Special Theory of Relativity

Karan Kumar

Class Policy

- Classes from 10:00 AM to 12:30 PM (5 min break at ~ 11:00 AM, 5 min break at noon).
- Up to four absences
- Lateness or leaving early counts as half-absence.
- Send email notifications of all absences to shpattendance@columbia.edu

Class Policy

- No cell phone uses during the class
- Feel free to step outside to the hallway in case of emergencies, bathroom, and starvations.
- Feel free to stop me and ask questions / ask for clarifications.

Curriculum

Lecture	Topic
~~1~~	~~Introduction~~
~~2~~	~~History of Particle Physics~~
3	Special Relativity
4	Quantum Mechanics
5	Detectors
6	Standard Model
7	Beyond the Standard Model
8	Neutrino Theory
9	Neutrino Experiment
10	Large Hadron Collider (LHC)
11	Higgs Boson and Beyond
12	Cosmology

I am presenting these lesson with as little bias as possible. I highly encourage you to take this information, continue researching as you progress in your studies, and form your own informed opinions.

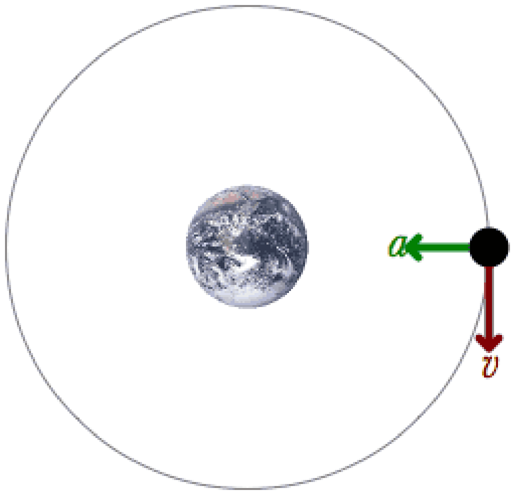
Book Reference

- Introduction to Elementary Particles Textbook by David J. Griffiths
- Introduction to special relativity by Robert Resnick.

INTRODUCTION

Special Theory of Relativity

- Classical physics feels natural to us. Once you learn Newton's second law, F = ma, it seems to match the way the world works. It sharpens our intuition but does not really challenge it.



Source: Wikipedia

Special Theory of Relativity

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- That is why in science we do not just follow our intuition, we follow the math and experiments even when the results seem surprising.
- Classical intuition mostly fails at Quantum level.

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- So whenever scientists come up with a new idea about particles or forces, the first check is: does it agree with special relativity? If it doesn't, then the idea is probably wrong.
- In that sense, special relativity sets the basic rules for all of physics.

Special Theory of Relativity

- But scientist do write theories that violate principle of special theory of relativity. They usually called Lorentz Invariance Violation. None of those have been proven.

Assessing Lorentz invariance violations [edit]

Early models assessing the possibility of slight deviations from Lorentz invariance have been published between the 1960s and the 1990s.[3] In addition, a series of test theories of special relativity and effective field theories (EFT) for the evaluation and assessment of many experiments have been developed, including:

- The parameterized post-Newtonian formalism is widely used as a test theory for general relativity and alternatives to general relativity, and can also be used to describe Lorentz violating preferred frame effects.
- The Robertson-Mansouri-Sexl framework (RMS) contains three parameters, indicating deviations in the speed of light with respect to a preferred frame of reference.
- The c^2 framework (a special case of the more general TH$\varepsilon\mu$ framework) introduces a modified dispersion relation and describes Lorentz violations in terms of a discrepancy between the speed of light and the maximal attainable speed of matter, in presence of a preferred frame.[4][5]
- Doubly special relativity (DSR) preserves the Planck length as an invariant minimum length-scale, yet without having a preferred reference frame.
- Very special relativity describes space-time symmetries that are certain proper subgroups of the Poincaré group. It was shown that special relativity is only consistent with this scheme in the context of quantum field theory or CP conservation.
- Noncommutative geometry (in connection with noncommutative quantum field theory or the noncommutative standard model) might lead to Lorentz violations.
- Lorentz violations are also discussed in relation to alternatives to general relativity such as loop quantum gravity, emergent gravity, Einstein aether theory, Hořava–Lifshitz gravity.

Source: Wikipedia

Einstein's Postulates

Special Theory of Relativity

- Einstein published the **Special Theory of Relativity** in 1905.

Einstein built the theory on two postulates:

- The principle of relativity
- The universal speed of light.

Outline

- The principle of relativity
- The universal speed of light
- The relativity of simultaneity
- Time Dilation
- Lorentz Contraction
- Relativistic Kinematics
- Spacetime

The Principle Of Relativity

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- According to the Special Theory of Relativity, the laws of physics are the same in all inertial reference frames — that is, in any system moving at a constant speed in a straight line.
- The principle of relativity says that the laws of physics are the same in any reference frame that moves at a constant speed, just as they are in one that is at rest.

The Principle of Relativity

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- The problem is that there’s no way to know if any system is truly at rest.
- And if you don’t know whether you’re at rest, then you also can’t define the velocity of other system.
- This makes it hard to say for sure whether a frame is inertial or not.

The Principle of Relativity

- Another way to check if you are in an inertial frame is to see if Newton’s First Law holds.

Question Time

Question: What does Newton's First Law of Motion state?

A) An object will always keep moving faster unless a force slows it down.
B) An object will remain at rest or move in a straight line at constant speed unless acted on by a net external force.
C) Every action has an equal and opposite reaction.
D) The force on an object is equal to its mass times its acceleration.

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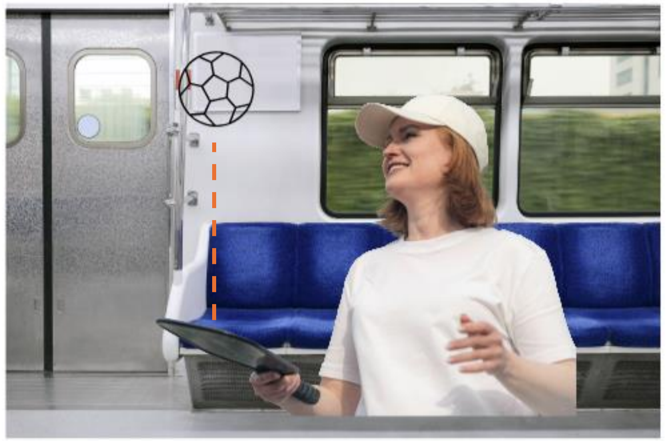
The Principle of Relativity

- Another way to check if you are in an inertial frame is to see if Newton’s First Law holds.
- But you can’t really find a perfectly frictionless surface. This is life, you live with it.

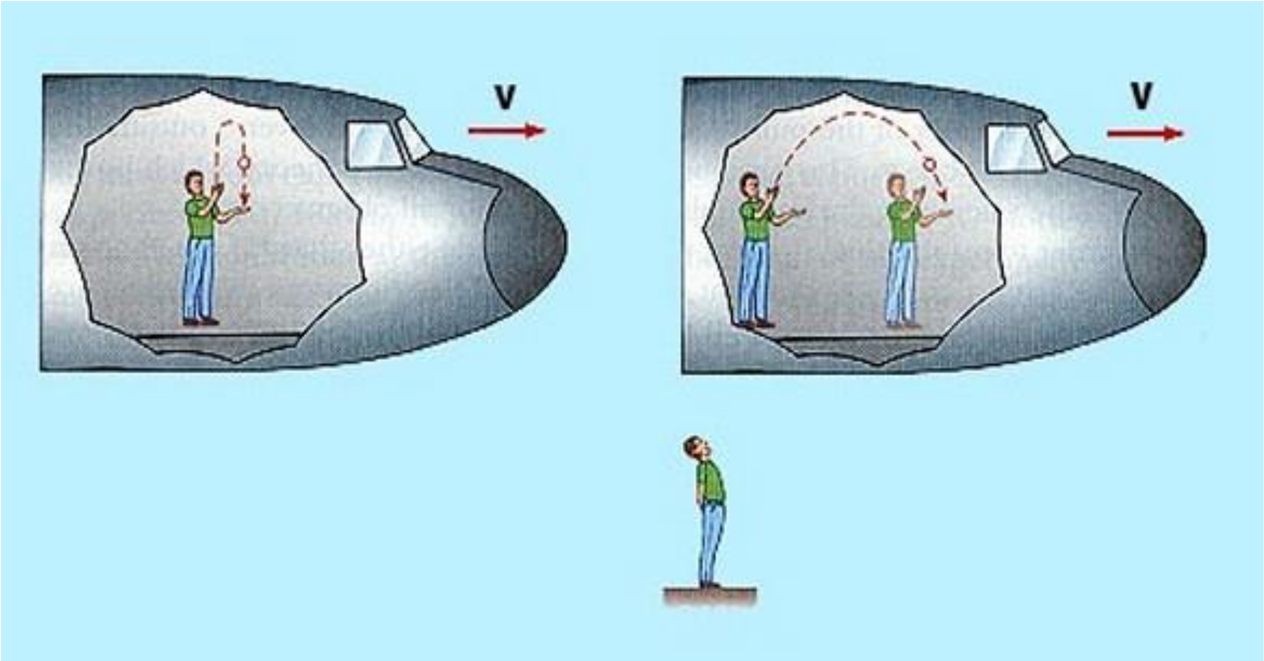
The Principle of Relativity

The ball follows the same trajectory when hit on a train moving at constant velocity as it does at rest, from the point of view of the hitter.



The Principle of Relativity

- But to an observer standing outside the train, the ball's path is different because it combines the train's motion with the ball's motion.



Source: https://www.physicsoftheuniverse.com/topics_relativity_light.html

The main idea is the **form of laws doesn't change**. If you looking from outside, you have take into account the velocity of the train but you are still applying the same equation of motion.

