STAT 443 - Time Series Project

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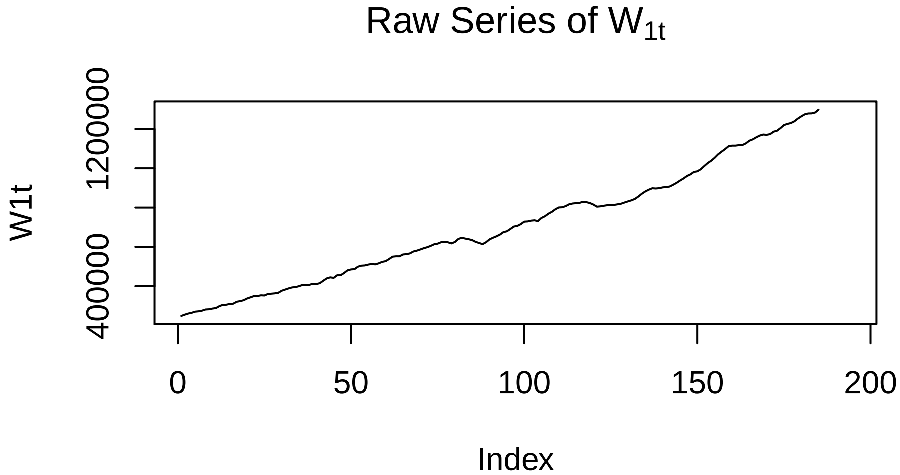
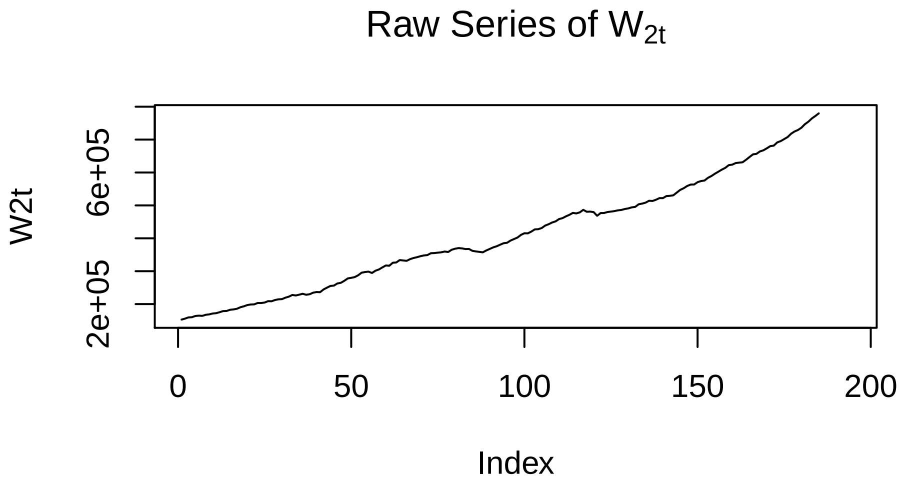
Group B

Question 1

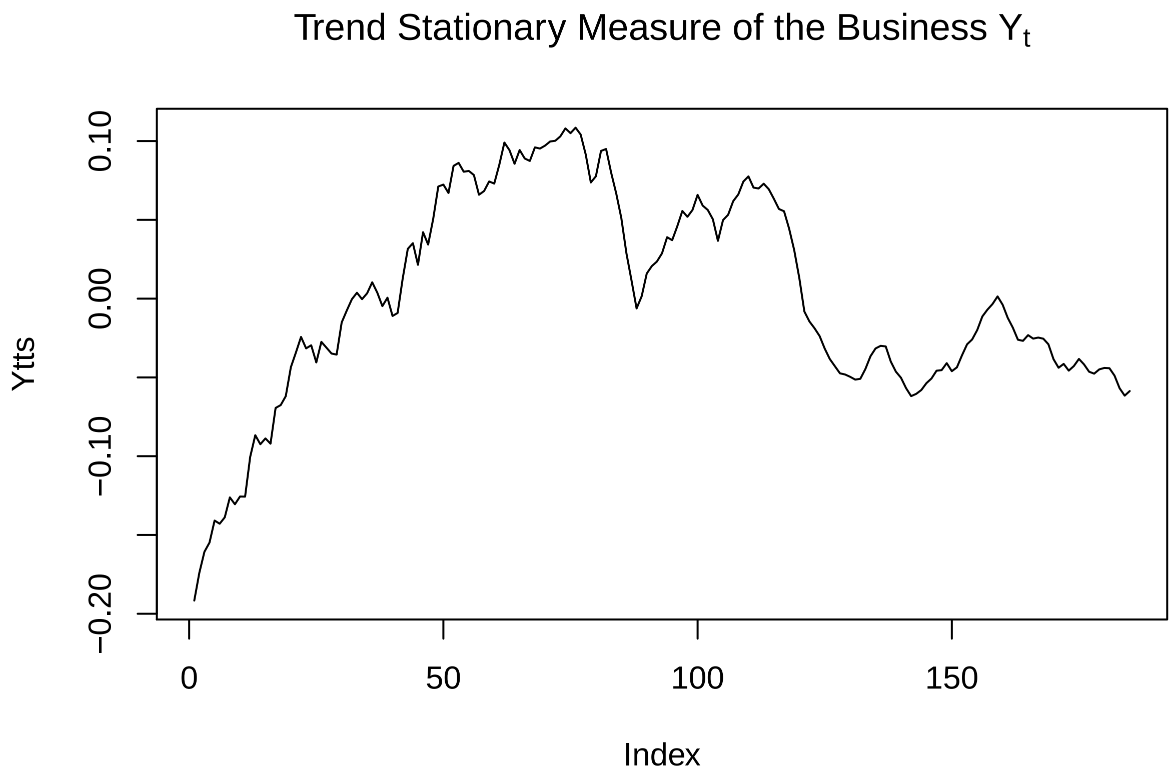




Here we can see that the shape of the curve between the Raw time series of seasonally adjusted personal expenditure and GDP have very little differences as both time series have an upward linear trend. In addition, the peaks and troughs of both plots occur at around the same time period. This indicates there may be some correlation between the seasonally adjusted personal expenditure and the GDP. However, there are many more tests necessary before making and assertions about the relationships between these two time series models.

