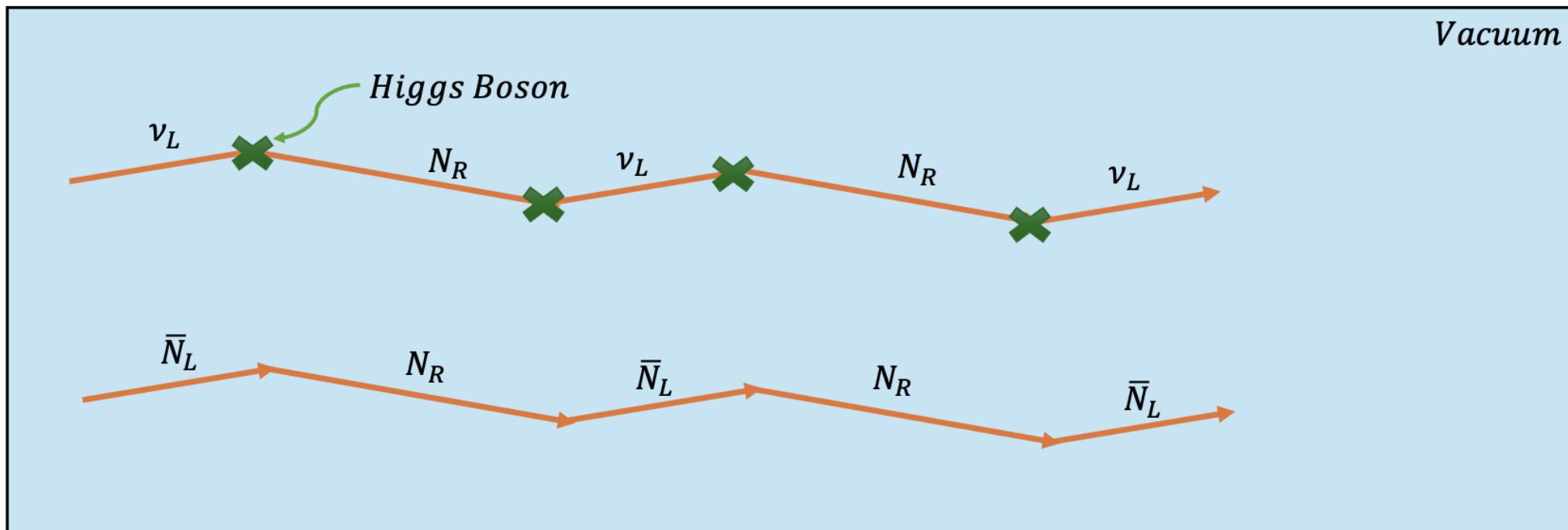
# How do Neutrinos Get Mass?

1. In these models, we add right-handed neutrinos ( $N_R$ )/left-handed antineutrino ( $\bar{N}_L$ ).
2. They don't have any Standard Model charges (can't interact with Standard Model particles).



