

# Real Analysis

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## 1 Limits and Continuity

### 1.1 Limits

The value of a function  $f$  at  $a$ , in the absence of any other information about  $f$ , gives absolutely no information about  $f$  for values close to  $a$ . A behaviour that is of significant interest is when  $f$  "approaches" a value (which is not necessarily  $f(a)$ ) at  $a$ , in the specific sense that by considering a sufficiently small neighborhood of  $a$ , all the images of  $f$  are arbitrarily close to  $a$ . We formalise this intuition in defining the limit of  $f$  at  $a$ .

**Definition** (Limit).  $l$  is the limit of  $f$  at  $a$ , or symbolically  $\lim_{x \rightarrow a} f(x) = l$  if and only if

$$\forall \epsilon > 0 \exists \delta > 0 : \forall 0 < |x - a| < \delta : |f(x) - l| < \epsilon.$$