Question 2

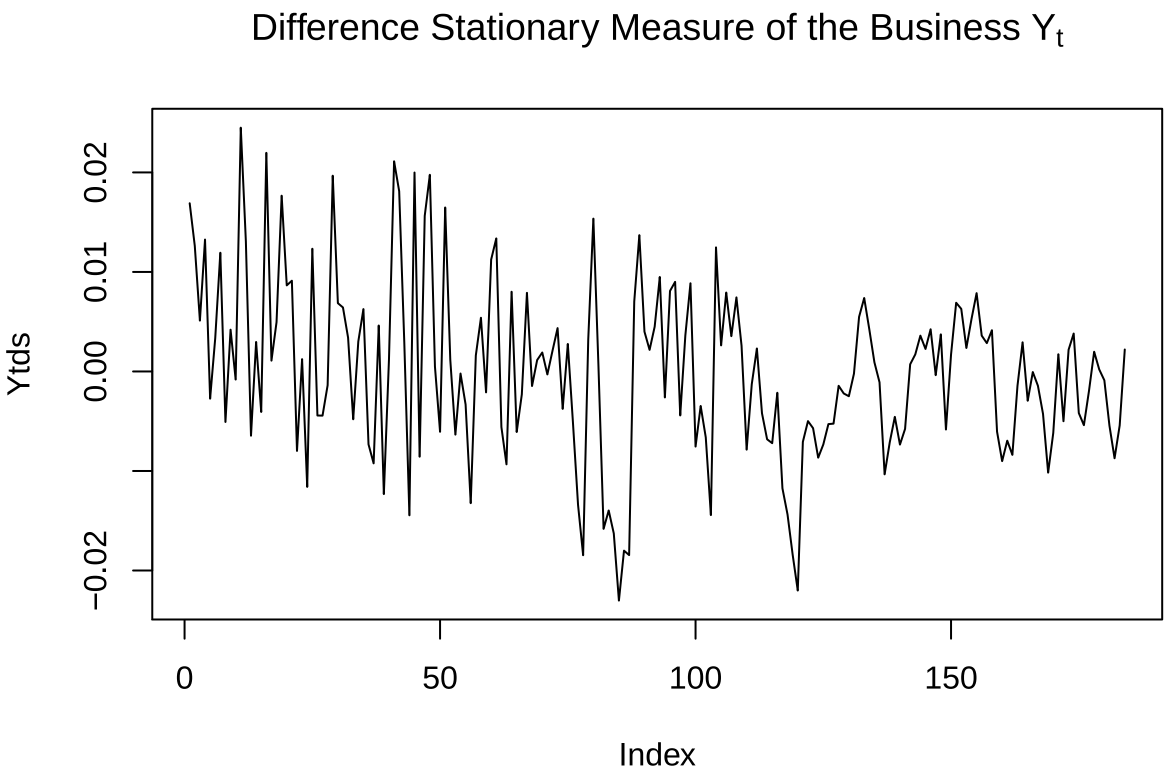
We define $X_t = ln(W_{1t})$. Since W_{1t} describes the GDP, its values are inherently positive, as a result it cannot be normally distributed. Therefore we use the $Log - Linearlaw$ which allows for the use of a Normal Distribution through the application of a natural logarithim. In addition, with the use of X_t, it can now be decomposed into a Trend component(T_t), Cyclical component(Y_t) and a Seasonal component(S_t) (i.e $X_t = T_t + Y_t + S_t$). The focus of this study is on the cyclical component Y_t and we will now attempt to isolate it.



The Trend Stationary model (TS) is designed as $X_t = \alpha + \mu t + Y_t$ where we conduct regression to obtain an estimate of the error term Y_t. With respect to the Trend Stationary model,Y_t is percieved as the difference between the raw economic time series W_{1t} and its long-run trend. When the regression was conducted our estimate for μ was $\hat{\mu} = 0.008263$, this indicates the quarterly growth rate for the GDP. As a result, the annual growth rate can be displayed as

$$\hat{\mu} \times 4 = 0.008263 \times 4 = 0.0331$$

Which implies our annual growth rate is 3.3%. We notice that the 76^{th} index has $Y_t \approx 0.1$ which indicates that in the late 1970's the GDP was 10% above its long-run value. We also notice that subsequently after the late 1970's there was a sharp drop in the early 1980's. This is correspondes to the actual Canadian economy in the 1970's. Canada was experiencing robust growth since the end of Word War II up until in the late 1970's. Subsequently Canada's economy began to sturggle, as many major powers started to battle inflation in the early 1980's.



The Difference Stationary (DS) model is designed as $\Delta X_t = \mu + Y_t$ and similar to the Trend Stationary model, we use regression to estimate Y_t. In this model, we interpret Y_t as the deviation of the actual growth rate ΔX_t from the average growth rate μ. In our regression, we obtain $\hat{\mu} = 0.0089863$ which is our estimated average quarterly growth rate. Thus

$$\hat{\mu} \times 4 = 0.036$$

This implies that the estimated average annual growth rate is 3.6%. This value is very close to the annual growth rate we calculated in the Trend Stationary Model. Notice in the 11^{th} index we see $Y_t \approx 0.02$ which means that the GDP grew 2 percentage points more than the average. Thus, in the early 1960's the economy grew at around 2.9%. In addition, we notice during the 85^{th} index that $Y_t \approx -0.02$ which implies that GDP growth decreased in the 1980's as it was growing 2 percentage points less than the average. Thus, in the early 1980's the GDP decreased by around 1%.

Question 3

Using the Bayesian Information Criterion for $k = 0, .., 6$

$$BIC(k) = ln(\hat{\sigma}_k^2) = \frac{ln(N) \times k}{k}$$

where N is the sample size for of our dataset,for TS $N = 185$ and DS $N = 184$. We obtain the following table

	$BIC(k)_{TS}$	$BIC(k)_{DS}$
$k = 0$	-5.453970	-9.469291
$k = 1$	-9.437384	-9.554756
$k = 2$	-9.527164	-9.529794
$k = 3$	-9.503044	-9.517196
$k = 4$	-9.492188	-9.488917
$k = 5$	-9.464028	-9.461730

	$BIC(k)_{TS}$	$BIC(k)_{DS}$
$k=6$	-9.436557	-9.437722

Here we see that for the TS model the k that minimizes the BIC is $k=2$ and the k that minimizes the BIC for the DS model is $k=1$. Given the restrictions on this project we claim that ideal Autoregressive model for both TS and DS models is an $AR(2)$.