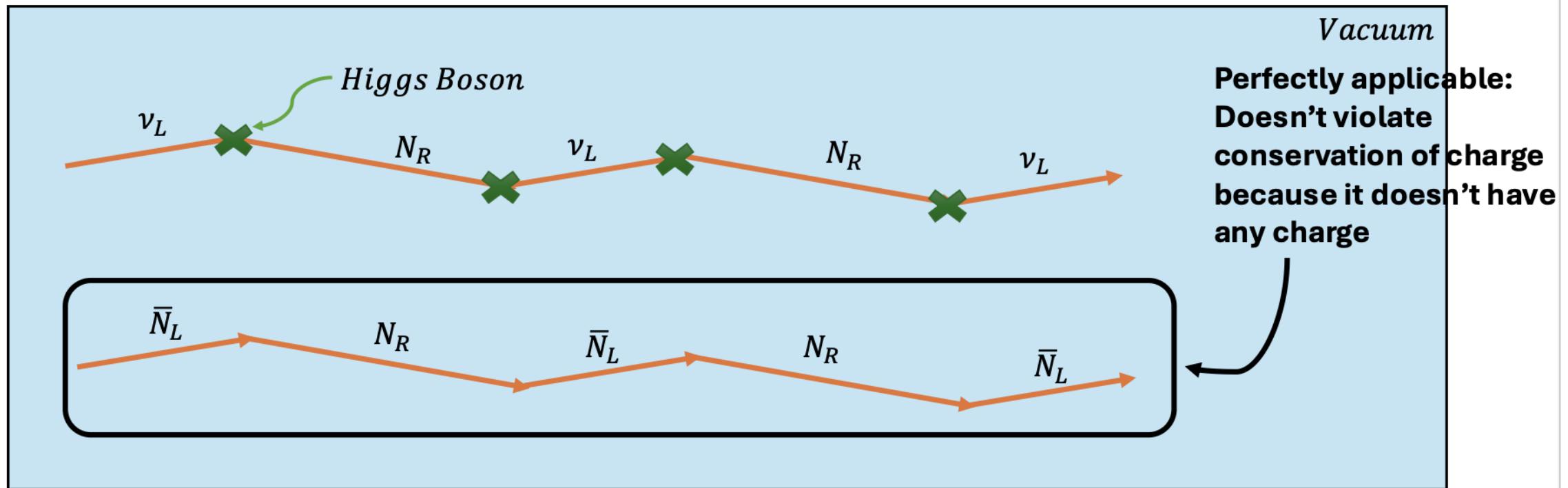
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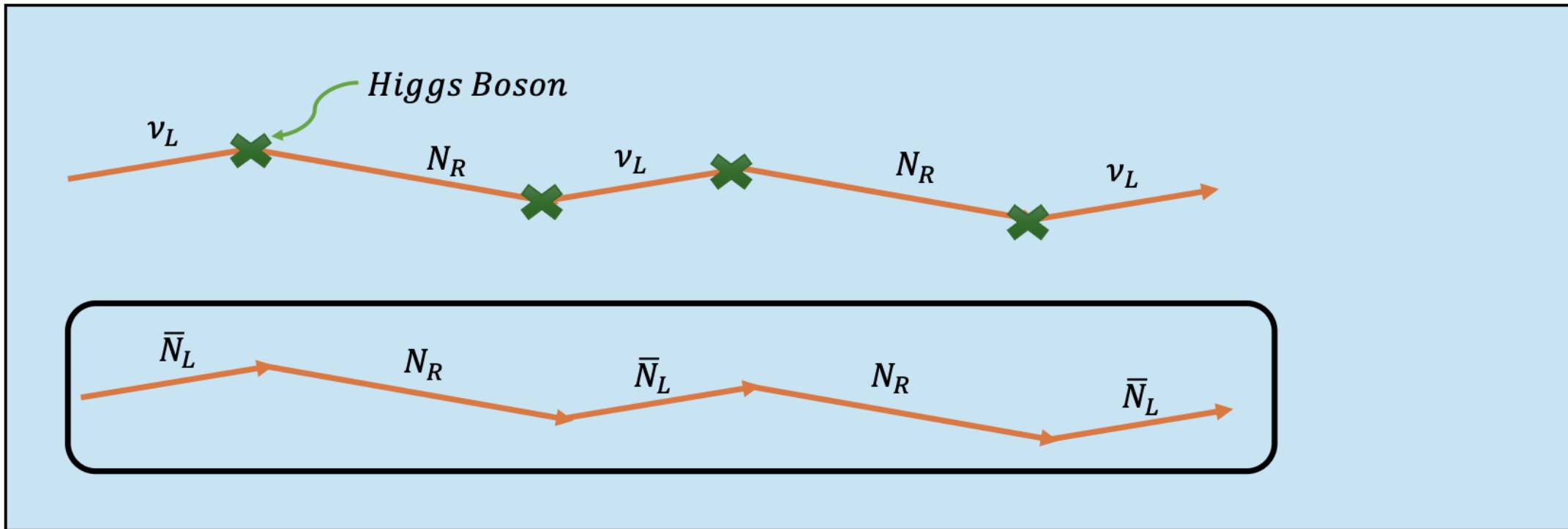
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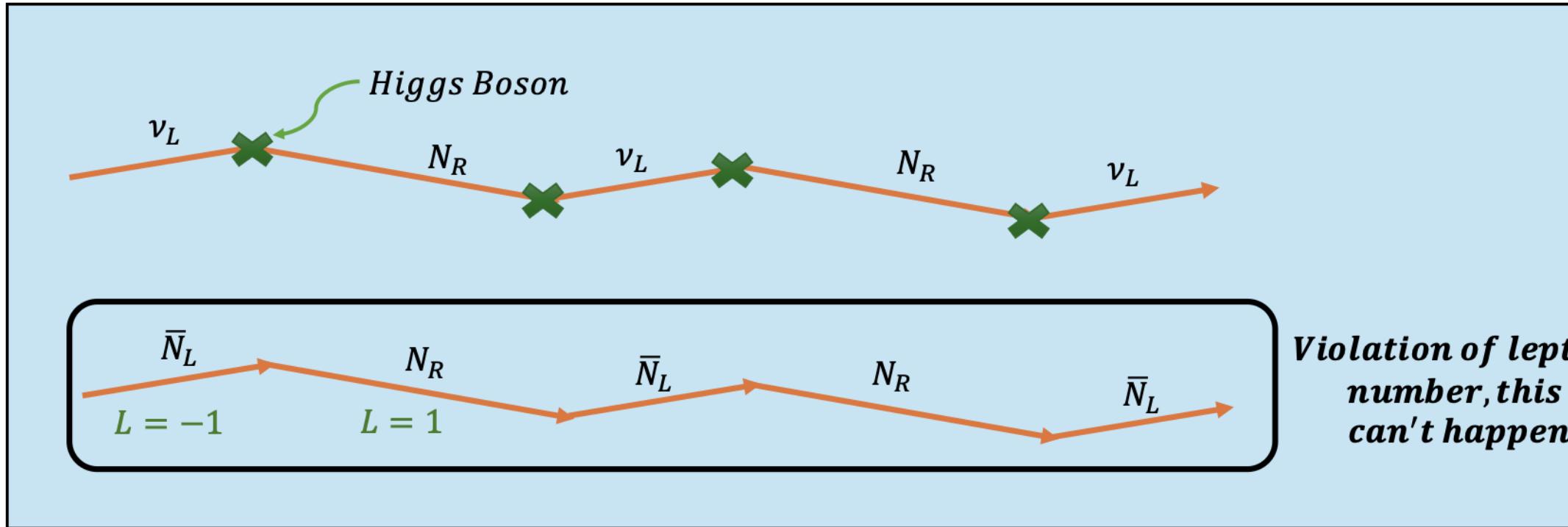
# How do Neutrinos Get Mass?