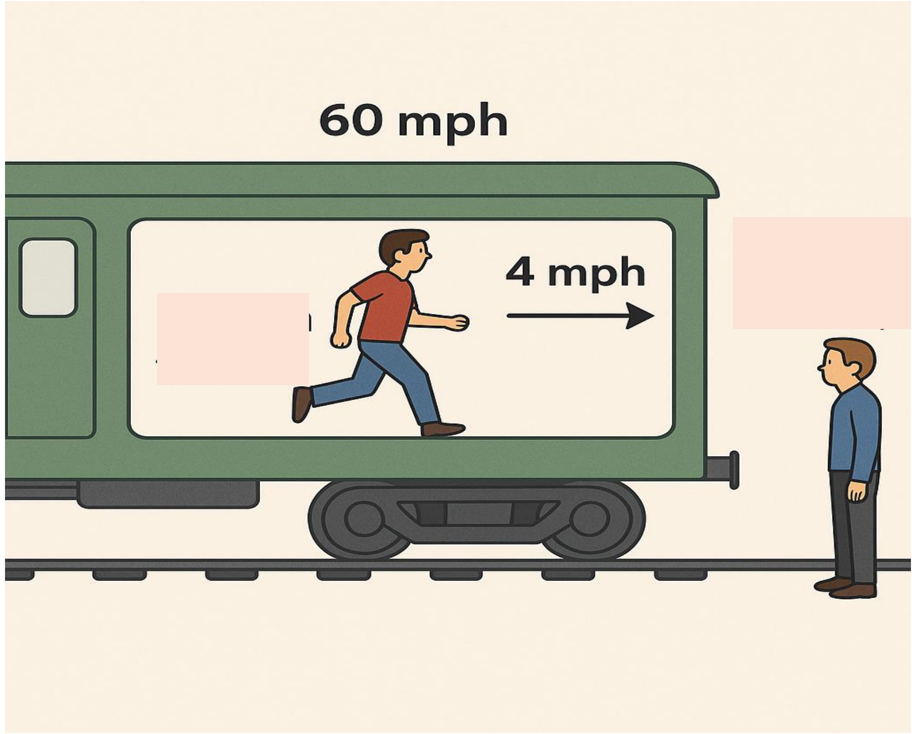
The Universal Speed Of Light

The Universal Speed of Light

- Einstein’s second postulate says that light always travels at the same speed in a vacuum, no matter how the source or the observer is moving.

The Universal Speed of Light

- This is very different from everyday motion. For example, if a train goes 60 mph and you walk forward inside it at 4 mph, someone standing outside sees you moving at 64 mph.

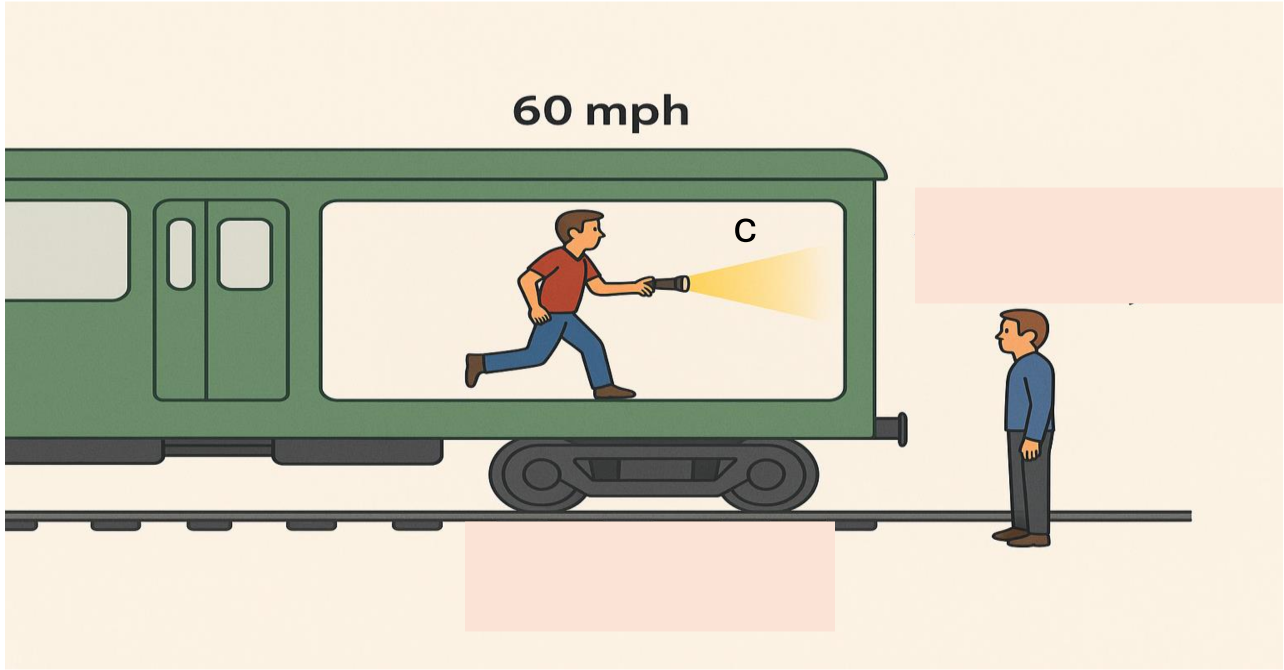


Source: Generated by AI

The Universal Speed of Light

- What would you expect the speed of light if you standing outside if you follow the previous scenario logic?

A. c (speed of light)
B. c + 60 mph
C. c – 60 mph
D. c/60 mph



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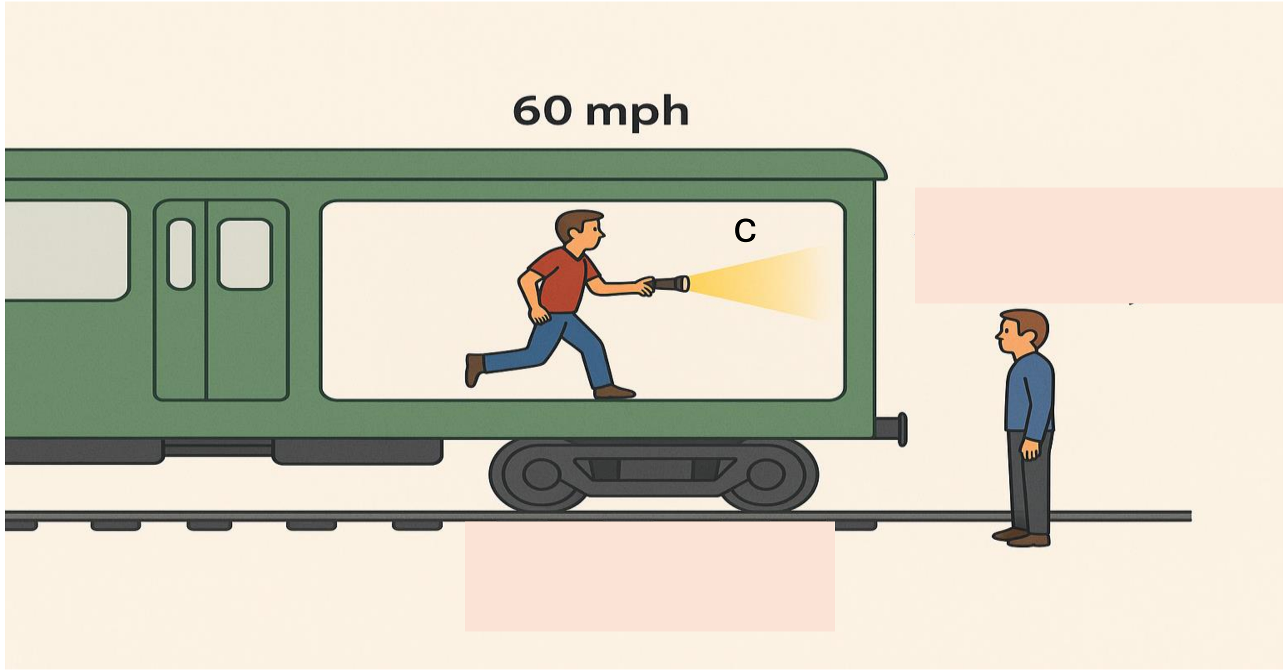

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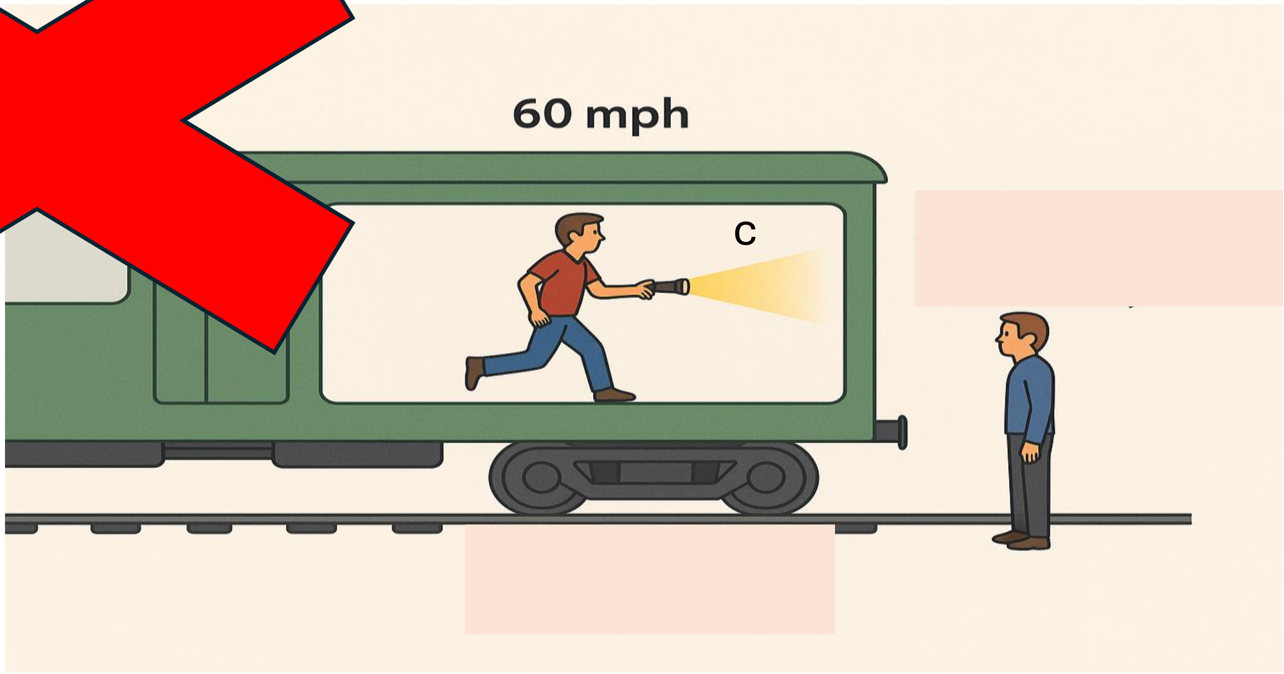
With light, this rule of “just add the speeds” doesn’t work. No matter how fast the source is moving, every observer measures the speed of light as the same.