We fit the TS and DS model to an AR(2) by running a regression on their respective Y_t

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2}$$

The TS model yielded the following regression results

$$\underset{t}{Y_t} = \underset{(18.43)}{1.290}\ Y_{t-1} - \underset{(-4.57)}{0.313}\ Y_{t-2} + a_t$$

$$n = 185, F-ratio = 5355,\ RSS = 0.012,\ R^2 = 0.9834$$

The DS model yielded the following regression results

$$\underset{t}{Y_t} = \underset{(3.99)}{0.297}\ Y_{t-1} + \underset{(0.78)}{0.057}\ Y_{t-2} + a_t$$

$$n = 184, F-ratio = 10.45,\ RSS = 0.0123,\ R^2 = 0.104$$

For the estimated TS model, we want to find $\gamma(0)^{\frac{1}{2}}$. We use the following equation:

$$\gamma(0)^{\frac{1}{2}} = \frac{\sigma}{\sqrt{1-\phi_1\rho(1)-\phi_2\rho(2)}}$$

Where we use $\hat{\sigma}^2 = \sqrt{\frac{RSS}{N-2}}$ as an estimate for σ^2, since we are using an $AR(2)$ model and our effective sample size is $N-2$. The ϕ_1 and ϕ_2 values can be substituted with the estimated coefficents from our intial regression $(\hat{\phi}_1, \hat{\phi}_2)$. In addition, we use a recursive formula to calculate the autocorrelation

$$\rho(k) = \hat{\phi}_1\rho(k-1) + \hat{\phi}_2\rho(k-2)$$

Where $\rho(0) = 1$ and $\rho(-k) = \rho(k)$ The recursive calculations provide the following results: $\rho(1) = 0.966$ and $\rho(2) = 0.929$. Thus, we can calculate $\gamma(0)^{\frac{1}{2}}$ as:

$$\gamma(0)^{\frac{1}{2}} = \frac{0.008}{\sqrt{1-(1.290)(0.966)-(-0.313)(0.929)}}$$

$$\gamma(0)^{\frac{1}{2}} = 0.038$$

Similar to the Autocorrelation function we use a recursive function to determine the infinte moving averages ψ_k.

$$\psi_k = \hat{\phi}_1\psi_{k-1} + \hat{\phi}_2\psi_{k-2}$$

Where $\hat{\phi}_1, \hat{\phi}_2$ are the coefficents we determined from our TS regression. In addition the following conditions hold

$$\psi_0 = 1 \text{ and } \psi_k = 0 \text{ for } k < 0$$

Using the recursive functions for both the Autocorrelation function and the Infinite Moving Averages for $k = 0, \ldots, 6$ we get the following table:

	ψ_k	$\rho(k)$
$k=0$	1.0000	1.0000
$k=1$	1.2899	0.9656
$k=2$	1.3506	0.9293
$k=3$	1.3382	0.8938

	ψ_k	$\rho(k)$
$k=4$	1.3031	0.8570
$k=5$	1.2618	0.8224
$k=6$	1.2195	0.7870

Question 4

Due to the nature of the model forecasting growth rates ΔX_{t+k} for DS models are simple. Recall that a DS model has the following expression

$$\Delta X_t = \mu + Y_t$$

Thus, if we were to obtain a forecast into k periods forward we have the following expression

$$E_t[\Delta X_{t+k}] = \mu + E_t[Y_{t+k}]$$

To determine $E_t[Y_{t+k}]$ we use the following recursive formula

$$E_t[Y_{t+k}] = \hat{\phi}_1 E_t[Y_{t+k-1}] + \hat{\phi}_2 E_t[Y_{t+k-2}]$$

Where $\hat{\phi}_1, \hat{\phi}_2$ are the coefficents we estimated from the DS regression we conducted in question 3.

In addition, to determine the confidence intervals of these forecasts, we need to determine the variance of the forecasts.This is given by:

$$Var_t[\Delta X_{t+k}] = Var_t[Y_{t+k}]$$

To determine the Variance of Y_{t+k} we use infinte moving averages ψ_k and the estimated sigma squared $\hat{\sigma}^2$.

$$Var_t[Y_{t+k}] = \sigma^2 \textstyle\sum_{j=0}^{k-1} \psi_j^2$$

Note, we use the same recursive formula to find the infinite moving averages $\hat{\psi}_k$ for the DS model that we used in Question 3 for the TS model. However, we use DS estimated ϕ values.

$$\psi_k = 0.297\psi_{k-1} + 0.057\psi_{k-2}$$

Therefore our 95% confidence interval for ΔX_{t+k} is

$$E[\Delta X_{t+k}] \pm 2 \times \sqrt{Var_t[\Delta X_{t+k}]}$$

$$(\mu + E_t[Y_{t+k}]) \pm 2 \times \sqrt{\sigma^2 \sum_{j=0}^{k-1} \psi_j^2}$$

For the TS model, the growth rate forecasts are not as simple.

Recall that the TS model has the following representation

$$X_t = \alpha + \mu t + Y_t$$

Thus, we have

$$\Delta X_{t+k} = \mu + Y_{t+k} - Y_{t+k-1}$$

And

$$E_t[\Delta X_{t+k}] = \mu + E_t[Y_{t+k}] - E_t[Y_{t+k-1}]$$

Similar to the DS model the forecasts for the Y_{t+k} values follows a recursive formula.

$$E_t[Y_{t+k}] = \hat{\phi}_1 E_t[Y_{t+k-1}] + \hat{\phi}_2 E_t[Y_{t+k-2}]$$

Where the $\hat{\phi}$'s are the estimated coefficents from the TS regressed model in Question 3.

In order to obtain the confidence intervals we need to determine the variance of the growth rate forecasts. Similar to the DS model we will need the TS estimated sigma squared $\hat{\sigma}^2$ and the TS estimated value for the infinite moving averages.

$$Var_t[\Delta X_{t+k}] = Var_t[Y_{t+k} - Y_{t+k-1}]$$
$$Var_t[\Delta X_{t+k}] = \sigma^2(1 + \sum_{j=0}^{k-1}(\psi_j - \psi_{j-1})^2)$$

Therefore our 95% confidence interval is

$$E[\Delta X_{t+k}] \pm 2 \times \sqrt{Var_t[\Delta X_{t+k}]}$$
$$(\mu + E_t[Y_{t+k}] - E_t[Y_{t+k-1}]) \pm 2 \times \sqrt{\sigma^2(1 + \sum_{j=0}^{k-1}(\psi_j - \psi_{j-1})^2)}$$

Using the outlined process, we obtain the following tables for DS and TS models.