*You can make a model where neutrinos get mass similar to other particles.*

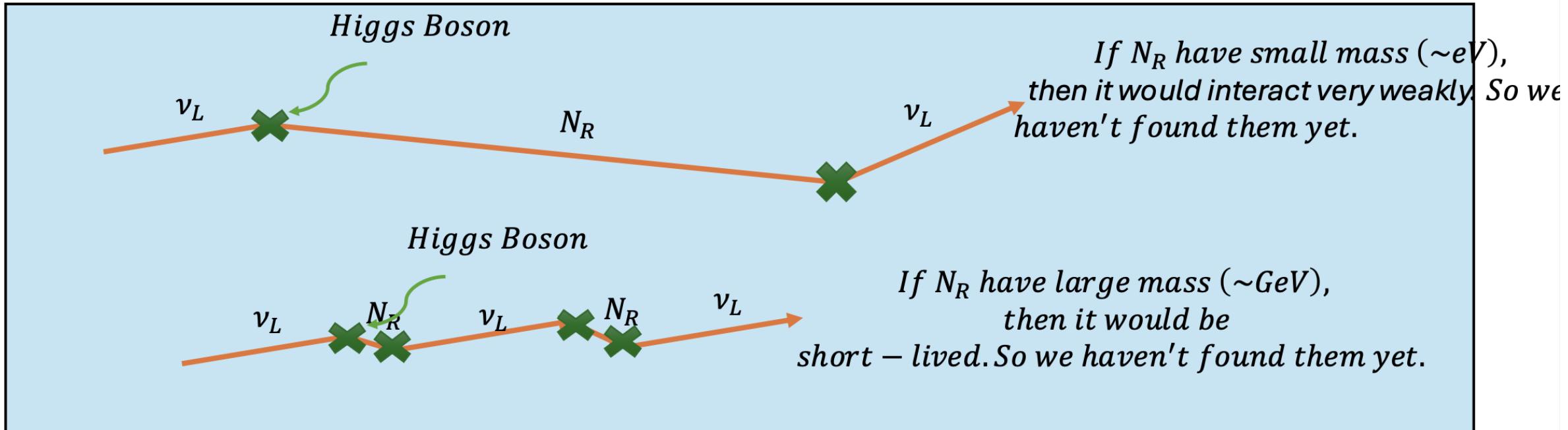


# How do Neutrinos Get Mass?

*To get rid of second interaction, you can impose Lepton Number Conservations.*



# How do Neutrinos Get Mass?



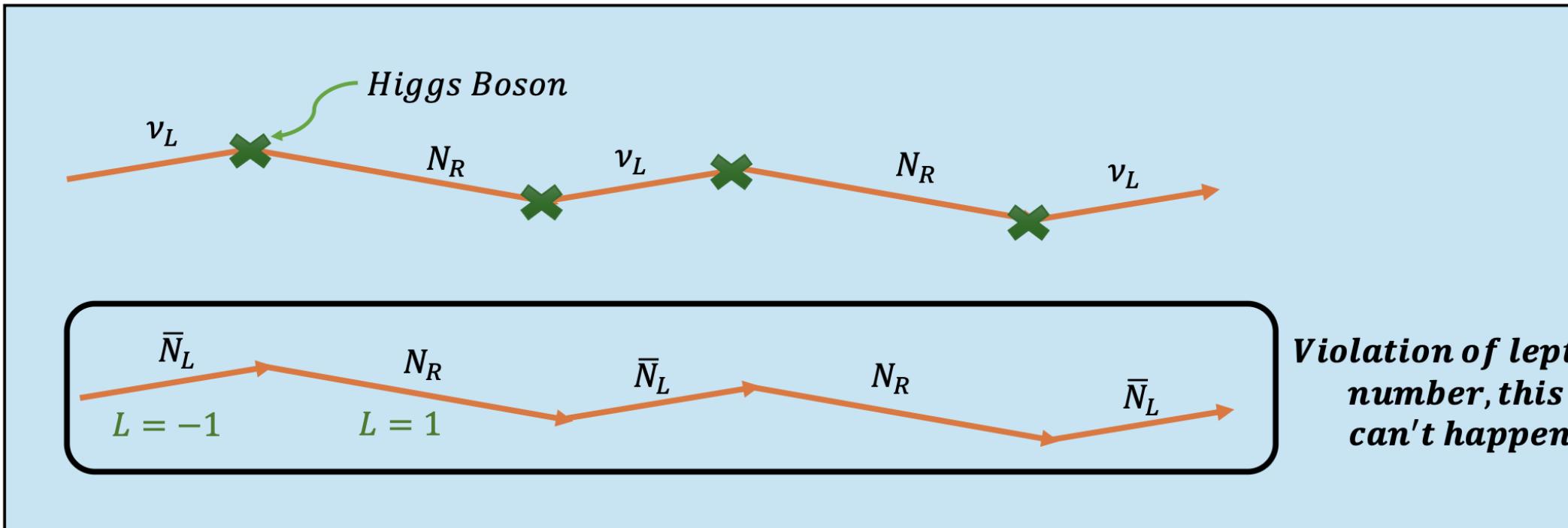
**If neutrinos get mass like this, then neutrinos are called Dirac particles.