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The Universal Speed of Light

- Einstein realized the problem was with Galileo's rule. He replaced it with a new formula, the **Einstein velocity addition rule**:

$$v = \frac{v_l + v_t}{1 + v_l v_t / c^2}$$

v_l : $velocity\ of\ light\ with\ respect\ to\ train$

v_t: velocity of train with respect to ground

v : velocity of light with respect to ground

The Universal Speed of Light

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v : velocity of light with respect to ground

$$v = \frac{c\ + v_t}{1 + c v_t / c^2} = c$$

If one of the speeds is c, this formula always gives the result c. That's why the speed of light is the same for everyone.

The Universal Speed of Light

But why nobody noticed this before?

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The Universal Speed of Light

But why nobody noticed this before?

$$v = \frac{v_l + v_t}{1 + v_l v_t / c^2}$$

This is why too small for everybody life. Only matter when we go close to speed of light.

The Universal Speed of Light

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- How did Einstein come up with the second postulate? He was trying to sort out puzzles in electricity and magnetism and eventually realized that light must have a universal speed.
- In fact, an important experiment called had already hinted at this before Einstein's work — the Michelson–Morley experiment of 1887.

The Universal Speed of Light

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- Maxwell's theory predicted that electromagnetic waves should travel at about $3 \times 10^8\ m/s$. That made it clear that light is an electromagnetic wave.
- But this raised a question: *relative to what?* For waves on water, the speed is relative to the water.
- So scientists assumed light must also travel through some medium, which they called the "ether."

The Universal Speed of Light

- If light really needed a medium called the “ether,” then Earth must be moving through it, since Earth rotates daily, orbits the Sun yearly, and the whole Solar System moves through the Galaxy.

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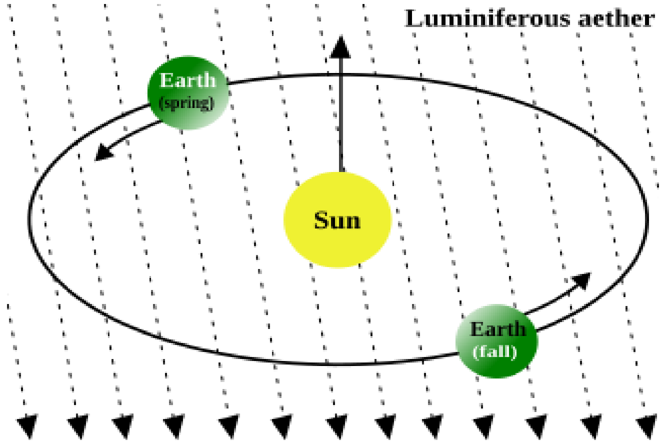
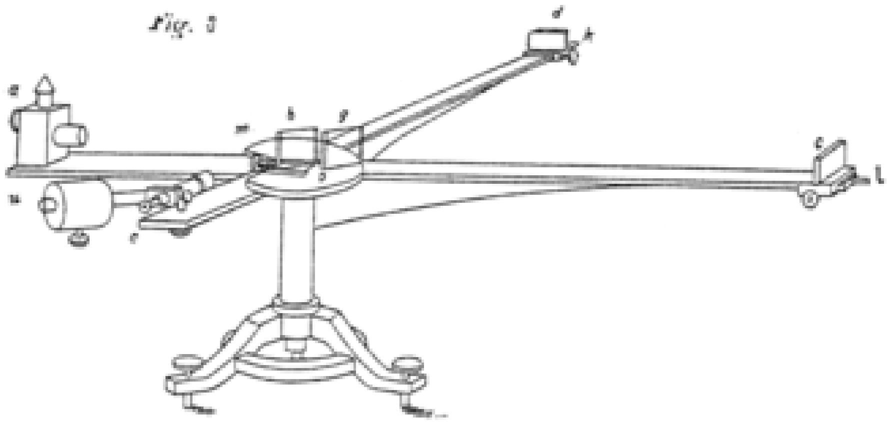
- It would be almost impossible for us to be perfectly at rest with respect to the ether.
- Just like a boat moving through a river feels the current pushing against it, Earth should feel an ‘ether wind.’
- But how strong is this wind, and which direction does it blow? To answer this, Michelson and Morley designed an experiment.

Michelson and Morley Experiment

The main idea behind this experiment was to calculate the speed of light in different direction.





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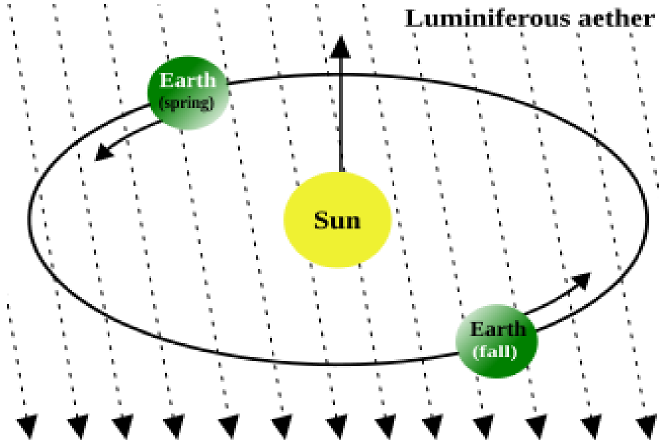
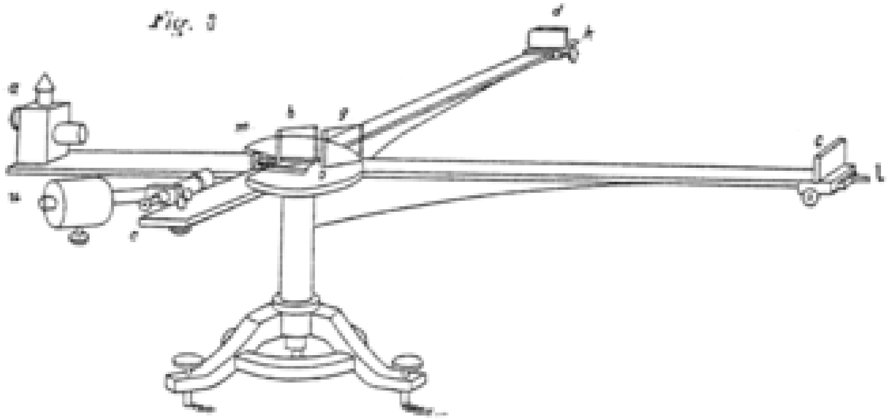
If ether really existed then speed of light should not be same in all direction.





Source: Wikipedia

Michelson and Morley Experiment

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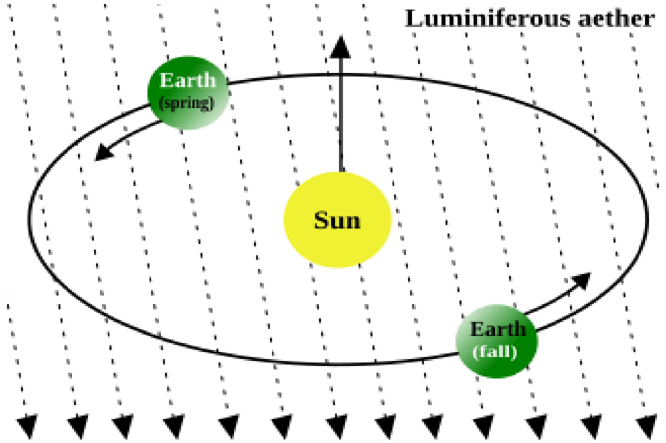
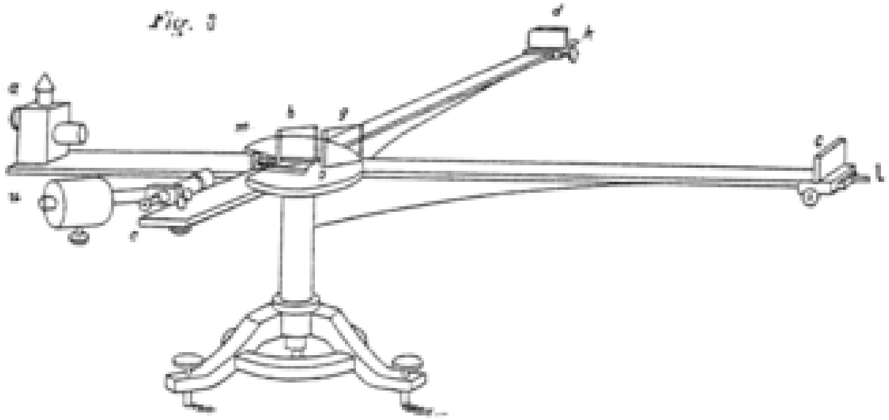
If ether really existed then speed of light should not be same in call direction.

What Michelson and Morley actually discovered is that light travels at the same speed in every direction.
The speed of light does not depend on Earth's motion. It isn't c relative to some ether — it's just c, everywhere and for everyone.





Source: Wikipedia

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- This is very counterintuitive. So you need to go beyond your intuition.
- The answer to how light can do this because of something call length contraction and time dilation.

The Relativity of Simultaneity

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- Special relativity shows that this common-sense idea breaks down — simultaneity is relative.