DS Forecasts with 95% Confidence Intervals

	$E[\Delta X_{t+k}]$	$LowerCI$	$UpperCI$
$k = 0$	0.011193	0.011193	0.011193
$k = 1$	0.009330	-0.007129	0.025789
$k = 2$	0.009215	-0.007954	0.026385
$k = 3$	0.009074	-0.008262	0.026410
$k = 4$	0.009025	-0.008339	0.026390
$k = 5$	0.009003	-0.008367	0.026373
$k = 6$	0.008994	-0.008377	0.026364
$k = 7$	0.008989	-0.008382	0.026360
$k = 8$	0.008988	-0.008383	0.026359

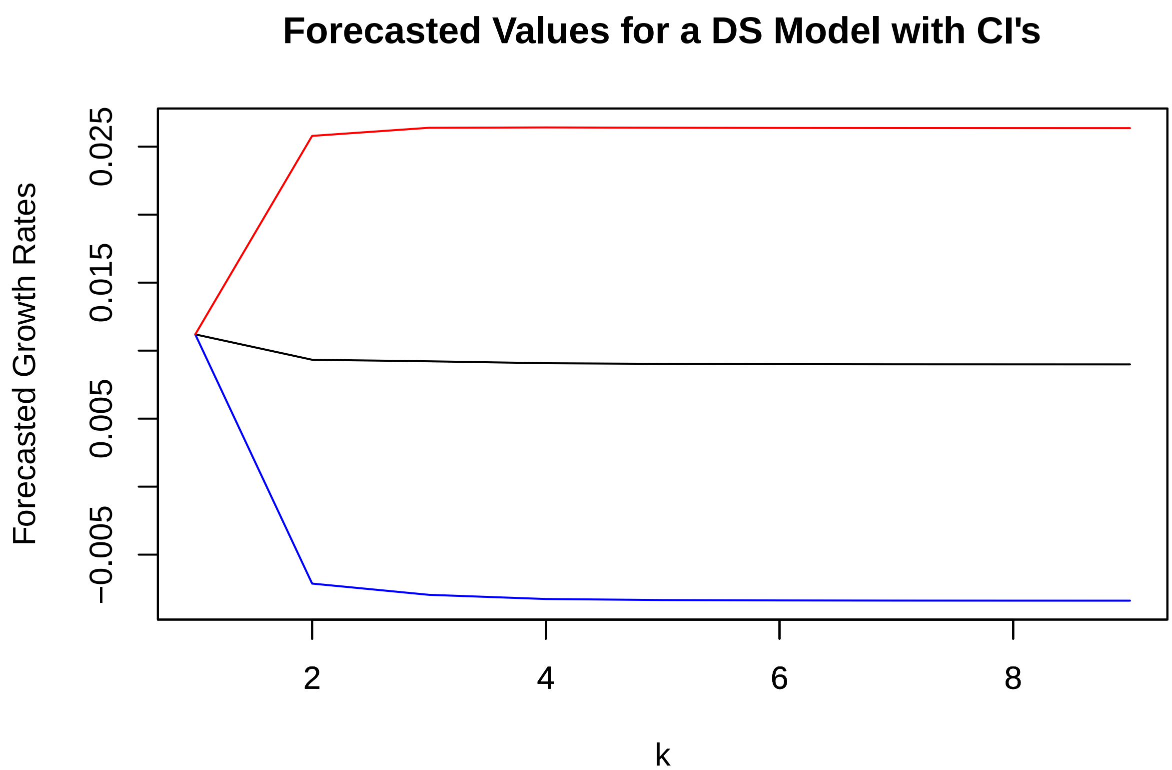


Figure 1: The Black line is our forecast, the blue line is the lower bounds CI and the red line is the upper bounds CI

TS Forecasts with 95% Confidence Intervals

	$E[\Delta X_{t+k}]$	$LowerCI$	$UpperCI$
$k = 0$	0.011193	0.011193	0.011193
$k = 1$	0.010546	-0.005679	0.026771
$k = 2$	0.010290	-0.006602	0.027183
$k = 3$	0.010163	-0.006758	0.027084
$k = 4$	0.010079	-0.006843	0.027002
$k = 5$	0.010010	-0.006922	0.026942
$k = 6$	0.009948	-0.006997	0.026894
$k = 7$	0.009890	-0.007070	0.026849
$k = 8$	0.009833	-0.007139	0.026806

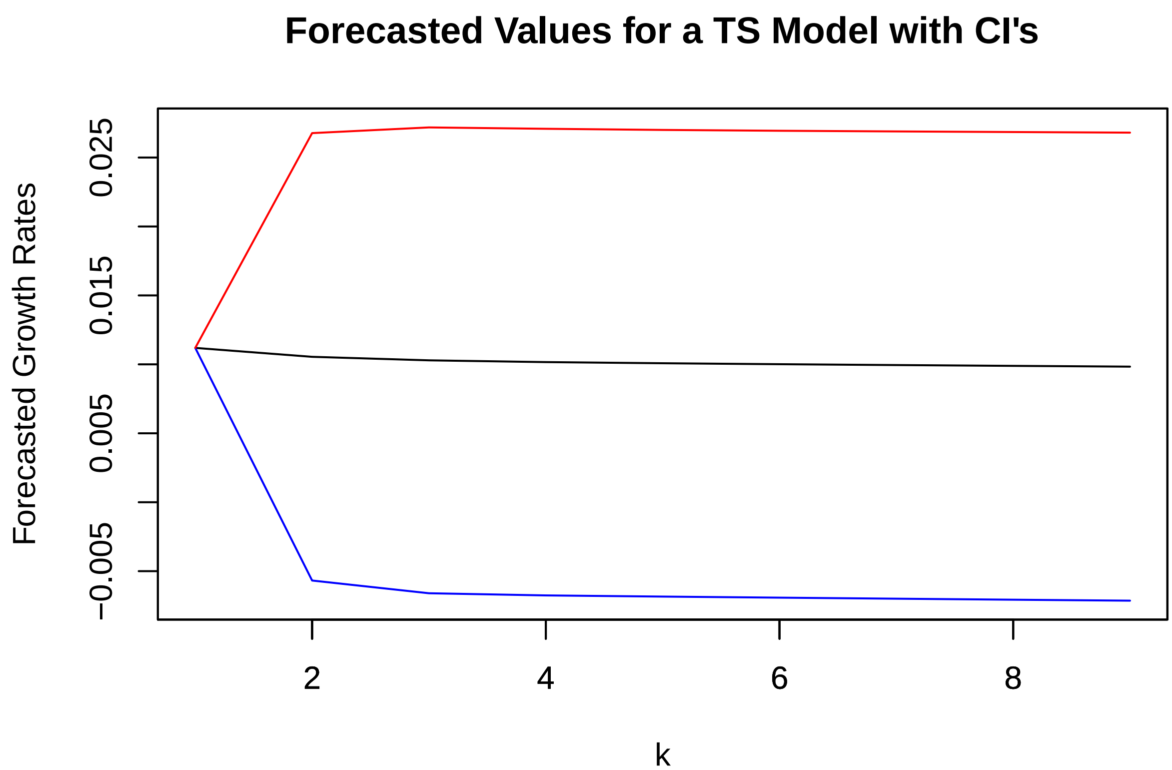


Figure 2: The Black line is our forecast, the blue line is the lower bounds CI and the red line is the upper bounds CI