**

# **What are the problems with this model?**

- ***It doesn't explain why the neutrino mass is so small.***
- ***We have no evidence that the Lepton Number Conservation is such an important symmetry that we had to impose it.***
- ***This doesn't give any new physics (boring model).***

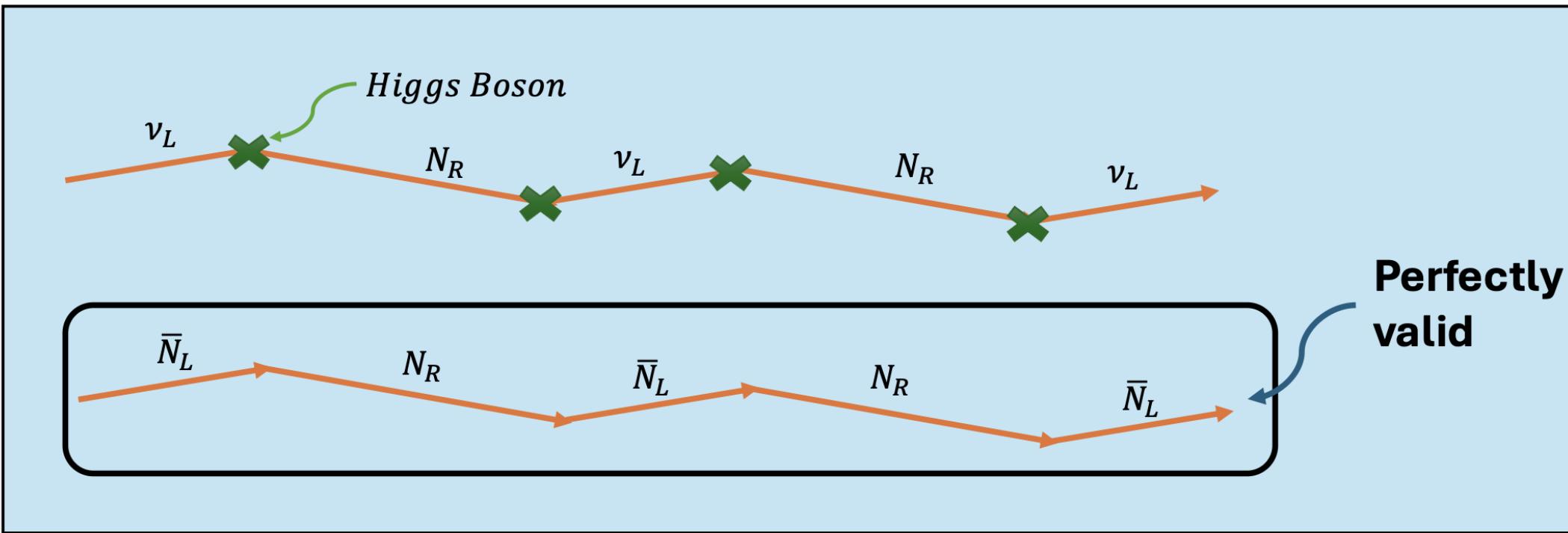
# How do Neutrinos Get Mass?