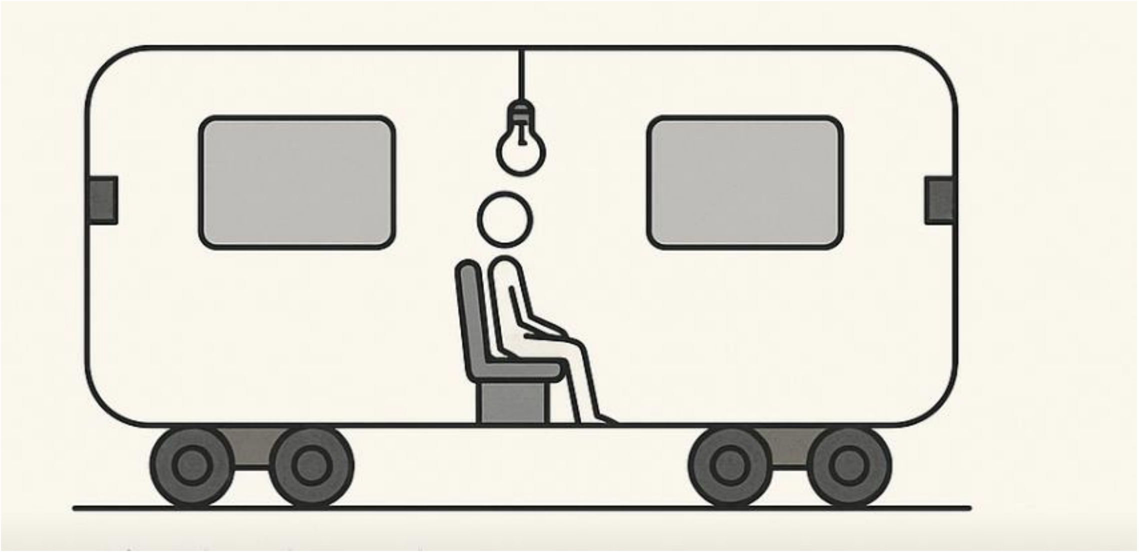
The Relativity of Simultaneity

Imagine you’re sitting in the middle of a train car. A lightbulb is hanging right above you at the center. When you switch it on, the light spreads out in all directions at the same speed.

There are two detectors: one at the **front** of the train, one at the **back**. Both are the same distance from the bulb.

These detector create sound when it light hit them.

When you turn on the lightbulb, light spreads out equally in all directions, traveling toward both detectors. The question is: which detector’s buzzer goes off first — the one in the front, or the one in the back?



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The Relativity of Simultaneity

Hint: Remember you are sitting in the train

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C) Both buzzers ring at the same time

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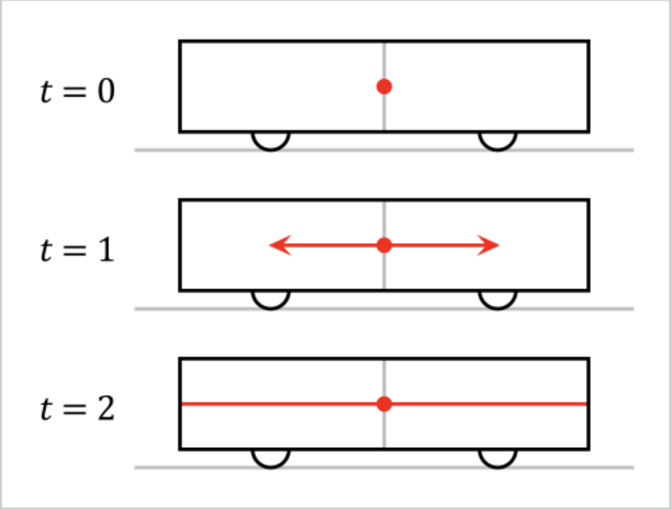


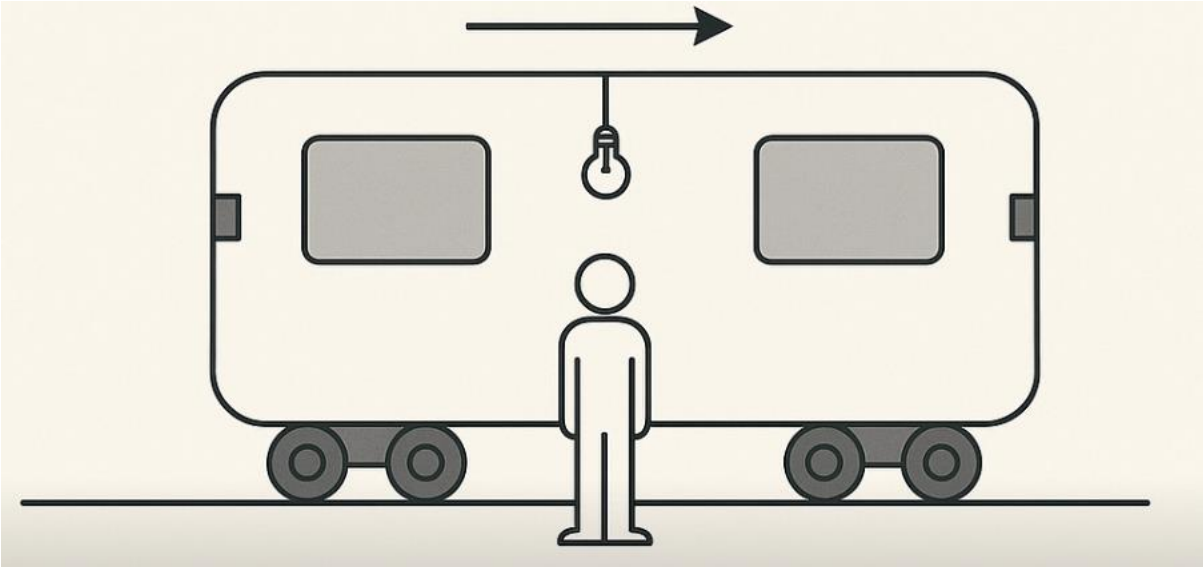
The train-and-platform experiment from the reference frame of an observer on board the train

Source: Wikipedia

If you're on the train: Since the bulb is in the middle, the light travels the same distance to each detector. So both buzzers ring at the same time.

The Relativity of Simultaneity

- What happen when you are standing outside the train? Would the both detector would ring at the same time?



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The Relativity of Simultaneity

Hint: Remember you are standing outside of the train

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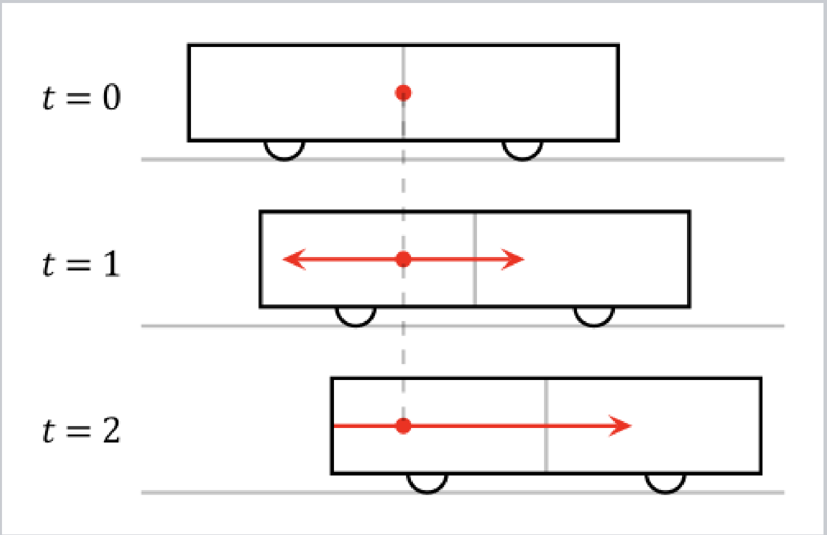
The Relativity of Simultaneity

From the perspective of an observer on the ground, the train moves forward while the light is traveling. That means:

- The **front of the train** is moving **away from the light**, so the light has a longer distance to cover to reach the front detector.
- The **back of the train** is moving **toward the light**, so the light has a shorter distance to cover to reach the back detector.

So the **back detector buzzes first** in the ground observer's frame.



Reference frame of an observer standing on the platform (length contraction not depicted)

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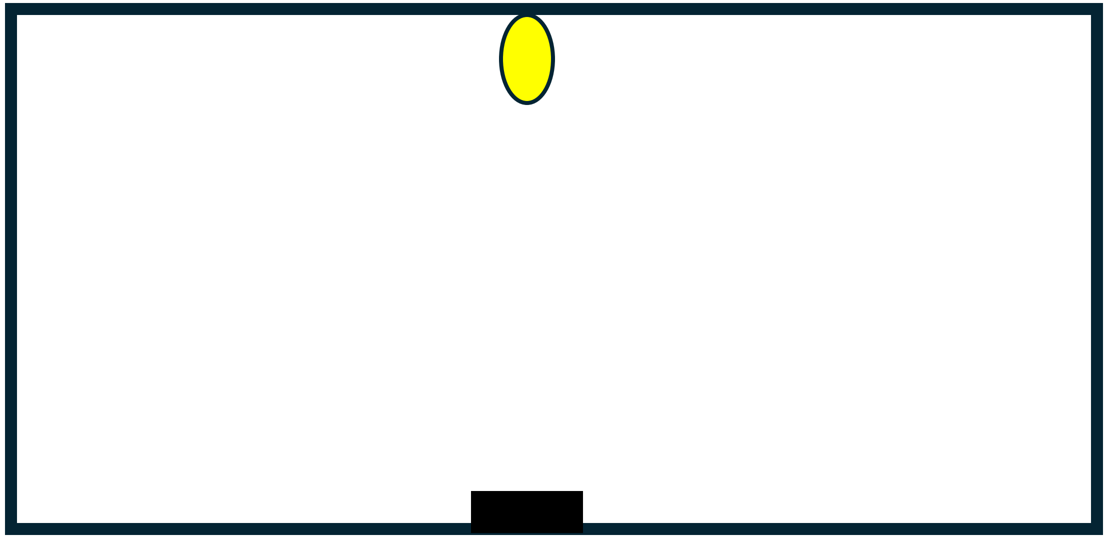
- This means that, from the ground’s point of view, the back buzzer goes off before the front buzzer.
- The conclusion is striking: two events that are simultaneous in one frame of reference may not be simultaneous in another.
- In everyday life, trains move far too slowly for us to notice this effect, which is why it feels so unfamiliar.

Time Dilation

Time dilation

A lightbulb is hanging at the ceiling in the middle of a train car. Right below the bulb, on the floor, there’s a detector that can sense the light.