Question 5

In the Dickey - Fuller test the following Hypothesis test occurs:

$$H_0\text{: } X_t \text{ is Difference Stationary}$$
$$H_1\text{: } X_t \text{ is not Difference Stationary}$$

using the code explained in the Exhibit (Question 5) we have the following results from this hypothesis.

$$\text{Statistic} = -2.6929$$
$$\text{P-value} = 0.2865$$

This implies that we should not reject H_0 and presume the series X_t is associated with a Difference Stationary Model.

Question 6

To determine certain characterisitcs of the DS and TS models we use a Box-Jenkins identification. This involves using the autocorrelation function $\rho(k)$ and partial autocorrelation ϕ_{kk} to determine if the respective Y_t's follow an $AR(p)$ or and $MA(q)$.

The following table shows the values from the TS's autocorrelation and partial autocorrelation functions.

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
$\rho(k)$	0.9656	0.9293	0.8938	0.8570	0.8224	0.7870
ϕ_{kk}	0.9656	-0.3131	-0.0552	-0.1223	0.0064	0.0317

To determine a cutoff point for these values we determine where the correlation values exceed in absolute terms

$$2 \times \frac{1}{\sqrt{T_{TS}}} = 2 \times \frac{1}{\sqrt{185}} = 0.1470$$

Therefore we can see that ϕ_{kk} has a cutoff value at $k = 2$ as it is the last point where $|\phi_{kk}| > 0.1470$. In addition, it does not seem that $\rho(k)$ has a cut-off point and is demonstrating characterisitics of a damped exponential.Thus by Box-Jenkins properties we can claim that the Y_t appears to be an $AR(2)$, exactly what we estimated in Question 3.

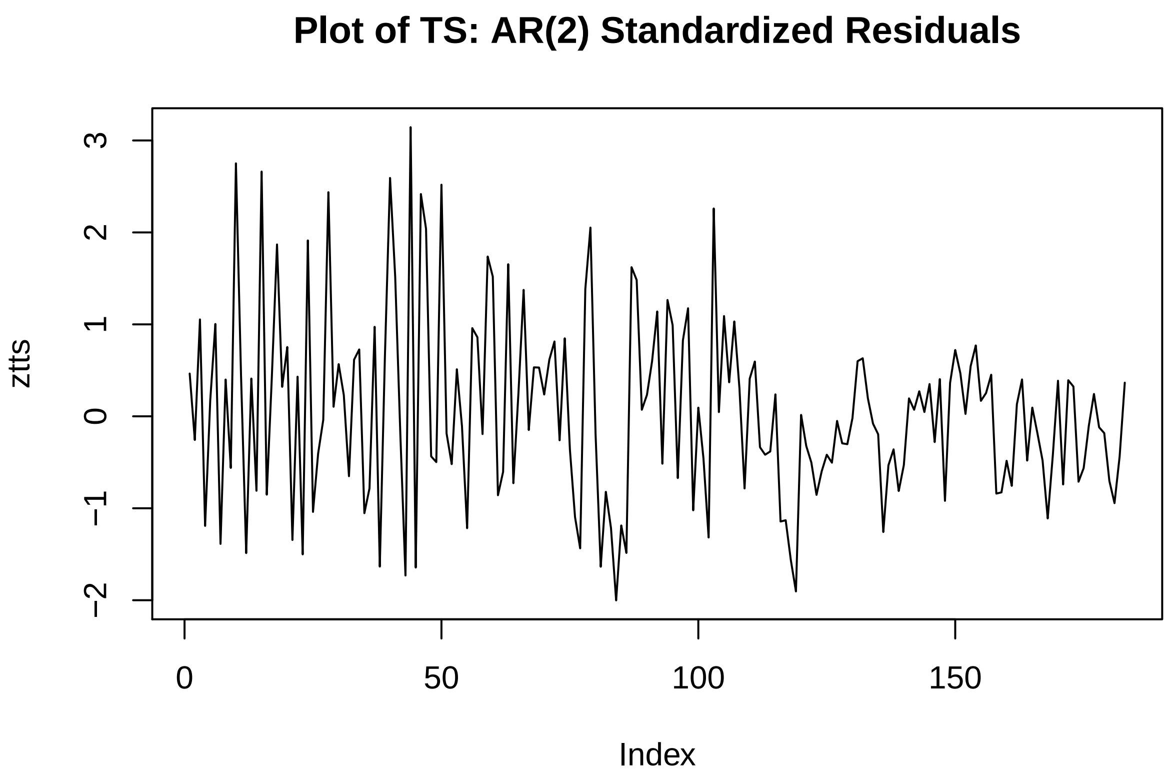
The following table shows the values from the DS's autocorrelation and partial autocorrelation functions.

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$
$\rho(k)$	0.3246	0.1539	0.1738	0.0819	0.0156	0.0725
ϕ_{kk}	0.3246	0.0574	0.1239	-0.0080	-0.0335	0.0647

Similar to what we did with the TS model, we determine a specific value that can act as the cut-off point for the autocorrelation and partial autocorrelation function.