*To get rid of second interaction, you can impose Lepton Number Conservations.*



# How do Neutrinos Get Mass?

*Since we don't have a good enough reason to impose Lepton Number Conservation symmetry, we can make model where break it.*



# How do Neutrinos Get Mass?

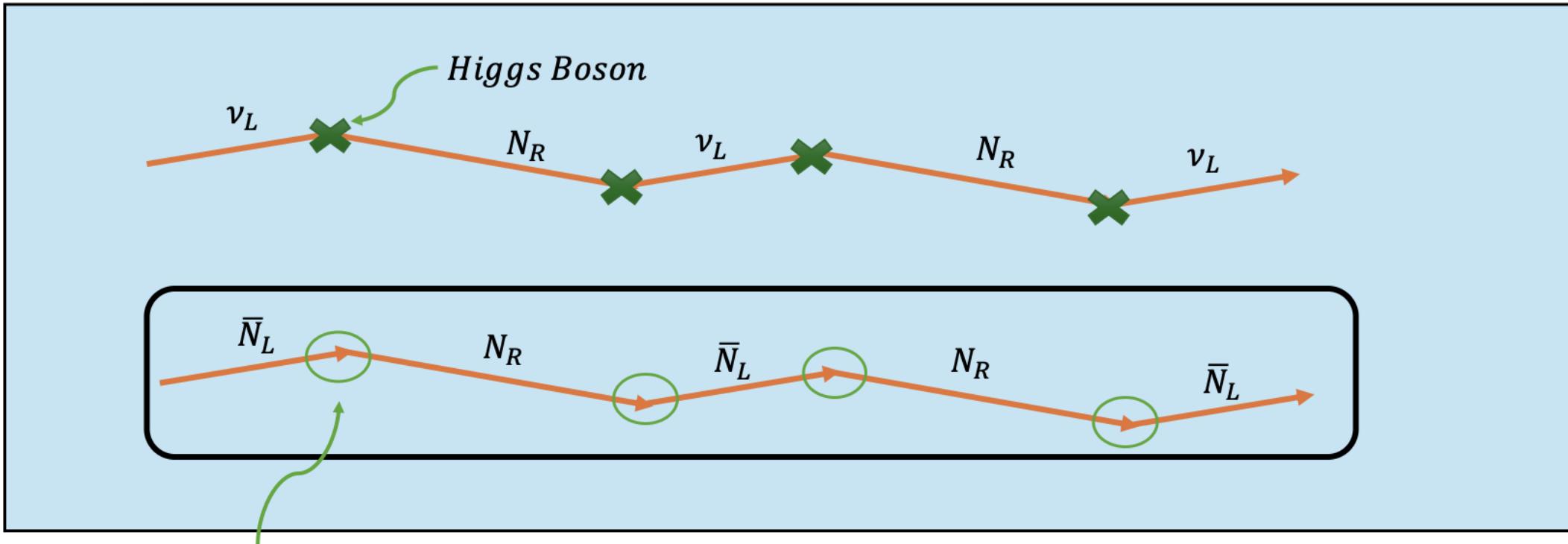
$$\text{Mass of neutrino} \propto \frac{1}{\text{Mass of } N_R}$$



Symmetry Magazine: Artwork by Sandbox Studio, Chicago with Ana Kova

*This model explain why the left – handed neutrino's mass is small.*

# How do Neutrinos Get Mass?



*Doesn't need Higgs Boson to generate mass, this is “New Physics.”*

***If neutrinos get mass through this model, neutrinos are called Majorana Particles.***