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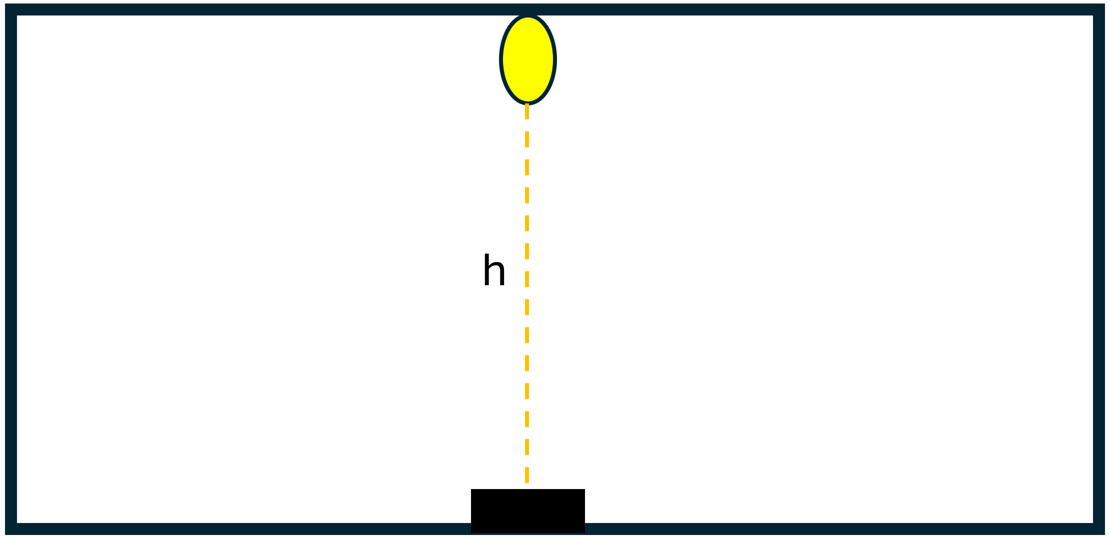

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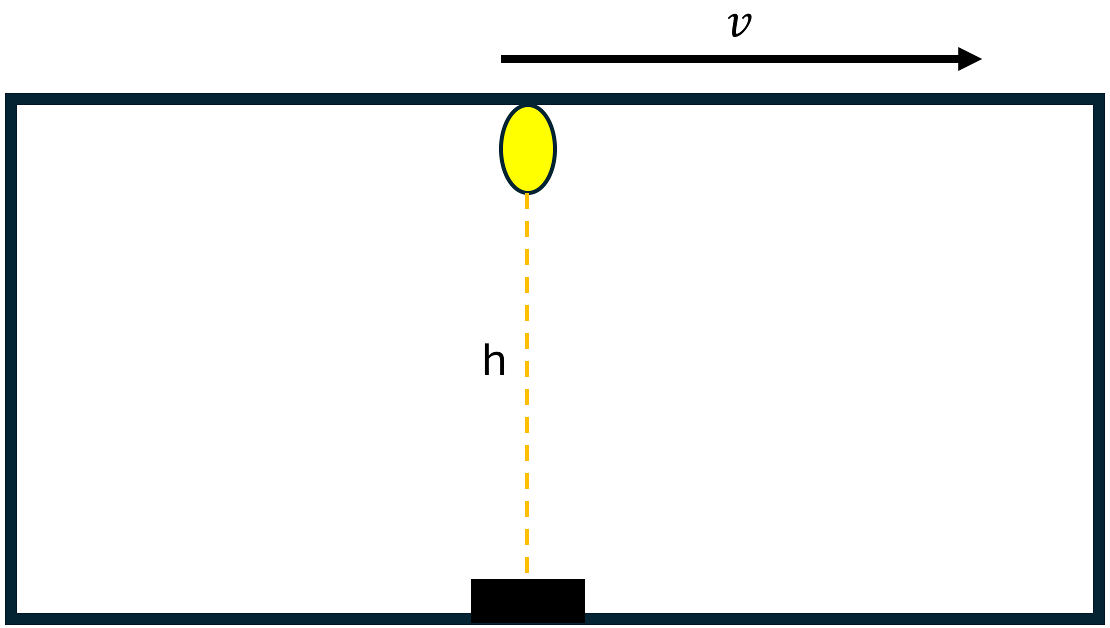

Light travel at speed of light (c) and it travel the distance h. How much time taken by the light? (From the point of observer in the train)

$$t_1 = \frac{h}{c}$$

Time dilation

What will happen for observer on the ground (outside of the train)?

Would observer the on the ground would measure the same time as observer in the train?

A. Yes
B. No

Time dilation

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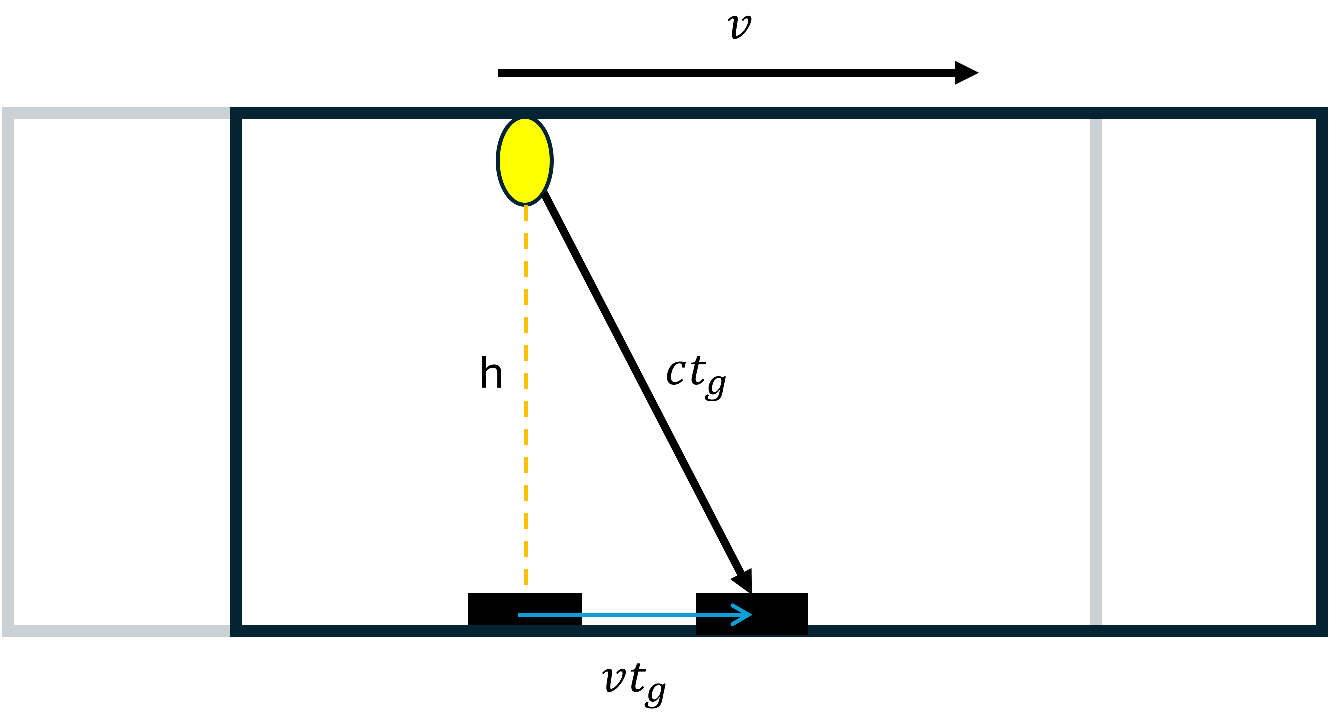
A. Yes

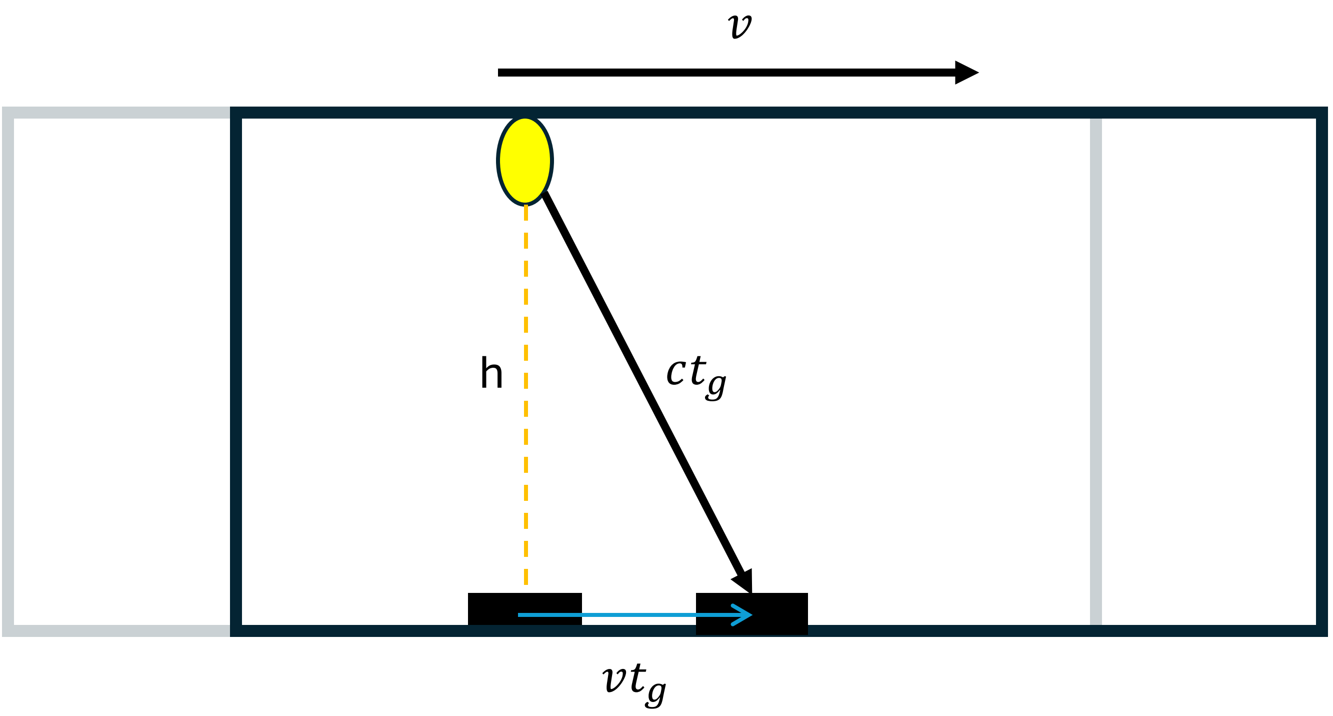
B. No

Time dilation

the train is moving forward while the light travels, so the light follows a diagonal path to reach the detector.



Exercise Time



Find t_g?