$$2 \times \frac{1}{\sqrt{T_{DS}}} = 2 \times \frac{1}{\sqrt{184}} = 0.1474$$

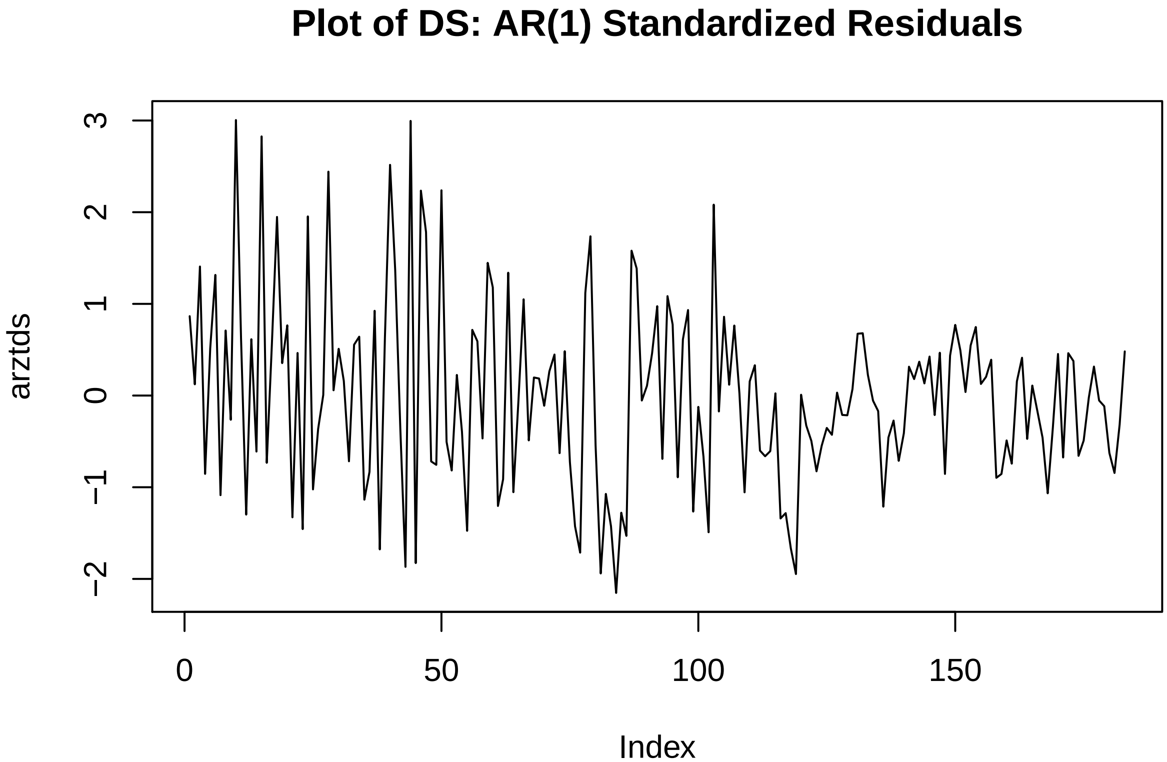
Based on this cut-off we see that the DS model can be associated with an $AR(1)$ model as $k = 1$ is the last value where $|\phi_{kk}| > 0.1474$. In addition, we can claim that the DS model can be associated with an $MA(3)$ as $k = 3$ is the last value where $|\rho(k)| > 0.1474$. However, based on the restrictions with this project we will assume that the DS model is associated with an $AR(1)$ and a $MA(1)$.



This plot displays that a majority of the values are between -2 and 2. Using the following standards from the Gaussian Law we can claim that the standardized residuals of the $AR(2)$ fit for the TS model follows a Normal Distribution.

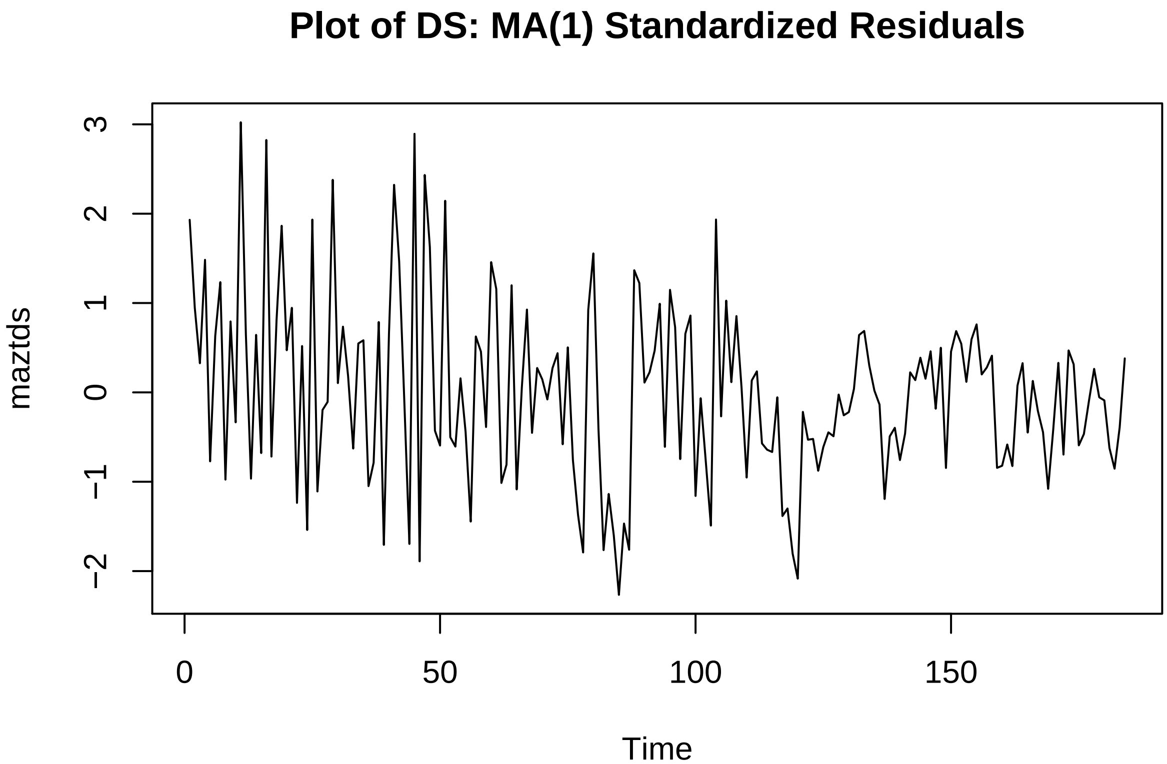
$$|\hat{z}_t| > 2 \text{ has 11 observations}$$

$|\hat{z}_t| > 3$ has 1 observation
$|\hat{z}_t| > 4$ has 0 ovservations



Similar to the AR(2) from the TS model, this plot demonstrates that a majority of the standardized residuals lie between -2 and 2. The following shows that the AR(1) model does not contain many outliers and therefore, by Gaussian Law standards we can claim normailty for this model.

$|\hat{z}_t| > 2$ has 9 observations
$|\hat{z}_t| > 3$ has 1 observation
$|\hat{z}_t| > 4$ has 0 ovservations



Similar to the previous 2 plots the $MA(1)$ model has standardized Residuals that lie between -2 and 2. The following Gaussian Law standards shows that outliers are limited in this model. As a result, we can claim normality for the $MA(1)$ model.

$|\hat{z}_t| > 2$ has 9 observations
$|\hat{z}_t| > 3$ has 1 observation
$|\hat{z}_t| > 4$ has 0 ovservations

Test for Serial Correlation

The test for Serial Correlation is also considered a test for independence between residual values for our models. This implies that if our models are correct then the residuals a_t should be independent of a_{t+k} for all $k \neq 0$. Therefore we would like to see our autocorrelation fuction to satisfy.

$$\rho_a(k) \equiv \frac{E[a_t a_{t+k}]}{\sigma^2}$$

We then use our estimates from each model to determine an estimated autocorrelation function. Which can apply for all of our models.

$$\hat{\rho}_a(k) \equiv \frac{\sum_{t=1}^{T-|k|} \hat{a}_t \hat{a}_{t+k}]}{\sum_{t=1}^{T} \hat{a}_t^2}$$

Now we introduce the Box-Piece Test where under the null hypothesis we have:

$$H_0 : \rho_a(k) = 0$$

Where $k = 1, 2, \ldots, M$ where we can define $M \approx \sqrt{T}$ Because the T value is close for both TS and DS model we can round down.

$$M \approx \sqrt{T} = \sqrt{185} \approx \sqrt{184} \approx 10.$$

Therefore we use the following statistic

$$Q = T \times (\hat{\rho}_{\hat{a}}(1) + \cdots + \hat{\rho}_{\hat{a}}(10)) \sim \chi^2_{10}$$

Using the following process we obtain the following important values for all of our models

	TS: $AR(2)$	DS: $MA(1)$	DS: $AR(1)$
Test Statistic	17.517	19.018	17.502
p-value	0.064	0.040	0.064

By looking at this table we can determine what models satisfy the null hypothesis.

We see that for the TS model with an $AR(2)$ fit, the statistic is less than the 5% critical value $\chi^2_{10} = 18.3$. Thus, we can say that the $AR(2)$ passes the diagnostic test as the null hypothesis is accepted.

The DS model with an $MA(1)$ fit, has a statistic that is greater than the 5% critical value $\chi^2_{10} = 18.3$. As a result, we reject the null hypthosis and claim that the $MA(1)$ model does not pass this diagnostic test.

The DS model with an $AR(1)$ fit, has a statstic that is less than the 5% crtical value $\chi^2_{10} = 18.3$. As a result, we accpet the null hypothesis and claim that the $AR(1)$ model passes this diagnostic test

Test for Overfitting

This test allows us to determine if we have fit a correct $AR(p)$ or a $MA(q)$ to our model. For example an $AR(2)$ will have the following representation:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + a_t$$

Thus AR(2+4) can be represented as

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_2 + \phi_3 Y_{t-3} + \cdots + \phi_6 Y_{t-6} + a_t$$

where we have 4 added terms.

Under the null hypothesis we have

$$H_0 : \phi_3 = \cdots = \phi_6 = 0.$$

This null hypothesis will give us the following test statistic

$$\Lambda = T \times ln(\tfrac{\hat{\sigma}_2^2}{\hat{\sigma}_6^2}) \sim \chi^2_4$$

Where $\hat{\sigma}_2^2$ is the estimated sigma squared for an $AR(2)$ and $\hat{\sigma}_6^2$ is the estimated sigma squared for an $AR(6)$.

Note the same process can be used for MA(1) and AR(1) models. Using this process for the TS and DS models, we get the following results.

	TS: $AR(2)$	DS: $MA(1)$	DS: $AR(1)$
Test Statistic	1.4924	2.8492	0.0108

For the TS $AR(2)$ model we have that its test statistic Λ is less than the corresponding critical value $\chi^2_4 = 9.49$. Thus we accept H_0, that there is evidence for $AR(2)$ being the correct model.

The DS $MA(1)$ model has test statistic Λ that is less than the corresponding critical value $\chi^2_4 = 9.49$. Thus we accept H_0, that there is evidence for $MA(1)$ being the correct model.

The DS $AR(1)$ model has test statistic Λ that is less than the corresponding critical value $\chi^2_4 = 9.49$. Thus we accept H_0, that there is evidence for $AR(1)$ being the correct model.

Jacrque-Bera test

This is another test in normality as we can use the estimated standardized residuals from each model to determine the respective model's skewness and kurtosis.

$$\hat{\kappa}_3 = \frac{1}{T}\sum_{t=1}^{T} z_t^3 \text{ and } \hat{\kappa}_4 = \frac{1}{T}\sum_{t=1}^{T} z_t^4$$

In addition we have a hypothesis test with the following conditions

$$H_0 : \kappa_3 = 0, \kappa_4 = 3$$

and the following test statistic

$$JB = T(\frac{\hat{\kappa}_3^2}{6} + \frac{(\hat{\kappa}_4 - 3)^2}{24}) \sim \chi_2^2$$

Using this process for all the available models, we have the following results.

	TS: $AR(2)$	DS: $MA(1)$	DS: $AR(1)$
Skewness	0.7388	0.4521	0.4944
Kurtosis	3.4421	3.3539	3.4081
Test Statistic	18.3362	7.2283	8.7715

In this scenario all statistics for each model follow a χ_2^2, and the 5% level of the critical value of χ_2^2 is 5.99. However, each of the model's JB statistic exceeded the critical value, as a result we claim that the AR(2) fit for the TS model, the MA(1) fit for the DS model and the AR(1) fit for the DS model do not showcase the characterstics of normality through the Jarque - Bera test.

Test for non-linear dependence

The nature of the TS and DS means that we have been working with linear models. As a result, we have not tested for non-linear dependence with our various models. We define $ARCH(6)$ where we have 6 lagged values of a_{t}^2, the respective resiudals in each model. $ARCH(6)$ is represented as:

$$a_t = z_t \times (\sigma^2 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_6 a_{t-6}^2)^{\frac{1}{2}}$$

Our null hypothesis under the ARCH test would be

$$H_0 : a_1 = a_2 = \cdots = a_6 = 0$$

In order to use this for our various models we regress each model's respective $\hat{a}_t^2$ on a constant and their respective lagged residuals. We then get the R^2 in each regression to use the following test statistic under our null hypothesis.

$$T \times R^2 \sim \chi_6^2$$

We use this framwork to obtain test statistics for all of our TS and DS models

	TS: $AR(2)$	DS: $MA(1)$	DS: $AR(1)$
Test Statistic	3.629529	5.384556	3.331799

In this test each model's test statistic follows the same distribution χ_6^2. The 5% critical value is 12.59. We can observe from the table above that each model's test statistic is less than 12.59. Therefore, for each model we accept H_0. Thus, through the $ARCH(6)$ test we can say that the AR(2) fit for the TS model, the MA(1) fir for the DS model and the AR(1) representation of the DS model, all can claim non-linear independence.