Hint: Pythagorean theorem

Time Dilation

$$(ct_g)^2 = (v\,t_g)^2 + h^2$$

$$c^2 t_g^2 \left(1 - \frac{v^2}{c^2}\right) = h^2$$

$$t_g = \frac{h}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow t_g = \gamma t_1$$

Time Dilation

$$t_g = \gamma t_1$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\gamma \geq 1$
If $v = 0$, $\gamma = 1$ (No relativistic effects.)
As v increases: $\gamma > 1$.

The time measured on the ground is **greater** than the time measured on the train.

Time Dilation

$$t_g = \gamma t_1$$

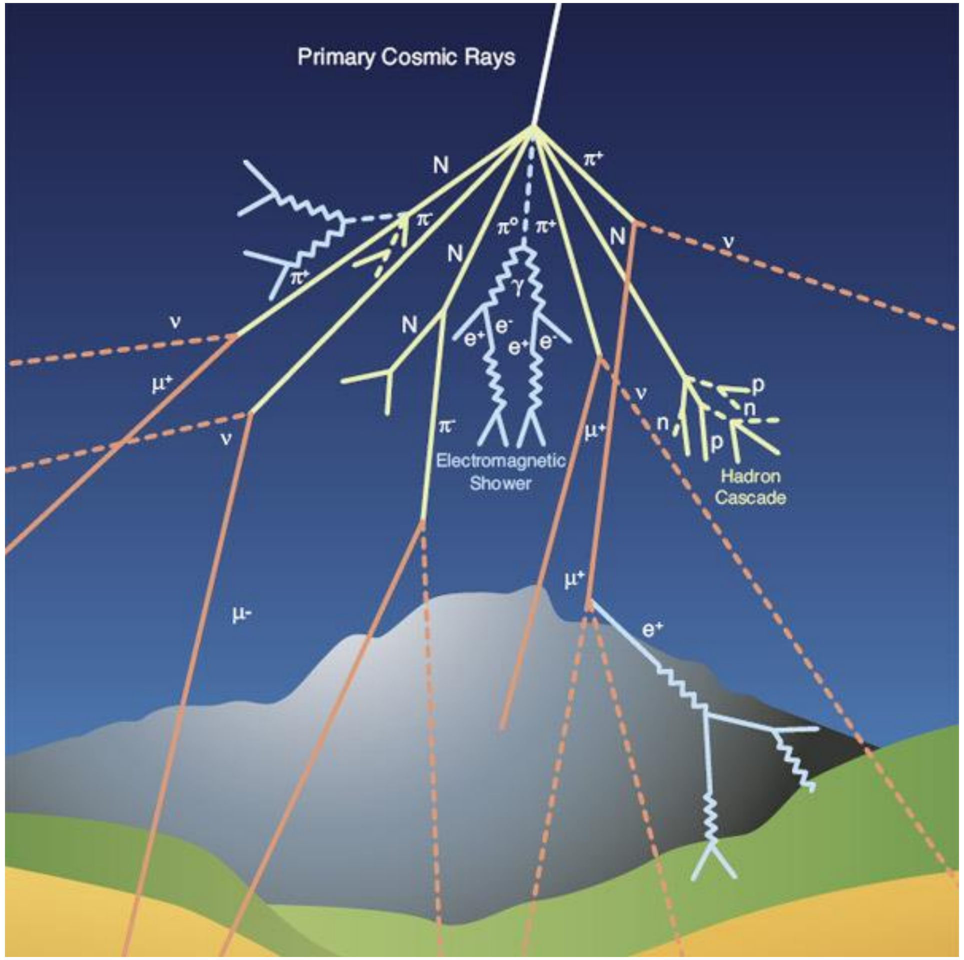
Moving clocks run slow by a factor of γ.

Time Dilation

Example: Moving Particles

Cosmic ray muons and time dilation

- **Where muons come from:**
  Cosmic rays (high-energy particles from space) hit the upper atmosphere and create showers of new particles, including **muons**.



Source: CERN

Time Dilation

Example: Moving Particles

Cosmic ray muons and time dilation

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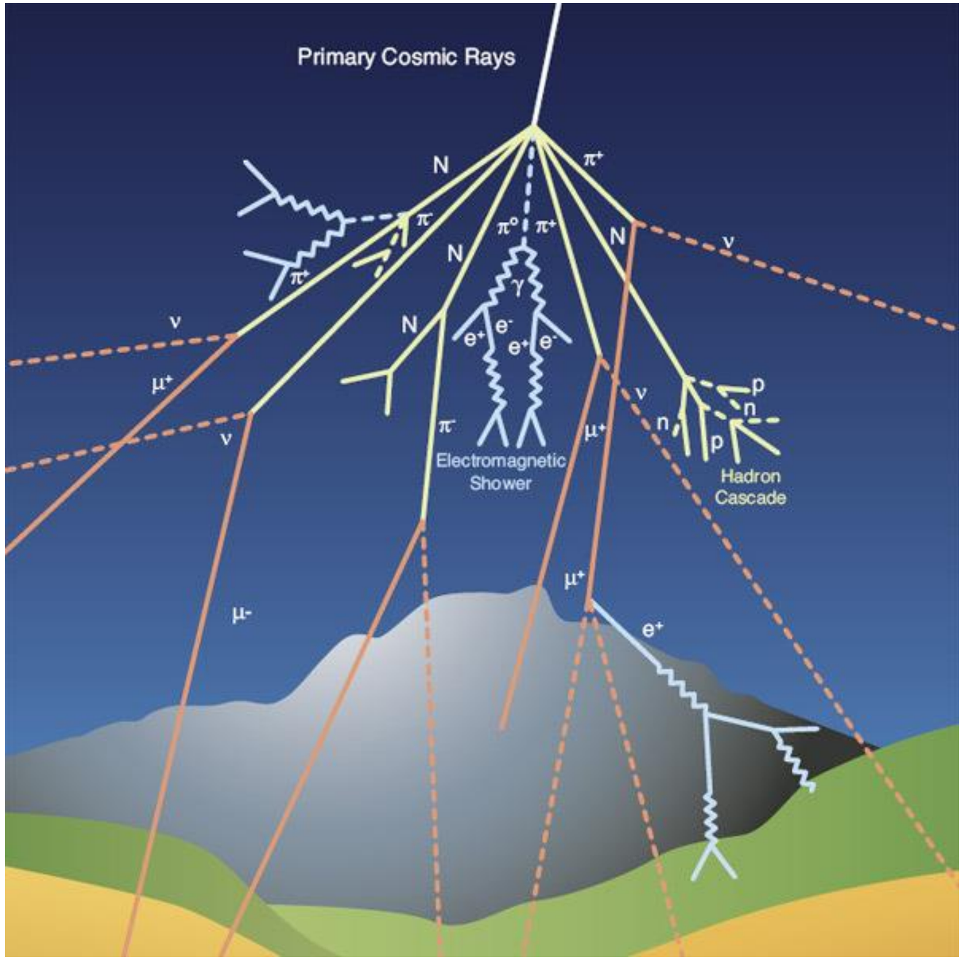
  Cosmic rays (high-energy particles from space) hit the upper atmosphere and create showers of new particles, including **muons**.
- **Muon lifetime at rest:**
  A muon lives only about 0.000002 s. Even if it moved at the speed of light, that short lifetime would only let it travel a few hundred meters.



Source: CERN

Time Dilation

Example: Moving Particles

Cosmic ray muons and time dilation

- **Where muons come from:**
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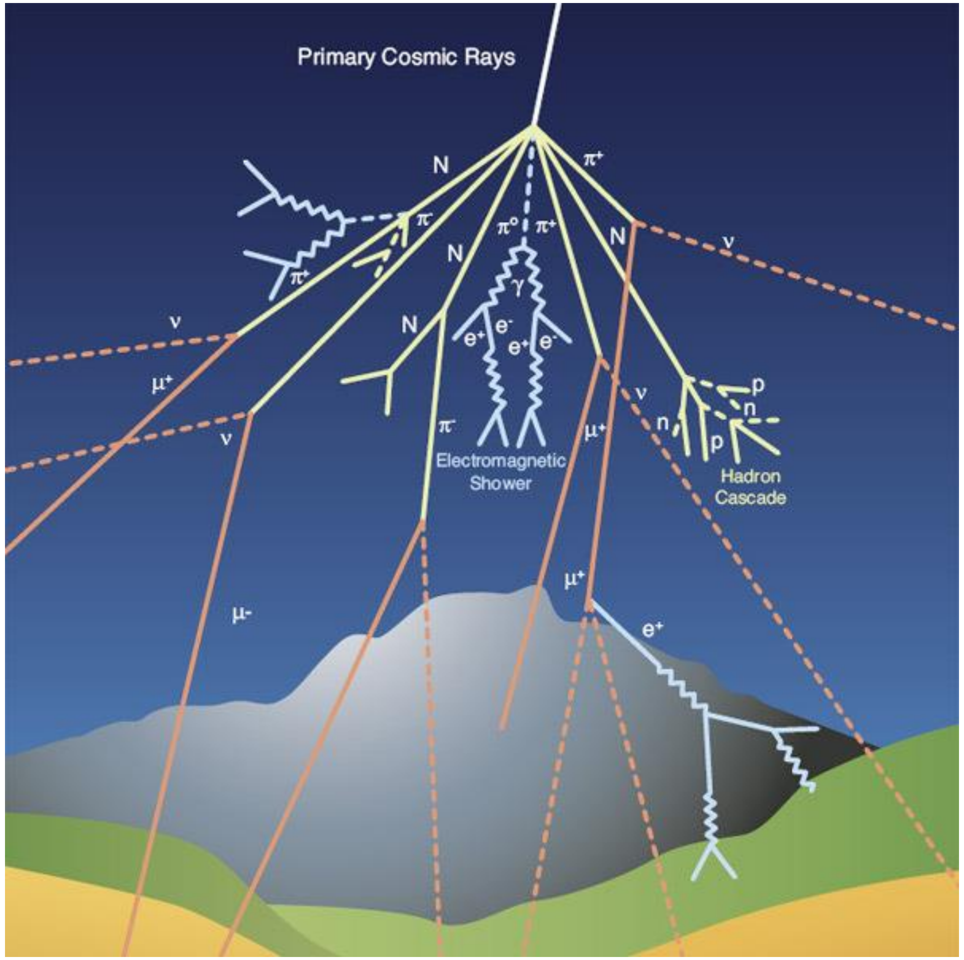
- **Problem:**
  The atmosphere is **tens of kilometers thick**. So if muons decayed at their normal lifetime, almost none should reach the surface of the Earth.



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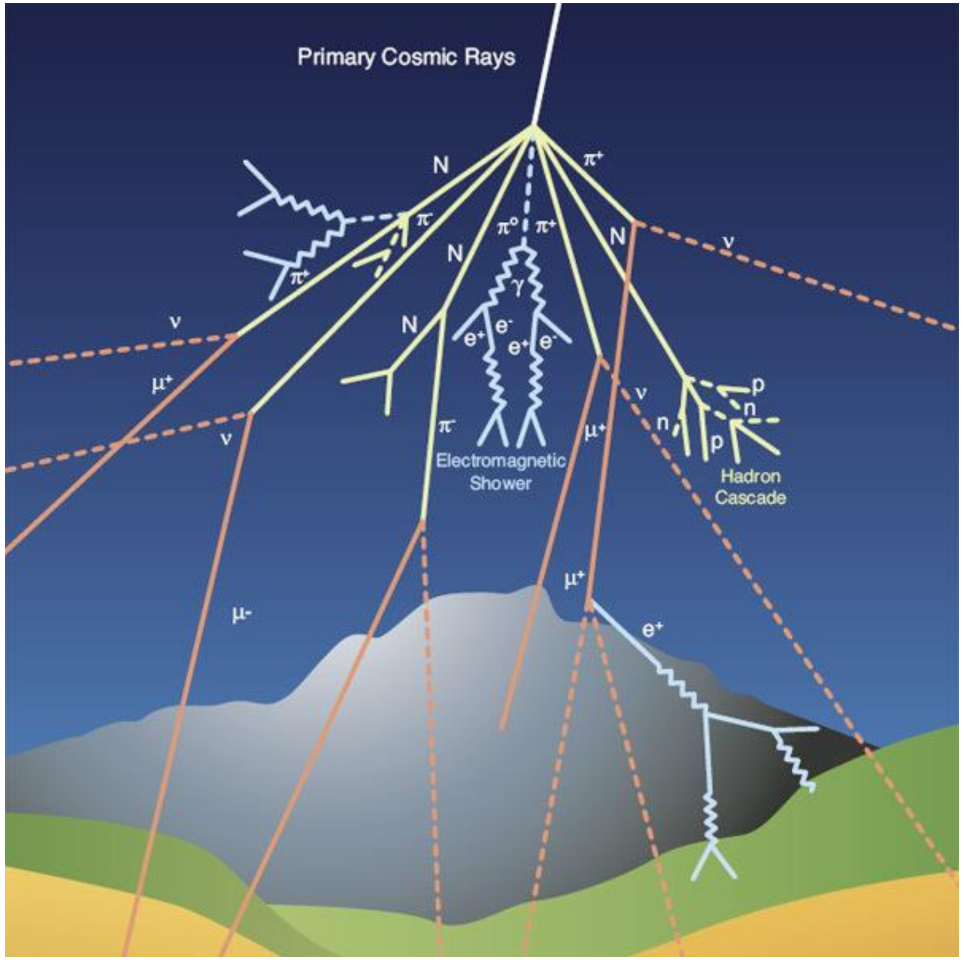
- **Problem:**
  The atmosphere is **tens of kilometers thick**. So if muons decayed at their normal lifetime, almost none should reach the surface of the Earth.
- **What we actually see:**
  Huge numbers of muons *do* reach the ground — far more than expected if we ignored relativity.



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Time Dilation

Solution to the Problem

- Because muons are moving near the speed of light, their internal clocks **run slow** (time dilation).

Time Dilation

Solution to the Problem

- Because muons are moving near the speed of light, their internal clocks **run slow** (time dilation).
- Their lifetime in our frame is stretched by the Lorentz factor γ, so they survive long enough to travel through the atmosphere.

Lorentz Contraction

Length Contraction

- To measure the length of an object means to locate its end points simultaneously.
- Because simultaneity is a relative concept, length measurements will also depend on the reference frame and be relative.

Length Contraction

the distance L by the amount the bulb has moved to the right during this time. Thus, $\Delta t_2 = (L - v\,\Delta t_2)/c$ or $\Delta t_2 = L/(c+v)$. The total time down and back, measured by S, is therefore

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2cL}{c^2 - v^2} = \frac{(2L/c)}{(1 - v^2/c^2)}.$$

This time interval is a nonproper one for it is measured by two clocks at two different places in S (at B_0 and B_2). The relation between the proper and nonproper time interval of the same two events (the sending and receiving of the light flash) is given by Eq. 2-13, $\Delta t' = \Delta t\sqrt{1 - v^2/c^2}$. If we substitute for $\Delta t'$ its value $2L'/c$ and for Δt its value

$$\frac{2L/c}{1 - v^2/c^2},$$

we obtain

$$\frac{2L'}{c} = \frac{2L}{c}\,\frac{\sqrt{1 - v^2/c^2}}{(1 - v^2/c^2)},$$

from which it follows that

$$L = L'\sqrt{1 - v^2/c^2}. \qquad \text{(2-14)}$$

Source: Introduction to Special Relativity by Robert Resnick

We can do the math behind it and found relation between length of objects between different observer. I am not going to bore you with that. You can do it as a take home exercise.

Length Contraction

$$L_g = \frac{1}{\gamma} L_t$$

An observer on the ground measures the moving boxcar to be shorter than the passenger inside the train measures it.

Moving objects are shorter **by a factor of 1/γ.**

Length Contraction

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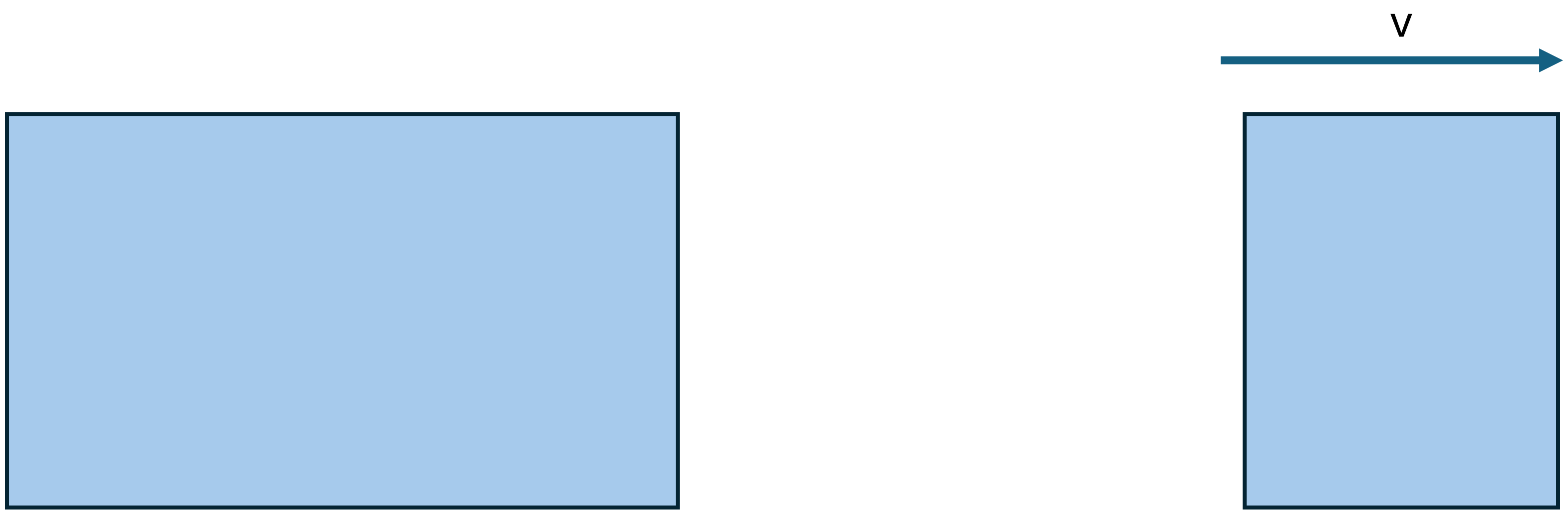
Moving objects are shorter **by a factor of 1/γ.**

When we say moving objects are shorter, we mean **they truly are shorter in that frame** — not just that they “look” short. It is not an optical illusion, it is a real physical effect.

Length Contraction

Direction matters in length contraction.

- **Length contraction** happens **only along the direction of motion**.
- Dimensions **perpendicular** to the motion are **unchanged**.



At Rest

In Motion

Relativistic Kinematics

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Relativistic Kinematics

- From the point of view of physics, **relativistic kinematics is one of the most important topics** in special relativity.
- In real experiments — especially in **particle physics** — we deal with **high-speed collisions**.
- To analyze these, we need to calculate **energies, momenta, and velocities** in different frames.
- Classical formulas break down at high speeds, so we need the **relativistic versions**.

Relativistic Kinematics

- Now Einstein knew that some of laws needed to be changed specifically law of conservation of momentum and conservation of energy. These laws were not conserved when you go higher speed in the classical form.

Relativistic Kinematics

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Old Idea:

He introduce the idea of the relativistic mass (mass that depend on the velocity)

$$m_r = \gamma m$$

But people literally started thinking that mass increases as you move with speed of light. But from experiment we know mass is invariant quantity.

Relativistic Kinematics

New Ways to Interpret:

When you move with speed of light, you increase the energy and momentum not mass.

$$E = \gamma m c^2 \,, \qquad p = \gamma m v$$

When γ = 1, $E = mc^2$, it is called rest mass. Rest mass is same in all inertial frame.

Relativistic Kinematics

- Einstein fixed the problems with classical momentum by redefining it.
- But countless experiments in particle physics have confirmed that **relativistic momentum and energy** are indeed conserved.

Relativistic Kinematics

Question

Which of the following is true about **kinetic energy** in classical physics?

A) It is the energy an object has because of its motion.

B) Its formula is:

$$T = \frac{1}{2}mv^2$$

C) It is always zero, no matter how fast the object moves.

D) It depends only on the object's position, not its speed.

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Relativistic Kinematics

Relativistic Kinetic Energy

$$T = Total\ Relativistic\ Energy - Rest\ Energy$$

$$T = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$

Relativistic Kinematics

- Even if an object is not moving, it still has energy just because it has mass.
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  $$T = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$
- **Relativistic kinetic energy** is **not the same** as the classical formula, though when the speed is much smaller than c, the two formulas are very close.