Question 8

Similar to question 2, we will linearize the raw time series data by taking the natural logarithim of the dataset. We then run regression by modelling it in the form that was given

$$ln(P_t) = \delta + ln(P_{t-1}) + a_t \text{ where } a_t \sim i.i.N[0, \sigma^2]$$

Again through the use of the Box-Pierce test we can utulize the autocorrelation function $\hat{\rho}_a(k)$ to determine if this model follows a random walk process.

Under the Null Hypothesis we have the following

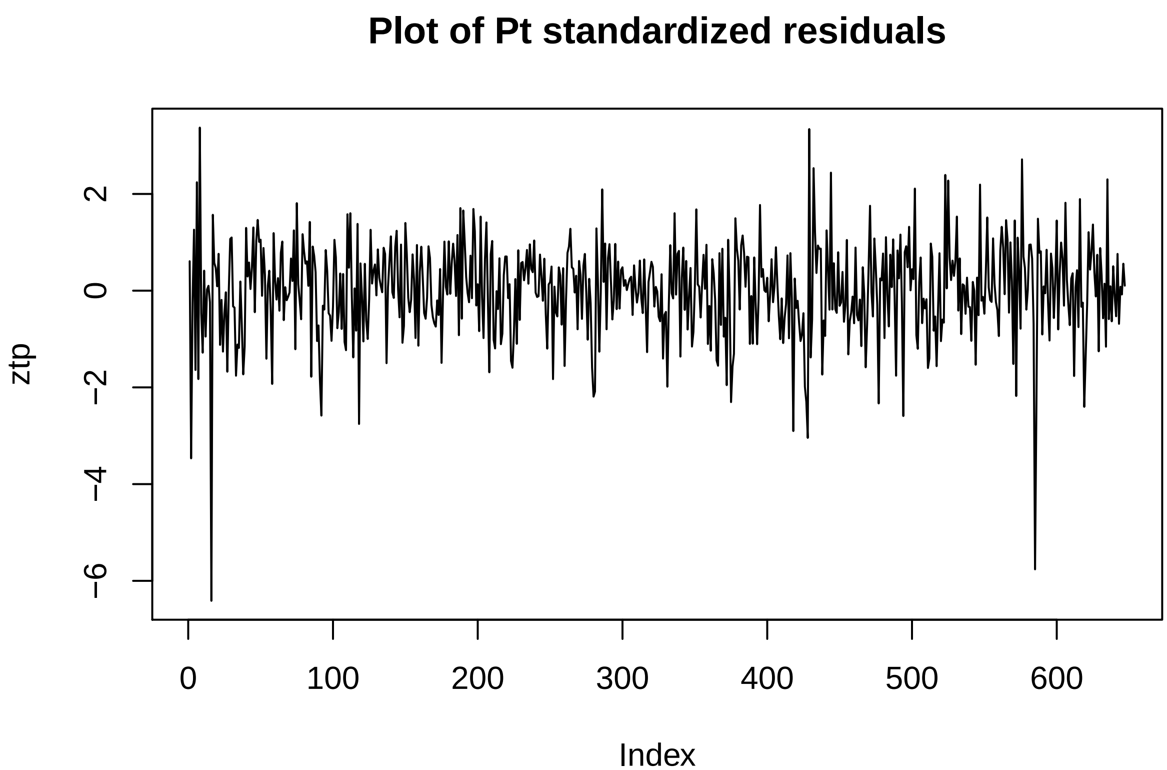
$$H_0 : \hat{\rho}_a(1) = \cdots = \hat{\rho}_a(k) = 0 \text{ where } k = 1, \ldots, M$$

We also pick M to be $M \approx \sqrt{T} = \sqrt{648} \approx 25$. Thus, here are the results of the Box-Pierce Test.

	$ln(P_t)$
Test Statistic	21.702
P-value	0.065

As we can see, the critical value from this test is $Q = 21.7$ and the 5% critical value for χ^2_{25} is 37.65. As a result, we can accept the hypothesis that $\hat{\rho}_a(1) = \cdots = \hat{\rho}_a(k) = 0$. Therefore, we claim that this model does not follow a Random walk as we require $\hat{\rho}_a(k) = 1$ in order for this time series to resemble a random walk.

The tests for normality are similar to those conducted in question 6. We can plot the standardized residuals to determine the nature of outliers.



Here we see that there are an abudnance of outliers and some of them are very large in magnitude. As a result, the characteristic of the plot displays that this model may not be following a Normal distribution.

However, let us try using the Jarque-Bera test where our null hypothesis is

$$H_0 = \kappa_3 = 0, \kappa_4 = 3$$

Using the same framework we conducted in question 6 we gain the following results.

	$ln(P_t)$
Skewness	-0.8079
Kurtosis	7.2270
JB Statistic	552.9232

We see that the statistic for this test is 552 which is absurdly larger than the 5% critical value for $\chi_2^2 = 5.99$. As a result, we reject H_0 which verifies our position with the plot of the standard residuals as we deny the notion that this model follows a Normal distribution.

Here are the first 15 values of the autocorrelation function for a_t^2. Using the acf function built into R we get the following results.

k	$\rho_a(k)$
1	1.0000
2	0.0745
3	0.0234
4	0.0144
5	0.0155
6	-0.0091
7	0.0460
8	-0.0455
9	0.1102
10	0.0729
11	0.0542
12	0.0416
13	-0.0103
14	0.0072
15	0.1180

Using the garchFit function from the built in R function we have the following results.

	Ω	α	β
Estimates	9.49e-05	0.060716	0.88844

Here we have Omega (Ω) > 0, and Alpha (α) , Beta ($Beta$) are non negative. In addition, $\alpha + \beta = 0.949 < 1$. Since all of these 3 conditions are satisfied we can claim that this model is stationary.

Sources

1 . Nicholson, Peter. (2003). *The Growth Story: Canada's Long-run Economic Perfomance and Prospect.*. http://www.csls.ca/ipm/7/nicholson-e.pdf. Accessed July 23, 2017.

2 . Sampson, M. (2013). *Time Series Analysis: Version 4.4.* Montreal, QC: Loglinear Publishing.