Relativistic Kinematics

- In classical physics, you can’t have a massless particle because with no mass it would have no momentum, no energy, and it couldn’t interact with anything.

$$F = ma = 0$$
$$p = mv = 0$$
$$K.E = \frac{1}{2}mv^2 = 0$$

Relativistic Kinematics

- But in relativity, there's a loophole.

Relativistic Kinematics

- Consider the function

$$f(x) = \frac{x^2 - 1}{x - 1}$$

At $x = 1$, what happen?

A) f(1)=0

B) f(1) is undefined because we get $\frac{0}{0}$, the formula you're using breaks down at that point.

C) The limit as x→1 is also undefined.

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Relativistic Kinematics

- Now lets look at relativistic energy and momentum.

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}\ , \mathrm{p} = \frac{\mathrm{mv}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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E and p become 0 when mass is 0 unless v=c.

A particle with zero mass but moving at the speed of light can still carry energy and momentum. However, the special theory of relativity alone cannot tell us how much energy or momentum such a particle has.

Relativistic Kinematics

All photons (particles of light) are **massless** and move at the **same speed — the speed of light**. So what makes one photon different from another?

- It’s **not** that one is heavier — they all have zero mass.
- It’s **not** that one is faster — they all move at the same speed.

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But we know a blue photon has more energy but a red photon has less energy.

Relativistic Kinematics

To answer that, we need **quantum mechanics**. In quantum theory, the energy of a photon is given by **Planck's formula**:

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where E is energy, h is Planck's constant, and v is frequency.

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A blue photon has more energy because it has a higher frequency.
A red photon has less energy because it has a lower frequency.

Spacetime

Spacetime

- Einstein’s special relativity already explained that space and time are not separate and absolute — they depend on the observer.
- But it was Hermann Minkowski who realized that the best way to understand this is to think of space and time as part of one single 4-dimensional world: spacetime.

Spacetime

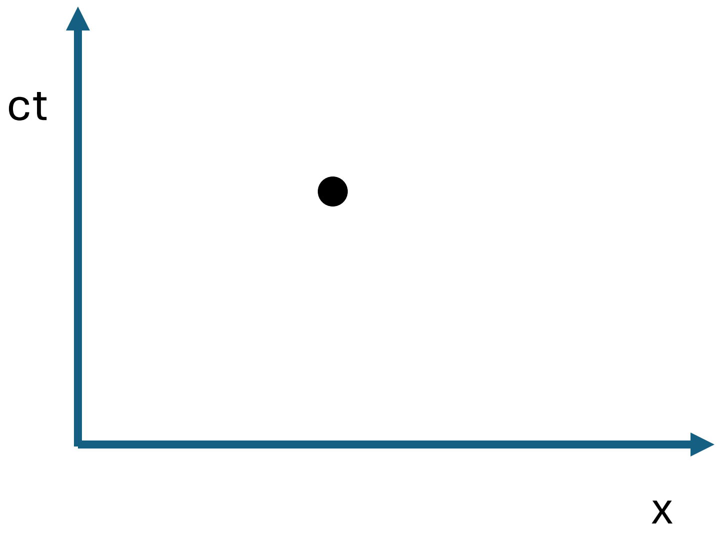
In physics, we talk about **events**.

- An event means something that happens at one exact place and at one exact time.
- Example: a glass drops and shatters on the kitchen floor at 6:30 pm.

Spacetime

We can draw events on a **spacetime diagram** (also called a Minkowski diagram):

- The **horizontal axis** shows **space (x)**.
- The **vertical axis** shows **time (ct)**.
- A single dot on the diagram = one event.



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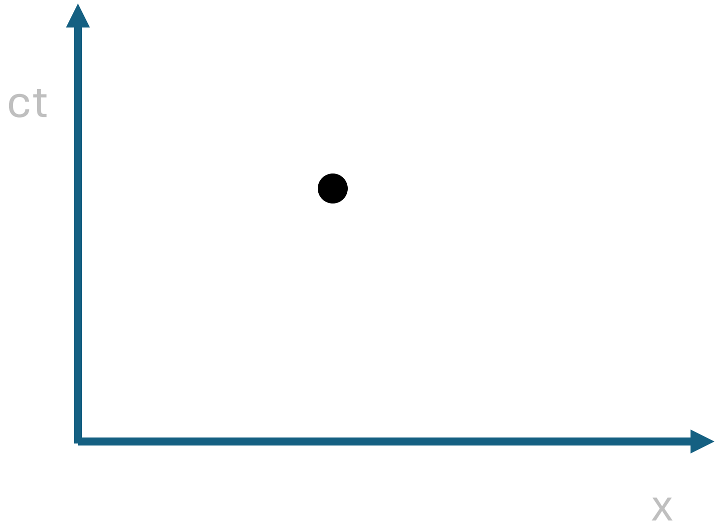
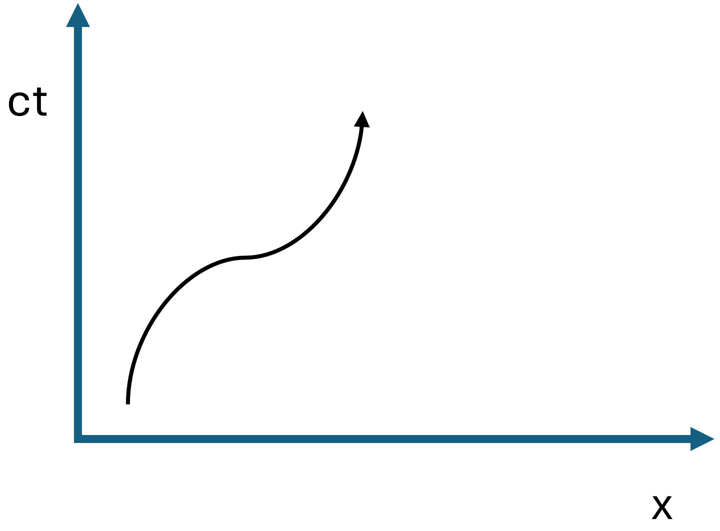
- A single dot on the diagram = one event.
- Connecting dots shows the motion of an object through spacetime (a **world line**).





Spacetime

If we live in **4-dimensional spacetime** (3 space + 1 time), then every **event** needs **four numbers** to describe it:

$$(ct,x,y,z)$$

- x,y,z → where it happened (space coordinates).
- t → when it happened (time, multiplied by c so it has the same units as space).

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So just like you need **3 numbers** (x,y,z) to describe a point in ordinary space, you need **4 numbers** to describe a point (event) in **spacetime**.

Spacetime

- When we put those four numbers together,
$$x^\mu = (ct, x, y, z)$$
we form a **four-vector**.
- It's just like a regular vector in 3D space (x,y,z), but now it has **four components**: one for time (ct) and three for space (x,y,z).

Spacetime

- **Start in the stationary frame S**

Suppose you have an event with coordinates

$$x^{\mu} = (ct, x, y, z)$$

That means the event happened at position (x,y,z)at time t.

- **Moving frame S'**

Now imagine another observer moving with velocity v in the x-direction.
This observer assigns new coordinates (ct',x',y',z') to the same event.

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How these coordinates are linked up?

Spacetime

The link between the two sets of coordinates is:

$$ct' = \gamma \left(ct - \frac{v}{c}x\right)$$

$$x' = \gamma(x - vt)$$

$$y' = y, \quad z' = z$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

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Time and space mix together: t' depends on both t and x.

This mixing is exactly what produces time dilation and length contraction.

Spacetime

Lorentz Transformation Matrix (boost in the x-direction):

$$\Lambda = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation rule:

$$x'^{\mu} = \Lambda^{\mu}_{\ \nu} x^{\nu}$$

- $x^{\nu} = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$ → spacetime coordinates in the original frame.
- $x'^{\mu} = (x'^0, x'^1, x'^2, x'^3) = (ct', x', y', z')$ → spacetime coordinates in the moving frame.

Spacetime

Even though the individual spacetime coordinates of an event (ct, x, y, z) **change** when we switch to a moving frame ($S \to S'$), there is one combination that always stays the same:

$$I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

or equivalently,

$$I = (ct)^2 - x^2 - y^2 - z^2.$$

This quantity is called the **spacetime interval**.

Spacetime

- The **position-time four-vector** $x^\mu = (ct, x, y, z)$ is the basic model for all four-vectors.
- Any **four-vector** a^μ is just a four-component object that transforms in the **same way** as x^μ when you change from one inertial frame to another.
- Mathematically:

$$a'^\mu = \Lambda^\mu_{\ \nu} a^\nu$$

where $\Lambda^\mu_{\ \nu}$ is the Lorentz transformation matrix.

Spacetime

1. Define the energy–momentum four-vector

$$p^\mu = \left(\frac{E}{c},\ p_x,\ p_y,\ p_z\right)$$

- Time component: E/c (energy divided by c)
- Space components: the usual momentum $\vec{p} = (p_x, p_y, p_z)$

---

2. Invariant quantity (analog of spacetime interval)

For the spacetime four-vector x^μ, the invariant is:

$$(ct)^2 - x^2 - y^2 - z^2.$$

For the energy–momentum four-vector p^μ, the invariant is:

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This invariant equals the square of the particle's **rest mass** m:

$$\left(\frac{E}{c}\right)^2 - |\vec{p}|^2 = (mc)^2$$

or equivalently:

$$E^2 = (pc)^2 + (mc^2)^2.$$

Spacetime

Maxwell's equations fit the rule

- In their usual 3D form ($\vec{E}, \vec{B}$), the equations look different to different observers → that's why we use the word **covariant**.
- But in their proper 4D tensor form with $F^{\mu\nu}$ and J^{μ}, the equations keep exactly the same form under Lorentz transformations.
- In that sense, they are **Lorentz invariant**.

Spacetime

Why this matters?

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- Since then, Lorentz invariance has been taken as a **fundamental requirement.**
- Special relativity tells us spacetime has a built-in symmetry: **Lorentz transformations** (rotations + boosts).
- Any correct physical law written in spacetime must respect this symmetry.

Questions to Think About

There are many fascinating paradoxes in the special theory of relativity. Two of the most famous ones are:

- **The Twin Paradox**
- **The Barn–Ladder Paradox**

Try looking these up and see if you can understand how relativity resolves them.